

An investigation into student understanding of vectors and superposition in the context of electric and magnetic fields

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Abstract

An exploratory study of student understanding of vectors was conducted in the context of electric and magnetic fields. In various tasks, students were asked to identify the direction of the electric field at points around an electric dipole. Students were also asked to draw the direction of the magnetic field vector at various points around two current carrying wires. In both contexts, only approximately one-half of students could correctly complete the task. The errors made by students indicate some basic difficulties in reasoning about vectors and in relating electric and magnetic field vectors to the field lines that are commonly used to represent them.

I. Introduction

Physics education research is the study of student learning and understanding of physics concepts. There are well documented student difficulties in a number of areas of the physics curriculum, and these difficulties are present regardless of institution, instructor, or textbook used [1]. The Physics Education Group (PEG) at the University of Washington (UW) has been investigating these difficulties for over 40 years. The group aims to identify students' thinking errors and to develop curricular materials to address them.

This paper reports on an exploratory study of student vector knowledge and ability to apply that knowledge in the context of electric and magnetic fields. Vector knowledge includes being able to add vectors (superposition) and understanding the idea of a vector field and limits (a vector representing the field at a *point*). Students with strong vector understanding should also be able to relate that knowledge to other representations, such as a field line diagram. We examined student performance on various tasks that required vector knowledge to complete successfully.

II. Instructional Context

At the UW, introductory physics is taught over a sequence of three quarters. The first quarter covers mechanics, the second quarter is electromagnetism, and the third quarter is waves, optics, and modern physics. The tasks examined in the present study are drawn from the electromagnetism course. Students at the UW begin the study of a topic by watching an introductory video from the SmartPhysics program [2] and then they attend lectures on the topic. The course includes both labs and *tutorial* sections

Following lecture(s) on a given topic, students attend a tutorial session related to the topic. These tutorials are small break-out sections of the larger lecture class in which students work in groups on material devised by the PEG to address known student difficulties. The materials used in this class come from *Tutorials in Introductory Physics* [3]. Before a tutorial session, students must complete a pretest which they will be referred back to during the tutorial session. One open-ended question on each exam is devoted to material coming specifically from the tutorials.

III. Task Involving Vectors in the Context of Electric Fields

The pretest for the electric potential difference (EPD) tutorial contains a question in which students are asked to identify the direction of the electric field at four points around an electric dipole (Figure 1). This question has been given over many years, first in a paper-based format in which students had to draw in electric field vectors at the requested points, and later in a computer-based format in which students select an arrow that points in the direction of the field.

Students take the pretest in the fourth or fifth week of a ten-week quarter. They have had lectures on electric charge, force, and field. They have also completed a tutorial on electric fields and flux, and have been introduced to field line diagrams.

To determine the correct answer to this pretest question, students need to know that the electric field points away from positive charges and towards negative charges. They should know that the field due to a point charge decays as $1/(r)^2$, where r is the radial distance from the charge. They

must know how to do vector superposition in order to determine the net field (due to contributions from both point charges) at each point. With these skills, students should be able to determine that the field points in the direction shown by vector A in Figure 1, at all points (1-4).

A. Task Results

The results from a paper-based administration and computer-based administration of the electric dipole question are shown in Table 1. The format of the test question does not seem to affect performance, as on both versions ~55% of students successfully identify the direction of the electric field at all four points. The two most common errors are also shown in the table. Around 10% of students choose zero for the field at the center between the two charges (point 2 in Figure 1). Between 15% and 25% of students choose (or draw) vector B (see Figure 1) at point 4. They seem to only be considering the contribution to the field from the negative charge, and/or fail to use vector superposition at that point.

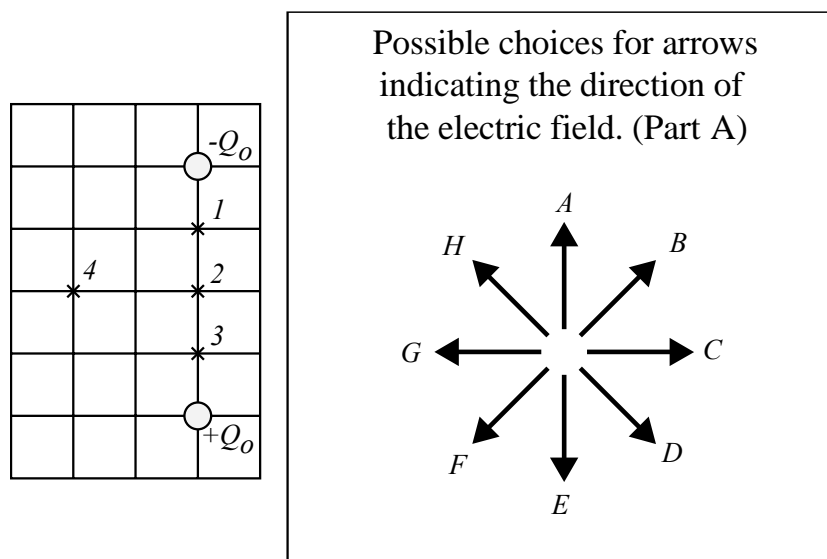


Figure 1: Computer-based version of the electric dipole question given on the EPD pretest. Two charges $+Q_0$ and $-Q_0$ are fixed in place at the positions shown. Four locations 1, 2, 3, and 4 are each marked with an 'x.' Students are told to select the arrow that most accurately indicates the direction of the electric field for each point. They have an option to select zero if the field is zero at any point.

Table 1: Performance on the electric dipole question. Results are shown from when the question was paper-based and from more recently when it was administered on the computer.

	1996-Paper Based (n=90)	Winter '14-Computer Based (n=337)
% Correct	55%*	55%
Zero at the center	10%	10%
Not tangent at point 4	25%	15%

* All percentages are rounded to the nearest 5%.

B. Task Modification

In order to determine how giving the field lines affects students' choices for the direction of the electric field at the various points around the dipole, a modified version of the EPD pretest was developed and administered in the summer of 2014 (Figure 2). Students first had to answer the dipole question with no field lines given.¹ After choosing the direction of the electric field at the four points in the no-field line case, students are asked to do the same thing, this time when field lines are given. Students could not go back and forth between the two questions.

Table 2 shows the results for the modified EPD pretest, along with the results from previous versions of the pretest in order to facilitate comparison. On all versions, around 55% of students correctly identify the direction of the electric field at all four points. One of the most common errors is saying there is no field at the point in the middle of the two point charges. When students were given the diagram with field lines however, very few students said there was no field at the center point. (We hypothesize that this is because there was a field line through the point.) For students who did make that error, they explained their choice by saying that the middle point was in between two opposite charges so the field must be zero there, or

they would say that there is no net force at that point (and so no field).

Another common error was choosing vector B (see Figure 2a and 2b) for point 4. Especially when students were given the field lines, a number of them (30%) said that the field line pointed in the direction of the field, and chose the arrow that curved with the field line at point 4. They were not thinking in terms of limits and failed to realize that the vector at a point represents the direction of the field at that point, not the direction over a region.

Performance on the two questions asked in summer 2014 was also compared. Only 45% of students answered both questions correctly. This indicates that nearly half of the students had trouble relating vector fields and field line diagrams.

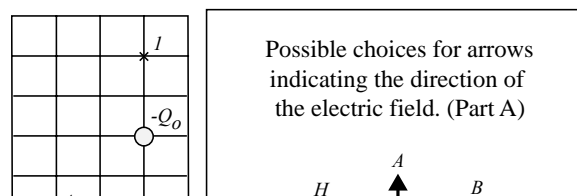


Figure 2a: The no-field line version of the electric dipole question. Students are asked to select the arrow that most accurately indicates the direction of the electric field at the points given (1-4). They must also explain their reasoning.

¹ The points asked about are slightly different than in previous versions, but this is relevant only to an investigation not discussed in this paper.

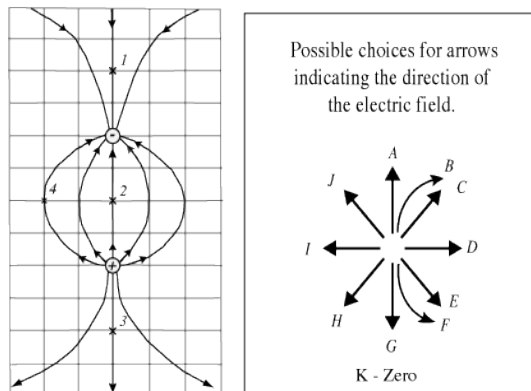


Figure 2 b: The field line version of the electric dipole question. Students are asked to select the arrow that most accurately indicates the direction of the electric field at the points given (1-4). They must also explain their reasoning.

Table 2: Results from four versions of the electric dipole question. The first two columns of results are also presented in Table 1.

	1996 Paper Based (n=90)	Winter '14 Computer-based (n=337)	Summer '14 Computer-based No Field Lines (n=54)	Summer '14 Computer-based Field Lines (n=54)
% Correct	55% *	55%	60%	55%
Zero at the center	10%	10%	20%	<5%
Not tangent at point 4	25%	15%	<5%	30%

*Percentages rounded to the nearest 5%.

IV. Task Involving Vectors in the Context of Magnetic Fields

To probe student ability to relate their vector knowledge to field line diagrams, and to also study their vector knowledge in a magnetism context, we used a question that appeared on the third exam of the quarter (Figure 3).

The test was paper-based, so students had to draw in their answers. In order to complete this task successfully, students must be able to identify the direction of the magnetic field from a current carrying wire (right hand rule); know how to do vector superposition; and understand what it means to be tangent to a point.

Results from this task are shown in Table 3. After lecture, lab, and tutorial instruction on electric and magnetic fields, forces, and field line diagrams, only about one half of the class could correctly answer this very basic question.

For the half of the class that drew an incorrect direction of the field at at least one of the points in the field line diagram, errors fell into a few major categories. A subgroup of students incorrectly stated that the field was zero at the point outside the drawn field lines (point A, Figure 3). These students seem to think that field lines encircle the field and outside the lines there is no field.

Other students had trouble with limits and drew an arrow indicating the direction of the field over region rather than at a specific

point (Figure 4). A similar difficulty was also discussed in the context of the electric dipole.

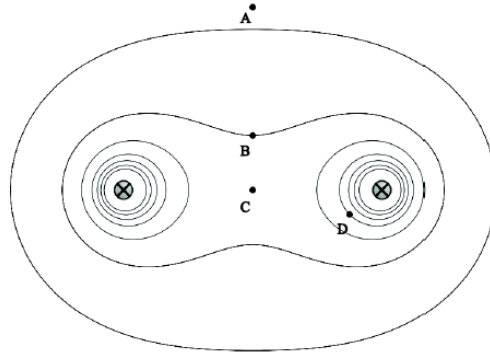


Figure 3: Diagram for the question on exam 3 of the electromagnetism course. Students are told the diagram shows magnetic field lines for a pair of long wires. Both currents are directed into the page. They are asked at each labeled point (A-D), to draw a vector to show the direction of the magnetic field.

Table 3: Results from the question on the third exam regarding the magnetic field lines of two current carrying wires (n=210). Students are asked to draw a vector showing the direction of the field at four points in the field line diagram.

Correct (at all four points)	45% *
Incorrect	50%
No answer	<5%

*Percentages rounded to nearest 5%.

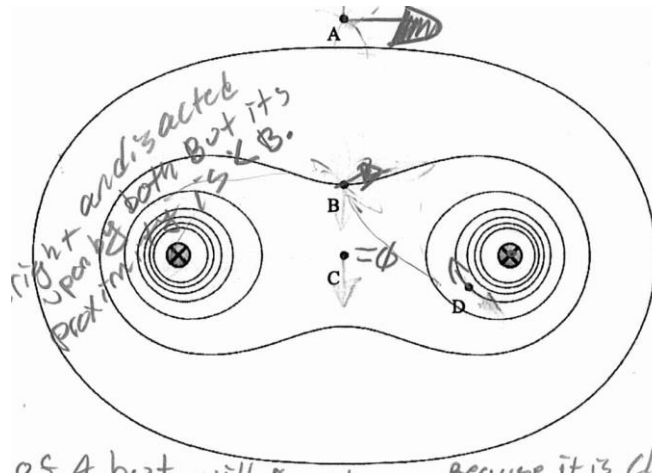


Figure 4 : Example of associating the vector at a point with the direction of the field over a region. Specifically, see point B.

The last major reasoning error observed was failing to relate the field line diagram with the net field of the two wires (Figure 5). Students who made this error seemed to think that the field lines close to the wire on the right represented the field due to that wire only, and the field lines close to the left wire represented its field only. To find the direction of the net field at point D (see Figure 5), they would draw one vector tangent to the field line (which they thought represented the field from the right wire) and another vector obtained with the right hand rule (which they thought was due to the left wire). They would then add these two vectors and state that their vector sum was the direction of the net field at point D. The problem with this reasoning is that the field lines already represent the net field, so the direction of the field at point D is the vector tangent to the field line at that point.

V. Conclusions

In both the context of electric and magnetic fields, students struggle with both vector mechanics and conceptual understanding. Here, “mechanics” refers to basic manipulations like superposition. Conceptual understanding includes

understanding that a field vector represents the direction of the field at a point, and further being able to apply one’s vector knowledge to different representations like field line diagrams.

The PEG works in a continuous, three-part cycle. They identify student misconceptions and difficulties by conducting research; they then develop curriculum to address the difficulties; and finally, they implement the new curriculum at UW. The process then begins again as the researchers examine performance with the modified curriculum.

Even after all relevant instruction, we have seen that students struggle with understanding and applying vectors. The next steps in this project are to develop tutorial materials to address this issue. One possibility is to give students a tutorial homework in which they are shown different point charge configurations and asked to draw the direction of the net field at various points, including a point right in the middle between the charges, and at a point outside of the charges. The research cycle would then start again as we analyzed the effect of the new material on exam performance.

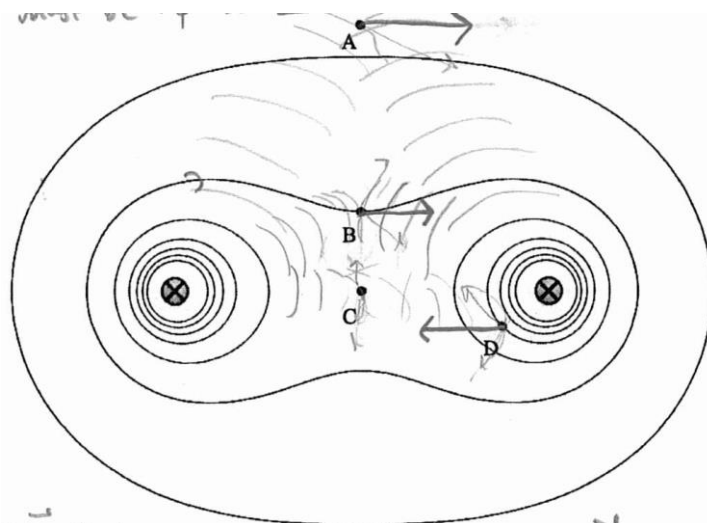


Figure 5: Example of failing to relate the direction of the net field with the tangent to the field line at point D.

[1] L.C. McDermott, “Melba Newell Phillips Medal Lecture 2013: Discipline-Based Education Research- A View From Physics,” *Am. J. Phys.* **82**, 729 (2014).

[2] G. Gladding, M. Selen, and T. Stelzer, www.smartPhysics.com (Macmillan, 2012).

[3] L. C. McDermott, P. S. Shaffer, and the PEG at UW, *Tutorials in Introductory Physics* (Pearson/Prentice Hall, Upper Saddle River, NJ, 2010).