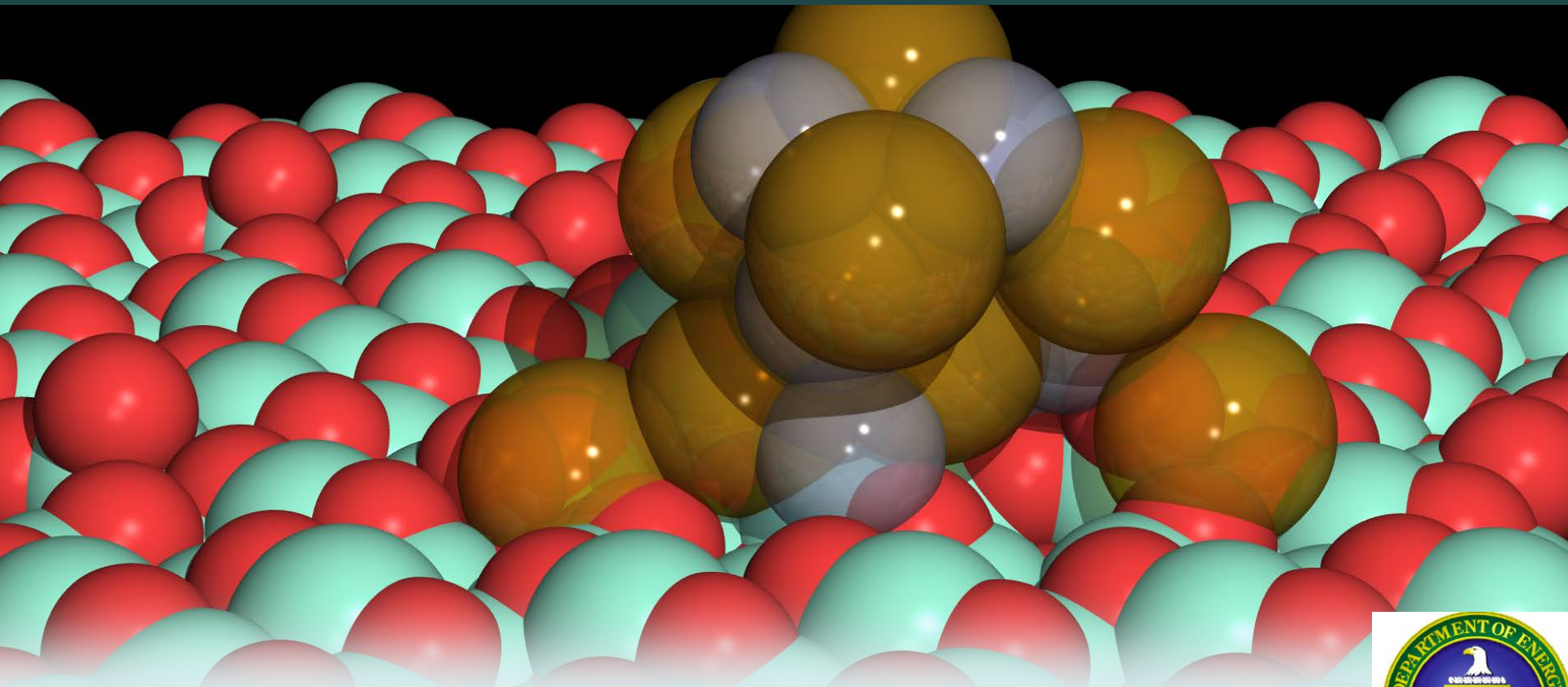


# Theory, Interpretation and Applications of X-ray Spectra\*

J. J. Rehr et al.





**A theoretical horror story**

**Starring**

**Fernando Vila  
&  
Anatoly Frenkel  
with**

**J. Kas, S. Bare & S. Kelly**

**Directed by J. J. Rehr**

**a Sequel**

## Part I: Application

*Operando* theory of nanocatalysts:

Structure, thermal properties, catalytic activity

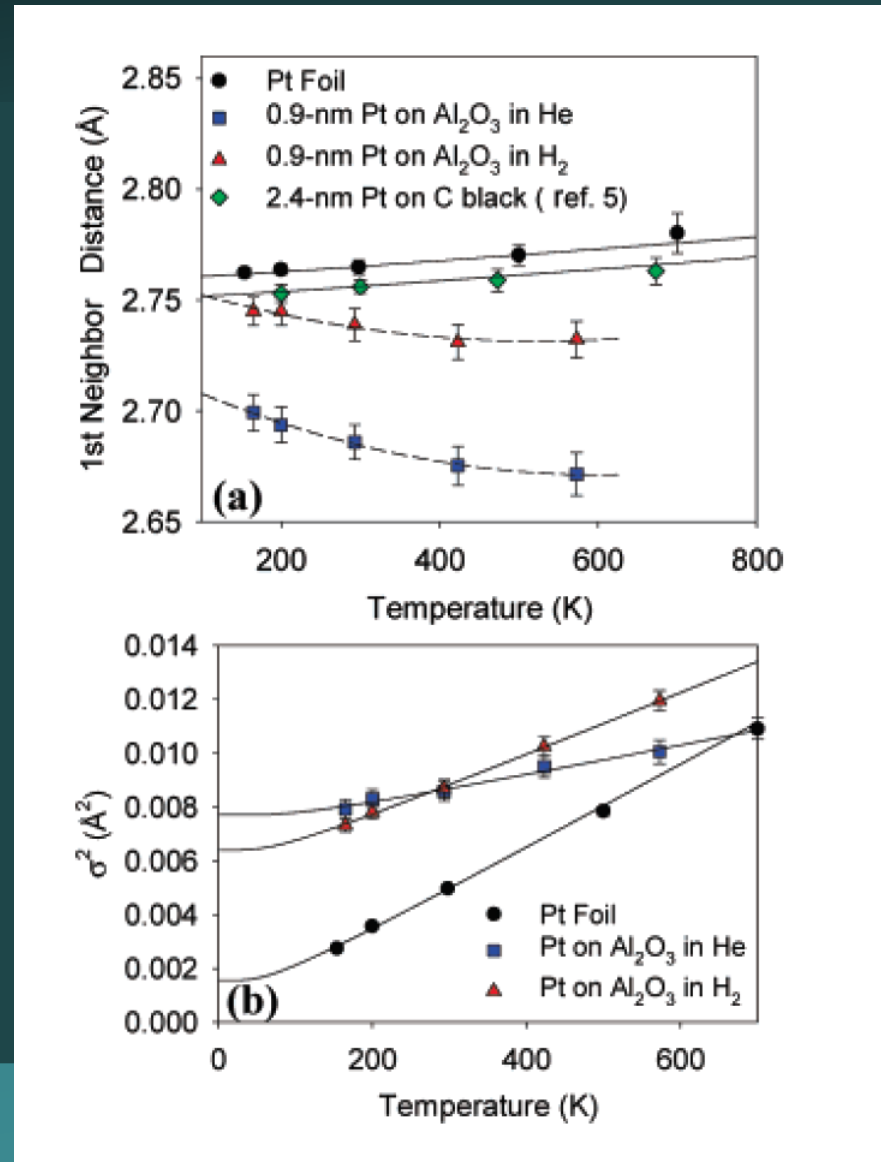
Pt Clusters on  $\gamma\text{-Al}_2\text{O}_3$

# Theoretical Challenge: Anomalous properties\* of $\text{Pt}_{10}/\gamma\text{Al}_2\text{O}_3$

- Pt-Pt nn  
Negative Thermal Expansion  
& Bond expansion in  $\text{H}_2$

- Anomalous Pt-Pt disorder

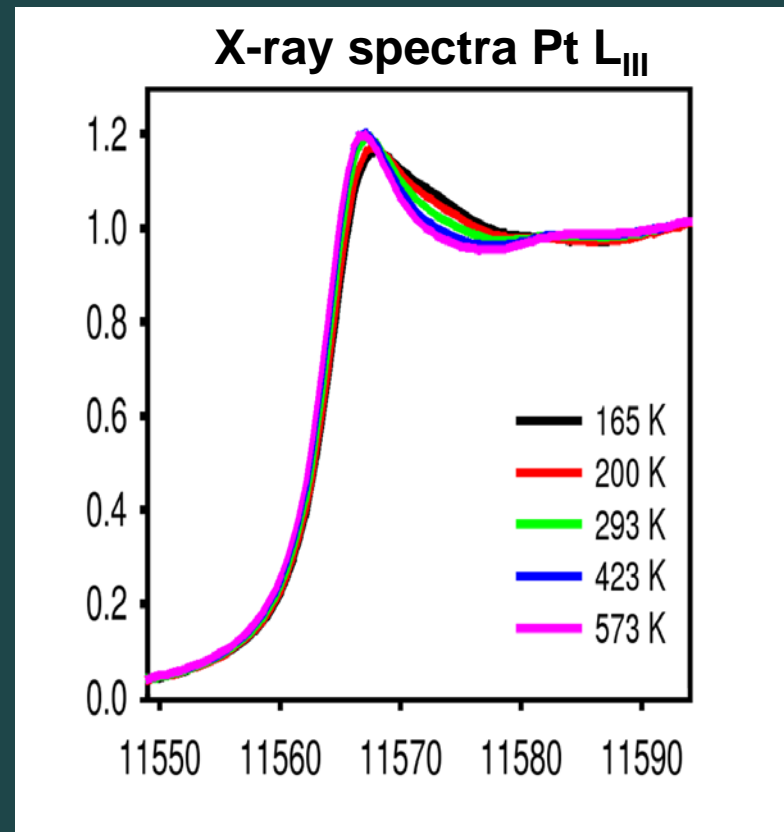
**NOT bulk-like !**



\*J.H. Kang, L. D. Menard, R. G. Nuzzo, and A. I. Frenkel. JACS 2006, 128, 12068

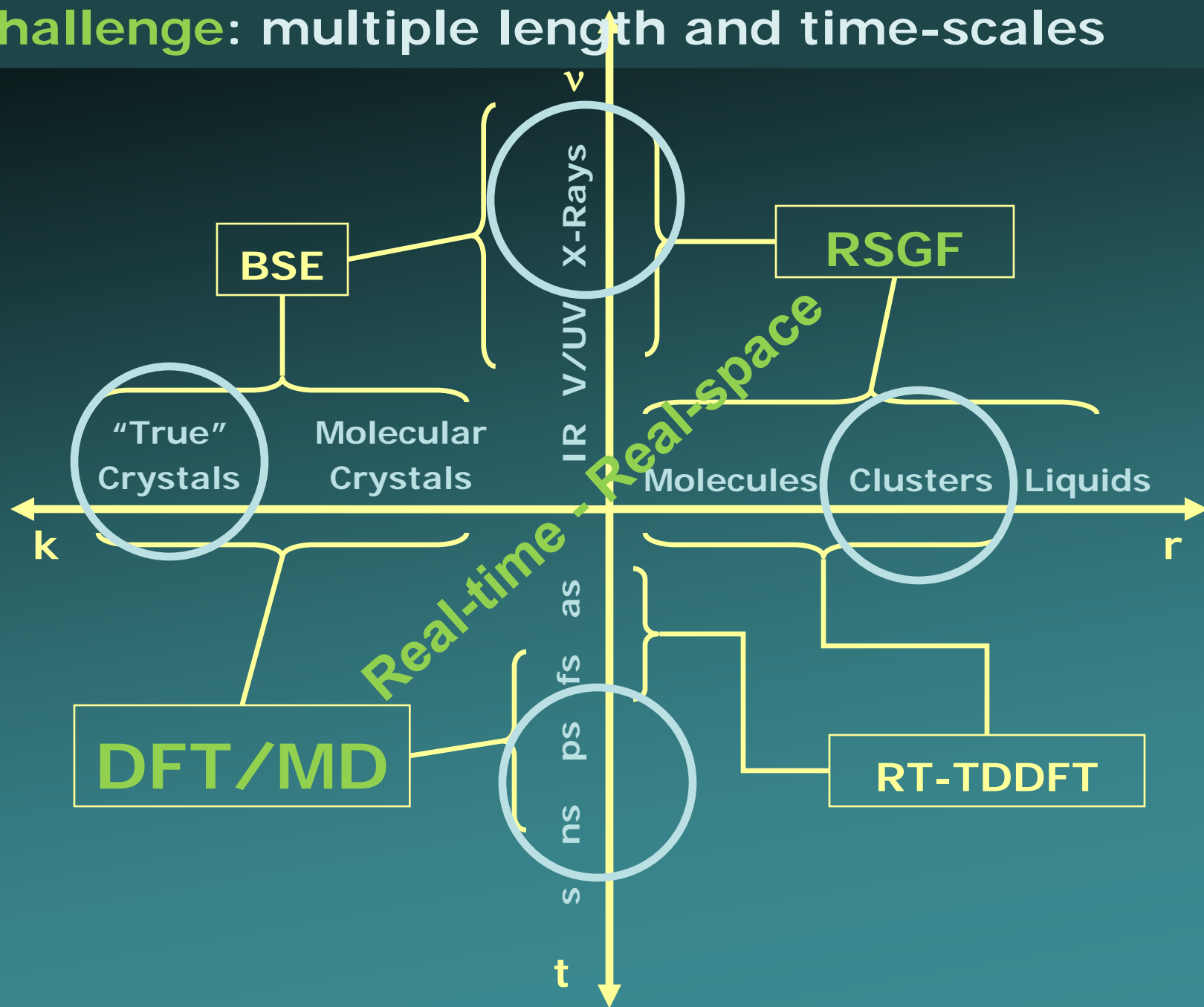
# More Anomalous properties\* $\text{Pt}_{10}/\gamma\text{Al}_2\text{O}_3$

- Increased edge intensity
- Redshift of XANES with increasing T
- Standard analysis fails!



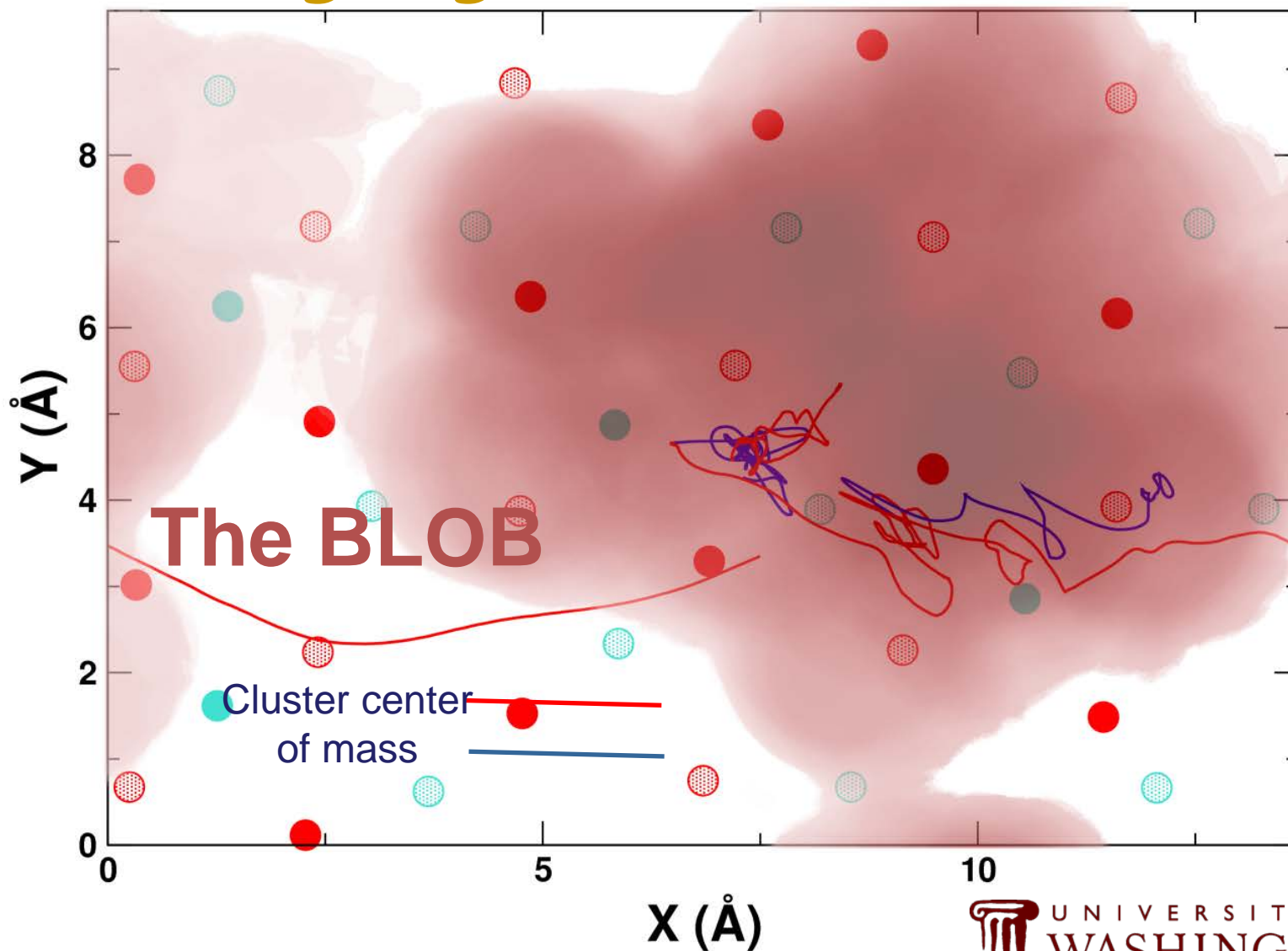
\*J.H. Kang, L. D. Menard, R. G. Nuzzo, and A. I. Frenkel. JACS 2006, 128, 12068

# Challenge: multiple length and time-scales





# What's going on?



# Dynamic structure in supported Pt nanoclusters: Real-time density functional theory and x-ray spectroscopy simulations

F. Vila,<sup>1</sup> J. J. Rehr,<sup>1,\*</sup> J. Kas,<sup>1</sup> R. G. Nuzzo,<sup>2</sup> and A. I. Frenkel<sup>3</sup>

<sup>1</sup>Department of Physics, University of Washington, Seattle, Washington 98195, USA

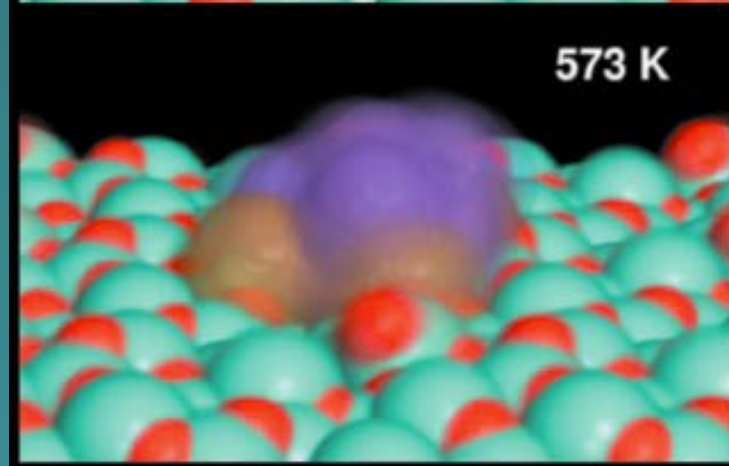
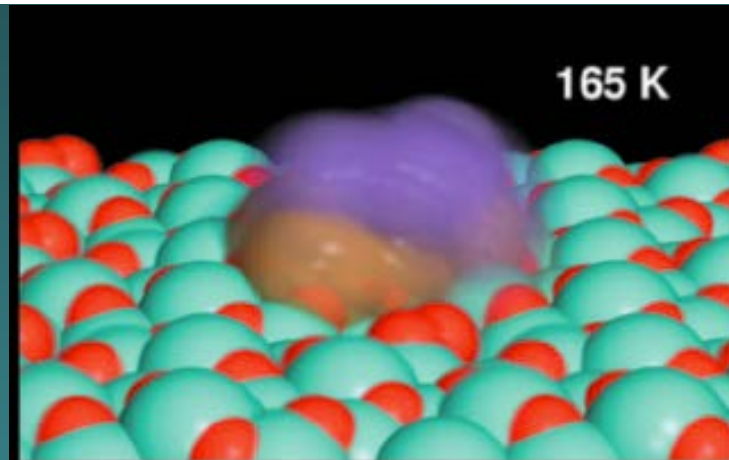
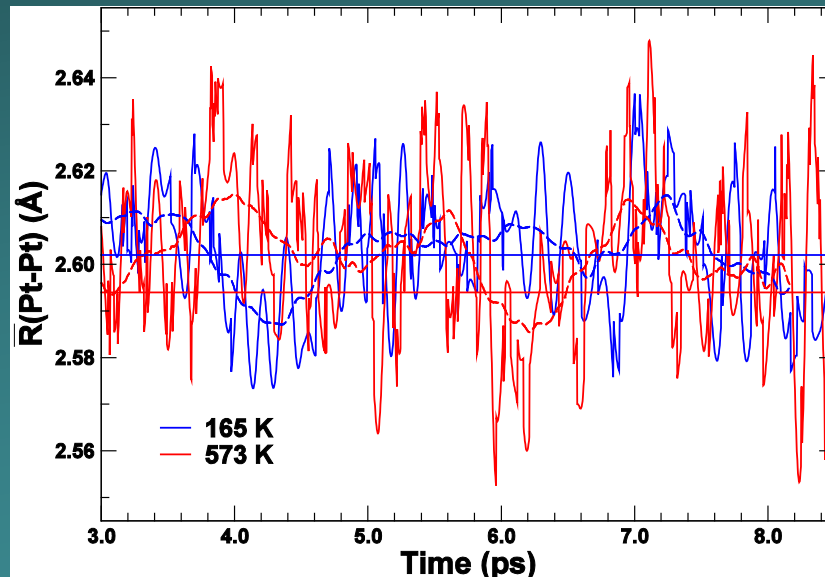
<sup>2</sup>Department of Chemistry, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

<sup>3</sup>Department of Physics, Yeshiva University, New York, New York 10016, USA

(Received 24 July 2008; published 11 September 2008)

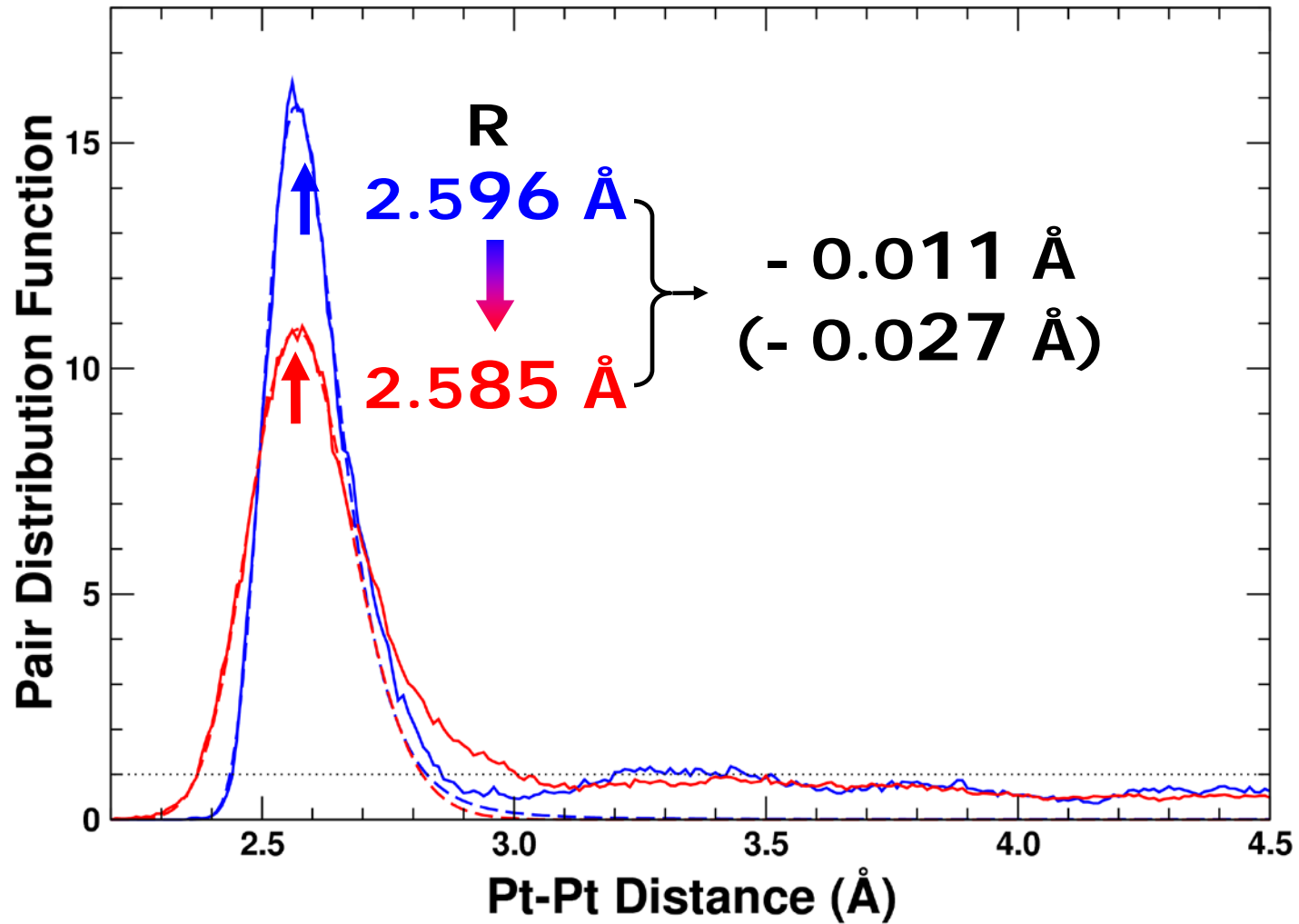
## Fuzzy "structure"

DFT/MD nn distance

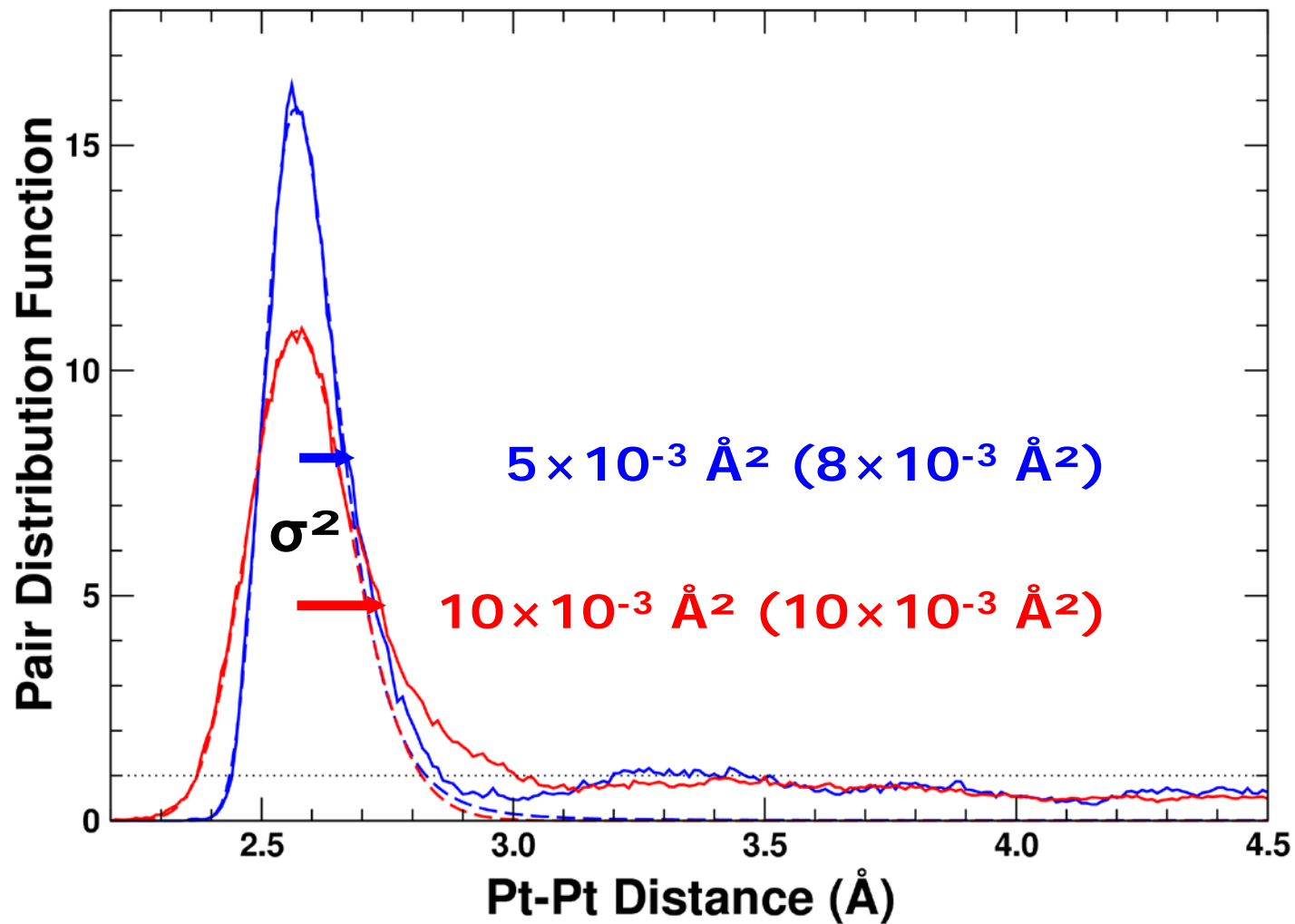




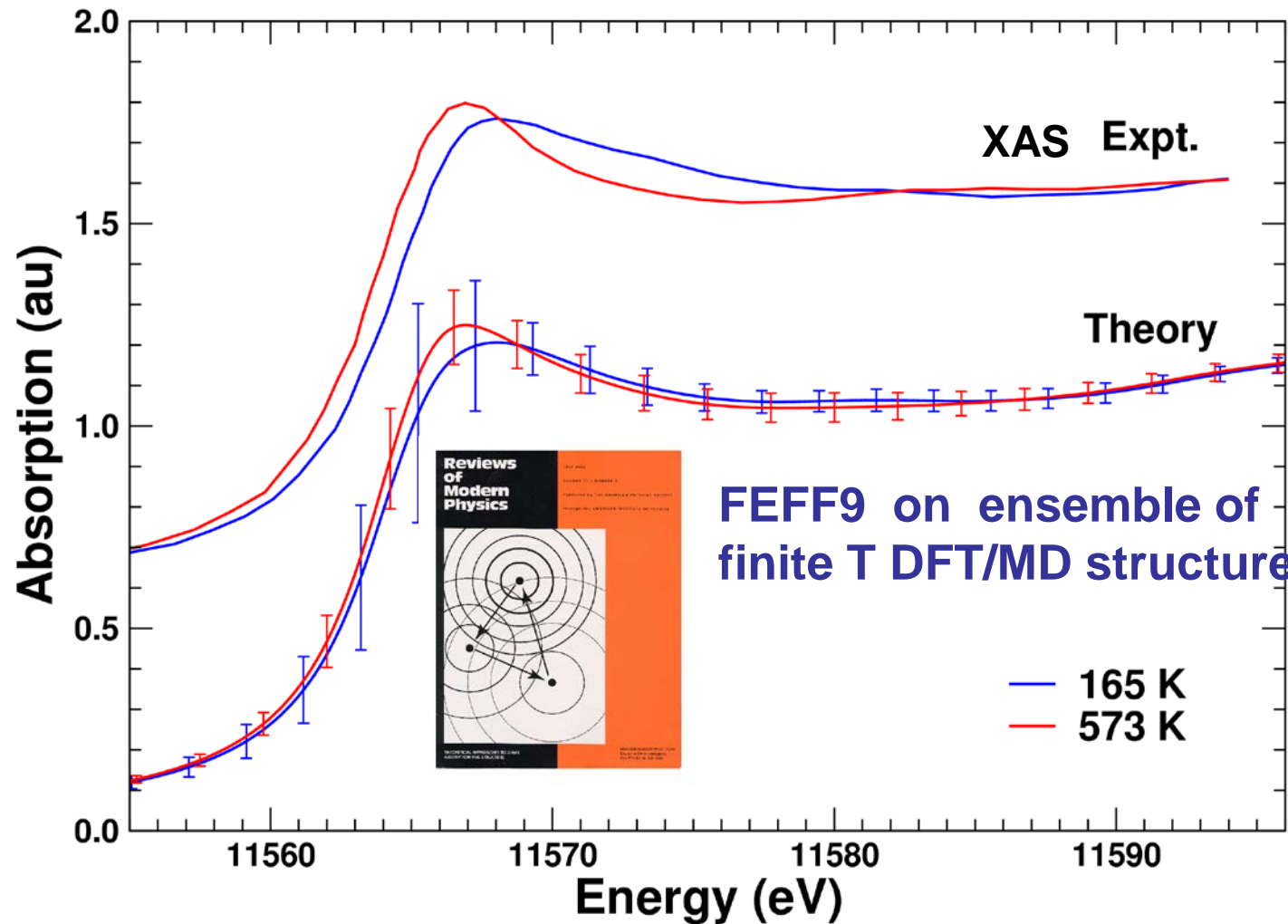
# Negative Thermal Expansion Explained



# High Pt-Pt Disorder Explained

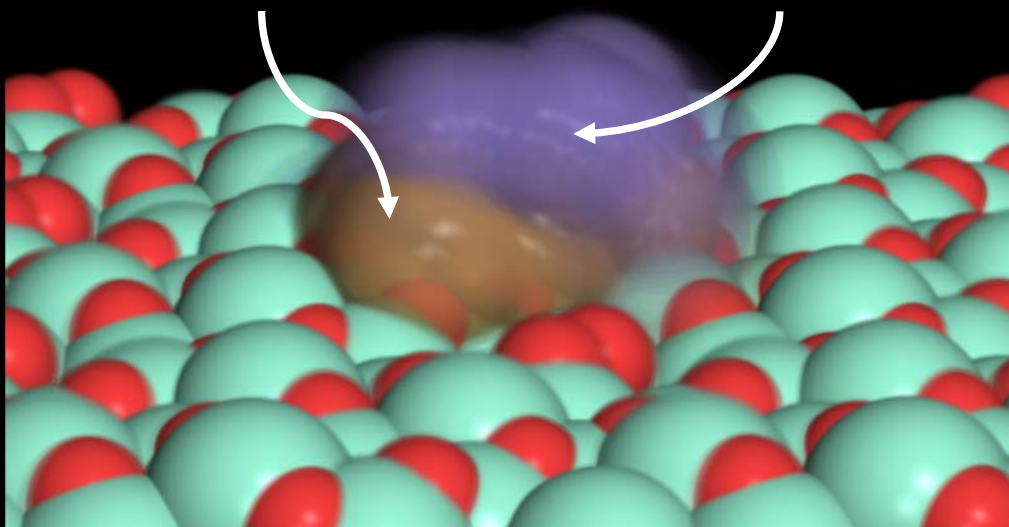


# Increased intensity and redshift explained

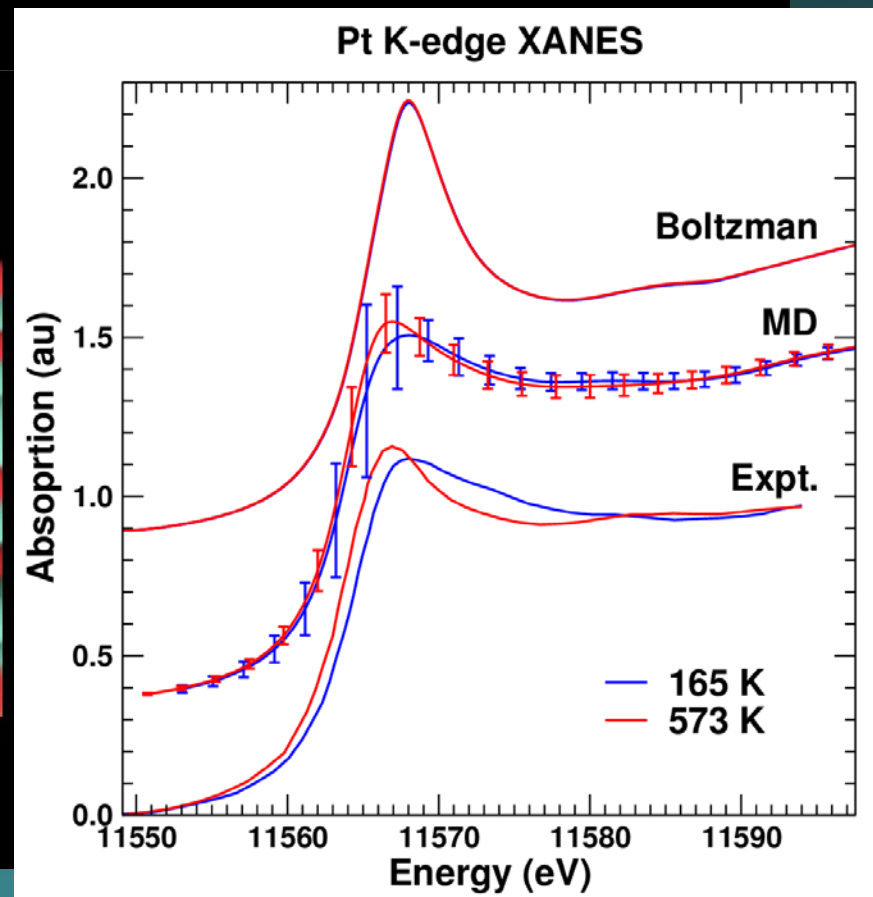


# Dynamic structural disorder

+ $\delta$  ("Oxidized")      - $\delta$  ("Metallic")



Pt<sub>10</sub> on  $\gamma$ -Al<sub>2</sub>O<sub>3</sub> @ 165 K



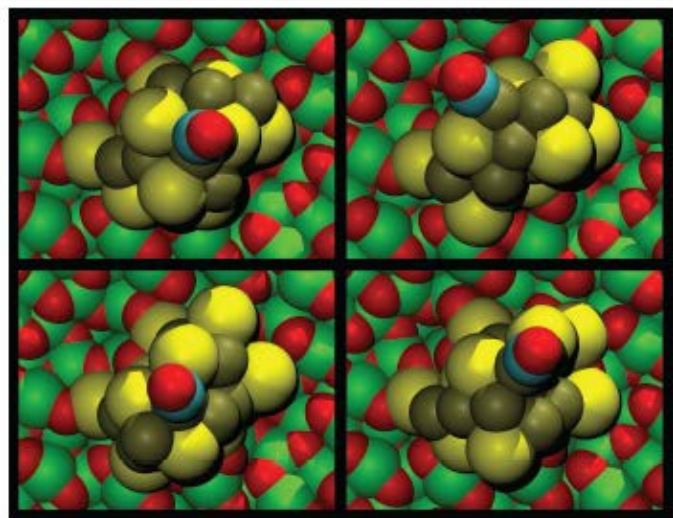
Need **Operando AIMD dynamics\*** to reproduce experiment

\*F. Vila, J. J. Rehr, J. Kas, R. G. Nuzzo, and A. I. Frenkel, Phys. Rev. B **78**, 121404(R), 2008

7 April 2014

Volume 140 Number 13

# AIP | The Journal of Chemical Physics



THE JOURNAL OF CHEMICAL PHYSICS 140, 134701 (2014)

## Dynamic structural disorder in supported nanoscale catalysts

J. J. Rehr and F. D. Vila

*Department of Physics, University of Washington, Seattle, Washington 98195, USA*

[jcp.aip.org](http://jcp.aip.org)

## Conclusion

Must consider  
dynamic structure  
to understand  
the nano-scale



# Part II: Theory and Interpretation of X-ray Spectra

- **GOALS:** *ab initio* theory  
Accuracy ~ experiment

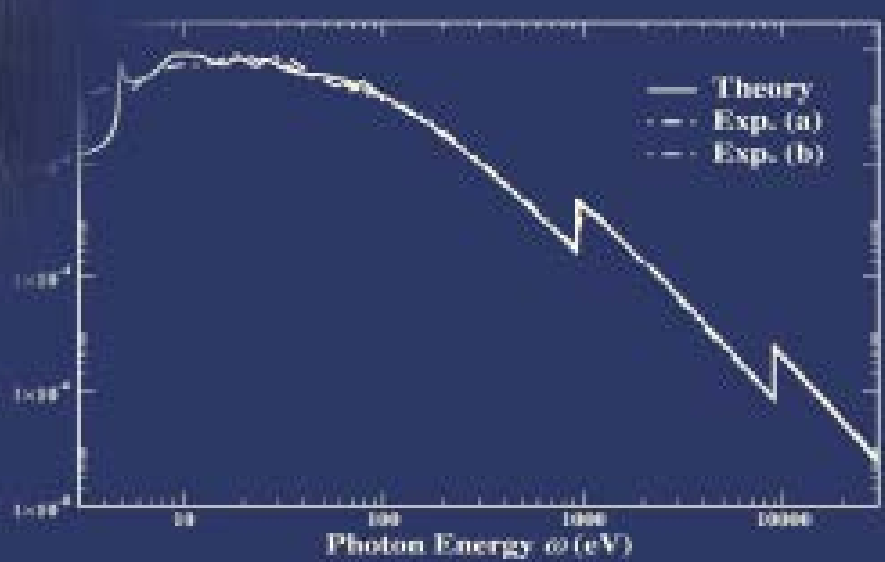
- **TALK:**

- I. Introduction History
- II. Real-space Green's function theory FEFF codes
- III. Improvements XAS, Optical spectra, etc, ...

# GOAL Theoretical X-ray Beamline



$$\mu \propto -\frac{1}{\pi} \text{Im} \langle i | d G(\omega) d^\dagger | i \rangle$$



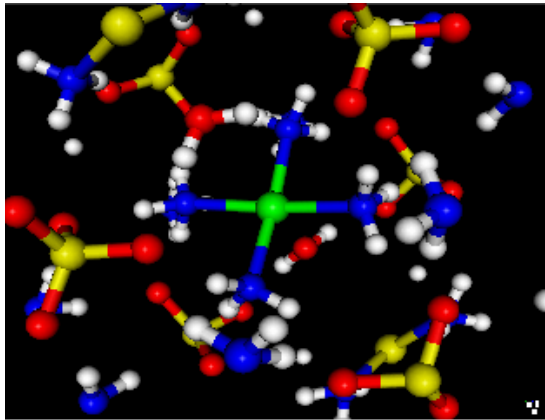
## Rehr Group

# Questions we want to answer:

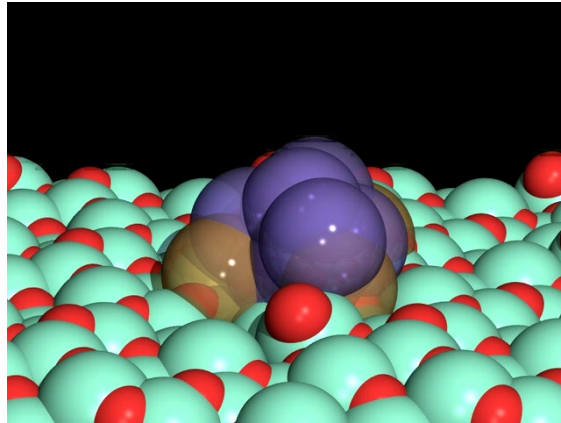
What are the *structure* and *dynamics* ?

and other *physical properties* ?

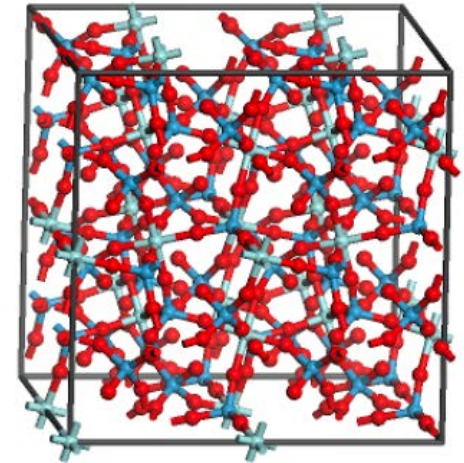
of complex systems ?



$\text{Cu}(\text{NH}_3)_4\text{SO}_4\text{H}_2\text{O}$

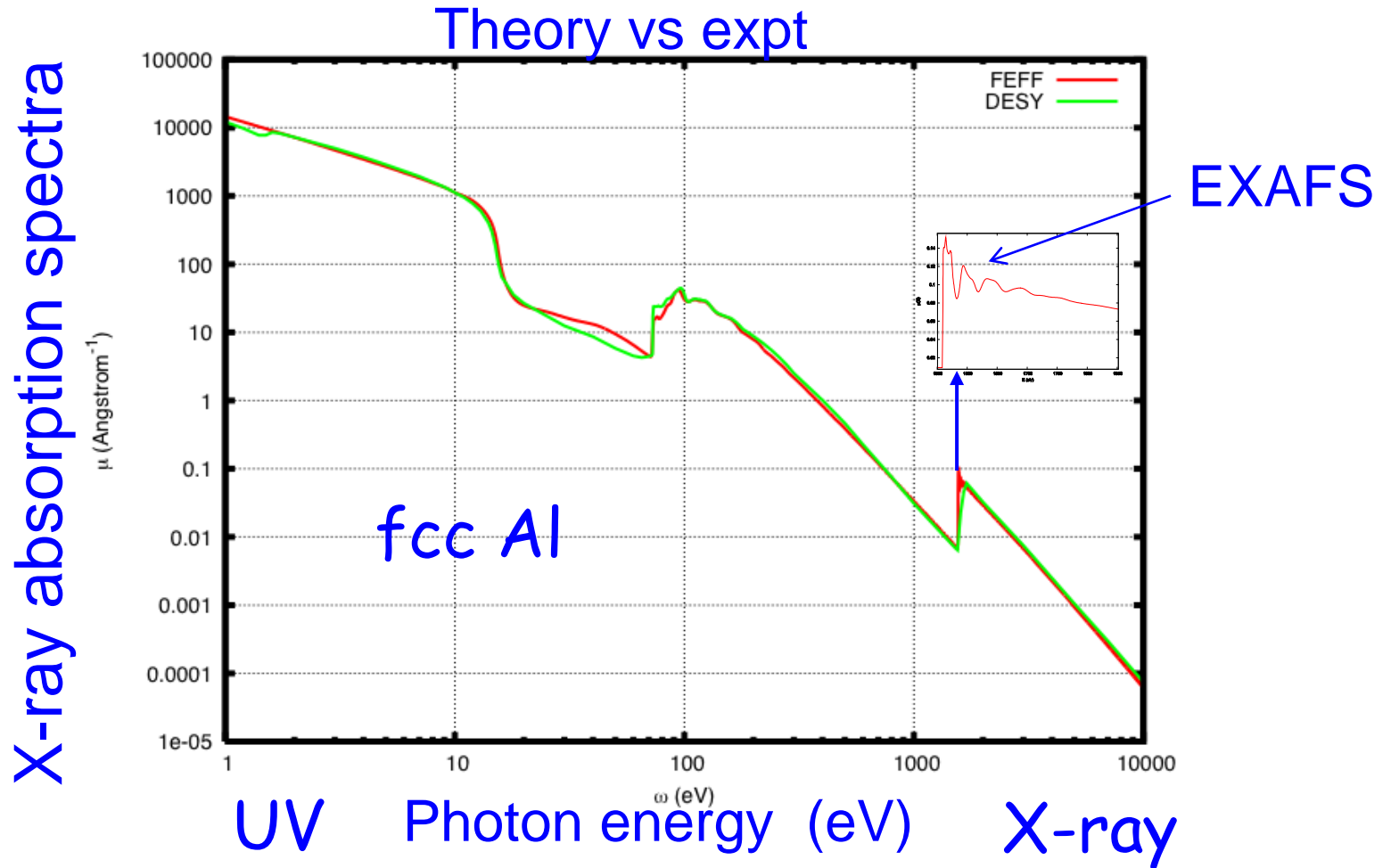


$\text{Pt}/\gamma\text{-Al}_2\text{O}_3$



Zirconium Tungstate

# Experiment: X-ray Spectroscopy



? What's in a spectrum ?

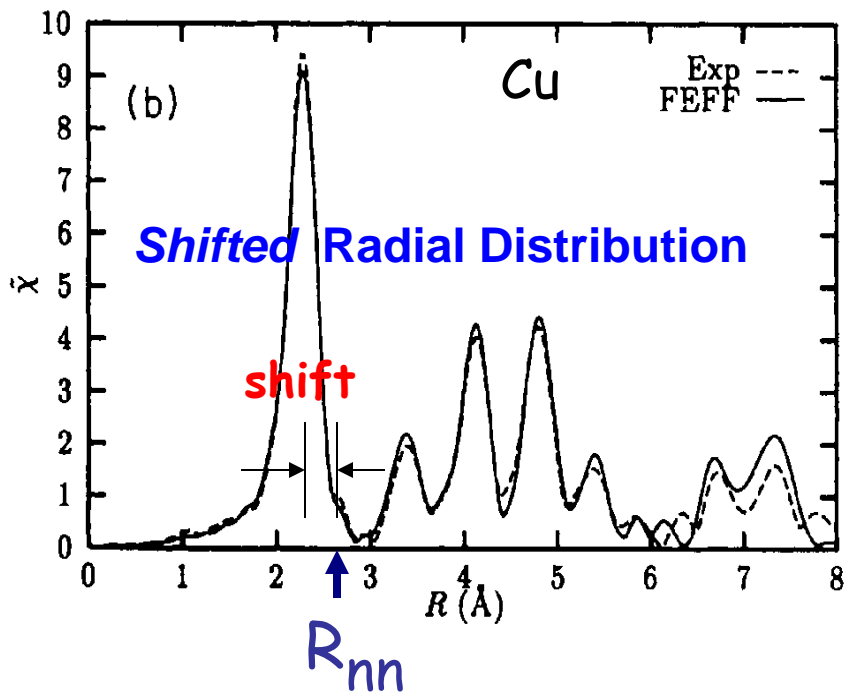
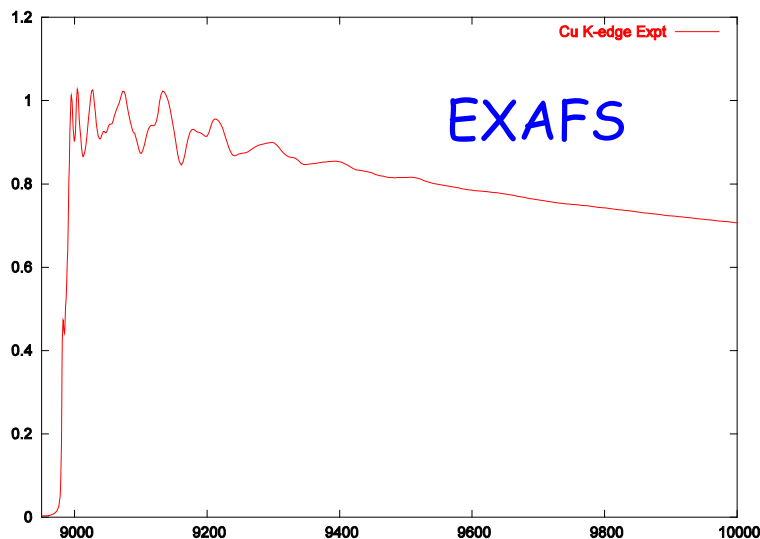
# Historical interpretation of EXAFS\*

\*Stern Sayers Lytle, UW 1971

Short range order theory



EXAFS Fourier Transform



→ X-ray Microscope!

**BUT** need to calibrate experiment with "Standard"



”Can you write an equation  
for the theory?”

P.A.M. Dirac  
(to R. Feynman)

# Answer: Exact EXAFS Equation\*

Effective Scattering Amplitude  $f_{\text{eff}}$

$$\chi(k) = S_0^2 \sum_R \frac{|f_{\text{eff}}(k)|}{kR^2} \sin(2kR + \Phi_k) e^{-2R/\lambda_k} e^{-2\sigma^2 k^2}$$

$S_0^2$  Many body amplitude factor

$\lambda_k$  Mean free path

$\sigma^2$  Mean square vib amplitude

EXAFS measures **local structure & disorder**

Distance  $R$       Coordination  $N$       Disorder  $\sigma^2$

\*JJR, RC Albers, CR Natoli, EA Stern, Phys Rev B34, 4350 (1986)

BUT: need many parameters !

**Question:** Can the EXAFS parameters

$$k \quad f_{\text{eff}} \quad \Phi_k \quad \sigma^2 \quad \lambda_k \quad S_0^2$$

be calculated theoretically ?

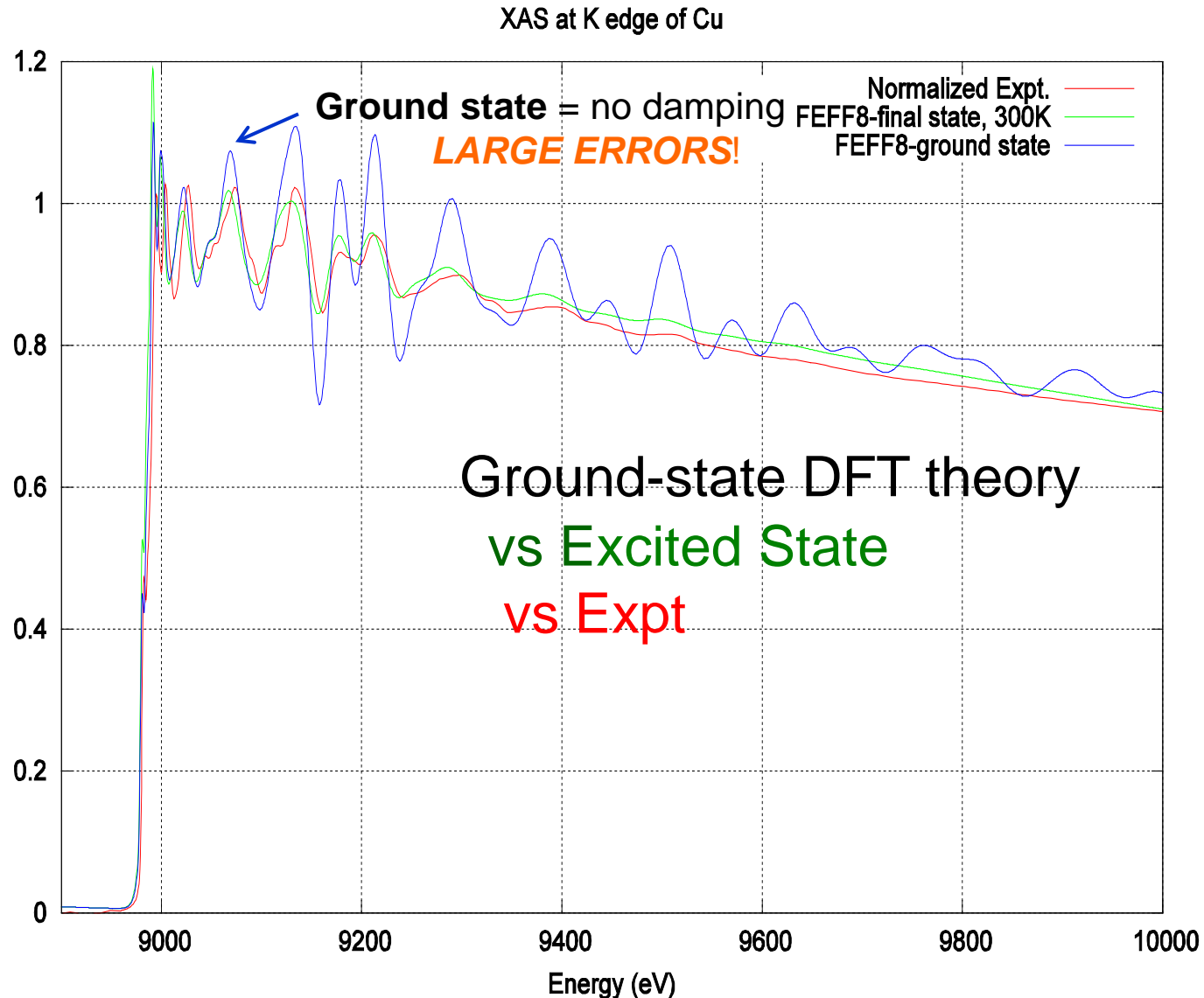
## Answer 1

"I always thought it was easier to measure  
x-ray absorption than to calculate it."

Hans Bethe

ca 1980

# Gotcha: Standard theory fails!





# Why?

Fermi Golden Rule for XAS  $\mu(\omega)$

$$\mu(\omega) \sim \sum_f |\langle \psi_f | d | \psi_i \rangle|^2 \delta(E_f - E_i - \hbar\omega)$$

**Too many quasi-particle** final states  $\psi_f$

$$\left[ \frac{p^2}{2m} + V'_{coul} + \Sigma(E) \right] \psi_f = E_f \psi_f$$

**Final state rule**

$$V'_{coul} = V_{coul} + V_{core-hole}$$

**Non-hermitian self-energy**  $\Sigma(E)$

(replaces DFT  $V_{xc}$ )

# Answer 2 (JJR)\* “Now\*\* it is often easier to calculate x-ray absorption than to measure it”

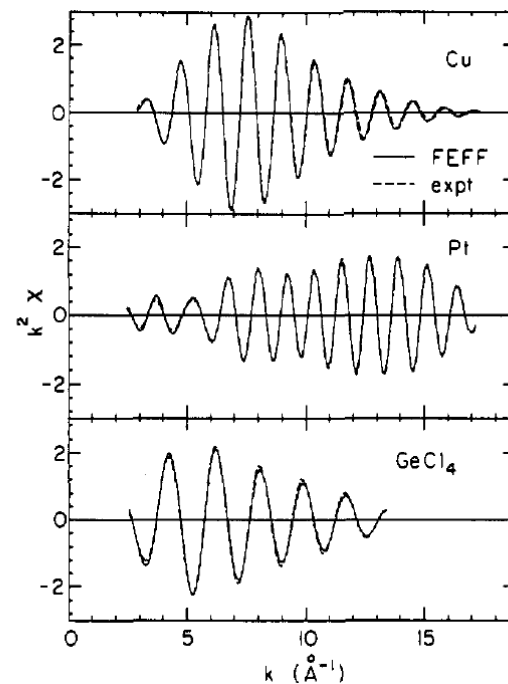
(\*\*if structure is known)

## Theoretical X-ray Absorption Fine Structure Standards

J. J. Rehr,<sup>\*,†</sup> J. Mustre de Leon,<sup>†,‡</sup> S. I. Zabinsky,<sup>†</sup> and R. C. Albers<sup>§</sup>

*Contribution from the Department of Physics, FM-15, University of Washington, Seattle, Washington 98195, and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545. Received November 13, 1990*

**Abstract:** Theoretical X-ray absorption fine structure (XAFS) standards are developed for arbitrary pairs of atoms through the periodic table ( $Z \leq 94$ ). These standard XAFS spectra are obtained from *ab initio* single-scattering XAFS calculation using an automated code, FEFF, which takes into account the most important features in current theories: (i) an exact treatment of curved-wave effects; (ii) approximate molecular potentials derived from relativistic atoms, (iii) a complex, energy-dependent self-energy; (iv) a well defined energy reference. FEFF also yields tables of XAFS phases and amplitudes as well as mean-free paths. Sample results are presented and compared with experimental results and with earlier work. We find that these theoretical standards are competitive with experimental standards, permitting XAFS analysis at lower wavenumbers and yielding distance determinations typically better than 0.02 Å and coordination numbers typically better than 20%. These standards also provide theoretical tests of chemical transferability in XAFS.



\*\* 10 years later - JACS 113, 5136 (1991)

# Reviews of Modern Physics

JULY 2000

VOLUME 72 • NUMBER 3

Published by THE AMERICAN PHYSICAL SOCIETY

through the AMERICAN INSTITUTE OF PHYSICS



THEORETICAL APPROACHES TO X-RAY  
ABSORPTION FINE STRUCTURE

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Use Prohibited Until 2005

How? (10 more years)

Quantitative

XAS theory

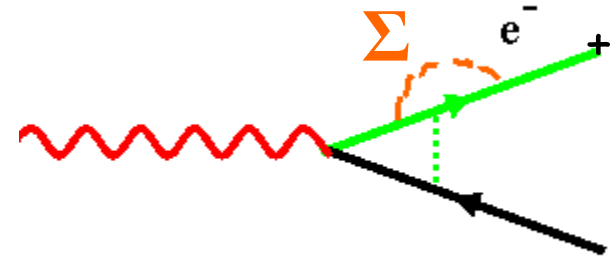
FEFF

J. J. Rehr & R.C. Albers

Rev. Mod. Phys. **72**, 621 (2000)

<http://leonardo.phys.washington.edu/feff/>

# Theoretical Tricks: Green's functions



- Real-space **Green's function** Theory
- Mean free path, Self-energy  $\Sigma$
- Screened core-hole

# Real-space Green's Function Theory

~~Golden rule via Wave Functions~~

$$\mu(E) \sim \sum_f |\langle i | \hat{\epsilon} \cdot \mathbf{r} | f \rangle|^2 \delta(E - E_f)$$



Paradigm shift

Golden rule via Green's Functions  $\mathbf{G} = 1/(E - h - \Sigma)$

$$\mu(E) \sim -\frac{1}{\pi} \text{Im} \langle i | \hat{\epsilon} \cdot \mathbf{r}' G(\mathbf{r}', \mathbf{r}, E) \hat{\epsilon} \cdot \mathbf{r} | i \rangle$$

*No sums over final states !*

# What's a Green's function?

Wave function in QM

$$H \Psi = E \Psi$$

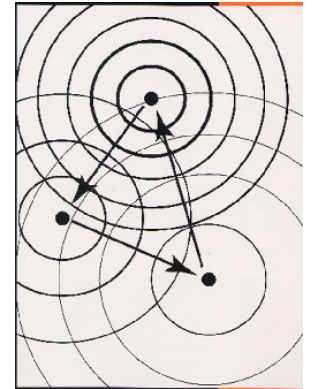
$\Psi(r)$  = Amplitude to find particle at  $r$

Green's function

$$(H - E) G = -\delta(r-r')$$

$G(r,r',E)$  = aka Propagator

= Amplitude to go from  $r$  to  $r'$



# Multiple-scattering theory of $G$

$$\mu(E) \sim -\frac{1}{\pi} \text{Im} \langle i | \hat{\mathbf{e}} \cdot \mathbf{r}' G(\mathbf{r}', \mathbf{r}, E) \hat{\mathbf{e}} \cdot \mathbf{r} | i \rangle$$



$$G = G^0 + G^0 t G^0 + G^0 t G^0 t G^0 + \dots$$

(MS path expansion - geometric series)

**Path**

$$= [1 - G^0 t]^{-1} G^0$$

"full MS"

**Full  
MS**

Ingredients:

$G_0$  free propagators  $t$ -matrix =  $e^{i \delta_l} \sin \delta_l \delta_{RR'} \delta_{ll'}$

# Implementation: FMS FEFF code

PHYSICAL REVIEW B

VOLUME 58, NUMBER 12

15 SEPTEMBER 1998-II

## Real-space multiple-scattering calculation and interpretation of x-ray-absorption near-edge structure

A. L. Ankudinov

*MST-11, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*

B. Ravel

*Ceramics Division, National Institute of Standards and Technology, Gaithersburg, Maryland 20899*

J. J. Rehr

*Department of Physics, University of Washington, Seattle, Washington 98195-1560*

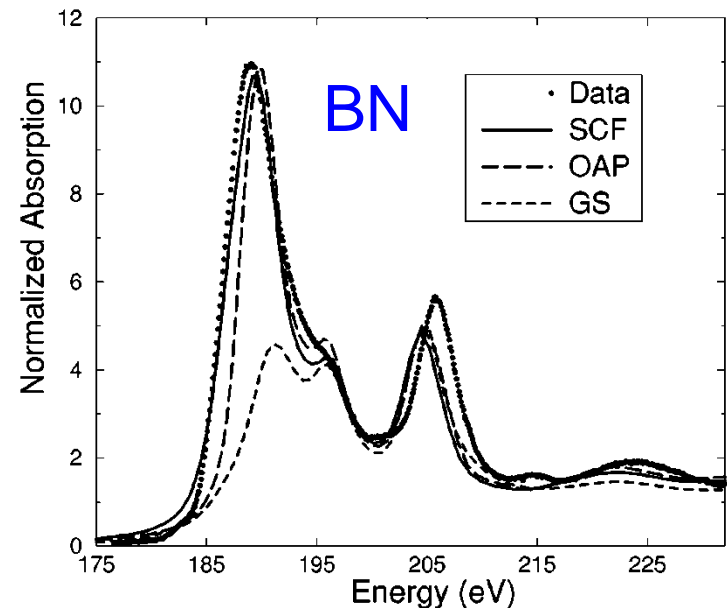
S. D. Conradson

*MST-11, Los Alamos National Laboratory, Los Alamos, New Mexico*

Core-hole, SCF potentials

*Essential!*

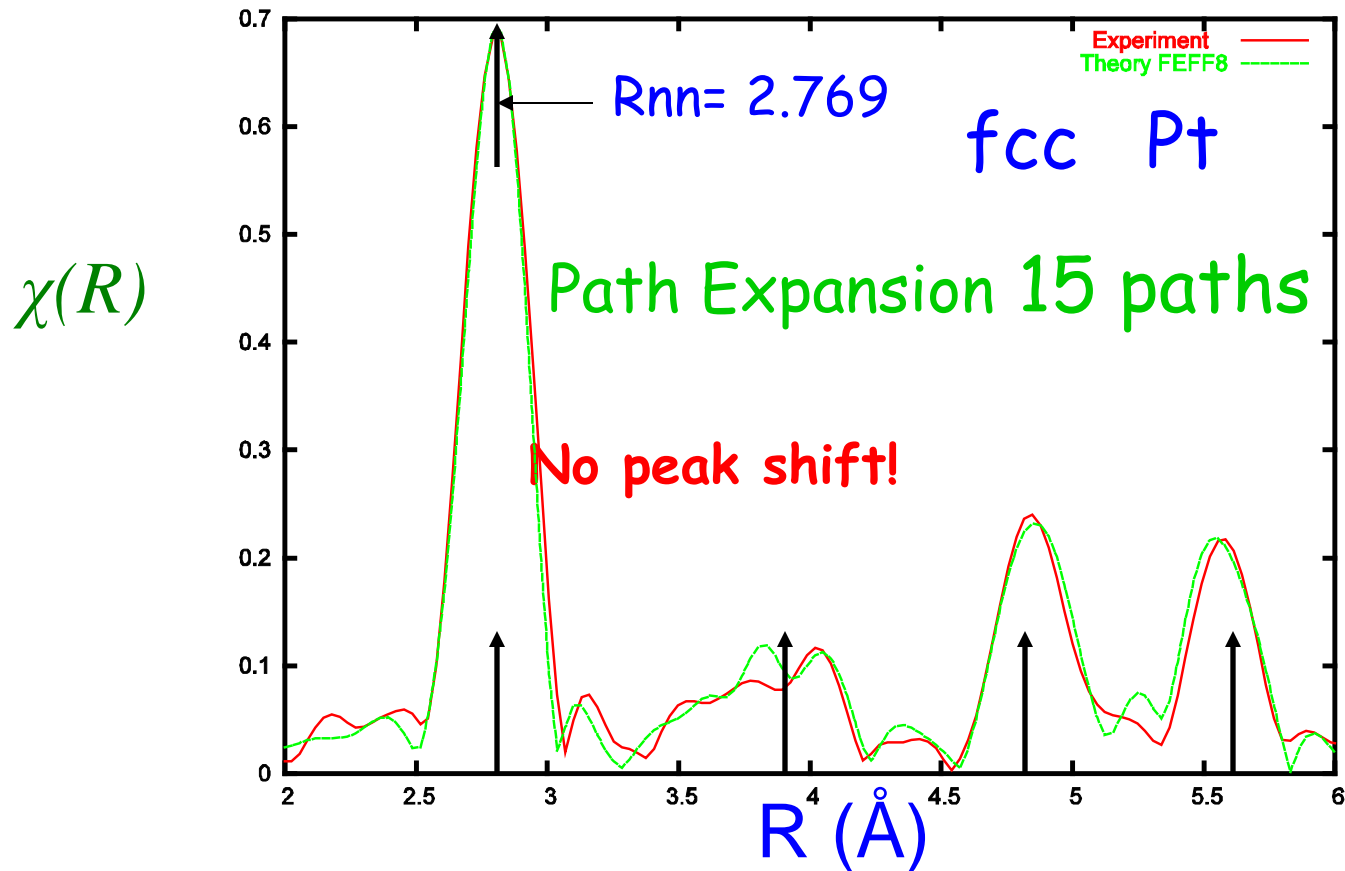
89 atom cluster





# Example: Pt EXAFS – MS path expansion

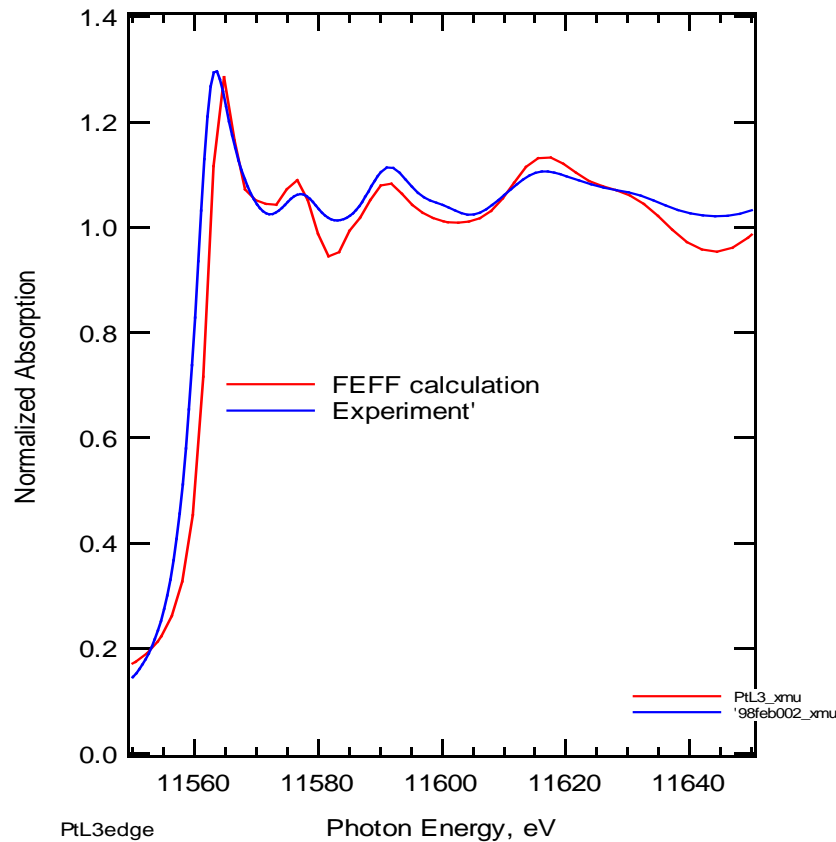
Phase Corrected EXAFS Fourier Transform \*



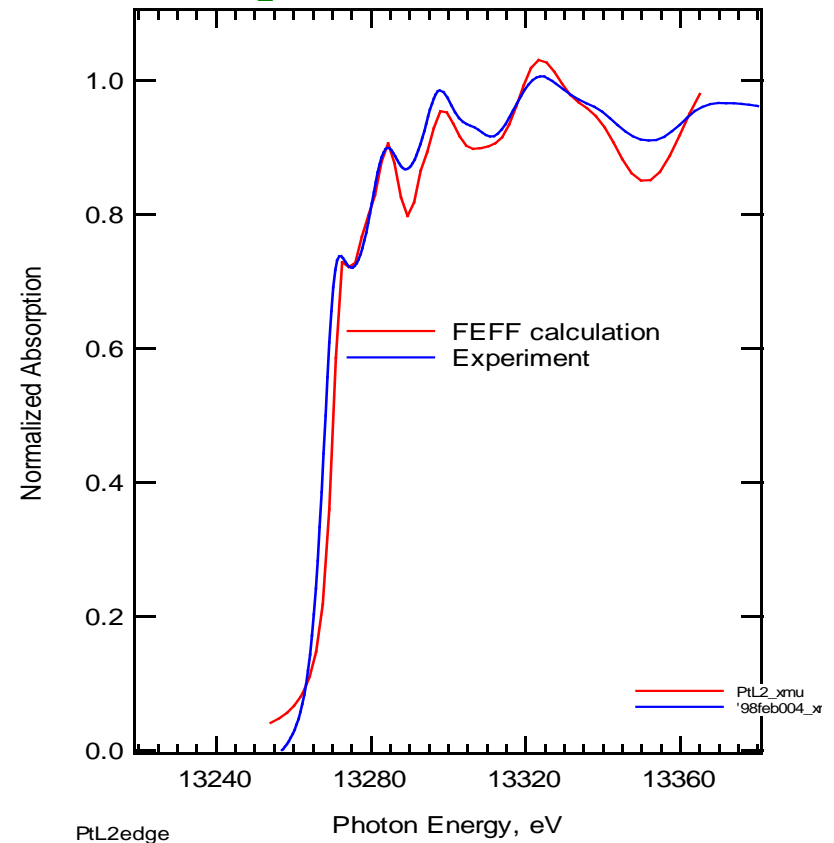
**\*Theoretical phases** → accurate distances to  $< 0.01 \text{ \AA}$

# Example: Pt XANES full multiple-scattering

## Pt L<sub>3</sub>-edge



## Pt L<sub>2</sub>-edge (S. Bare, UOP)



- Good agreement: *Relativistic* FEFF8 code reproduces all spectral features, *including absence of white line at L<sub>2</sub>-edge*.
- *Self-consistency essential*: position of Fermi level strongly affects white line intensity.

# Green's Functions and Parallel Computation

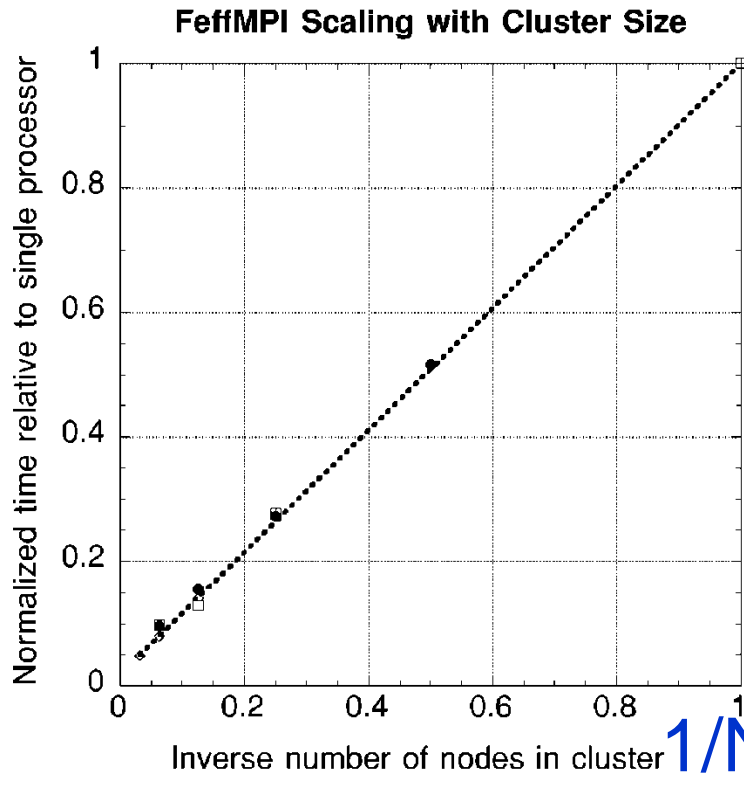
PHYSICAL REVIEW B, VOLUME 65, 104107

## Parallel calculation of electron multiple scattering using Lanczos algorithms

A. L. Ankudinov,<sup>1</sup> C. E. Bouldin,<sup>2</sup> J. J. Rehr,<sup>1</sup> J. Sims,<sup>2</sup> and H. Hung<sup>2</sup>

<sup>1</sup>*Department of Physics, University of Washington, Seattle, Washington 98195*

<sup>2</sup>*National Institute of Standards and Technology, Gaithersburg, Maryland 20899*



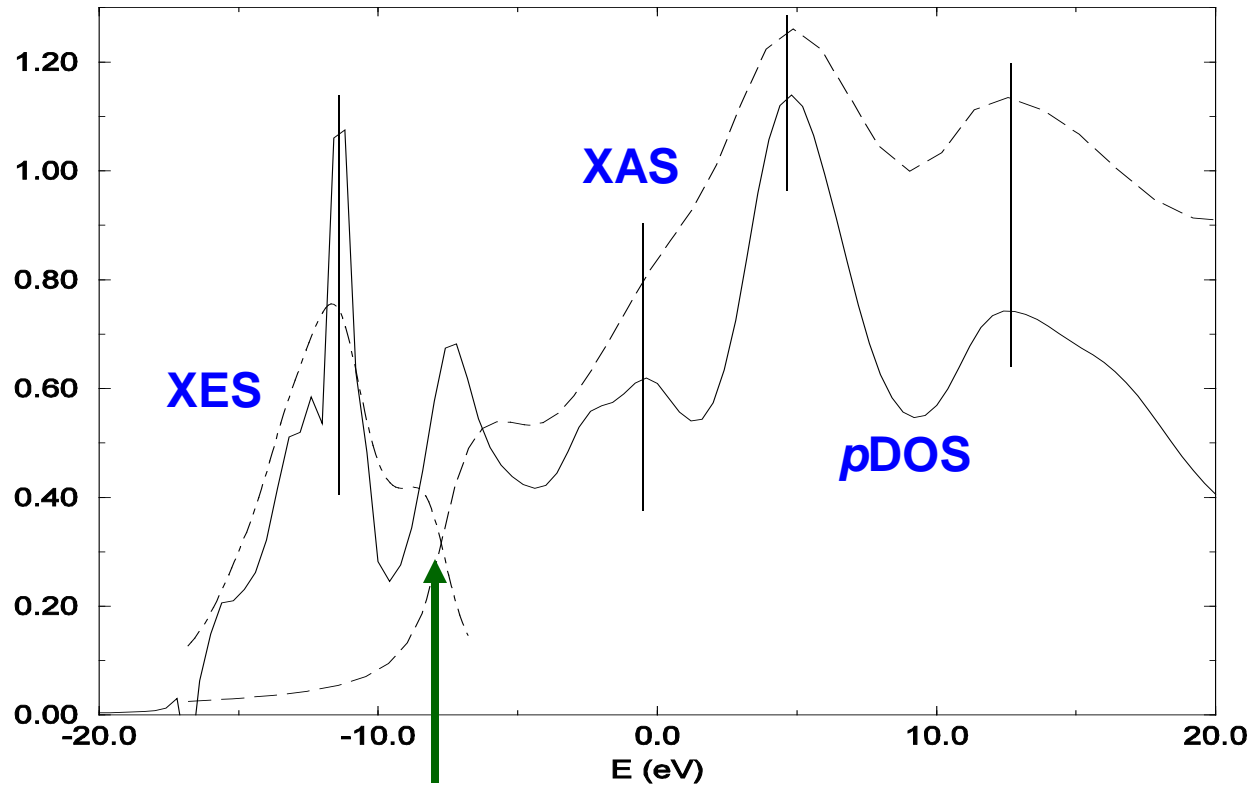
$$\mu(E) \sim -\frac{1}{\pi} \text{Im} \langle i | \hat{\epsilon} \cdot \mathbf{r}' G(\mathbf{r}', \mathbf{r}, E) \hat{\epsilon} \cdot \mathbf{r} | i \rangle$$

**Energy  $E$**   
is just a parameter !

“Natural parallelization”  
Each CPU does one energy

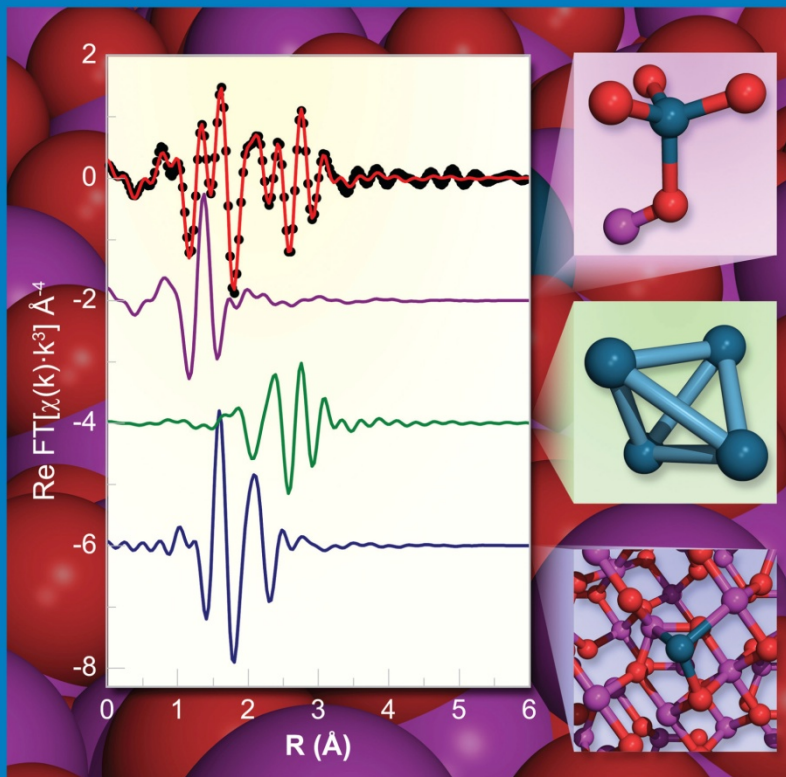
# Interpretation of XANES : Excited State Electronic Structure

## Cu $p$ DOS vs XAS and XES



Fermi energy  $E_F$

Final state energy  $E$



Theory versus  
Experiment:  
Characterization of  
Supported Rhenium  
Catalysts  
(see page 5A)

## Application

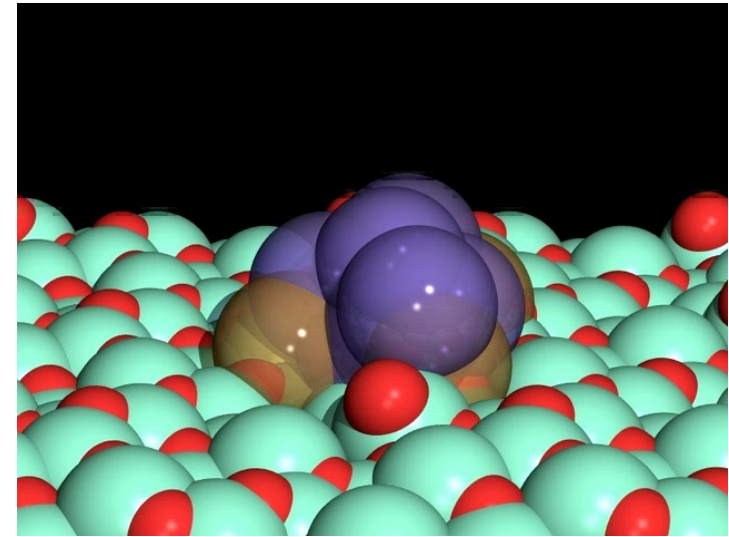
# EXAFS analysis of Re-catalysts

Simon R. Bare, et al.,  
*J. Phys. Chem.* 115,  
5740 (2011)

NANOMATERIALS, INTERFACES, HARD MATTER

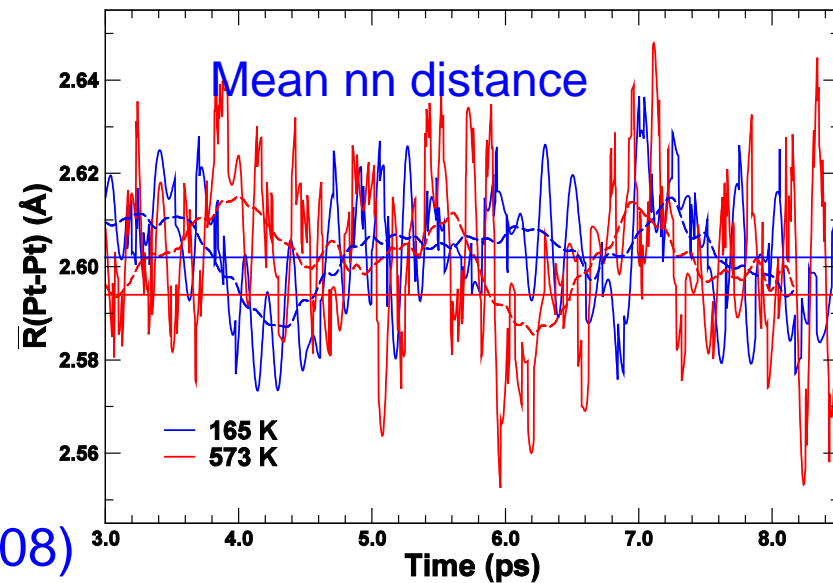
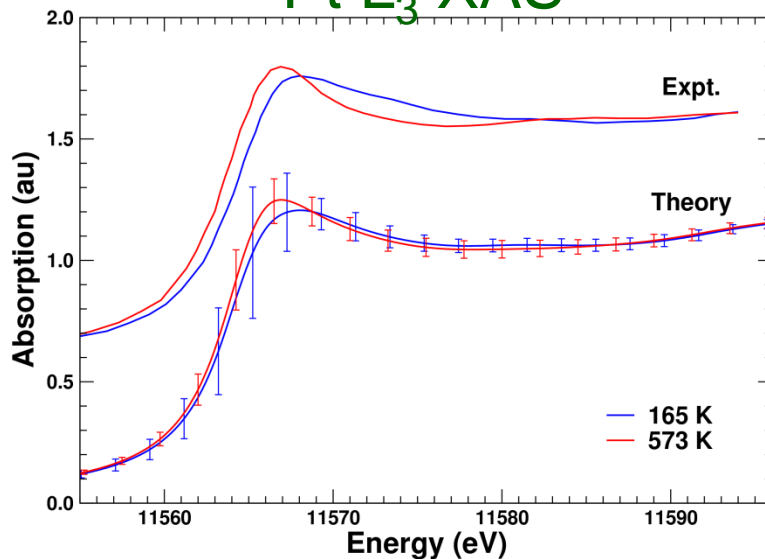
# Application: Pt catalysts

Finite Temp, Real-time DFT/MD  
calculations of supported  
Pt nano-catalysts\*



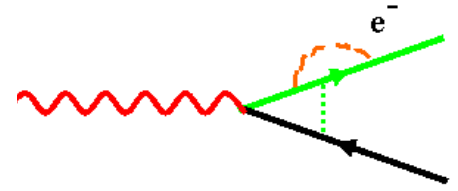
2500 3 fs time-steps

## Pt L<sub>3</sub> XAS



\*F. Vila et al. Phys Rev B **78**, 121404(R) (2008)

# What's Next: Even better theory



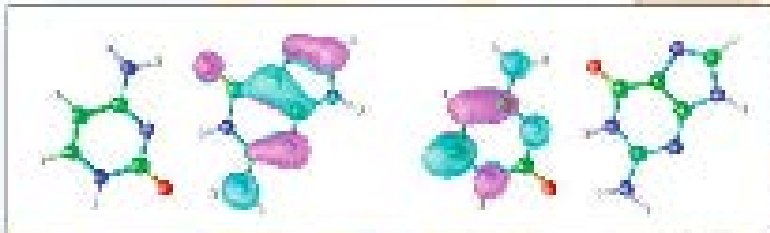
- A. Other spectra: NIXS, RIXS, Optical, Compton
- B. *Ab initio* mean free paths  $\lambda_k$
- C. *Ab initio* Debye Waller factors  $\sigma^2$
- D. Multi-electron excitations  $S_0^2$

# COMPTES RENDUS DE L'ACADÉMIE DES SCIENCES

Année 10  
Fascicule 6

Julien Jorès, Acad.  
1000 pages

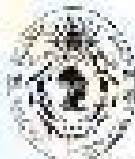
## PHYSIQUE



### DOSSIER

Theoretical spectroscopy (Spectroscopie théorique)  
Quantum theory (Mécanique quantique)  
Linda Young

ACADÉMIE DES SCIENCES - PARIS



(10 more years)

Quantitative XAS

FEFF9

JJR et al., Comptes Rendus  
Physique **10**, 548 (2009)

*in Theoretical Spectroscopy*  
L. Reining (Ed) (2009)



# Application: Optical Spectra

## The Rediscovery of the 'French Blue' diamond

1 François Farges<sup>1,2</sup>, John Vinson, John J. Rehr<sup>3</sup> and Jeffrey E. Post<sup>4</sup> - DOI: 10.1051/epj/2012105

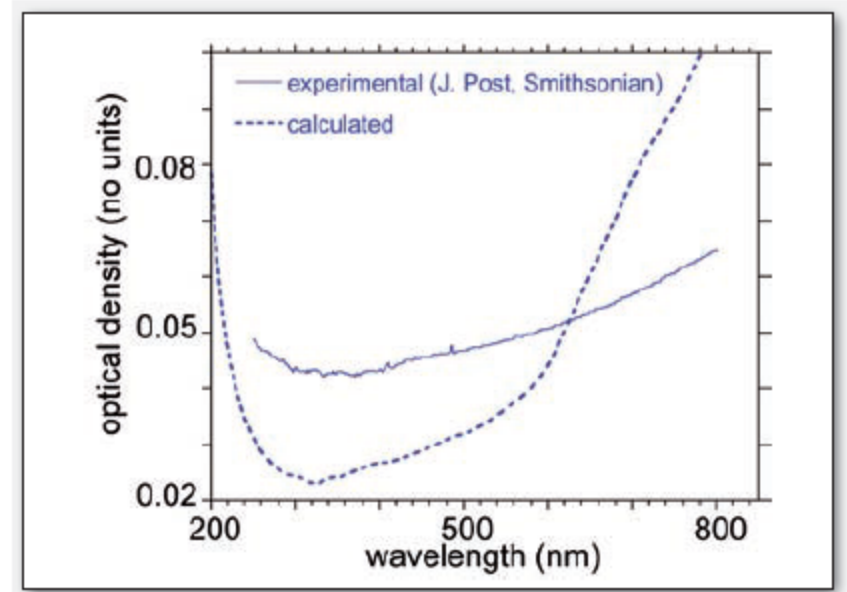
2 Laboratoire de minéralogie et de cosmochimie du Muséum (LMCM) and CNRS UMR 7300 - Muséum national d'Histoire naturelle

3 Dept. of Environmental and Geological Sciences - Stanford University, CA 94305-2115 - USA.

4 Dept. of Physics - University of Washington - Seattle, WA 98195-1560 - USA.

5 Department of Mineral Science - Smithsonian Institution - Washington, DC 20560 - USA.

► FIG. 2: Optical spectroscopy spectrum measured for the 'Hope' diamond (courtesy of Smithsonian Institution, Washington, DC, USA) together with a computer spectrum assuming a boron atom inside a 69-carbon atom cluster.

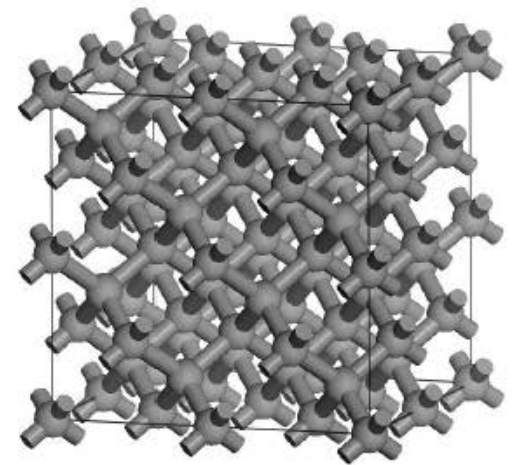
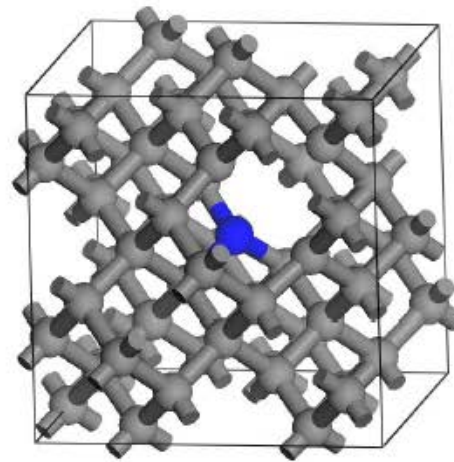


Europhysics News 43, 1 (2012)

# Optical Absorption of N-Doped Diamond

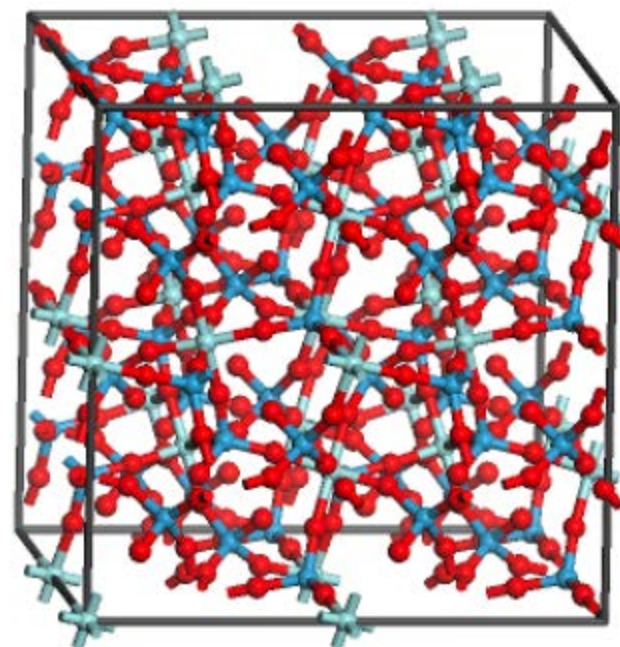
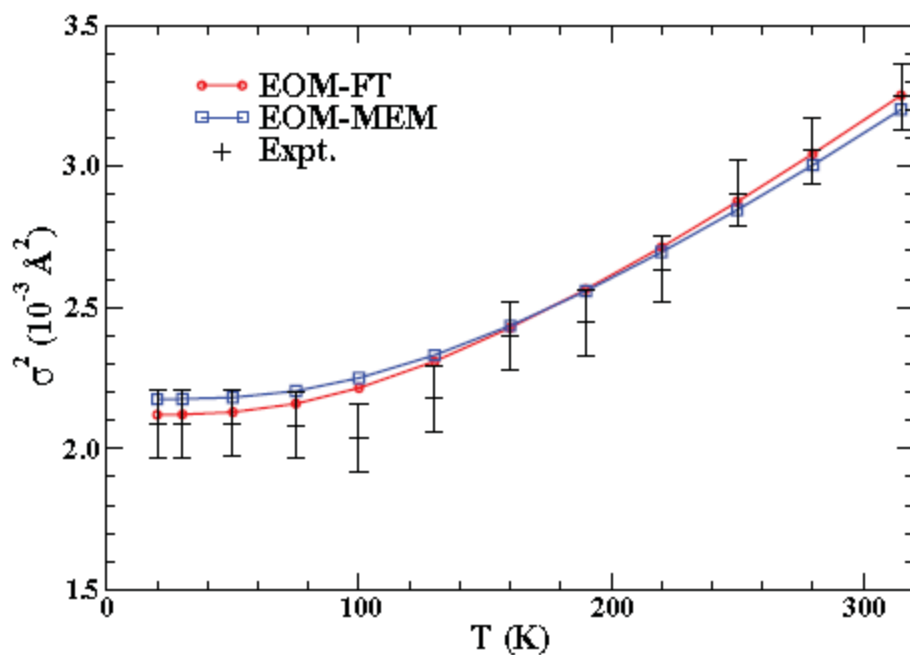
Presentation by: Winnie H. Liang

INT REU: University of Washington 2014



# Debye-Waller factors $\sigma^2$ in Zr tungstate

## Mean square vibration amplitudes









That's all folks!