Probing the Proton: Axial Form Factors

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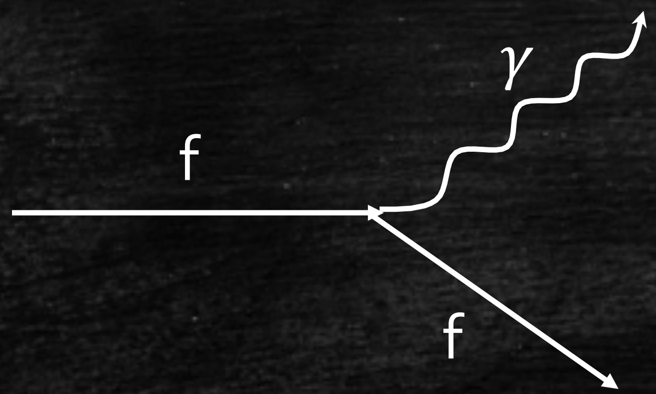
INT REU Program

Contents

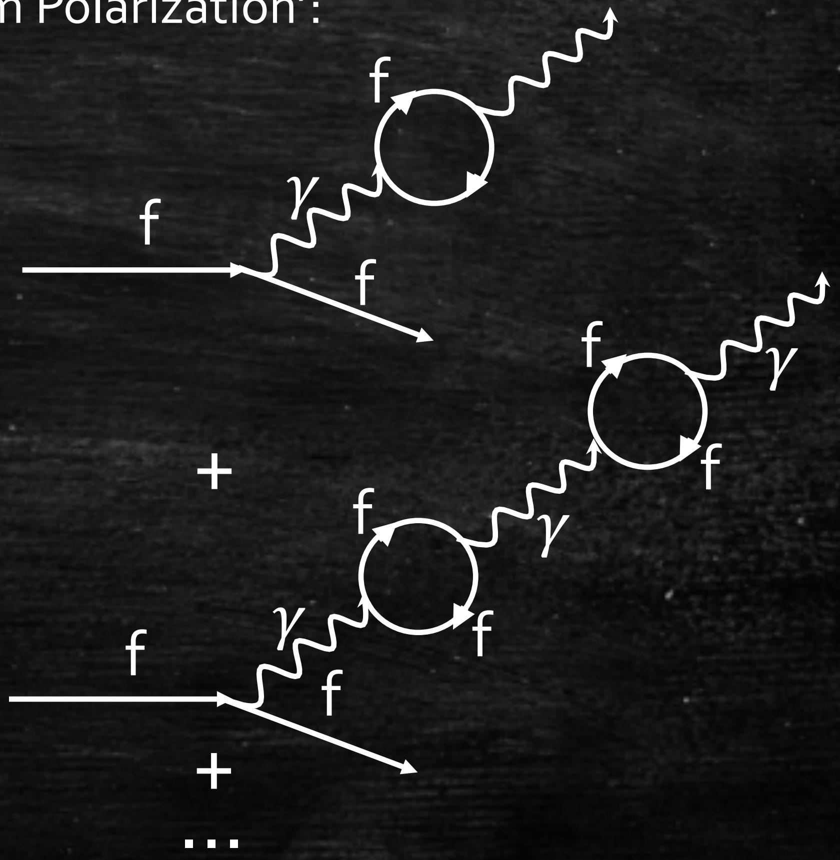
- Why QCD Isn't *QED*
- What's Inside the Proton?
- Modeling the Proton:
  - Electromagnetic Form Factors
  - Axial Form Factors

QED

- QED couples fermions to electromagnetic fields:
- Fundamental QED Vertex:
  - 2 ψ terms -> 2 fermions
  - 1 A^{μ} term -> 1 photon



There are an infinite amount of other "perturbations" that yield the same initial and final states. Of special importance is 'Vacuum Polarization':



QED

Corrections due to vacuum polarization ”absorbed” into a running coupling term, which increases with momentum. Note that the corrections in QED are small!

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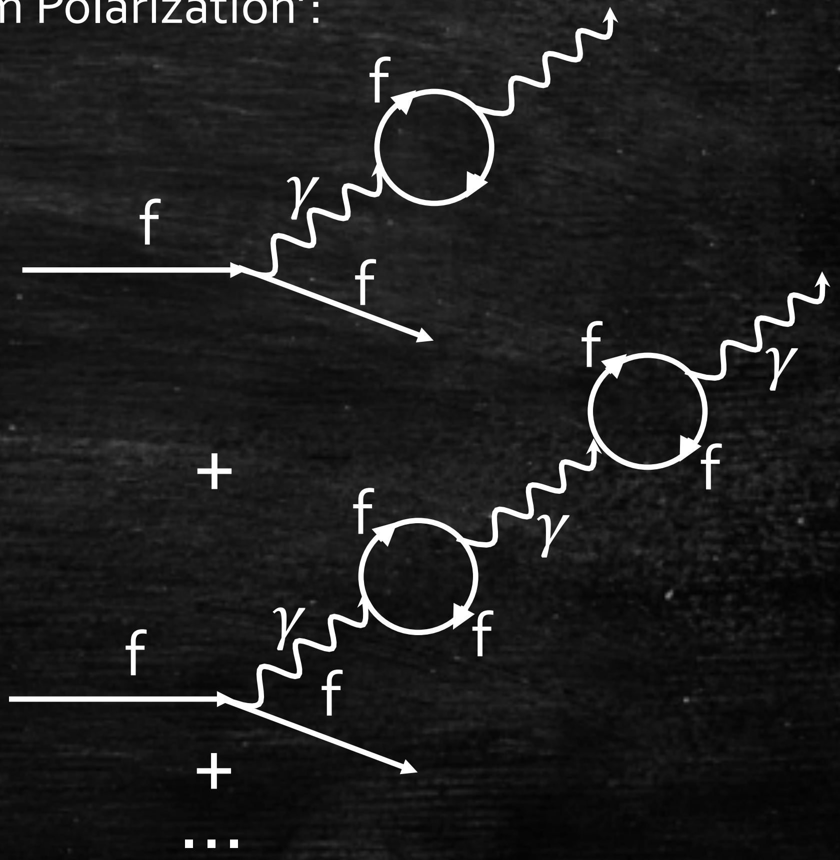
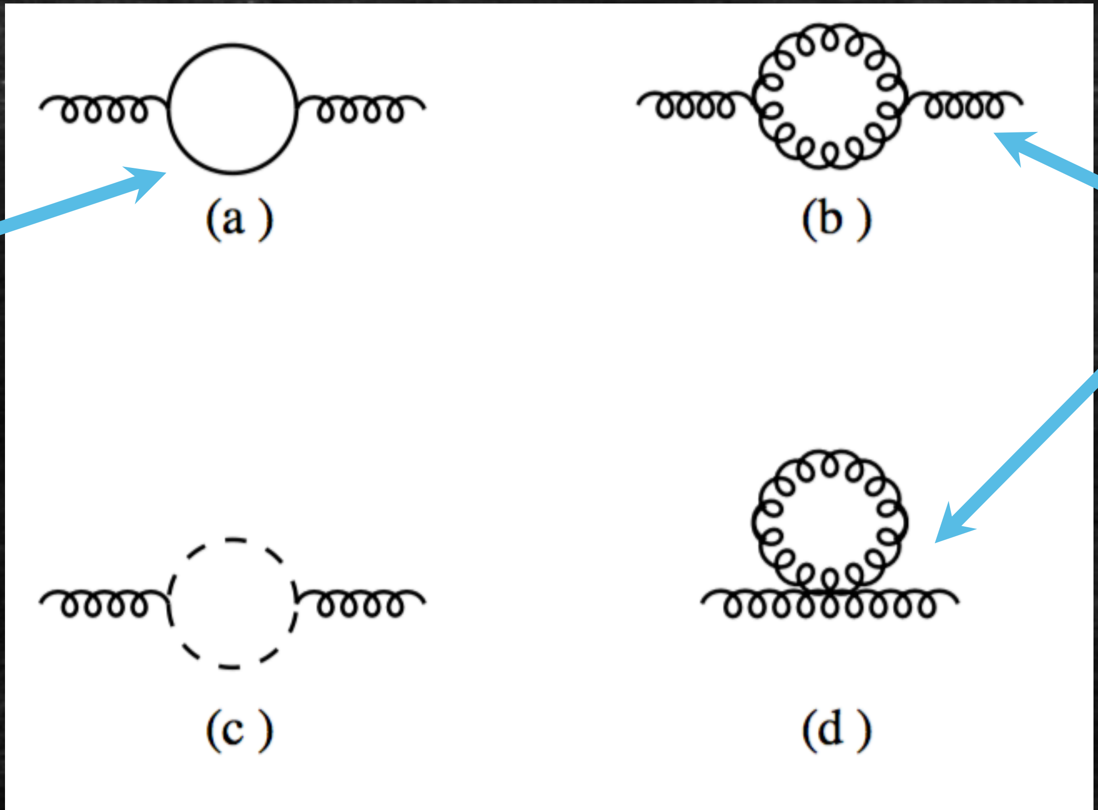


QCD

- Similar 3-particle fundamental vertex to QED ($\bar{q}, q, A_\mu$)



From Thomas & Weise – The Structure of the Nucleon

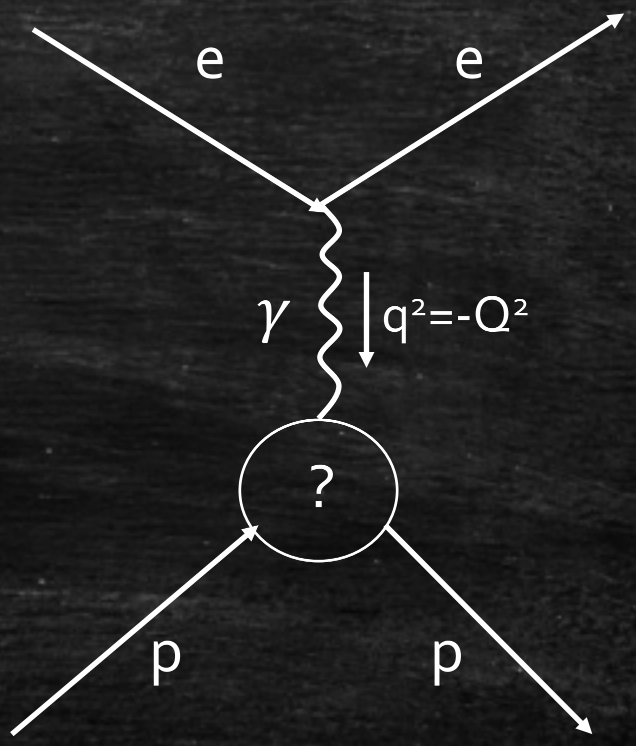
- Unlike in QED, there are also 3- and 4-gluon self-interactions
- This is because quarks carry color charge!

QCD

Unlike QED, these perturbative corrections cause the running coupling constant to vary *inversely* with momentum. There is a large bare coupling value which makes perturbative calculations difficult!

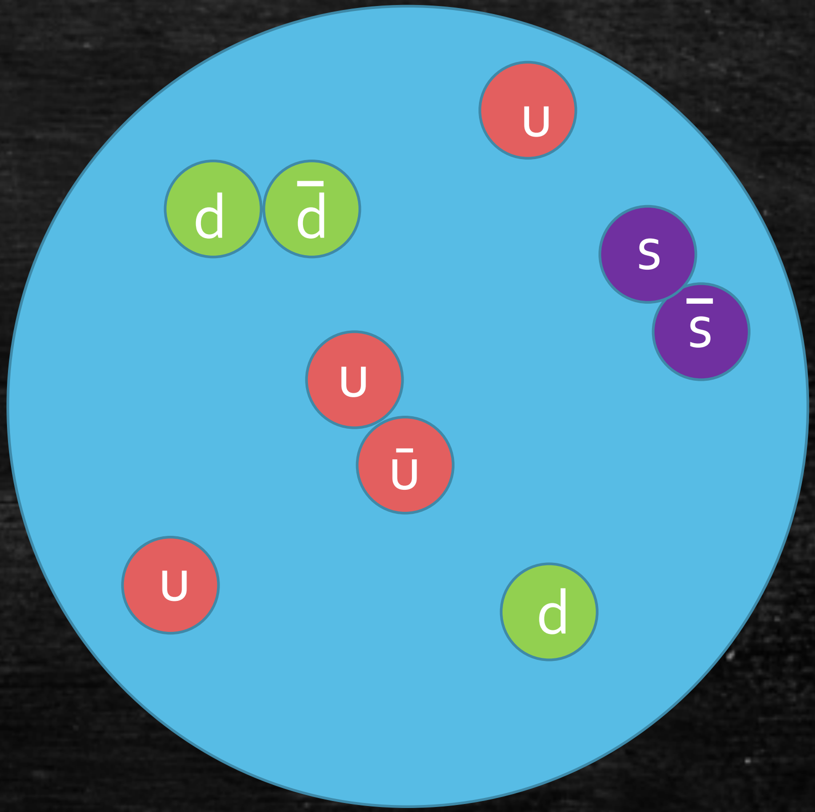
History of Scattering Experiments

- 1908-1913: Rutherford's 'Gold Foil' experiments demonstrated the existence of the atomic nucleus.
- 1950s: Robert Hofstadter at SLAC discovered the existence of subnucleon structure, which later led to quark and QCD theory
- 1987: The European Muon Collaboration built muon scattering experiments which led to the Proton Spin Crisis



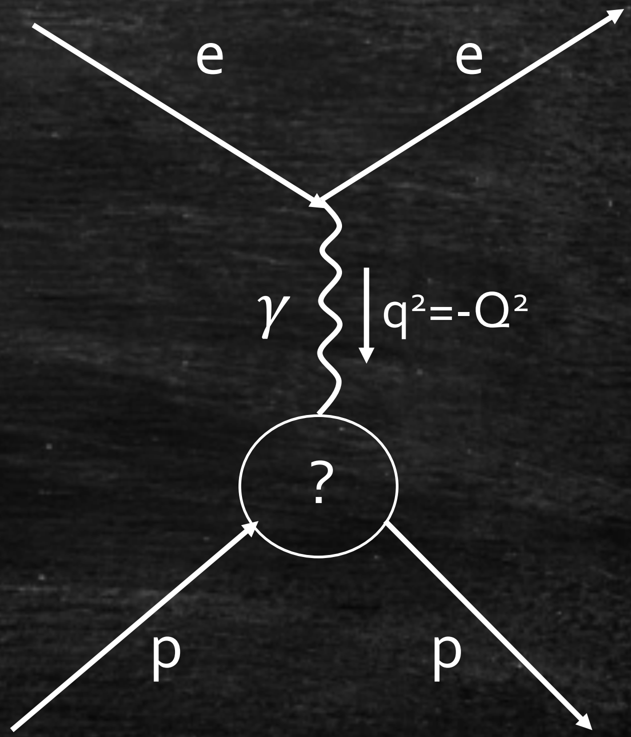
The Proton and its Spin

- Three valence quarks bound by gluons, each with a different color charge
- An infinite "sea" of quarks and gluons, where quarks are balanced out with antiquarks
- This "sea" of quarks and gluons do not contribute to the proton's charge, but they contribute to ~70% of its spin – the "Proton Spin Crisis"



Quantifying the Proton's Structure

- Electric and Magnetic Form Factors G_E and G_M are determined in Deep Inelastic Scattering experiments
- G_E and G_M are related to charge and magnetization densities and vary with Q^2. Larger Q^2 -> More precision

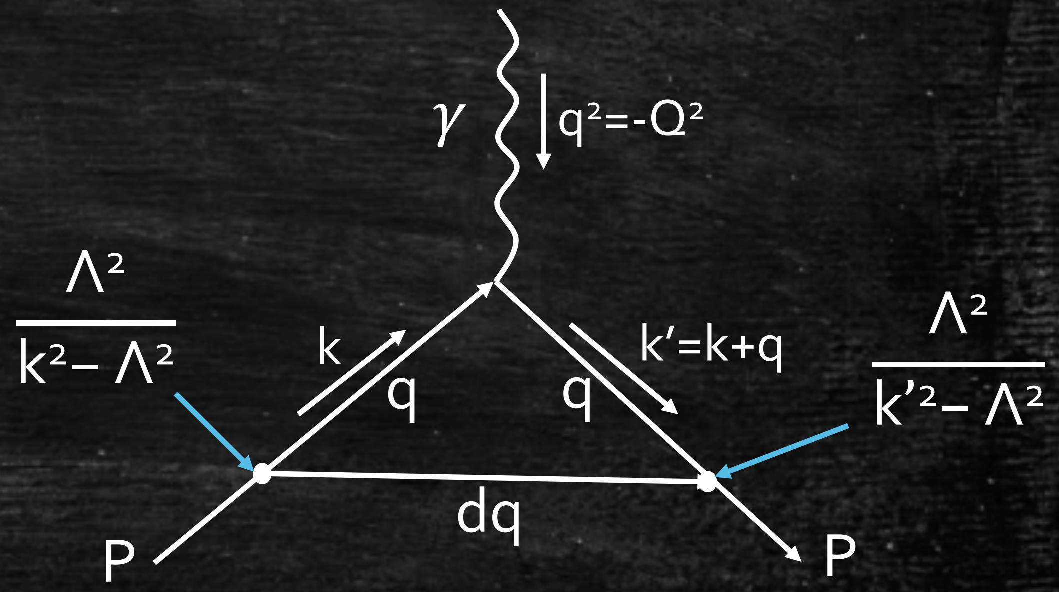


$$J^{\mu} = \bar{u}(p')\left[F_1\gamma^{\mu} + F_2\, i\, \frac{\sigma^{\mu\nu q\nu}}{2M}\right] u(p)$$

$$G_E = F_1 - \frac{Q^2}{4M^2}F_2$$
$$G_M = F_1 + F_2$$

A Quark-Diquark Model with Scalar Vertices

- The proton-quark-diquark vertices are model parameters
- This form allows simple covariant calculation without having to deal with divergences & renormalization in QFT
- To completely solve the Feynman diagram, the diquark propagator must be analyzed more deeply



Quark-Diquark Model Spin-Flavor Wavefunction

To find the diquark spin dependence:

1) ψ_{s+f} spin-state decomposition:

- Vector diquark: $\langle \uparrow\uparrow_{dq} \downarrow_q | \psi_{s+f} \rangle$, with coefficient A_V
- Scalar diquark: $(\langle \uparrow\downarrow_{dq} \uparrow_q | - \langle \downarrow\uparrow_{dq} \uparrow_q |) | \psi_{s+f} \rangle$, with coefficient A_S

2) Match charge-dependence to expected behavior with a lone quark charge operator $\hat{C}$

For the proton:

1) $|\psi_{s+f}\rangle = A_S |u_q S_{dq}\rangle + A_V\left(|uqTdq\rangle + \sqrt{2}\,|dqTdq\rangle\right)$

2) $\langle \psi_{s+f} | \hat{C} | \psi_{s+f} \rangle = \frac{+2}{3} A_S + \left(\frac{+2}{3} + \frac{-1}{3}(2)\right) A_V$

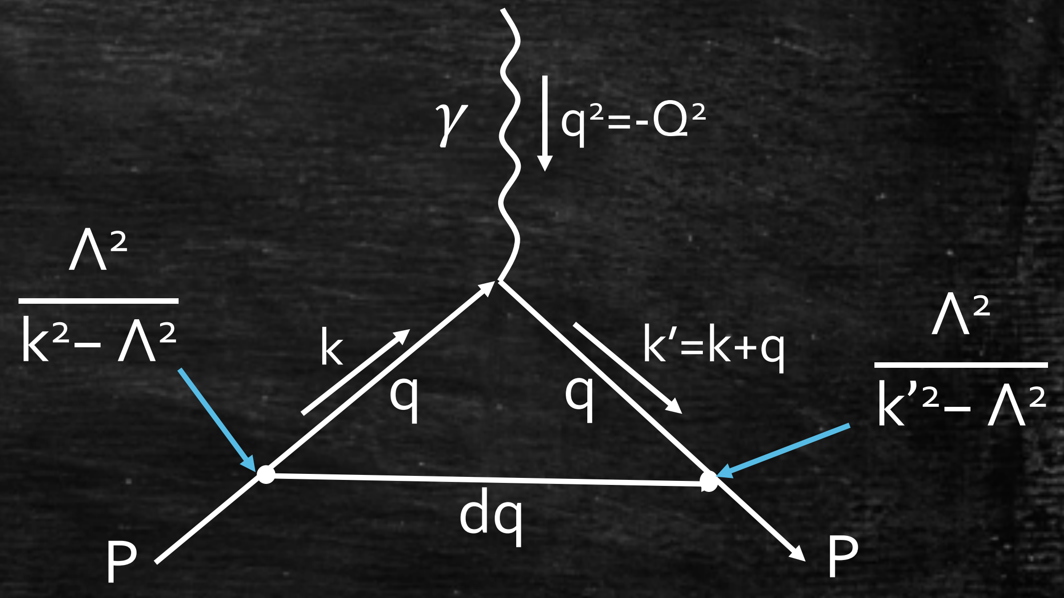
The second term cancels, leaving only the scalar contribution! Thus, $A_S = \frac{+3}{2}$.

Similar calculation with u and d quarks switched leads to the neutron spin contributions, with $A_S = \frac{-1}{2}$ and $A_V = \frac{+1}{2}$

A Quark-Diquark Model with Scalar Vertices

Using the above scalar and axial vector contributions to the total current, it is possible to solve the Feynman diagram completely. These equations are solved and fitted to the form,

$$J^{\mu} = \bar{u}(p')\left[F_1\gamma^{\mu} + F_2\, i\,\frac{\sigma^{\mu\nu q\nu}}{2M}\right] u(p)$$



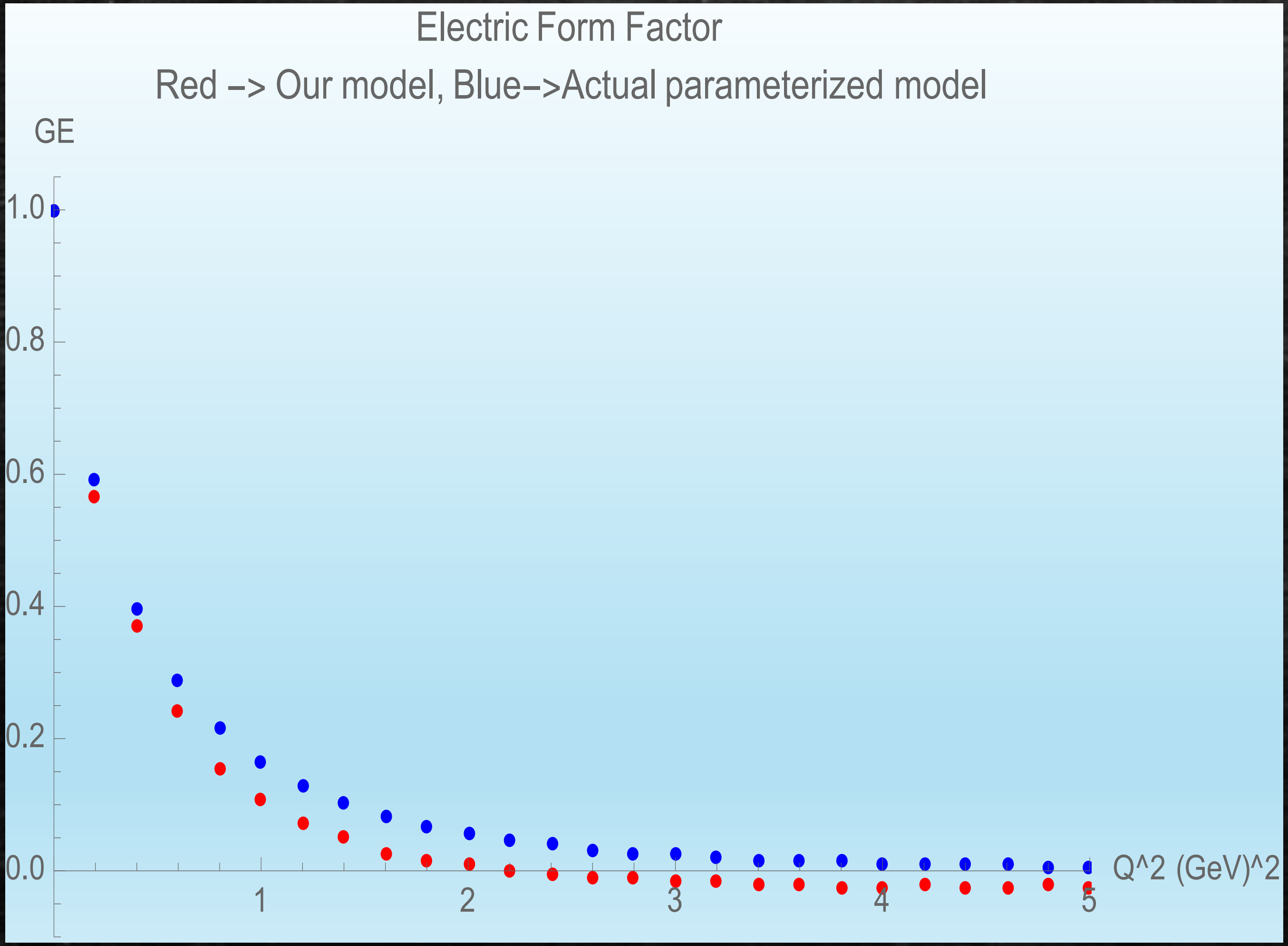
Scalar diquark:

$$J^{\mu} = \frac{\Lambda^2}{k^2-\Lambda^2}\,\frac{\Lambda^2}{k'^2-\Lambda^2}\,\frac{i}{(p-k)2-md^2}\,\bar{u}(p')\left[\frac{i(k'+m)}{k'^2-m^2}\gamma^{\mu}\frac{i(k+m)}{k^2-m^2}\right] u(p)$$

Axial diquark:

$$J^{\mu} = \frac{\Lambda^2}{k^2-\Lambda^2}\,\frac{\Lambda^2}{k'^2-\Lambda^2}\,\frac{ig_{\nu\rho}}{(p-k)2-md^2}\,\bar{u}(p')\left[(-i\gamma^{\nu}\gamma^{5})\frac{i(k'+m)}{k'^2-m^2}\gamma^{\mu}\frac{i(k+m)}{k^2-m^2}(-i\gamma^{\rho}\gamma^{5})\right] u(p)$$

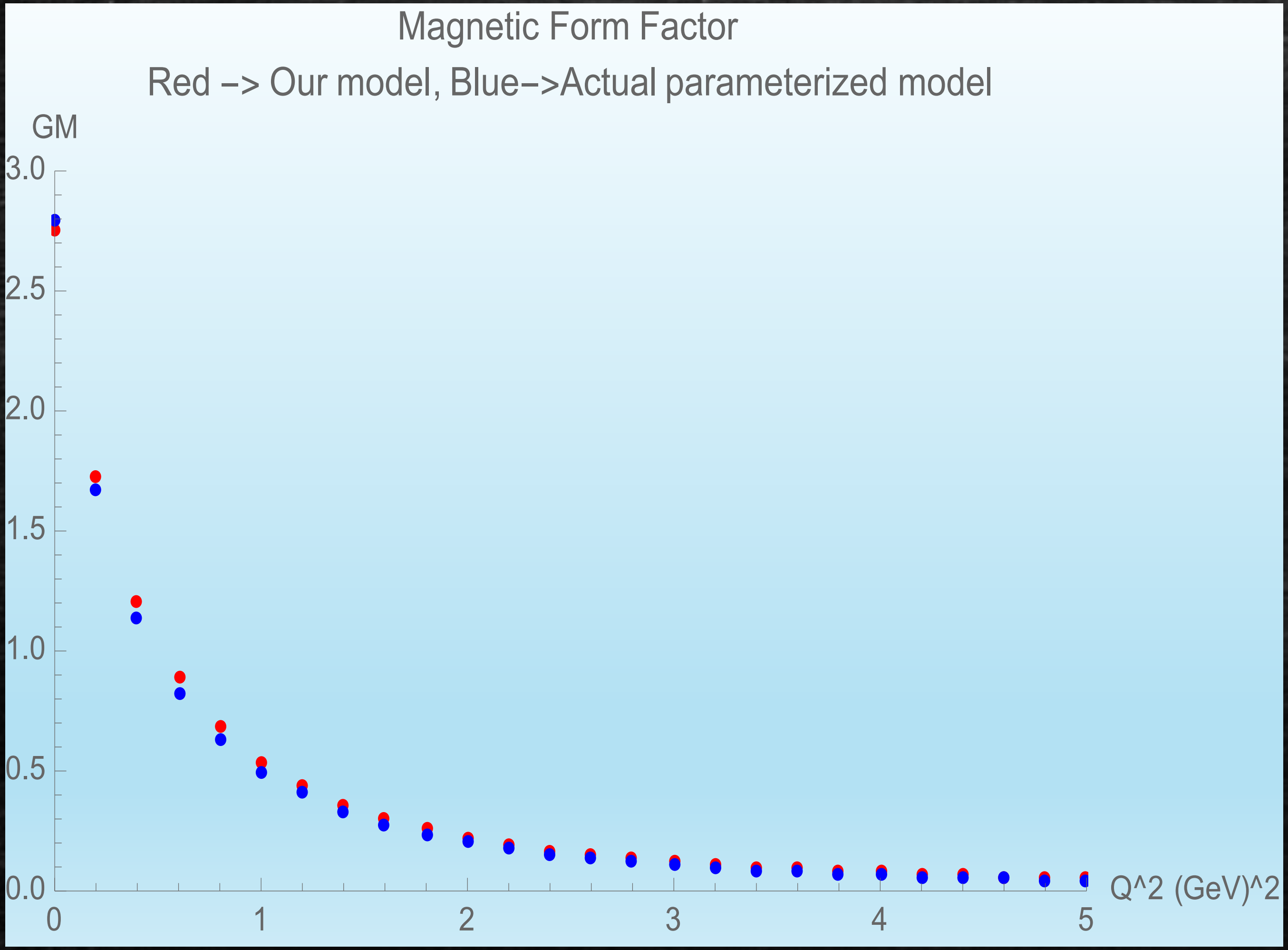
Quark-Diquark Model: Electric Form Factor



Fitted parameters (GeV):

- m_{proton}=.939
- m_{quark}=.434
- $m_{diquark}$=.718
- Lambda=.483
- χ^2~.05

Quark-Diquark Model: Magnetic Form Factor



Fitted parameters (GeV):

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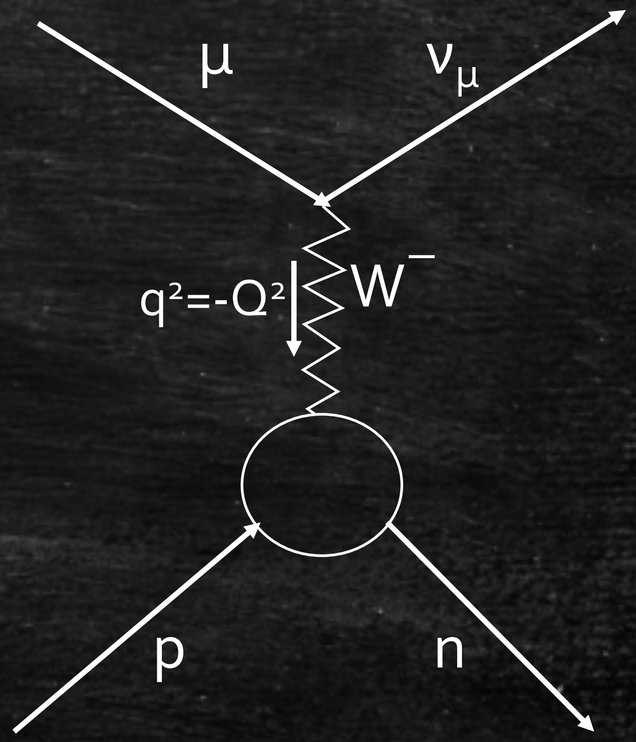
Scattering with Weak Interactions

- Parity violation in weak interactions allows further charge, spin & isospin distribution data
- Additional form factors:
  - Axial FF (G_A): axial vector ("vector" where transformations do not flip its sign)
  - Pseudoscalar FF (G_P): pseudoscalar ("scalar" where transformations flip its sign)

$$J^{\mu} = \bar{u}(p')\left[(\mathrm{GA}\gamma^{\mu}\gamma^{5} + G_{\mathrm{P}}\gamma^{5}q^{\mu})\,\frac{\vec{\tau}}{2}\right]u(p)$$

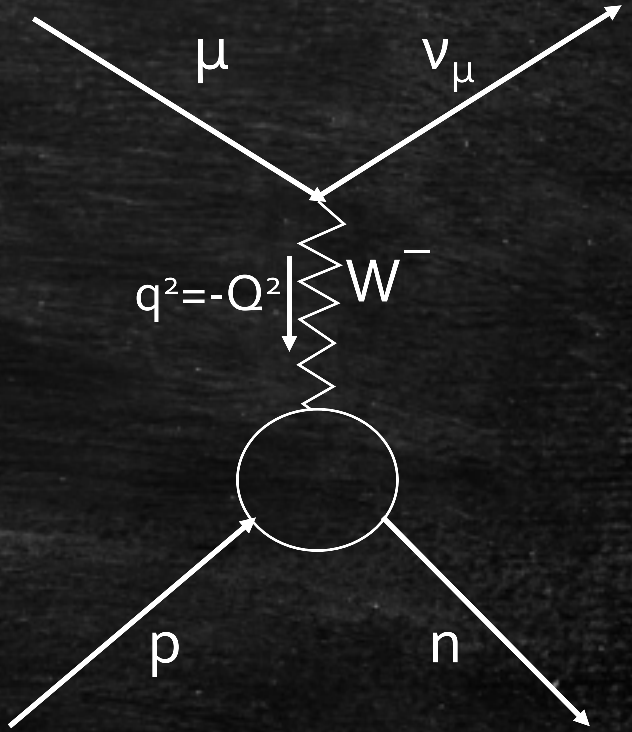


Scattering example: muon capture

Axial Form Factors

Axial Form Factors:

- Provide information on spin-isospin distributions
  (i.e. they can discriminate between ‘upness’ and ‘downness’)
- Provide insight into the differences between proton and neutron structure
  - Isospin symmetry violation
  - Strangeness contributions to charge & magnetization
- Not well-measured due to the complicated nature of weak scattering experiments. Thus, they are currently modeled only by a simple dipole form,
  $G_D(Q^2) = \left(1 + \frac{Q^2}{\Lambda}\right)^{-2}$



Scattering example: muon capture

Axial Form Factors

Again, we must find scalar and axial diquark contributions

1) Start with the proton flavor wavefunction,

$$|\psi_f\rangle = \frac{1}{\sqrt{2}}|u_q S_{dq}\rangle + \left(\frac{1}{\sqrt{6}}|uqTdq\rangle + \frac{1}{\sqrt{3}}|d_q T_{dq}\rangle\right)$$

2) Define 'upness' and 'downness' operators $\hat{u}$ and $\hat{d}$, with eigenvalues equal to the number of u or d quarks in $|\psi\rangle$, and leave behind a scalar or axial wavefunction ψ_s or ψ_a

i.e.

- $\hat{u}\,|u_q S_{dq}\rangle = \psi_s$
- $\hat{d}\,|u_q S_{dq}\rangle = 0$

3) Compute $\langle\hat{u}\rangle$ and $\langle\hat{d}\rangle$

$$\langle\hat{u}\rangle =: \Delta u = \langle\psi_f|\hat{u}|\psi_f\rangle = \frac{1}{2}\psi_s + \frac{1}{6}\psi_a$$

$$\langle d\rangle =: \Delta d = \langle\psi_f|\hat{d}|\psi_f\rangle = \frac{1}{6}\psi_a$$

Normalize →

$$\Delta u = \frac{3}{2}\psi_s + \frac{1}{2}\psi_a$$

$$\Delta d = \psi_a$$

Axial Form Factors

$$\Delta u = \frac{3}{2}\psi_s + \frac{1}{2}\psi_a \; ; \; \Delta d = \psi_a$$

$$A^\mu = \bar{u}(p')\left[G_A \gamma^\mu \gamma^5 \frac{\vec{\tau}_3}{2}\right]u(p),$$

Following the right equation, the $\vec{\tau}_3$ term is the third Pauli Spin Matrix. Thus, the proton wavefunction is given by

$$\psi_f = \Delta u - \Delta d = \frac{3}{2}\psi_s - \frac{1}{2}\psi_a$$

$$\vec{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

+1 for u quarks

-1 for d quarks

Axial Form Factors

$$\Delta u = \frac{3}{2}\psi_s + \frac{1}{2}\psi_a \; ; \; \Delta d = \psi_a$$

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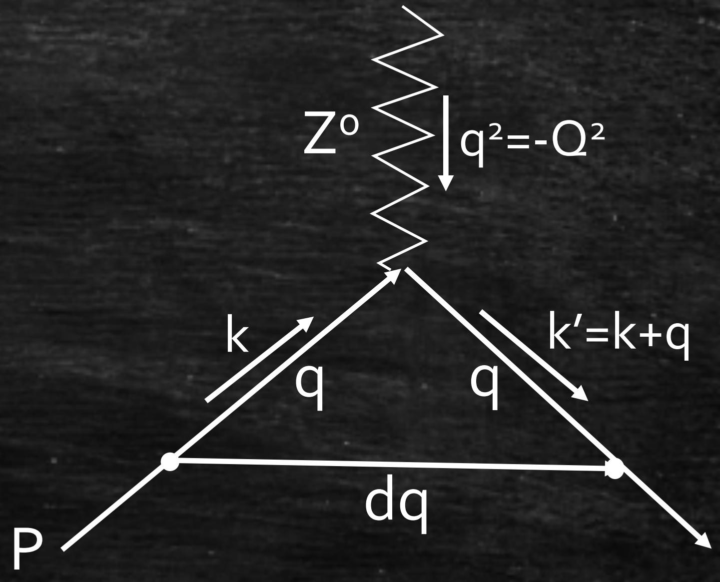


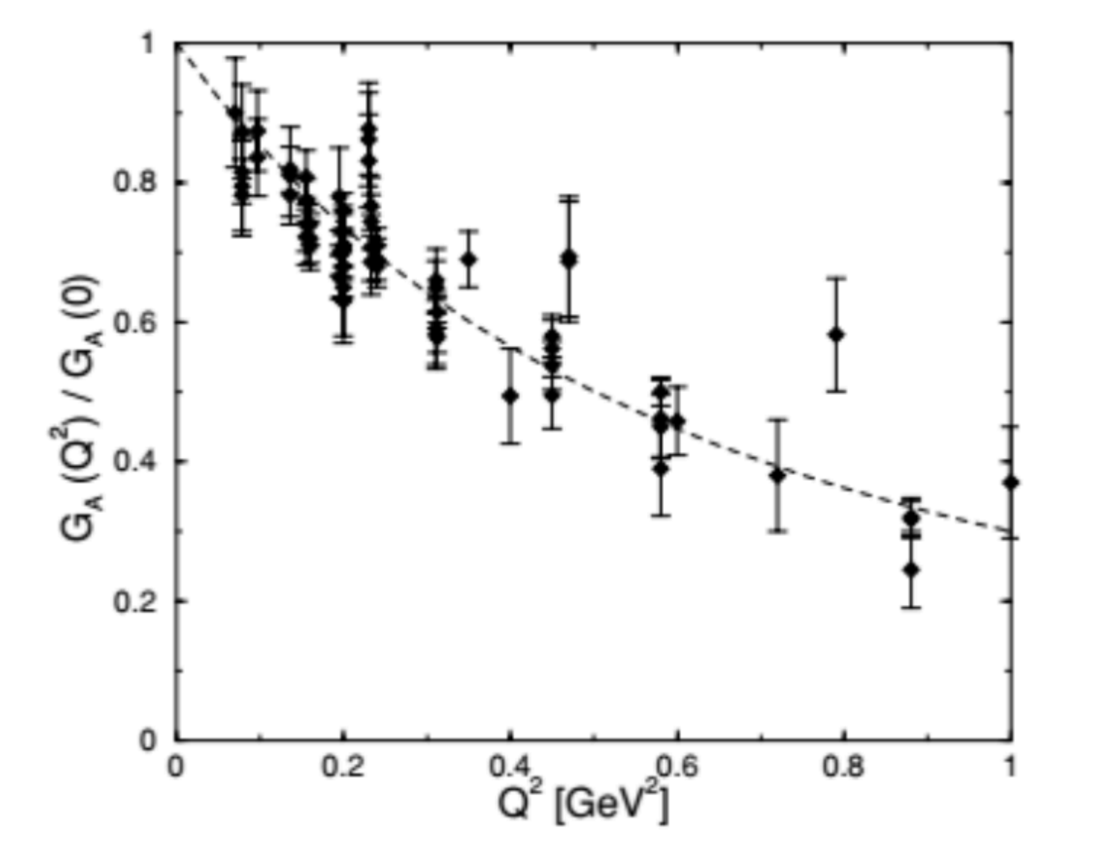
Scalar diquark: $$J^{\mu} = \frac{\Lambda^2}{k^2 - \Lambda^2}\,\frac{\Lambda^2}{k'^2 - \Lambda^2}\,\frac{i}{(p-k)2 - md^2}\,\bar{u}(p')\left[\frac{i(k'+m)}{k'^2 - m^2}\gamma^{\mu}\gamma^{5}\frac{i(k+m)}{k^2 - m^2}\right]$$

Axial diquark: $$J^{\mu} = \frac{\Lambda^2}{k^2 - \Lambda^2}\,\frac{\Lambda^2}{k'^2 - \Lambda^2}\,\frac{ig_{\nu\rho}}{(p-k)2 - md^2}\,\bar{u}(p')\left[(-i\gamma^{\nu}\gamma^{5})\frac{i(k'+m)}{k'^2 - m^2}\gamma^{\mu}\gamma^{5}\frac{i(k+m)}{k^2 - m^2}(-i\gamma^{\rho}\gamma^{5})\right]u(p)$$

Expected Findings

We expect similar but more complicated Q^2 dependences, like we see for G_E and G_M.

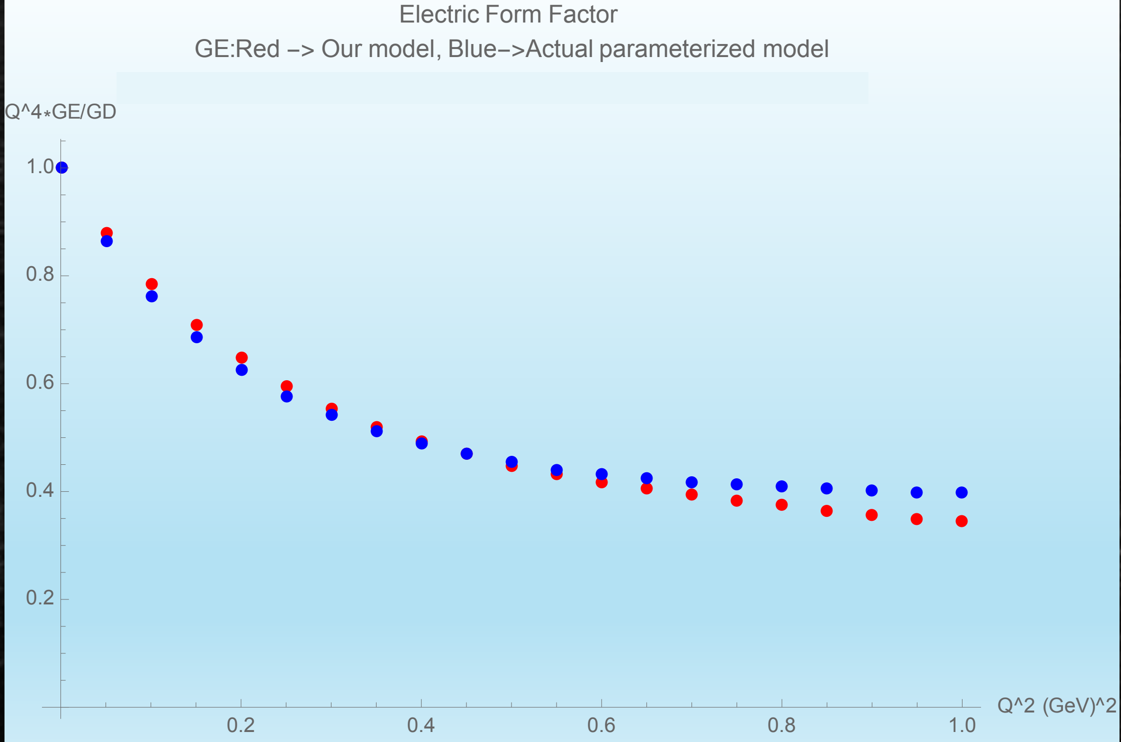
Using the simple G_D Form Factor



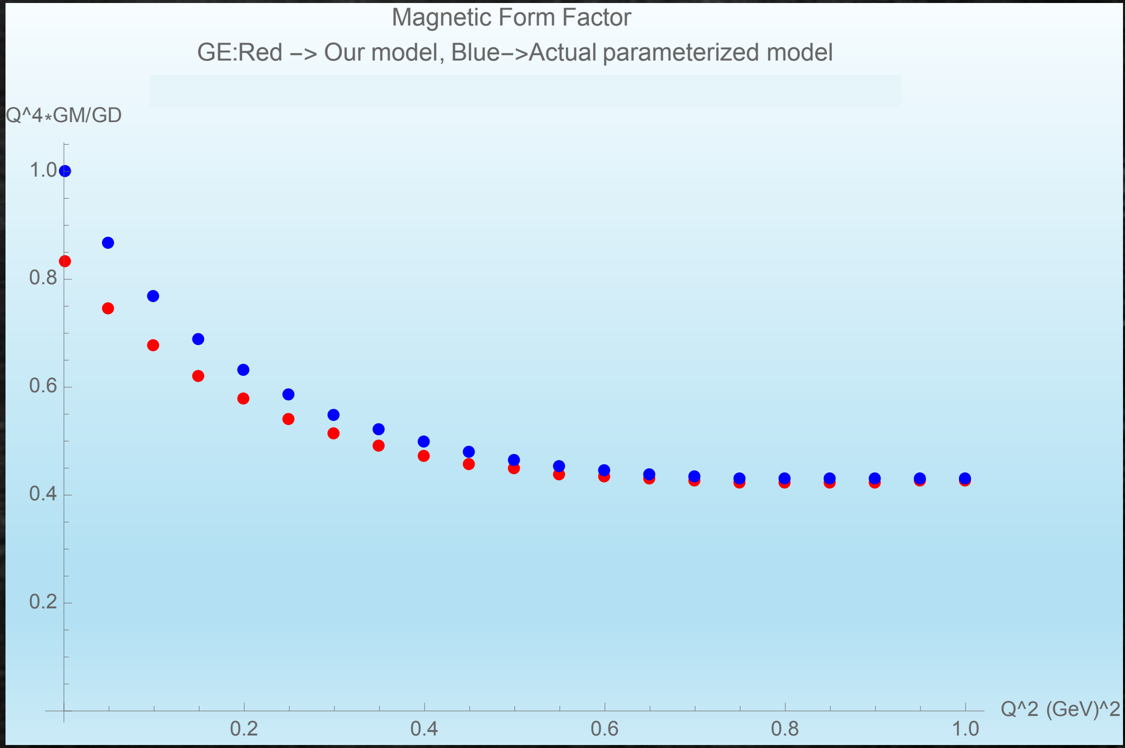
From Meissner et. al. "Axial Structure of the Nucleon"

Expected Findings: Accurate Low-Q^2 Data

Electric Form Factor



Magnetic Form Factor



Conclusion

- QCD is extremely difficult to calculate perturbatively!
- Electric, magnetic, and axial form factors are related to proton structure and can lead to further insights into the *Proton Spin Crisis*
- The new quark—diquark model seeks to improve the existing axial form factor model for further insights into the proton’s spin structure

Thanks to…

- INT and NSF for allowing me to do physics without having to pay to do it!
- Tim Hobbs, Xilin Zhang and Gerald Miller for being fantastic mentors, and for allowing me to not only *help out* but to *do* a challenging and exciting nuclear theory project – the only downside of my time here is that it was only 10 weeks!
- Deep and Gray for putting this whole program together
- Other REUs for great times hiking, climbing, swimming, and eating our way through Eastern Washington

Citations

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