Predicting Proton Axial Form Factors with a Quark-Diquark Model

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Abstract

A quark-diquark model is used to predict the nucleon Axial Form Factor more accurately and rigorously than the currently accepted expression of a simple dipole form. The model uses scalar vertex functions at the nucleon-quark-diquark vertices with parameters chosen to accurately match nucleon electric and magnetic form factors. These parameters are then used to independently predict the proton axial form factor.

1 Introduction

Since early scattering experiments and the inception of the parton model, the internal structure of the nucleon has remained elusive. Improved theories and experiments determined that the quark is a fundamental particle of the nucleon, and is bound to the nucleon by gluons. These interactions are governed by the laws of quantum chromodynamics (QCD) which requires all strongly bound matter to exist in an unbroken color charge symmetry, i.e., a color-singlet state. Due to the computational complexity of QCD, however, the exact quark-gluon structure of the nucleon has not yet been precisely determined. The naive quark-parton model states that the nucleon is made up of three quarks, bound together by gluons.

In reality the three bound quarks make up only the valence structure of the nucleon, while there also exist an infinite number of "sea" quarks and gluons. Because they exist in quark-antiquark pairs, the "sea" quarks and gluons do not contribute to the nucleon's valence charge structure. On the other hand, they do contribute to the nucleon's total spin. Neutrino Scattering experiments performed in 1988 at the European Muon Collaboration (EMC) showed that the valence quarks make up roughly 30% of the nucleon's total spin. The exact machinery behind this phenomenon is still of great interest, thus the "EMC Effect" remains an exciting theoretical and experimental research topic.

Form factors are useful tools in characterizing the charge and angular momentum distribution of the nucleon through scattering experiments. These experiments involve a lepton current and a nucleon current, with a photon exchanged between the current vertices. The proton absorbs the photon momentum, so the final nucleon momentum equals the sum of initial nucleon and photon momenta, as denoted by $p' = p + q$. The proton current is decomposed electric and magnetic Sachs form factors, $G_E(Q^2)$ and $G_M(Q^2)$, respectively. These form factors quantify the nucleon's electric and magnetic charge makeup, where larger scattering momenta $q^2 = Q^2$ correspond with higher spatial precision, providing insight into small-scale nucleon charge and magnetization distributions. Various models exist which accurately characterize these form factors.

Further understanding of nucleon structure can be provided through weak scattering experiments, including quasielastic neutrino-nucleon scattering, charged pion electroproduction and radiative muon capture [1]. These experiments are quite similar to those of electromagnetic scattering, but utilizing a weak boson propagator rather than a photon. These experiments are used to characterize an additional class of form factors, made up by the axial form factor G_A, pseudoscalar form factor G_P, and anapole form factor G_T. These form factors are not quantified as completely as the Sachs form factors due to experimental complexities. The axial form factor has mostly been characterized with a phenomenological dipole form factor distribution, which has only been examined at low-Q^2 up to 1 Gev^2. Upcoming experiments should provide additional data in the near future. The importance of these form factors will be discussed in section 3.

A new model is proposed to allow further physical insight and rigor to the behavior of G_A. The model is a quark-diquark approximation with scalar vertices of similar form to the dipole approximation. The strength of the model is shown by its ability to independently predict experimental axial form factor data and predict higher-Q^2 behavior as additional data are found. In order to make this prediction, the model is fitted and normalized according to G_E and G_M data. The fitted parameters and normalization are then carried over to the neutral weak current calculation to predict G_A.

2 Electromagnetic Form Factors

The scalar model follows a quark-diquark approximation with nucleon-quark-diquark vertices parametrized by, $\Lambda^2/(k^2 - \Lambda^2)$, where Λ is an ultraviolet regulator and k is the lone quark momentum, as shown in figure (1). The main advantages of the chosen vertex form are twofold - Λ encodes information about the strength and cutoff of the model approximation, and the k dependence in the denominator enforces convergent behavior in the loop calculation. Thus, the model is manifestly covarient. Furthermore, because of the similar structure of the scalar vertices to the dipole form factor, at high Q^2 the two models can be compared directly.

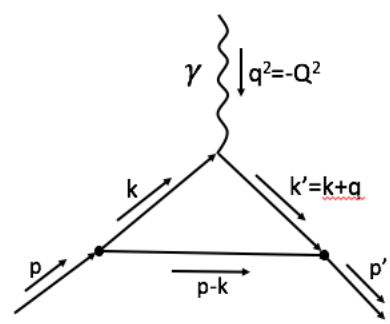


Figure 1: Feynman Diagram for electromagnetic current calculation in a quark-diquark model. Here, p and p' are the incoming and outgoing nucleon momenta, k and k' are the lone quark momenta, and $p - k$ is the diquark momentum. The total model calculation involves both flavor singlet (scalar) and triplet (axial vector) diquark contributions.

The Sachs Form Factors G_E and G_M are calculated by first finding the Pauli and Dirac form factors, F_1 and F_2. F_1 and F_2 are calculated by decomposing the electromagnetic current according to,

$$J^\mu = \bar{u}(p')[F_1\gamma^\mu + F_2 \frac{i}{2M}\sigma^{\mu\nu}q_\nu]u(p). \tag{1}$$

G_E and G_M can be determined from F_1 and F_2 by,

$$G_E = F_1 - \frac{Q^2}{4M^2} F_2$$

and

$$G_M = F_1 + F_2.$$

In order to find F_1 and F_2 from our model, we must solve the electromagnetic current from the Feynman diagram in figure (1) and match like terms to equation (1). The electromagnetic currents are given by the covariant equations,

$$J_S^\mu = \frac{\Lambda^2}{k^2 - \Lambda^2} \frac{\Lambda^2}{(k+q)^2 - \Lambda^2} \frac{i}{(p-k)^2 - m_d^2} \bar{u}(p')[\frac{i(\not{k} + \not{q} + m)}{(k+q)^2 - m^2} \gamma^\mu \frac{i(\not{k} + m)}{k^2 - m^2}]u(p), \tag{2}$$

$$J_A^\mu = \frac{\Lambda^2}{k^2 - \Lambda^2} \frac{\Lambda^2}{(k+q)^2 - \Lambda^2} \frac{ig_{\nu\rho}}{(p-k)^2 - m_d^2} \bar{u}(p')[(-i\gamma^\nu \gamma^5) \frac{i(\not{k} + \not{q} + m)}{(k+q)^2 - m^2} \gamma^\mu \frac{i(\not{k} + m)}{k^2 - m^2} (-i\gamma^\rho \gamma^5)]u(p), \tag{3}$$

where (1) and (2) correspond to scalar spin-0 and axial spin-1 diquarks, respectively.

Following the Feynman variable approach to integrate over k^2 and matching like terms [3], we find that:

$$F_{1S}(q^2) = \frac{\Lambda^2}{8\pi^2} \int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3} (\frac{\Delta}{2} + q^2(v+y)(1-v-y-x) + (Mx+m)^2)\, dV, \tag{4}$$

$$F_{2S}(q^2) = \frac{\Lambda^2}{8\pi^2} \int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3} 2M(Mx+m)(1-x)\, dV, \tag{5}$$

$$F_{1A}(q^2) = \frac{\Lambda^2}{8\pi^2} \int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3} (\Delta + q^2(v+x+y)(1-v-y) + 2(Mx+m)^2 + 4xMm)\, dV, \tag{6}$$

$$F_{2A}(q^2) = \frac{\Lambda^2}{8\pi^2} \int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3} (-4Mx)(M(1+x) + 2m)\, dV, \tag{7}$$

where $dV = dv\, dw\, dx\, dy\, dz$, $\Delta = \Lambda^2(w+v) + q^2(v+y)(v+x+y-1) + M^2 x(x-1) + m^2(y+z) + m_d^2 x$, m is the lone quark mass, M is the proton mass, andm_d is the diquark mass.

The total contributions from equations (4-7) to the total form factors F_{1tot} and F_{2tot} are calculated based on $SU(2)$x$SU(2)$ spin-flavor considerations [2]. The proton and neutron spin-flavor wavefunctions are,

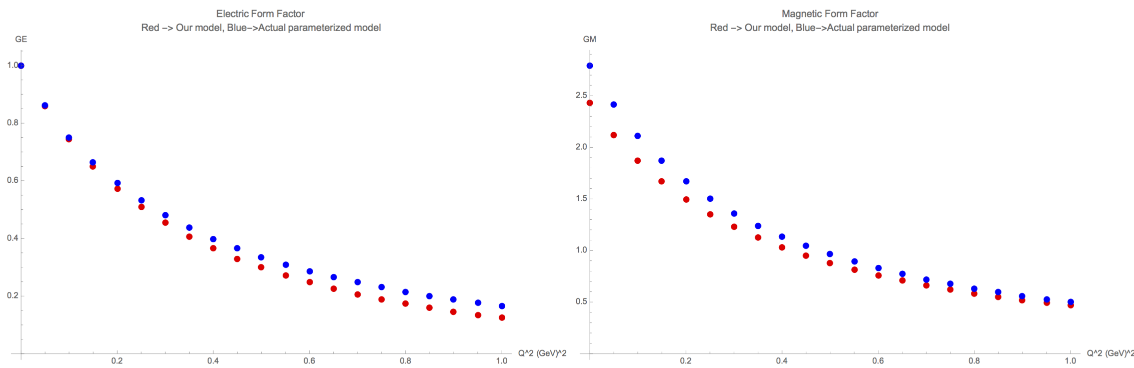
$$|\Psi_{sf,p}\rangle = |uS\rangle \tag{8}$$

$$|\Psi_{sf,n}\rangle = \frac{1}{\sqrt{2}} |uS\rangle + \frac{1}{\sqrt{6}} |uT\rangle - \frac{1}{\sqrt{3}} |dT\rangle\,, \tag{9}$$

respectively. In the preceding notation, $|uS\rangle$ denotes a nucleon state featuring an up lone quark and a flavor singlet diquark, while $|dT\rangle$ represents a down quark and a flavor triplet diquark, etc. Note that the proton contains a purely scalar diquark contribution involving equations (4-5), although the axial contributions in equations (6-7) will be useful when considering axial form factors.

m	m_d	Λ	$m_{d,a}$	Λ_a
.419	.590	.670	.753	.293

Table 1: Fitted Parameters for Scalar and Axial Vector Contributions



(a) Proton Electric Form Factor, G_E

(b) Proton Magnetic Form Factor G_M

Figure 2: Proton Form Factors evaluated using data from table (1)

Once the scalar- and axial vector- diquark contributions are determined, it is possible to solve the Feynman diagrams for F_1 and F_2 covariantly using the Feynman integral approach over the Feynman variables v, w, x, y, and z in equations (4-7). The integrations are performed numerically by fitting to phenomenological data from Kelly [4]. The variables Λ, m and m_d, are free parameters in the model. The results are shown in table (1).

The proton electromagnetic form factors are well-fitted from the model, as shown in figure (2). There were difficulties, however, in fitting the proton and neutron form factors at the same time. This discrepency would likely be minimized with the inclusion of the pion cloud. For the following analysis of the weak axial form factors, the proton wavefunction will be sufficient.

3 Weak Axial Form Factors

The weak form factors are of utmost importance in understanding the proton spin problem. In particular, the isovector axial form factor G_A is useful in determining the nucleon's spin-flavor distribution. An accurate measurement of G_A is useful in isolating strangeness and parity violation in the nucleon [5]. G_P is most useful to determine pion cloud contributions to the nucleon's quark distribution, while also testing the Partially Conserved Axial Current (PCAC) hypothesis through the Goldberger-Treiman relation [6, 7]. G_T is taken to be zero due to vanishing second-class currents [8].

At Q^2 between 1-5 GeV^2 G_A is well-approximated by the dipole form, $G_D = g_a(1 + \frac{Q^2}{M_A})^{-2}$. Here, g_A is the weak axial-vector coupling constant, which is an important fundamental quantity in electroweak theory and is well-determined experimentally to $g_A = 1.28$ [9], and M_A is the "axial mass" and is measured to $M_A = 1.069$ [1]. Due to experimental challenges, the experimental data on the axial form are currently only known up to $Q^2 = 1GeV^2$. With higher-momentum data coming out in the near future with -, it is likely that the dipole approximation will not hold and new models will be necessary to better predict G_A.

Using the quark-diquark model explained above, we can also calculate neutral current interactions in order to predict G_A and G_P. The scalar diquark fitted quantities, m, m_d, and Λ, as well as the F_1 normalizations are carried over and used to predict the weak form factors.

The axial form factors are related to the axial current by,

$$A^\mu(q^2) = \bar{u}(p')[(\gamma^5 G_A(q^2) + 2M\gamma^5 q^\mu G_p(q^2))\frac{\vec{\sigma}_3}{2}]u(p) \tag{10}$$

Solving figure (3), the axial currents for the scalar and axial vector diquarks are given by,

$$A^\mu = \frac{\Lambda_a^2}{k^2-\Lambda_a^2}\frac{\Lambda_a^2}{(k+q)^2-\Lambda_a^2}\frac{i}{(p-k)^2-m_{d,a}^2}\bar{u}(p')[\frac{i(\not{k}+\not{q}+m)}{(k+q)^2-m^2}\gamma^\mu\gamma^5\frac{i(\not{k}+m)}{k^2-m^2}]u(p), \tag{11}$$

$$A^\mu = \frac{\Lambda_a^2}{k^2-\Lambda_a^2}\frac{\Lambda_a^2}{(k+q)^2-\Lambda_a^2}\frac{ig_{\nu\rho}}{(p-k)^2-m_{d,a}^2}\bar{u}(p')[(-i\gamma^\nu\gamma^5)\frac{i(\not{k}+\not{q}+m)}{(k+q)^2-m^2}\gamma^\mu\gamma^5\frac{i(\not{k}+m)}{k^2-m^2}(-i\gamma^\rho\gamma^5)]u(p), \tag{12}$$

We first calculate only the axial form factors, $G_A(q^2)$. Decomposing into Feynman variables and matching to the axial current form in equation (10), we find that the scalar and axial vector contributions are given by,

$$G_{AS}(q^2) = \frac{\Lambda_a^2}{8\pi^2}\int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3}(\frac{\Delta}{2} - q^2(v+y)(1-v-y-x) + (Mx+m)^2)\,dV, \tag{13}$$

$$G_{PS}(q^2) = \frac{\Lambda_a^2}{8\pi^2}\int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3}2M(xM(x+2v+2y)+m)\,dV, \tag{14}$$

$$G_{AA}(q^2) = \frac{\Lambda_a^2}{8\pi^2}\int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3}(-\frac{\Delta}{2} - q^2(v+y-x)(1-v-y) + x^2M^2 + 2m^2)\,dV, \tag{15}$$

$$G_{PA}(q^2) = \frac{\Lambda_a^2}{8\pi^2}\int_0^1 \frac{\delta(v+w+x+y+z-1)}{\Delta^3}2M(xM(-1+x+2v+2y)-4m)\,dV, \tag{16}$$

for scalar diquark Axial and Pseudoscalar, followed by vector diquark Axial and Pseudoscalar form factors. Note that $m_{d,a}$ and Λ_a take on different values than the electromagnetic model, while Δ follows the same form as equations (4-7) but with $m_{d,a}$ and Λ_a rather than m_d and Λ.

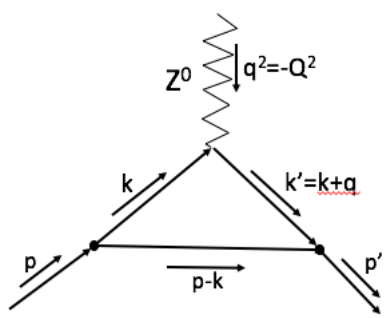


Figure 3: Feynman Diagram for axial current calculation in a quark-diquark model. Here, p and p' are the incoming and outgoing nucleon momenta, k and k' are the lone quark momenta, and $p-k$ is the diquark momentum.

As before, the scalar and axial vector diquark contributions to the total weak current are not intuitively obvious. A similar non-relativistic prescription to that of the electromagnetic form factors must be used starting with the flavor wave function and following symmetry arguments. The axial wavefunction for the proton is composed,

$$|\Psi_{a,p}\rangle = \frac{3}{2}\,|\Psi_s\rangle - \frac{1}{2}\,|\Psi_a\rangle\,, \tag{17}$$

where $|\Psi_s\rangle$ and $|\Psi_a\rangle$ denote the scalar and axial vector diquark contributions, respectively. As in the electromagnetic form factor analysis, these ratios are carried over to perform covariant calculations in our model.

The axial form factors depend on both scalar and axial vector diquark contributions, as opposed to the electromagnetic form factors which only make use of scalar contributions. By fitting the axial form factor to experimental $low - Q^2$ data from [1], it is possible to determine values for the axial as well as scalar diquark model properties. Currently, various fitting schemes are being tested, and a tentative plot is shown in figure (4).

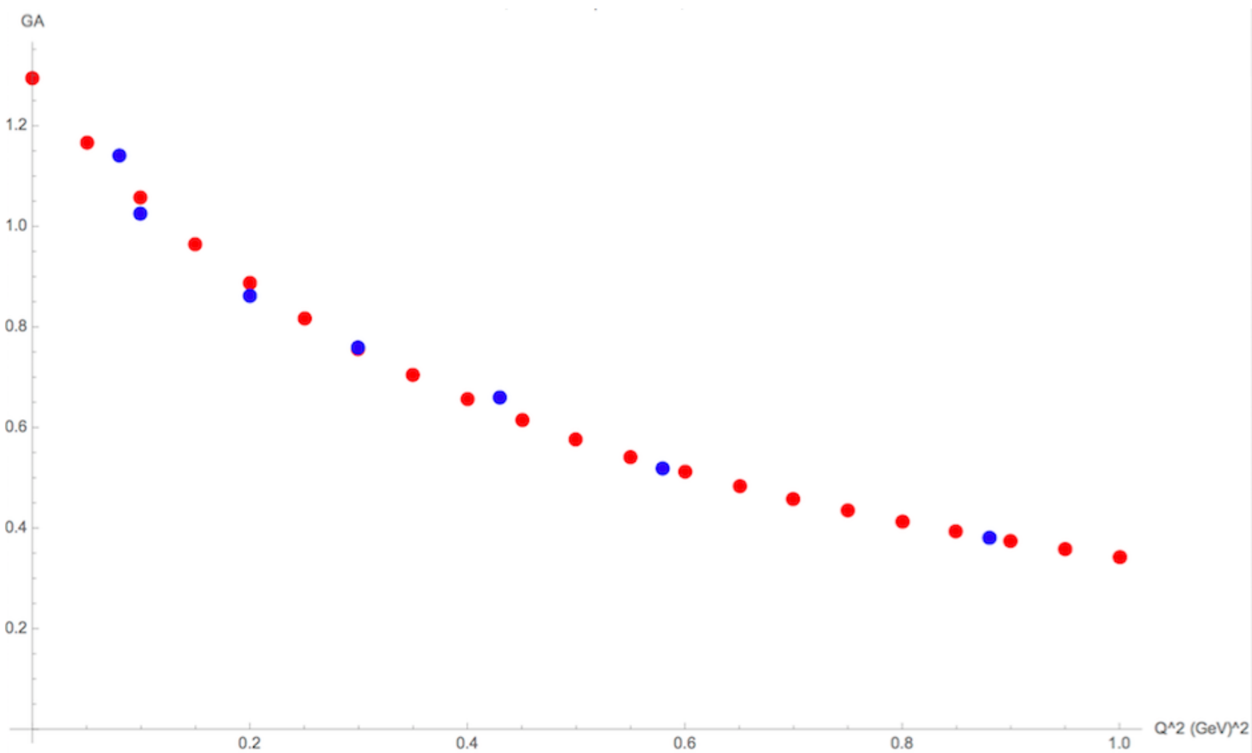


Figure 4: Tentative plot of axial form factor, G_A, data as calculated from the model. The red dots mark those predicted from the model, while the blue dots represent experimental data.

4 Current and Future Work

We are testing further fits of the Axial and Pseudoscalar Form Factors. In addition, we are investigating error estimates for data fits at various points of the project. Of great interest is the inclusion of the Goldberger-Treiman relation in our analysis. The Goldberger-Treiman relation provides additional information about the relationship between the Axial and Pseudoscalar Form Factors, and allows us to peek into the nature of axial current conservation. Because axial form factors are relativelypoorly understood, additional insight into the relations between them would be of great theoretical interest. Any further understanding of weak form factors brings us one step closer to solving the 'Proton Spin Puzzle'.

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