Metasurface-based spin-selective optical cavity

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Abstract

We seek to develop a low-contrast metasurface-based optical cavity that breaks degeneracy between left- and right-handed polarization modes. Using birefringent materials, we propose such a cavity design, and develop a formalism to determine left- and right-handed polarization eigenmodes. In this polarization eigenbasis, the cavity operates as two decoupled resonators, each with a distinct path length. We discuss our methodology for designing silicon nitride metasurface-based birefringent optical elements, and work completed to facilitate the search for appropriate design parameters.

1 Introduction

Optical cavities form an important part of the optical physicist's toolbox. By augmenting the photon lifetime and light-matter interaction within the cavity volume, cavities provide an effective method for enhancing weakly coupled phenomena in AMO physics, quantum optics, gravitational astronomy, and many other disciplines.[1] Moreover, through the engineering of cavity geometry and materials, cavities may be built to support specific longitudinal and transverse modes, or to manipulate more exotic properties such as spin and orbital angular momentum.[2]

Constructing cavities with explicitly polarization-dependent behavior presents novel avenues for experimentation. High-finesse cavities demonstrating "natural" linear birefringence (*i.e.* with a preferred linear polarization axis, perhaps arising from small symmetry-breaking defects in cavity components or alignment) are commonly observed. However, photons are most naturally understood in terms of intrinsic spin, which may be correctly associated with the circular polarization states of classical light.[3] Hence, careful control of circular polarization is useful when addressing problems fundamentally concerned with photon spin. Applications useful to spintronics, such as producing exiton-polaritons with defined spin, or in quantum information science, where information might be encoded in photon spin, require both such control as well as efficient spin-photon coupling, suggesting the usefulness of a cavity sensitive to circular polarization states.

We seek to develop a novel cavity whose behavior is dependent on the circular polarization of the enclosed light. In particular, we wish to design a cavity with defined circular polarization modes, exhibiting an arbitrary frequency splitting between left- and right-handed modes within the cavity volume. We are interested in pursuing a metasurface-based cavity design to allow for tight control of polarization, as well as to allow for miniaturization of the system. We propose a cavity design and introduce a formalism to determine the cavity polarization modes; we also discuss our progress toward developing appropriate metasurface-based optical elements.

2 Background

2.1 Jones calculus and birefringence

For a plane wave with momentum $\mathbf{k} = k\hat{\mathbf{z}}$, we may describe $\mathbf{E}(z,t) = \mathbf{E_0}e^{i(kz-\omega t)}$ as follows:

$$\mathbf{E}(z,t) = \begin{pmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{pmatrix} e^{i(kz-\omega t)} \tag{1}$$

Note that $\mathbf{H}(z,t)$ is completely determined by $\mathbf{E}$ and the wave impedance of the medium. Hence, the Jones vector

$$\begin{pmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{pmatrix} \tag{2}$$

fully characterizes the amplitude, phase, and polarization of light, provided it comprises a transverse wave in a pure polarization state. Moreover, polarization optical elements may be expressed as $\mathbb{C}^{2\times 2}$ Jones matrices operating on Jones vectors. For example, a horizontal linear polarizer has the form

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{3}$$

Birefringent materials, such as quarter-wave plates and half-wave plates, have different optical path lengths for orthogonal "fast" and "slow" polarization axes, hence impart a different phase on transmitted light depending on linear polarization. For a quarter-wave plate with a horizontal fast axis, vertically polarized light gains an additional $\pi/2$ phase. The Jones matrix representation is therefore

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{4}$$

Generally, for some birefringent material imposing some phase difference β, with a fast axis forming an angle θ with horizontal, the Jones matrix is

$$R(\theta)\begin{pmatrix} 1 & 0 \\ 0 & e^{i\beta} \end{pmatrix} R^\dagger(\theta), \tag{5}$$

where $R(\theta)$ is a rotation matrix.

2.2 Birefringent metasurfaces

As first demonstrated by Arbabi *et al.*, metasurfaces constructed from quasi-periodic arrays of subwavelength elliptical posts demonstrate tunable birefringence.[4] Specifically, by adjusting the major and minor diameters of the posts, specific phases ϕ_x, ϕ_y may be imposed on transmitted or reflected light. Hence, allowing a rotation of post axes by some angle θ, this platform allows for Jones transformations of the form

$$M = R(\theta)\begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix} R^\dagger(\theta). \tag{6}$$

Arbabi *et al.* show that any symmetric, unitary Jones matrix may be written in the form of (6). In fact, we will show that symmetry and unitarity are necessary conditions for any local transformation equivalent to (6), *i.e.* that may be constructed using birefringent posts. Let

$$D = \begin{pmatrix} e^{i\phi_x} & 0 \\ 0 & e^{i\phi_y} \end{pmatrix} \tag{7}$$

and note that D is unitary. We observe that

$$M^T = [R(\theta)DR^\dagger(\theta)]^T = R(\theta)DR^\dagger(\theta) = M \tag{8}$$

(this is true for any diagonal matrix D, even if not unitary), and that

$$\begin{aligned} MM^\dagger = M(M^T)^* &= MM^* && (9) \\ &= R(\theta)DD^*R^\dagger(\theta) && (10) \\ &= I. && (11) \end{aligned}$$

Hence all transformations that may be constructed using the birefringent post platform must be both unitary and symmetric.

2.3 Circularly polarized light

Light with horizontal and vertical components $\pi/2$ out of phase, *i.e.* with a normalized Jones vector

$$\sigma_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}, \tag{12}$$

is termed left (right)-hand circularly polarized. Importantly, we will speak of right (left)-handed light when the polarization rotates (counter)clockwise viewed in the direction of propagation.

Because circular polarization is a quantity with handedness, we expect some behavior under transformations: (i) handedness will not change under proper rotations in $\mathbb{R}^3$, and (ii) handedness will reverse under improper rotations in $\mathbb{R}^3$. Since handedness is defined in the direction of propagation, (i) must be true. In defense of (ii), consider a reflection across the x, y-plane, $z \to -z$. Then

$$\mathbf{E}(z,t) \to \mathbf{E}(-z,t) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix} E_0 e^{i(-kz-\omega t)}. \tag{13}$$

If we rotate (*e.g.* π about the x-axis) into the direction of propagation, we have

$$\mathbf{E}(-z,t) \overset{\text{rot}}{=} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \mp i \end{pmatrix} E_0 e^{i(kz-\omega t)}, \tag{14}$$

and we observe $\sigma_\pm \to \sigma_\mp$.

Finally, we note that for incident light normal to some reflective surface, there cannot be any preferred transverse axis: any phase imposed by reflection must be uniform. Hence, circularly polarized light normal to a reflective surface *changes handedness on reflection*, in addition to gaining a uniform π phase shift; the latter is a direct consequence of boundary conditions at the surface and may be calculated from the Fresnel conditions.

3 Proposed optical cavity

3.1 Spin-preserving mirror

It is useful to preserve circular polarization upon reflection within the cavity. Given the effect of reflection on Jones vectors, we may represent the change of basis with the following matrix,

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{15}$$

noting that this transformation is identical to that of a half-wave plate. Of course, (15) is not entirely accurate, since our reflection certainly does not have a preferred basis; nonetheless, up to a transverse rotation (a freedom provided for circularly polarized light if we allow a uniform phase shift), it suffices.

To preserve the handedness of incident light, we must add phase democratically (before and after the reflection) to account for P. In practice, the solution comprises a quarter-wave plate Q directly before the mirror: through one reflection, the light passes through the quarter wave plate twice, hence the total transformation is

$$QPQ = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = I \tag{16}$$

and the incident Jones vector is preserved.

3.2 Cavity design

We seek to use birefringent materials to impose polarization-dependent cavity lengths, and hence generate a circular polarization splitting within the cavity volume.

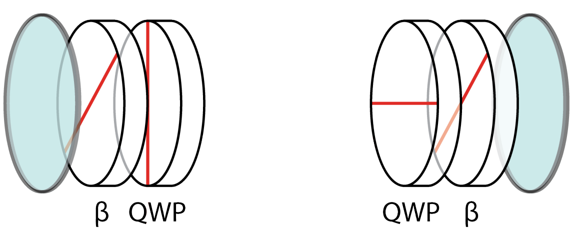

Figure 1: Spin-selective Fabry-Perot cavity with birefringent materials. Cavity comprises of two quarter-wave plates, "β plates", and mirrors. Red lines indicate fast axes.

Our design employs (from the center outward) two quarter-wave plates oriented with fast axes at $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ respectively, two wave plates with path difference β both oriented with fast axes at $\pi/4$ with respect to $\hat{\mathbf{x}}$, and two mirrors, forming a birefringent Fabry-Perot cavity with a longitudinal axis parallel to $\hat{\mathbf{z}}$; see Fig. 1. We observe several symmetries consistent with supporting circularly polarized modes, *i.e.* the same as those identified in Section 2.3. Specifically, we note that the cavity is self-similar (up to a transverse rotation) after a rotation of π about any transverse axis, and that the "orientation" of the cavity reverses after a reflection through the z-axis (see Fig. 2). With some minor qualification, these symmetries support circularly polarized modes in the interior of the cavity.

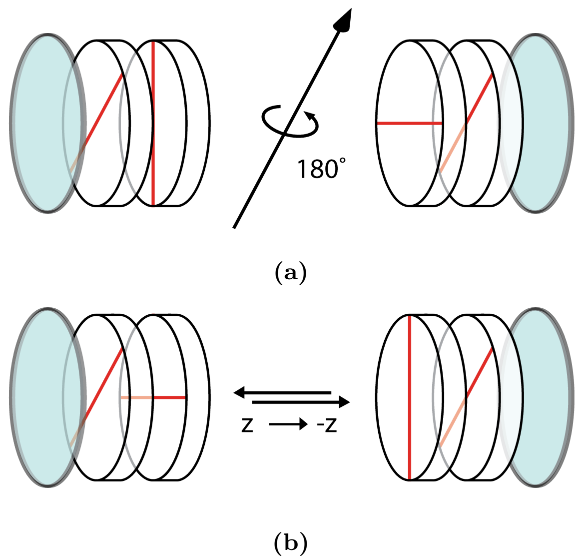

Figure 2: Cavity symmetries. (a) π rotation about fast axis of β plates. (b) Reflection through z axis.

3.3 Operator description of cavity

We seek to determine the modes of the cavity, as well as determine the round trip optical path length experienced by each mode. Jones calculus fails to address path length beyond a relative phase, hence we seek to develop a formalism to address both polarization and mode function.

For simplicity, consider the cavity in Fig. 1 with planar mirrors. In this case, the cavity is entirely determined by propagation through free-space and the birefringent materials. Let $|u(z)\,\hat{\kappa}\rangle$ be a state vector with a mode function $u(\mathbf{r}, z)$ and transverse polarization vector $\hat{\kappa}$. We define a propagation operator $\hat{U}(z)$ such that

(i) $|u(z)\,\hat{\kappa}\rangle = \hat{U}(z)\,|u(0)\,\hat{\kappa}\rangle$.

(ii) $\hat{U}(0) = 1$. Follows from (i).

(iii) $\hat{U}(z_1)\hat{U}(z_2) = \hat{U}(z_1 + z_2)$. Propagation should be additive.

(iv) $[\hat{U}(z_1), \hat{U}(z_2)] = 0$. Follows from (iii).

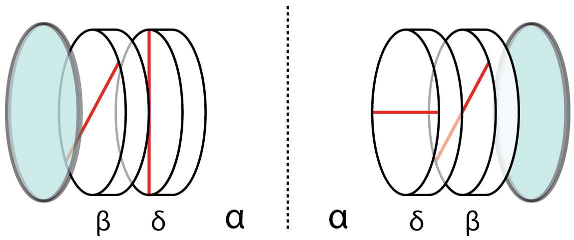

Figure 3: Cavity lengths. Propagation distances are labeled α, β, and δ, where the latter two are imposed orthogonal to the fast axis of their respective elements.

It is also convenient to allow for a change of basis via a local rotation operator. First we note that if two transverse polarizations $\hat{\kappa}, \hat{\nu}$ are non-parallel, then some state $|u_1(z)\,\hat{\kappa}\rangle + |u_2(z)\,\hat{\nu}\rangle$ effectively comprises a vector field $|\mathbf{u}(z)\rangle$. For an orthonormal polarization basis $\hat{\imath}$, $\hat{\jmath}$, we denote

$$|\mathbf{u}(z)\rangle = \begin{pmatrix} u_1(z) \\ u_2(z) \end{pmatrix} := |u_1(z)\,\hat{\imath}\rangle + |u_2(z)\,\hat{\jmath}\rangle\,. \tag{17}$$

Then we may define the expected *local* rotation operator,

$$R(\theta) := \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{18}$$

whereby at each point $(\mathbf{r}, z)$, the field is rotated by θ into a new orthogonal polarization basis.

Finally we define propagation in some birefringent region aligned with our polarization basis

$$\hat{Q}(z_i, z_j) = \begin{pmatrix} \hat{U}(z_i) & 0 \\ 0 & \hat{U}(z_j) \end{pmatrix}. \tag{19}$$

Measured from the center plane, the cavity roundtrip operator is as follows:

$$\begin{aligned} \hat{T} = \hat{Q}(\alpha+\delta,\alpha) R\left(\frac{\pi}{4}\right) \hat{Q}(2\beta,0) R^\dagger\left(\frac{\pi}{4}\right) \\ \hat{Q}(\alpha+\delta,\alpha+\delta) R\left(\frac{\pi}{4}\right) \hat{Q}(2\beta,0) R^\dagger\left(\frac{\pi}{4}\right) \\ \hat{Q}(\alpha,\alpha+\delta) \end{aligned} \tag{20}$$

where α is the distance between the center of the cavity and the quarter-wave plates, $\delta = \lambda/4$, and β is the additional path length imposed by the birefringent plates (other distances are assumed to be zero; see Fig. 3). Note that the operator for the β plate is doubled for each pass before and after the mirror, and that it is rotated transversely by $\pi/4$.

To find the polarization modes, we seek the eigenmodes of the roundtrip operator. Expanding (20), we find

$$\begin{aligned} \hat{T} = \frac{1}{2}\left(\hat{U}(4\alpha+2\delta) + \hat{U}(4\alpha+4\beta+2\delta)\right) I_2 + \\ \frac{1}{2}\hat{U}(4\alpha)\left(\hat{U}(4\beta)-1\right)\begin{pmatrix} 0 & \hat{U}(3\delta) \\ \hat{U}(\delta) & 0 \end{pmatrix} \end{aligned} \tag{21}$$

Hence, we find (normalized) eigenmodes of

$$|\mathbf{u}_\pm\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \pm\hat{U}(\delta) \\ 1 \end{pmatrix} |u(0)\rangle \tag{22}$$

with eigenvalues of

$$\frac{1}{2}\hat{U}(4\alpha+2\delta)\left(1+\hat{U}(4\beta) \pm \left(\hat{U}(4\beta)-1\right)\right). \tag{23}$$

Observe that for $\delta = \lambda/4$, the eigenmodes in (22) are exactly the left and right circular polarization states of light. Moreover, for each polarization mode, the eigenvalue is exactly

$$\hat{U}_+ = \hat{U}(4\alpha+4\beta+2\delta), \quad \hat{U}_- = \hat{U}(4\alpha+2\delta), \tag{24}$$

i.e. some propagation through distinct round-trip path lengths. In effect, a polarization mode resonates in one of two decoupled Fabry-Perot cavities, each of different length. If we were to explicitly define $\hat{U}(z)$, then we could solve $|u_\pm(0)\rangle$ as eigenmodes of $\hat{U}_\pm$; see [5] for a discussion of the propagation operator in the paraxial limit. Alternatively, we can view (24) as stroboscopic pictures of Hamiltonian operators, an approach which has been shown effective by Sommer and Simon for solving the eigenmodes of a variety of cavity geometries.[6] Most importantly, for appropriate values of β, we may expect different wavelengths for each polarization mode, effectively splitting the modes by circular polarization.

It is relevant to note that if both modes are to be circularly polarized, it is necessary that $\delta = \lambda/4$ for *both* wavelengths; else at least one mode will be slightly elliptical. Given the chromatic sensitivity of metasurface optics, this problem is perhaps a small detail in the larger engineering challenge of producing a metasurface cavity capable of supporting multiple non-degenerate modes.

4 Metasurface-based optics

4.1 A phase picture of optical elements

Metasurfaces may alter both the phase and (by controlling transmission and reflection) amplitude of incident light. In practice, we are concerned with manipulating the phase through metasurface optics;

through the careful design of metasurface elements, we seek to impose a "phase profile" on transmitted or reflected light. To this end, it is practical to describe various optical elements in terms of their effect on phase, rather than appeal to the geometric or ray optics perspective.

In the phase picture, a lens with some focal length f is designed to cause the constructive interference of an incident plane wave at a point a distance f from the lens. Hence, we can calculate an appropriate phase profile:

$$\phi(\mathbf{r}) = -k\sqrt{\mathbf{r}^2 + f^2}, \tag{25}$$

where $\phi(\mathbf{r})$ is the phase imparted to some wavefront incident to the lens at position $\mathbf{r}$. If we allow for birefringence, then the phase profiles for quarter- and half-wave plates are as expected:

$$\text{QWP:} \quad \phi_x = \frac{\pi}{2};\ \phi_y = 0 \tag{26}$$

$$\text{HWP:} \quad \phi_x = \pi;\ \ \phi_y = 0 \tag{27}$$

It is often possible to "sum" phase profiles of multiple classical elements into one metasurface device. In the case of our cavity design, this proves impossible if our metasurface is comprised of locally birefringent posts. Recall from Section 2.2 that any Jones transformation that may be constructed using this platform must be both unitary and symmetric. Of course, at least two metasurface elements must be included in our cavity, since the desirable modes would exist within the cavity volume, between these elements. Consider the Jones matrices for a quarter-wave plate and the rotated β plate. The composed transformation,

$$\begin{aligned} R(\pi/4)\begin{pmatrix}1 & 0\\ 0 & e^{i\beta}\end{pmatrix}R^{\dagger}(\pi/4)\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix} &= \\ \frac{1}{2}\begin{pmatrix}1+e^{i\beta} & i\left(1-e^{i\beta}\right)\\ 1-e^{i\beta} & i\left(1+e^{i\beta}\right)\end{pmatrix}&, \end{aligned} \tag{28}$$

is not symmetric, hence requires exactly two metasurfaces, and a total of four for the cavity as a whole.

4.2 Simulation and parameter search

Arbabi *et al.* have implemented arrays of elliptical, subwavelength high-contrast posts to exhibit birefringence. The group claims the posts act as truncated "weakly coupled low-quality factor resonators", and thereby impart a phase on incident light.[4] Our group seeks to implement this metasurface platform using silicon nitride, a low-contrast material, for two principle reasons: (a) low-contrast devices are less lossy than their high-contrast counterparts, and (b) nitride metasurfaces are compatible with existing CMOS fabrication infrastructure. However, we have found some suggestion that the fields confined to neighboring posts may be more strongly coupled in the low-contrast case. Hence, a "weakly-coupled resonator" picture, though perhaps not well-developed in its own right, may be even less adequate in modeling the behavior of low-contrast metasurfaces.

In practice, engineering a phase profile into a metasurface design consists of mapping local phase to the relevant post parameters. We determine this mapping with a brute-force approach: we simulate post behavior over a large space of relevant parameters and determine the resulting phase and amplitude; given some desired phase, we then search for parameters with minimum phase error and maximum amplitude. For a birefringent metasurface using elliptical posts, the optimized parameters include:

- major and minor diameter
- post thickness (height)
- lattice constant

We assume that posts are weakly-coupled, and for each set of parameters use S^4, a Rigorous Coupled-Wave Analysis (RCWA) solver, to determine the imposed phase and amplitude for both horizontal and vertical polarizations.[7] It is relevant to note that RCWA assumes a periodic array of identical posts, which may not effectively model post behavior in a non-uniform array if posts are not weakly coupled, as noted above. For some phase profile and lattice constant/post thickness parameters, we first optimize post diameter; we then determine the optimal lattice constant and post thickness, noting that these parameters must be uniform due to fabrication considerations (see Fig. 4). Finally, these parameters are used to construct Finite-Difference Time-Domain (FDTD) simulations of the entire metasurface device. Two completed metasurface devices (fabricated using electron-beam lithography) are shown in Fig. 5.

A large portion of my work concerned optimizing and extending code for post simulation and parameter search. Using the S^4 package, I wrote code to iterate the RCWA post simulation over all parameters listed above; I also developed scripts to efficiently search the parameter space to map coordinates to a given phase profile. This code was computationally expensive, hence the above scripts were parallelized and made available for use on Hyak, the University of Washington supercomputing cluster.

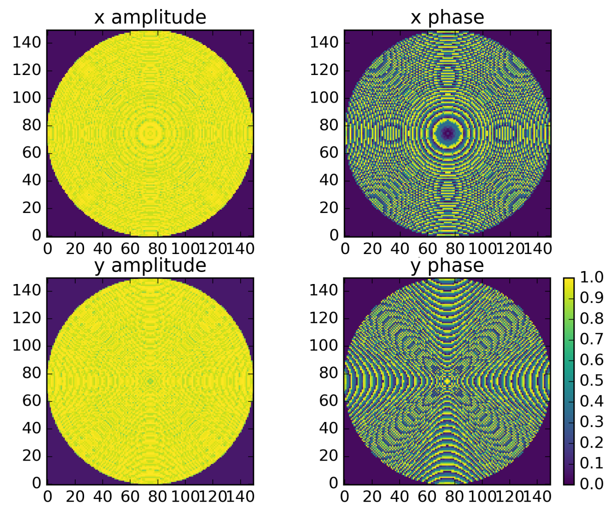

Figure 4: Optimization results. Optimal phase and amplitude selected for a lens with polarization-dependent focal length.

5 Conclusion

We propose a design for a spin-selective optical cavity, utilizing linearly birefringent materials to impose different optical path lengths for left- and right-handed polarization modes. The existence of circularly polarized modes within the cavity volume is supported by requisite cavity symmetries, namely invariance under certain rotations and a lack of mirror symmetry. To determine the polarization modes within the cavity, we introduce an operator formalism in terms of propagation matrices operating on a vectorized mode function. With this methodology, for a planar birefringent Fabry-Perot cavity we determine cavity polarization eigenmodes corresponding to the left- and right-handed polarization states of light, each with a distinct path length and hence (for an appropriate parameter β of birefringence) a spin-selective splitting in the mode spectrum.

It remains to explicitly calculate eigenmodes for more complicated cavity geometries; however, since the cavity appears to operate as two distinct, uncoupled resonators in the polarization eigenbasis, existing methods for such a calculation seem promising.

We discuss progress toward constructing low-contrast metasurface-based optical elements to realize the cavity design. Our methodology presently consists of a brute-force parameter search for an optimal metasurface design; this task is computationally intensive and requires parallelized code and significant computational resources. Future work in the group may seek to develop a more robust model of the local and semi-local behavior of metasurface posts, either in the resonator picture or in some other perspective. Once appropriate parameters are established, we seek to simulate the cavity using FDTD methods, and then work toward a fabrication and characterization of the cavity.

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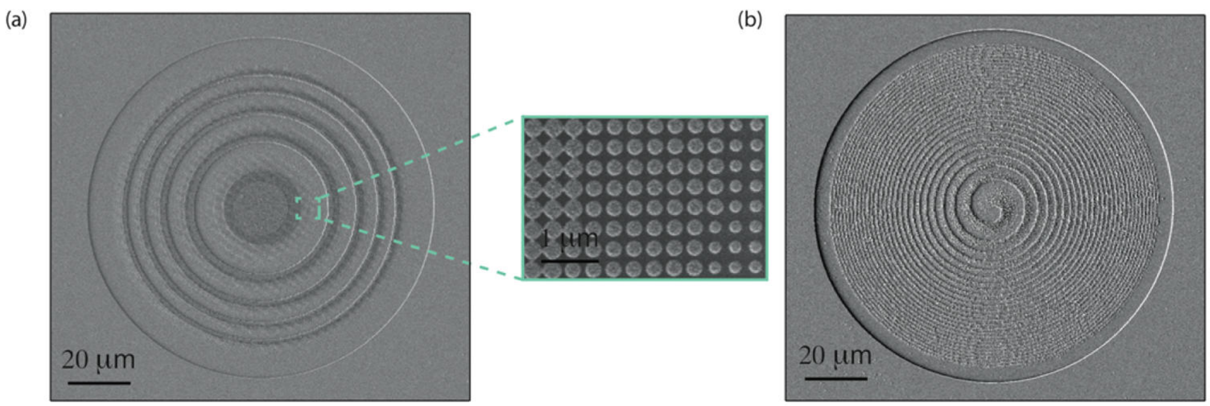

Figure 5: Low-contrast metasurface devices. SEM. (a) lens, (b) vortex beam generator. Note that these devices are not birefringent.[8]