Determination of the Light Front Wave Function for Quasi-Elastic Electron-Deuteron Scattering

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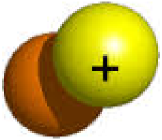
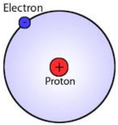
Definition of Deuteron and Reason for Research Interest

Deuteron is a bound state of two nucleons, a neutron and a proton. Much like hydrogen was to the electromagnetism force, Deuteron is to the strong force as it is a bound state of two particles which interact primarily through one dominating force.





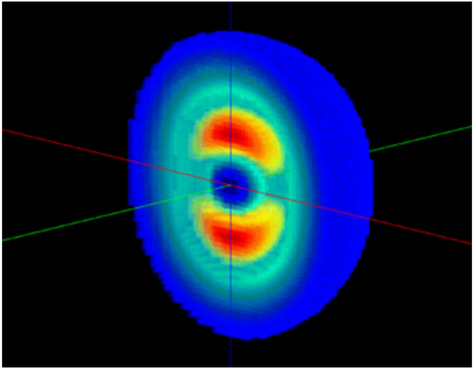
Spin Momentum Description of Deuteron

"Anything that is not explicitly forbidden is mandatory."
Feynman

Deuteron is the only system of two nucleons which is known to exists in a bound state. This tells us that some mechanism in nature excludes proton-proton as well as neutron-nuetron bound states from forming. Modern theories believe that this is due to spin-dependence of nuclear force excluding a bound state which has constituent particles with spins anti-aligned. Due to Pauli's exclusion principle, these identical particle bound states are not physical. This tells us that Deuteron exists primarily in the $S = 1$ state.

Orbital Momentum Description of Deuteron

As the below image indicates via its lack of spherical symmetry, Deuteron does not exist purely in a $L = 0$ state but has a component of its angular momentum in another state, the $L = 2$ state. Numerically the amplitude of the wavefunction in the $L = 2$ state is between $4 - 7\%$.



Tensor Polarized and Unpolarized Deuteron

Modern attempts at understanding asymmetry in the nuclear force look at differences in what is known as tensor polarized and unpolarized Deuteron. The polarized version has a greater amplitude of angular momentum in the $L = 2$ state. This causes the quarks to be come contralized about the origin so that scientists may examine short range mechanisms in the strong force.

What is Light-Front Quantum Mechanics?

In any specified
or generic reference frame,
light front quantum mechanics is
done through the following bases,

$$x^{+} = t + z \qquad x^{-} = t - z$$

$$\vec{x}_{\perp} = \vec{x}_{\perp}.$$

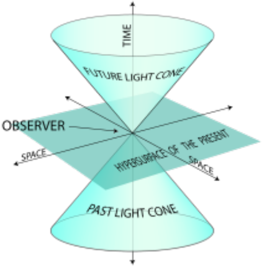


Figure : Light Cone

Due to the speed of light being fixed in all reference frames, physical descriptions of the same system are the same for all reference frames. This hints at the relativistic nature of light front dynamics.

Examining nature of x^+ and x^-

If we examine the
path of a photon with momentum
in $-\hat{z}$ direction, we notive that
it propogate along constant x^-.

x^-
is then more intuitively understood
as a position-like variable
while x^+ acts as time-like variable.
The four momentum of a particle
in light front variables is then

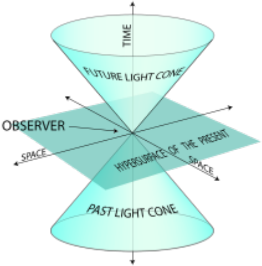


Figure : Light Cone

$$p = (p_0 + p_3, \vec{p}_\perp, p_0 - p_3)$$

Calculating Light Front Wavefunction and Cross X

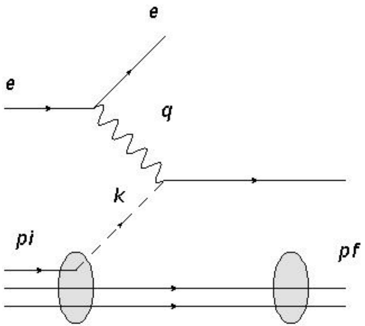


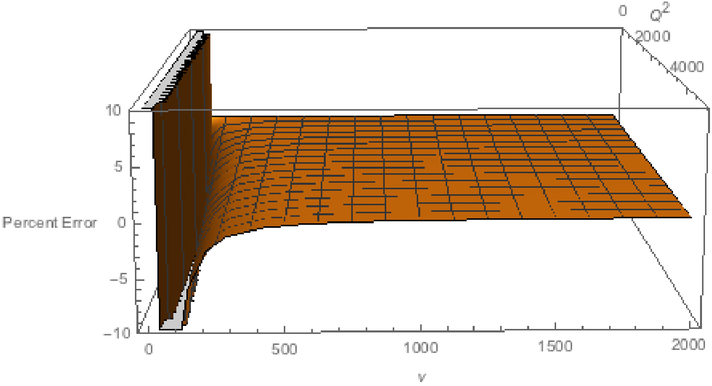
Figure : Electron-Nucleus Scattering

Consider the quasi-elastic interaction and where $q = (\nu, 0, 0, -\sqrt{Q^2 + \nu^2})$ in equal time. The cross section is given by

$$\sigma \sim \sum_f \int \frac{d^3 p_f}{E_f} \int d^4 p \delta((q + p_i - p_f - p)^2 - m^2) |\langle p, f | J(q) | i \rangle|^2.$$

Let's examine the approximations

$$(q + p_i - p_f - p)^2 - m^2 \approx -Q^2 + q^- k^+$$



Calculating Light Front Wave Function and Cross X

If we consider the virtual photon to exist in a low uncertainty wavepacket, for large values of photon momentum ν, we find that the wave has a small uncertainty in position. As we are interested in the reaction where the bound state splits into two nucleons, we can approximate the interaction via some anhilation operator

$$|\langle p, f|J(q)|i\rangle| \to |\langle f|b(k)|i\rangle|.$$

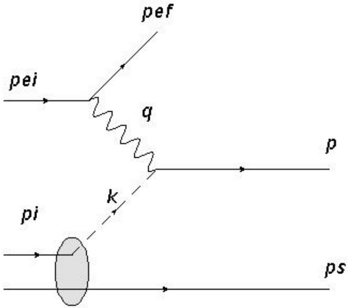
Sparing the details of the calculation we find the following relation.

$$\sigma \sim \int d^2k_\perp |\langle i|b^\dagger_{k^+=mx}(k_\perp)b_{k^+=mx}(k_\perp)|i\rangle|^2$$

Where $x = \dfrac{Q^2}{2 * m * \nu}$ and m is the mass of a nucleon. We can identify the term in brackets as the mod-square of the momentum state wavefunction.

A Scalar Approximation

Consider the following first-order Feynman diagram



We know for the scalar particle approximation that the cross section has the form

$$\sigma = \frac{1}{4\pi^2 Flux} \int d^4 p_s d^4 p \delta^4(p_i + q - p - p_s)\delta(p^2 - m^2)\delta(p_s^2 - m^2)|\mathcal{M}|^2$$

A Scalar Approximation

By comparing the above expression for the differential cross section of this system with the expression for the in the light front approximation, we find that the transfer matrix elements of the system, we find that

$$|\psi_{LF}|^2 = \langle i|b^\dagger(k)b(k)|i\rangle = |\mathcal{M}|^2 \tag{1}$$

Applying the Feynman rules to the diagram, we find the following values for the matrix elements

$$|\mathcal{M}| = \frac{g^2}{Q^2}\frac{1}{M(M-2\sqrt{p_s^2+m^2})}.$$

We can identify the $\frac{g}{Q^2}$ term as a momentum description of the virtual photon and the remaining term as the momentum description of the nucleon.

$$\psi_{LF}(p_s) = \frac{g}{M(M-2\sqrt{p_s^2+m^2})}.$$

Sanity Check

Let's expand the light front wavefunction for small spectator momentum as well as rewrite one of the M's in terms of the nucleon mass and let the constants be equal to some normalalization constant A.

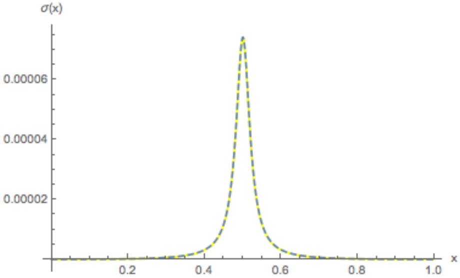
$$-b\psi_{LF} \approx A + \frac{p_s^2}{m}\psi_{LF}$$

If we were to take the potential to be a delta function, we would notice that this expression is exactly the Schrodinger equation.

Bethe-Salpeter Wave Function

The Bethe-Salpeter wave function is the exact relativistic wavefunction of a two particle bound state. It is given below

$$\psi_{BS}(k_\perp) = -g\frac{\theta[x(1-x)]}{M^2 - \frac{k_\perp^2 + m^2}{x(1-x)}}$$



If we recall

$$\sigma \sim \int d^2k_\perp |\langle i| b^\dagger_{k^+=mx}(k_\perp) b_{k^+=mx}(k_\perp)|i\rangle|^2$$

we can compare the x dependence of these cross sections. They are plotted above.

Future Work

-Spin of particles can be included to calculation. When considering the nuclear interactions, this becomes important due to spin dependence of strong force.
-Higher order perturbations in matrix elements of Feynman diagrams can be calculated giving more realistic wave function.
-Shape of hadrons can be included in form factors.
-Neglected terms in light front approximation can be included if cross section is being calculated.

Acknowledgements

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