Investigating student understanding of forces in the context of pressure

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Abstract

This paper describes an investigation about the reasoning students use when comparing the pressure at different points in liquids. In many cases, the comparison can be made by analyzing the forces exerted in the fluid and connecting those forces to the pressure. Two tasks were used to probe student abilities to use forces in fluids. The results from these tasks revealed difficulties that students may have reasoning about forces in general, and also identifying forces acting on a portion of a fluid. These difficulties may be related to errors that students make when answering questions about pressure.

I. Introduction:

The Physics Education Group (PEG) at the University of Washington (UW) has investigated student understanding of many different topics in introductory physics, including hydrostatic pressure and buoyancy. The group has developed tutorials both on pressure in a liquid and buoyancy as a part of their *Tutorials in Introductory Physics* [1]. The pressure tutorial has been shown to improve student performance on questions about pressure comparisons in liquids, although the tutorials are continually reassessed and edited based on current research results [2].

This paper describes an investigation of student ability to make a comparison of the pressure at two points in different liquids, a task that has been shown to be hard for many students [3]. In the tutorial, students are introduced to pressure using free-body diagrams and Newton's laws. If students were to apply the methods used in the tutorial to compare the pressures in two different liquids, they would first need to have a good understanding of how to identify and rank forces. They would also need to be able to treat a section of the liquid as an object on which forces are acting. We assigned two tasks to probe students' abilities in these areas. The first task asks students to identify and rank forces on solid objects that are not in a fluid. The second task requires students to treat a section of a liquid as an object and compare forces acting on it to those acting on solid objects.

II. Context for Research

The tasks used in this study were given in the first quarter of a calculus based introductory physics class at the UW, which covers mechanics and fluids. The course includes lecture, laboratory, and tutorial components. The lecture component is treated much like a flipped classroom, in that students begin each topic by watching an online video before attending lectures on the topic. After the lecture, students attend a tutorial section related to the topic. In the tutorial, students work in small groups on materials developed by the PEG to address specific student difficulties with the topic. The materials come from *Tutorials in Introductory Physics*. Before the tutorial, students must complete a pretest on the material that will be covered. In addition, one question on each exam specifically covers material learned in tutorials.

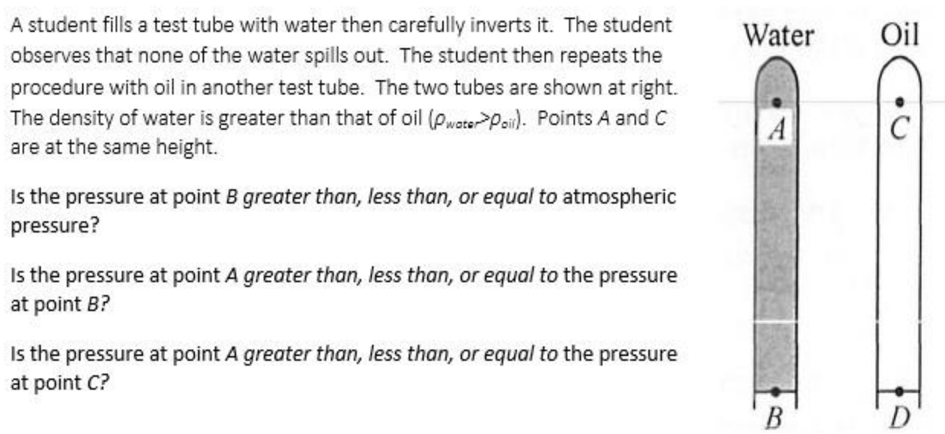


Figure 1: the test tube problem

III. Motivation

We began this investigation by analyzing a three part problem given on the final exam during two different quarters at UW. One question required students to compare the pressure at two points at the same height but in different liquids. Although students were not required to explain their answers, their responses helped us identify possible reasons that students may be having difficulty with this type of question.

A. Description of question

The problem describes two inverted test tubes, one filled with water and the other with oil. The students are told that the density of water is greater than that of the oil. They are then asked three questions about the pressures at different points.

The first question asks students to compare the pressure at a point on the lower surface of the water to atmospheric pressure. In order to answer this question correctly, they need to realize that the pressure at the surface of a liquid that is open to air is always equal to atmospheric pressure, regardless of the physical location of the surface.

The second question asks for a comparison of the pressure at a point in the water above the surface and the pressure at the point on the surface. There are several ways to answer this question. Students could use the pressure equation, $P=P_o+\rho gh$, to make this comparison, where P gives the pressure at a depth h relative to a location of known pressure in a liquid with density ρ. This approach requires students to know that pressure increases with depth, and since the point is above the surface, the pressure must be less than that at the surface. Another possible approach involves drawing a free-body diagram for the section of water between the two points. Since the water is in equilibrium, the net force must be zero, and the magnitudes of the forces at each surface of the section can be compared using Newton's Second Law. Comparing the magnitudes of these forces allows for a comparison of the pressure at each point (see Figure 2A).

The third question asks students to compare the pressure at a point in the water to the pressure at a point of equal depth in the oil. In order to answer this question, students could use the above equation to realize that the pressure in the water changes more per unit of distance than the pressure in the oil, and conclude that the pressure in the water at that point must be less than the pressure in the oil. Another way to find the correct answer would be to draw free body diagrams for the section of water and the section of oil, which are both in equilibrium, and use Newton's laws to compare the forces on each (see Figure 2B). Since the forces on the bottom of the sections are equal (the pressure at the bottom surface must be atmospheric pressure for both liquids), and the weight of the water is greater than the weight of the oil, the force on the top of the water must be less than the force at the top of the oil. This means the pressure at the point in the oil must be greater than the pressure at the point in the water.

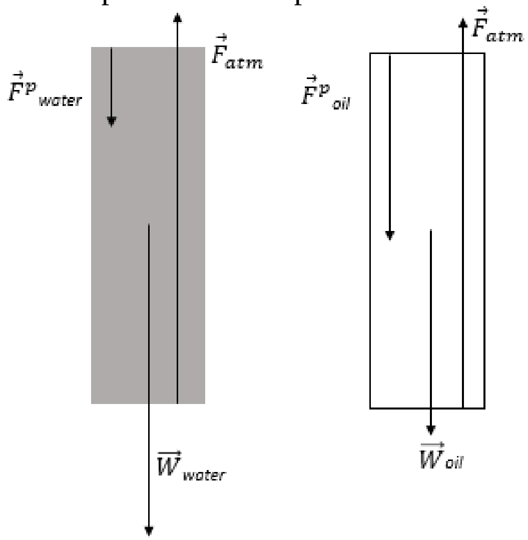


Figure 2: Free-body Diagrams for the sections of water (left) and oil (right) in the test tubes. A) Since the force from the atmosphere must balance the sum of the force from the water above point A and the weight of the water, the force from above must be less that the force from the atmosphere. B) Since the weight of the water is greater than the weight of the oil, and the upward forces from the atmosphere are the same on each liquid, the downward force due to pressure in the fluid must be less in water than in oil.

B. Results of question

The results of the test tube question are given in Table 1. Since the results from multiple sections were very similar, they have been combined into one table. On the first two questions, more than half of the students answered correctly. Only about a fifth of the students answered the third question correctly.

Table 1: Results from test tube question in figure 1. Results have been rounded to the nearest 5%. The correct responses are shown in bold.

	Question 1	Question 2	Question 3
Greater Than	25%	20%	70%
Less Than	15%	**75%**	**20%**
Equal	**60%**	5%	10%

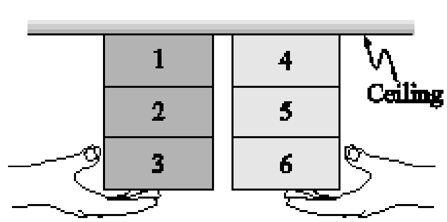
The most common incorrect answer choice on the first question, given by a quarter of the students, was that the pressure at the surface is greater than atmospheric pressure. This might suggest that the students understand the depth dependence of pressure, but may think that pressure is atmospheric at the top of the container, not at the surface open to the air.

The results for the second question also may suggest that many students understand that pressure is dependent on depth. The most common incorrect choice was that the pressure is greater at the top of the tube. Previous research has reported that some students treat depth as the distance along a tube from the opening, so they say that the pressure increases as this distance increases [4]. Students may be using somewhat similar reasoning in this question to conclude that pressure is greater at the top of the tube than at the surface.

Finally, a very small percentage of students answered the third question correctly. The most common choice was that the pressure is greater in the water than in the oil. According to previous research, some students say that the weight of fluid above the point is what causes the pressure [5]. This reasoning would lead students to the conclusion that the pressure is greater in the water. Students may also know that pressure is related to density, but incorrectly associate a greater density with a greater pressure [6]. Even if students attempt to reasoning through the problem with a correct conceptual understanding, they could have difficulties with the many steps of reasoning required that would cause them to answer incorrectly.

C. Summary

The questions posed in this problem show us that many students do not have a complete understanding of pressure. The authors of the



Two stacks of books are held at rest against the ceiling as shown. Two students each push a stack of books upward with a force of equal magnitude F. Books 1, 2, and 3 have a greater mass than books 4, 5, and 6. All books are the same size.

System S consists of books 2 and 3; system T consists of books 5 and 6.

Draw separate free body diagrams for System S and System T.

Is the magnitude of the force of block 1 on system S *greater than, less than, or equal to* the magnitude of the force of the hand on System S?

Is the magnitude of the force of block 1 on system S *greater than, less than, or equal to* the magnitude of the force of block 5 on system T?

Figure 3: The stacked books problem

tutorials have shown that many students are able to answer questions about situations that are very similar to the ones they have seen in class correctly [7]. However, this problem suggests that students may have difficulty in applying the ideas they have learned to more difficult situations. In particular, the students do not always choose answers that are consistent with the fact that the liquids are in equilibrium. This could be because students are having difficulty applying what they previously learned about Newton's laws in a new situation, or it could be because of unresolved difficulties in understanding those ideas in the first place. In order to probe student thinking in greater detail, we decided to develop two tasks. The first task, described in section IV, would probe student difficulties with applying forces. The second task, described in section V, would probe students' abilities to treat a portion of a liquid as an object on which forces can act.

IV. First Task: Newton's Laws and Equilibrium

The first task was intended to probe students' abilities to use forces and Newton's laws. It was designed to be analogous to the test tube situation, but using solid objects instead of liquids and asking about forces instead of pressure. We assigned this task as a free response question on a mid-term exam after all instruction on forces, and required explanations. The results revealed some important aspects of student reasoning about forces that may be related to their reasoning about pressure.

A. Task Description

We designed the first task, shown in Figure 3, to determine if students could correctly identify forces on solid objects and compare them using Newton's laws. In the problem, two stacks of books are held against a ceiling with equal forces pushing up on them. One stack of books has more mass than the other, but the books are all the same size. Students are first asked to draw the free body diagrams for two systems. Each system consists of a combination of the two lower books from either stack. Students should draw three forces on each system: an upward force from the hand, a downward force from the book above the system, and the weight of the system (which is directed downward).

The first question, designed to be analogous to asking about the pressure at two different points in the same liquid, asks for a comparison of the downward force from the upper book on one system to the upward force of the hand on that same system. Students need to use Newton's Second law to realize that the since system is at rest (so there is no acceleration), the net force must be zero. Since there are two forces directed downward and one force directed upward, the force of the upper book on the system must be less than the force of the hand on the system.

The second question, designed as an analogy to comparing the pressure at two points in different liquids, asks for a comparison of the forces from each of the upper books on the system below. Students are given that both systems are at rest, the force of the hand is equal on each stack, and that the weight is greater for the more massive stack of books. Students can then use Newton's second law to reason that the force of the upper book on the system is less in the more massive stack of books.

B. Task Results

The results from the test tube problem are shown in Table 2. Although less than half of the students answered the second question correctly, a greater percentage of students answered these questions correctly compared to the questions about the test tubes. This result indicates that they have a better understanding of forces in the context of solid objects than pressure in a liquid. However, there were a number of issues with student reasoning, some of which led to the correct answer and some which did not. These issues could be related to the students' abilities to make pressure comparisons later on.

Table 2: Results from stacked books problem shown in Figure 3. The correct answers are given in bold.

	Question 1	Question 2
Greater Than	<5%	45%
Less Than	**90%**	**45%**
Equal	10%	10%

(N=115)

1. Difficulty treating multiple objects as a single system

In this problem, students were asked to treat a combination of two books as one system, and then were asked about the forces acting on that system. Some students failed to reason consistently with this system. For example, some labels on their free body diagrams showed the force of the hand acting only on the bottom book, rather than acting on the entire system. Other labels showed the force from the upper book as only exerting a force on the top book in the system. There were also some students who drew separate weights for both books in the system. These errors did not always correlate with incorrect reasoning in subsequent questions, but nevertheless indicate some difficulty in this area. Students who have difficulties in treating two solid objects as a system may also have difficulty reasoning about the forces on a portion of a fluid.

2. Incorrect association of Normal Force and Weight

Many students made an incorrect assumption about the relationship between the weight of an object and the normal force by that object on the object below it. A small number of students explicitly demonstrated this error in their free body diagrams, either by labeling the force of the upper book on the system as the weight of the book above the system, or by drawing both a contact force and a weight from this book on the system. Other students did not show this relationship explicitly on their diagrams, but demonstrated the same type of reasoning in their explanations for both questions. These students treated the normal force exerted downwards by the object on the system as equivalent to the weight of the object. This reasoning leads to a correct answer for the first question but an incorrect answer for the second question. Most of these students seemed to think that the normal force and the weight would always be the same magnitude. This is similar to the idea raised in the pressure question that the weight of fluid above a point is what causes the pressure at that point.

Alternatively, some students seemed to realize that the normal force depends on more than just the weight, but they did not fully understand how to reason about this in their arguments. They argued that the upward force exerted by the hand must be greater on the heavier stack. They then said that the downward force exerted by the book above the system must be greater since the force by the hand is greater. This error shows that they have difficulty reasoning about the relationship of the normal force to the weight of the object even if they know that other forces can be involved as well.

Overall, most of the students who made errors regarding the treatment of the normal force seem to be ignoring the force exerted on the stack by the ceiling. This force, in addition to the weight of the books, contributes to the normal forces. Ignoring the effect of the ceiling is reminiscent of an error students make in pressure comparisons, in which they fail to account for the forces exerted by the tank on the fluid [8].

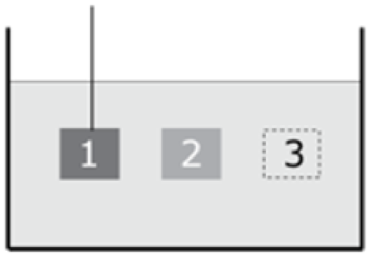
3. Incorrect application of Newton's Second Law

One of the most fundamental errors made by students was an incorrect application of Newton's second law, $F_{net}=ma$. This law says that the net force exerted on an object is equal to that object's mass multiplied by its acceleration, so if the acceleration of the object is zero, there is no net force. However, some students used this equation in the second question to say that the force exerted by block 1 on the system S is greater than the force by block 4 on system T.

The first error in using the equation this way is that, in general, the equation does not refer to a particular force exerted by an object, but instead to the net force. The second error in using the equation to answer this question is that there is no acceleration for any of the books, which should lead students who use this equation to say that the forces exerted by each of the upper blocks is zero. Very few students referenced the acceleration in their explanation, and those who did said the acceleration was the same, but not zero, in both stacks. It seems like these students do not know exactly how to reason through the question. They may have simply found an equation with the right variables used it. Students need to understand the circumstances under which Newton's second law can actually be applied before they can understand the concept of pressure as it is taught in the tutorial.

C. Summary

The purpose of the task shown in Figure 3 was to investigate students' mastery of the concepts they learned about forces in order to gain a better understanding of the extent to which they can later apply these concepts when learning about pressure. The results from the task revealed that many students do not have the level of understanding about forces necessary to apply to pressure, so more work needs to be done in this area. Some students made errors in their reasoning about forces that are very similar to the errors seen in pressure tasks, and a better understanding of forces may help these students develop better reasoning in both areas. Incorporating a problem similar to this one in instruction could possibly be beneficial in helping students develop their understanding of forces, but this would need to be followed up with research on the extent to which they can apply that understanding to the concept of pressure later.



Three cubical objects are all at the same height above the bottom of a tank of water, as shown.
Cube 1, which sinks, is suspended in place by a string.
Cube 2, when submerged in water, remains at the location it is placed.
Cube 3 is a cubical section of water as shown.
All three cubes have identical size and shape.

1. Rank the buoyant forces on each cube, B1, B2, and B3, according to magnitude from largest to smallest. If any of the buoyant forces are zero, state that explicitly.
2. Is V_1, the volume of water displaced by cube 1 *greater than, less than*, or *equal to* V_2, the volume of water displaced by cube 2?
3. Is the magnitude of the force by the water on the top of cube 1 ($F_{1,top}$) *greater than, less than*, or *equal to* the magnitude of the force by the water on the bottom of cube 1 ($F_{1,bottom}$)?
4. Is the magnitude of the force by the water on the top of cube 3 ($F_{3,top}$) *greater than, less than*, or *equal to* the magnitude of the force by the water on the bottom of cube 3 ($F_{3,bottom}$)?
5. Is the magnitude of the total force by the water on cube 1 ($F_{1,total}$) *greater than, less than*, or *equal to* the magnitude of the total force by the water on cube 3 ($F_{3,total}$)?

Figure 5: Forces in a Liquid task

V. Second Task: Ranking Forces in a Liquid

The second task that was developed was intended to investigate students' ability to treat a portion of a liquid as an object. The ability to visualize this object is crucial for understanding pressure. We assigned this task as part of an online pretest given after the lecture on static fluids.

A. Task Description

In this task, there are three cubes in a tank of water. The first cube sinks in water but is suspended by a string, the second cube is neutrally buoyant, and the third cube is a section of the water in the tank. All three cubes are the same size and are at the same level. Students are asked two sets of questions about this situation.

The first set of questions starts by asking for a ranking of the buoyant forces on each cube. In order to answer this part correctly, students could recognize that since the cubes are all the same size, they displace the same volume of water and thus the buoyant forces must all be equal. Next, students are asked to compare the volume of water displaced by each of the solid cubes. This allows us to investigate whether or not students make the connection between volume of fluid displaced and the buoyant force.

The second set of questions is about comparing forces by the water on the top and bottom faces of the various cubes. First, students are asked to compare these forces for the cube that sinks in water. In order to answer this question correctly, they need to realize that since the pressure is greater at the bottom of the cube, that force must be greater. Another way to answer is related to the buoyant force. If they know that the total force of the water (the buoyant force) on the cube is directed upward, students could make the correct comparison without explicitly talking about the pressure. Then, students are asked to compare the force exerted by the water on the top and bottom of the cube of water. Using either of the reasons that were used for the comparison using the solid block, students could reason that the force on the bottom of the cube must be greater.

Finally, students are asked to compare the net force by the water on the first and third cubes. Since the difference in the forces on the top and bottom is the same for each cube, the net force must be the same for each cube. When compared to the ranking of the buoyant force, this question also reveals whether or not students know that the buoyant force is the same as the net force of the water on an object.

B. Task Results

The answers students gave on this task revealed many different ideas that they hold about the buoyant force. This section will focus only on the results that relate to understanding students' ideas about pressure. These results are from questions 1 and 4.

Results from the first question, which required students to rank the buoyant force on each of the three blocks, are shown in Table 3. Since the second and third blocks are the most similar, we looked at differences in how students treated each of those blocks. Almost all aspects of these two cubes are the same: the density, volume, depth, and behavior in the water (both cubes are neutrally buoyant). Since the only differences in the cubes are the fact that one is a solid while the other is made of water, any differences in how students treat each cube would most likely be due to this fact. These results show that approximately 35% of the students made an incorrect comparison of the buoyant force on each cube, and thus fail to treat the cubical section of water as an object.

Table 3: Results from question asking for a comparison of the Buoyant Force acting on Cube 2 and Cube 3 (see Figure 3, question 1). Results have been rounded to the nearest 5%, and the correct answer is shown in bold.

Equal	**60%**
Not equal	35%
Blank	10%

N=100

In addition, some students demonstrated explicit difficulty in treating the water as an object when comparing the forces by the water on the top and bottom of the cube of water (see Figure 3, question 4). Between 10 and 15% of students seemed to indicate in their explanations that the water cannot exert any forces on this cube simply because it is made of water. These students gave explanations like, "it's just water," or "it's part of the system," and used those statements to justify the idea that there are no forces exerted on the water. These students definitely have difficulty in talking about forces acting on the water. Most of them did not correctly compare the buoyant forces on the solid and liquid cubes. However, a small number (<5%) of students correctly compared the buoyant forces but indicated difficulty with the water as an object in this situation.

C. Summary

The second task was designed to find out if students have difficulties in treating a section of water as an object and in identifying the forces acting on it. The results showed that a significant percentage of students failed to treat the forces acting on the cube of water in the same way they treated the forces acting on a solid object. This finding suggests that students would likely have difficulty working through the pressure tutorial, in which they must compare forces acting on various sections of a fluid. The idea of treating a liquid as an object does not appear to be something that all students can recognize, and needs to be addressed before those students can effectively learn about pressure using the tutorial.

VI. Conclusion

The tutorial on pressure requires students to identify forces acting on pieces of a liquid and to compare the magnitudes of those forces. It has previously been assumed that students will begin instruction in pressure with a sufficient understanding of these ideas to make sense of the questions in the tutorial. However, the results from the tasks in this study reveal that many students struggle with various aspects of the reasoning required to make pressure comparisons in liquids. Students have difficulty comparing forces even before liquids are introduced, and also demonstrate difficulty in considering the forces acting on a piece of a liquid. Improving student abilities in these areas may lead to improved understanding of pressure in general.

Future research should include finding ways to address student difficulties in these areas, by developing and testing materials to be used in tutorials. For example, a problem similar to the one used in the first task could be useful in further developing students' understanding of forces and thus benefit their understanding of pressure as well. A problem like this could be incorporated into a tutorial or as a tutorial homework problem. Similarly, we would also want to address the idea that water can be treated as an object more explicitly. After developing curriculum to address the difficulties students have with comparing forces in liquids, that curriculum would need to be tested by implementing the ideas and evaluating their effects on student reasoning.

VII. Acknowledgements

I would like to thank Peter Shaffer and the rest of the Physics Education Group for welcoming me into their group this summer and assisting me with my project. Also, thanks to Nikolai Tolich, the instructor of the class I worked with. In addition, I would like to thank DJ Wagner and the student researchers at Grove City College for helping me find background information on students' conceptions of pressure. I would also like to thank Deep Gupta, Gray Rybka, Linda Vilett, and Cheryl McDaniel for organizing the summer REU program. Finally, I would like to thank the NSF for funding the program.

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