Modifying NMR Probe Electronics in the Muon g-2 Experiment

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The intent of the Muon g-2 experiment is to test the completeness of the Standard Model by measuring with extreme precision the anomalous magnetic moment $a_\mu \equiv \frac{1}{2}(g-2)$ of the muon and comparing that experimental value to theoretical prediction. The experiment calculates the anomalous magnetic moment by using direct detection to measure the precession of the muon magnetic moment about the axis of a uniform magnetic field and using approximately 400 NMR probes to measure the Larmor precession of protons in the same magnetic field. The experiment was most recently performed at Brookhaven National Laboratory, where it achieved a measurement of $a_\mu = 11\,659\,208(6) \times 10^{-10}$ with an uncertainty of 0.54 ppm. The Muon g-2 experiment is now being moved to Fermilab, where the g-2 collaborators aim to improve the anomalous magnetic moment measurement to have an uncertainty of 0.14 ppm. To achieve this accuracy, modified NMR probes are being designed and constructed at the University of Washington's Center for Nuclear Physics and Astrophysics (CENPA). The electronic components of the NMR circuitry are currently about 15-20 years old and a new design using state of the art components is under construction.

1. INTRODUCTION

The magnetic moment μ of a muon with charge e and mass m is directly related to its spin $\mathbf{S}$ by

$$\boldsymbol{\mu} = \gamma \mathbf{S} \tag{1}$$

where the gyromagnetic ratio $\gamma = g_\mu \frac{e}{2m}$ and g_μ is a dimensionless constant called the g-factor. As a point-like fermion, relativistic quantum mechanics expects the muon to have a g-factor of 2 before radiative corrections are taken into account. After these corrections are applied, the Standard Model predicts deviation from the value of 2. This deviation is expressed in the anomalous magnetic moment, and is defined as $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$. High precision measurements of the muon's anomalous magnetic moment, compared with commensurately high precision theoretical predictions, are an excellent way to test the completeness of the Standard Model.

The electron's anomalous magnetic moment has been measured with more precision than the muon's, and its experimental value agrees extremely well with theoretical predictions. However, only QED couplings to virtual particles are significant in the case of the electron. Because the muon is over 200 times more massive than the electron, it couples more strongly to virtual fields and the contributions from the strong and weak forces are also significant. A tau particle would couple even more strongly than a muon, but the muon's longer average lifetime makes it a more suitable particle for the experiment.

2. THE MUON G-2 EXPERIMENT

The Muon g-2 Experiment was first performed at CERN in 1961, where the anomalous magnetic moment was measured to a precision of 4300 ppm [1]. Over the next 15 years, CERN performed the experiment three more times until a precision of 10 ppm was reached. Brookhaven National Laboratory (BNL) then took over g-2 in a series of five experiments from 1997 to 2001, where the experimental setup was designed to improve the original precision measurements taken at CERN. The experiment is currently being moved to Fermilab, with expected data collection beginning in 2016. The storage ring from BNL was carefully moved to Fermilab in 2013 by truck and barge. Meanwhile, the international g-2 collaboration has been improving upon the old design and building new, improved components to the experimental set-up. The University of Washington's Center for Nuclear Physics and Astrophysics (CENPA) has been building and testing new NMR probes.

At BNL, a beam of protons accelerated in the Alternating Gradient Synchrotron (AGS) to momentum 24 GeV/c was focused to 1 mm on a nickel target. The pions produced were collected into a beamline decay channel in which about 38% of pions decayed into muons. Muons with momentum 3.1 GeV/c were injected via a superconducting magnetic inflector into a storage ring with a vertical 1.45 T uniform magnetic field provided by a continuous superconducting magnet, where a pulsed "kicker" ensured the muons were placed in a centered, stable orbit in the horizontal plane of the ring. An electric quadrupole provided vertical stability for the muons [1].

There are three frequencies of interest in the muon storage ring. The first is the *spin precession frequency* ω_s, which is the rate of precession of the muon's magnetic moment about the axis of the magnetic field, which can be seen in Figure 1

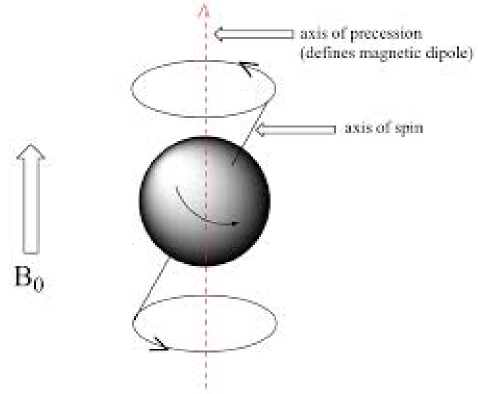

FIGURE 1: From [6]: The spin precession frequency is the rate of precession of the muon's magnetic moment about the axis of the magnetic field B_0.

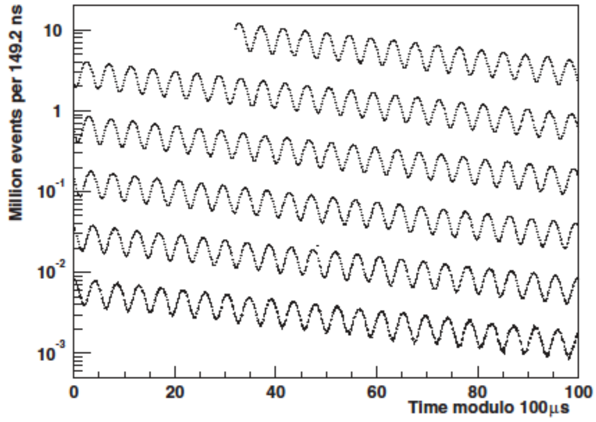

FIGURE 2: From [1]: Positron count rates show oscillations due to a preferred emission angle in muon decay.

and expressed as

$$\boldsymbol{\omega}_s = -\frac{gq\mathbf{B}}{2m} - (1-\gamma)\frac{q\mathbf{B}}{\gamma m}. \quad (2)$$

Secondly, there is the *cyclotron* frequency, which is the rate at which the muons travel around the storage ring. This frequency is given by

$$\boldsymbol{\omega}_c = -\frac{q\mathbf{B}}{m\gamma}. \quad (3)$$

Finally, the *anomalous precession frequency* is defined to be the difference between the spin precession and cyclotron frequencies:

$$\boldsymbol{\omega}_a \equiv \boldsymbol{\omega}_s - \boldsymbol{\omega}_c = -\left(\frac{g-2}{2}\right)\frac{q\mathbf{B}}{m} = -a_\mu \frac{a\mathbf{B}}{m}. \quad (4)$$

From the rest frame of the muons, the electric quadrupole used for stabilization is a motional magnetic field $\tilde{\mathbf{B}}$ and therefore has an effect on the spin precession frequency, which would then change the anomalous precession frequency to be

$$\boldsymbol{\omega}_a = -\frac{q}{m}\left[a_\mu \mathbf{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\frac{\tilde{\mathbf{B}}\times\mathbf{E}}{c}\right)\right]. \quad (5)$$

At the "magic" momentum of 3.1 GeV/c, $\gamma = 29.3$ and the last term, which includes the motional magnetic field, drops out [1]. So muons are injected into the storage ring with the "magic" momentum and the electric quadrupole has no effect on the precession rate of the muon magnetic moment. Therefore, a precision measurement of the anomalous magnetic moment depends on measuring $\boldsymbol{\omega}_a$ and $\mathbf{B}$ with extreme accuracy.

For the g-2 Experiment, the anomalous magnetic moment is expressed as

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p} \quad (6)$$

where ω_p is the Larmor frequency of protons in the uniform magnetic field and $\frac{\mu_\mu}{\mu_p}$ is the ratio of the muon's and proton's magnetic moments, which can be extracted from precision measurements of hyperfine levels in muonium to be $\frac{\mu_\mu}{\mu_p} = 3.18334539(10)$ [8].

The average lifetime of a muon at rest is about 2.2μs. However, for the g-2 Experiment observers where the muon has $\gamma = 29.3$, time dilation extends that average lifetime to about 64μs. The muon undergoes the decay

$$\mu^+ = e^+ + \overline{\nu_\mu} + \nu_e \quad (7)$$

where the positron has a preferred emission angle due to parity violation. Detectors placed around the storage ring measure oscillations in positron count rates because of the parity violation, where the frequency of oscillation is related to ω_a [1].

The Larmor precession frequency of free protons, alternatively, is measured using NMR probes using pulsed NMR techniques. There are 366 fixed probes around the outside of the storage ring and 17 are placed in a vacuum-sealed trolley which can move around inside the ring. Figure 3 shows a schematic for the NMR probes.

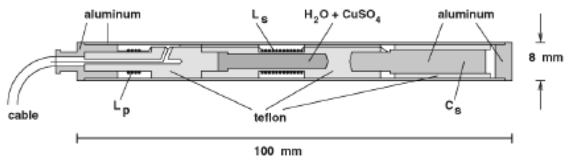

FIGURE 3: From [1]: Schematic for NMR probes at BNL.

At Brookhaven, a small water sample was held in the central chamber, which can be seen in Figure 3 labeled as "H_2O + $CuSO_4$". At Fermilab, this chamber will hold petroleum jelly instead of water to minimize leakage. Around the chamber is a small coil. The magnetic moment of a proton in the petroleum jelly sample lines up with the magnetic field in the storage ring; arbitrarily, call this the z direction. A "$\frac{\pi}{2}$" pulse is sent to the surrounding coil which introduced a second, smaller, oscillating magnetic

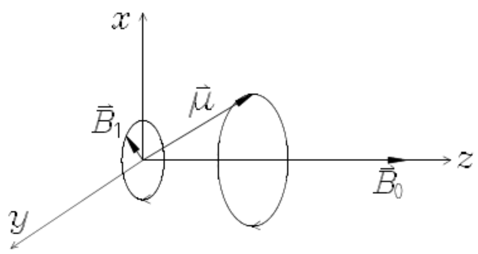

FIGURE 4: From [7]: An applied oscillating magnetic field B_1 flips the magnetic moment μ down into the $x-y$ plane.

field, represented by B_1 in Figure 4, which "flips" the magnetic moment down into the $x-y$ plane. Once there, the oscillating magnetic field is turned off and the magnetic moment relaxes back into the z direction while precessing about the axis of the magnetic field at the proton Larmor frequency $\omega_p = \gamma_p B_0$ where γ_p is the proton gyromagnetic ratio. This relaxation is called the *free induction decay*. The relaxation precession induces a current in the same coil which was previously transmitting the $\frac{\pi}{2}$ pulse. This outgoing free induction decay (FID) signal ideally looks like an exponential decay curve modulated by a sine wave.

3. NMR PROBE CIRCUITRY

By the time Fermilab begins running the g-2 experiment, the electronics will be at least 15 years old. The circuit used in the BNL experiments can be seen in Figure 5, where Figure 9 (at end) is a simplified block diagram of the schematic. Producing the same function with modern, state-of-the-art components is an excellent opportunity to simplify the design of the circuit while increasing flexibility and possible applications.

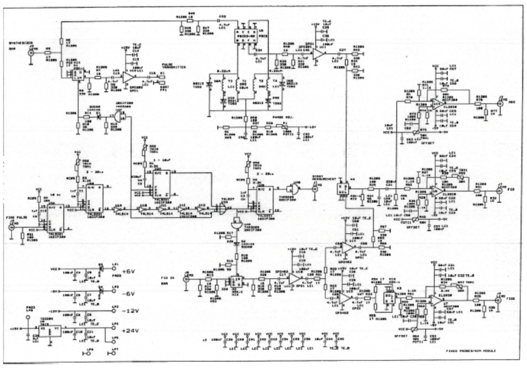

FIGURE 5: From [5]: Schematic of original circuit.

A 61.74 MHz sine wave is produced from a synthesizer and mixed with a 4 Hz "fire pulse" square wave to be transmitted to the NMR probe as the $\frac{\pi}{2}$ pulse. The fire pulse goes through a series of logic gates and integrated circuits before splitting into three parts, one of which enters the mixer with the synthesizer. Another part goes through more logic gates before signalling to the computer to start collecting a measurement, and the third part is mixed with the returning induced signal from the NMR probe. The logic gates act as a timer governing when the coil in the NMR probe is transmitting a signal versus when the circuit is receiving an induced signal from the NMR probe, and when the measurement of that returning signal should begin.

The returning signal is then split and alternately mixed with the synthesizer or mixed with itself. The returning signal has a frequency of the Larmor precession for protons in a 1.45 T magnetic field, which is approximately 61.79 MHz. A mixer produces a signal with frequencies which are the sum and difference of the two input signals. Since the frequencies of the signal from the synthesizer and the returning signal are so similar, the output of the mixer has a very high frequency (123.53 MHz) and a very low frequency (50 kHz). Low frequencies are easier to work with, so a low-pass filter is placed directly after the mixer which effectively blocks the high frequency output, leaving only the 50 kHz FID signal. This can also be used to analyze the frequency by the "zero count" method, which counts every time the signal crosses zero and thereby calculates the frequency. By mixing the returning signal with itself, on the other hand, the output shows the envelope of the FID (FIDE); ideally, this should look like an exponential decay.

Between leaving the synthesizer and being mixed with the returning signal, the 61.74 MHz sine wave goes through a splitter with a "phase adjustment" circuit. This splits the signal in two and adjusts the phase of each to be orthogonal. This can then be fine-tuned so that the synthesizer signal is at either zero or a maximum when the returning pulse starts, and consequently the FID is at either a zero or a maximum when it is analyzed to retain as much information about the induced signal as possible.

When modifying the circuit, the first step is to reproduce the basic function of the circuit with minimum components. To this end, the zero-cross and FIDE outputs are removed and can be added in later with software. The phase adjustment circuit and "Start Measurement" output are also removed and may be added in later to fine tune the final output FID. Without these components the circuit simplifies to Figure 10 (at end).

4. CONSTRUCTING THE CIRCUIT

To make a prototype of the circuit, we ordered components from the RF-components distributer website Mini-Circuits®. We looked exclusively at connector-type products, meaning the components are self-contained and are connected to other components by cables; an example component can be seen in Figure 6. We ordered three low noise amplifiers, three Level 7 mixers, three 20 dB attenuators and three 10 dB attenuators, a 2-way 180° power splitter/combiner, all of which had male SMA connectors (seen in figure 8). It was also necessary to purchase female-female SMA cables to connect the components and SMA-BNC adapters to connect the circuit to external sources and outputs, like the synthesizer and an oscilloscope.

When designing and building the circuit, there were

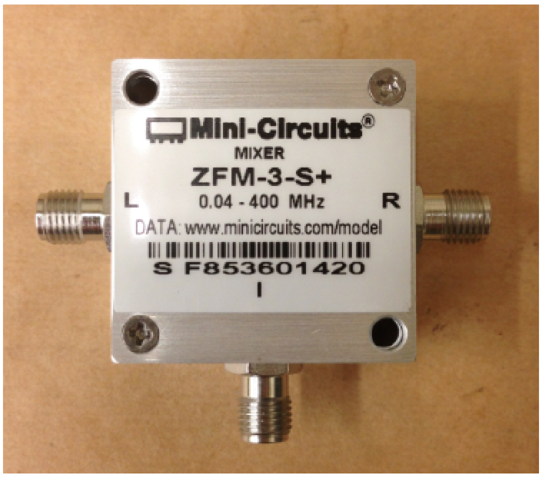

FIGURE 6: Mini-Circuits® connector-type ZFM-3-S+ mixer.

several considerations to keep in mind. Each component has maximum ratings which define its operational parameters, as seen in Table 1 (at end of paper). Each component needs to operate with low power as well as include 60 MHz in its frequency range. All components have a standard 50 Ω impedance.

The amplifiers require that the input RF signal is less than +5 dBm, which is equivalent to about 3 mW and corresponds to an incoming voltage about 0.397 V. The amplifiers also need the supply current to remain below 60 mA. The mixers are Level 7, which means that the incoming local oscillator (LO) signal power must be less than +7 dBm or about 5 mW, which corresponds to an incoming voltage of approximately 0.5 V.

Additionally, the incoming RF signal power must be less than 50 mW, or have an incoming voltage of less than about 1.58 V. Finally the intermediate frequency current can only be 40 mA maximum. The restrictions on the attenuators and power splitter are more lax; incoming signal power must be less than 0.5 W for attenuators, and less than 1.0 W for the splitter [4].

Adding amplifiers and attenuators to the circuit ensure that the restrictions are adhered to while each signal remains appropriately strong. Between two mixers, in particular, it may be possible to make a direct connection with suitably small power and voltage. However, it is still preferable to add an attenuator and amplifier in series between the two mixers to act as a buffer for any reflected signal from imperfect impedance matching. A component's impedance will never be exactly 50 Ω, and buffers help minimize noise from those imperfections.

In addition to the in-line attenuators ordered from Mini-Circuits®, it was necessary to include attenuators already found at CENPA. These attenuators were not in-line but rather self-contained black boxes with input and output BNC ports. The strength of attenuation could be adjusted by a dial to any level 0-10; a lower number corresponds to higher attenuation. Level 0 completely attenuates the signal.

The final circuit with minimal components is seen in Figure 11 (at end). This circuit does not mix the signal from the logic box with the returning signal to regulate timing, nor does it include a low-pass filter after the final mixer to get rid of the high-frequency part of the output.

5. FUTURE WORK

Now that a simplified, cheaper, state-of-the-art circuit is shown to work, it can be improved and modified to be appropriate for an experimental setting like Fermilab's g-2 experiment. A first step would be to organize and fix the components to a baseboard. In g-2 about 400 probes are used, so the electronics should be compact and easily transportable.

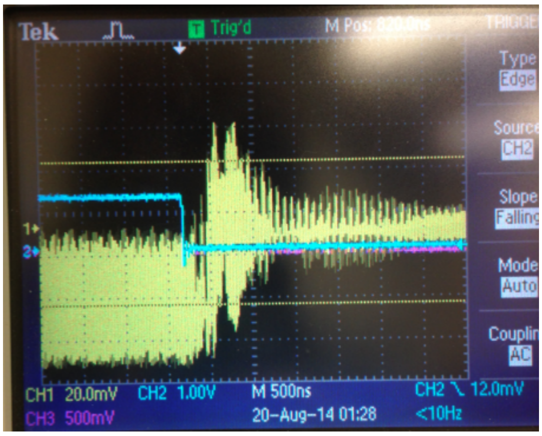

FIGURE 7: Returning signal from the modified circuit.

In Figure 7, the blue signal shows the logic pulse as it emerges from the logic box. When it ends, the $\frac{\pi}{2}$ pulse, which can be seen in the first half of the yellow signal, also ends. After the transmitted pulse ends there is about 250 ns of noise where the coil is transitioning from transmitting a pulse to receiving one a hint of the free induction decay; while some exponential decay can be seen, the time scale is too short to definitively identify it as a FID. To see how much of a FID signal the circuit is actually receiving, power in the circuit should be increased. In the future we can use more logic circuitry or software to cut out noise; this will take the place of the "Start Measurement" part of the original circuit.

There are several other applications to add to the circuit in the future for added flexibility. A next step would be to manipulate the logic box to control the length of the $\frac{\pi}{2}$ pulse. A shorter pulse than desired would not tip the magnetic moment all the way into the $x-y$ plane, while a pulse longer than desired would flip the magnetic moment too far and give it a negative z component. More direct control over the length of the pulse would provide another method of fine-tuning the FID, which would naturally make it easier to analyze and extract ω_p.

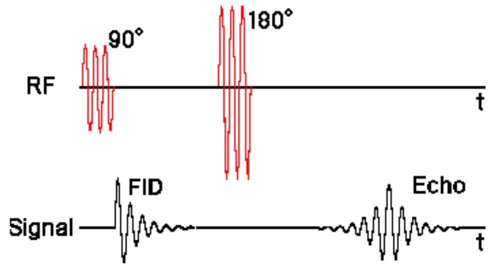

FIGURE 8: From [3]: A $\frac{\pi}{2}$ or 90° pulse followed by a π or 180° pulse, and their resultant signals.

Another addition would be to add the capability to perform the spin-echo method of pulsed NMR. To perform a spin-echo sequence, a $\frac{\pi}{2}$ pulse is applied followed by a π pulse. This gives not only a FID but also an "echo" signal after the π pulse. The timing of the two pulses and their induced signals is shown roughly in Figure 8. The spin-echo method is useful for identifying field inhomogeneities by measuring spin relaxation times T_1 and T_2, which are related to the relaxation of parallel and perpindicular components the magnetic moment (relative to the external magnetic field) respectively.

6. REFERENCES

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[8] W. Liu et al., Phys. Rev. Lett. 82, 711 (1999).

Component	**Model**	**Frequency Range**	**Restrictions**
Amplifier	ZFL-1000LN+	0.1-1000 MHz	Input RF +5 dBm max, Supply current 60 mA max
Mixer	ZFM-3-S+	0.04-400 MHz	LO Power +7 dBm, RF power 50 mW max, IF current 40 mA max
Attenuator	VAT-20+	DC-6000 MHz	0.5 W max
Attenuator	VAT-10+	DC-6000 MHz	0.5 W max
Power Splitter/Combiner	ZFSCJ-2-1+	1-500 MHZ	1W max

TABLE 1: Components used in creation of modified circuit, with the frequency range and relevant restrictions of each.

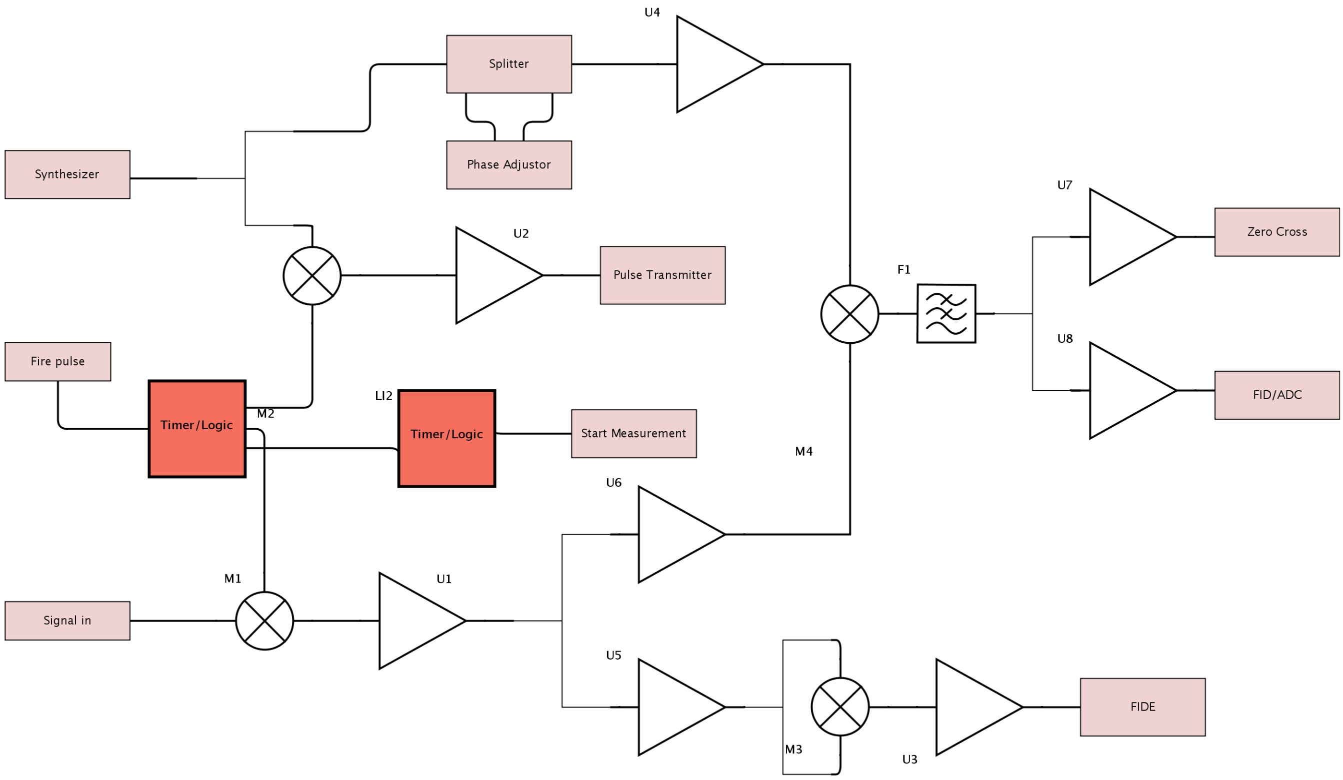

FIGURE 9: Block diagram of original circuit.

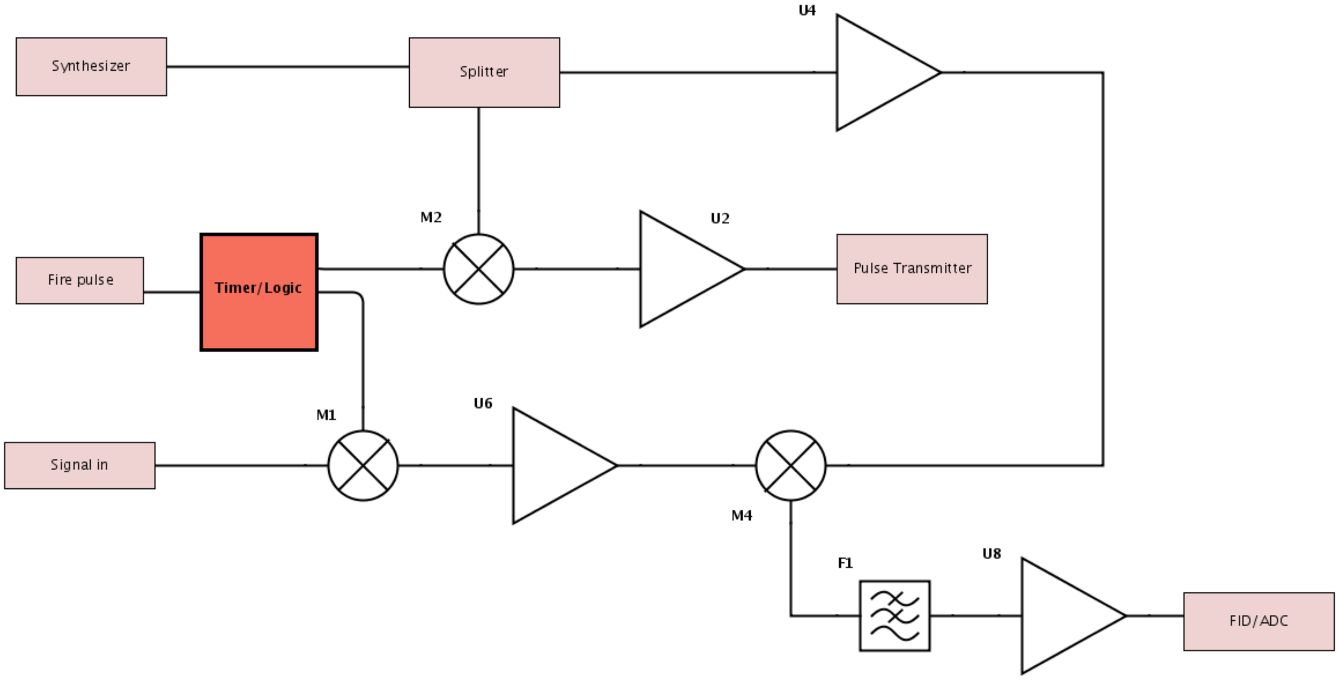

FIGURE 10: Block diagram for planned modified circuit.

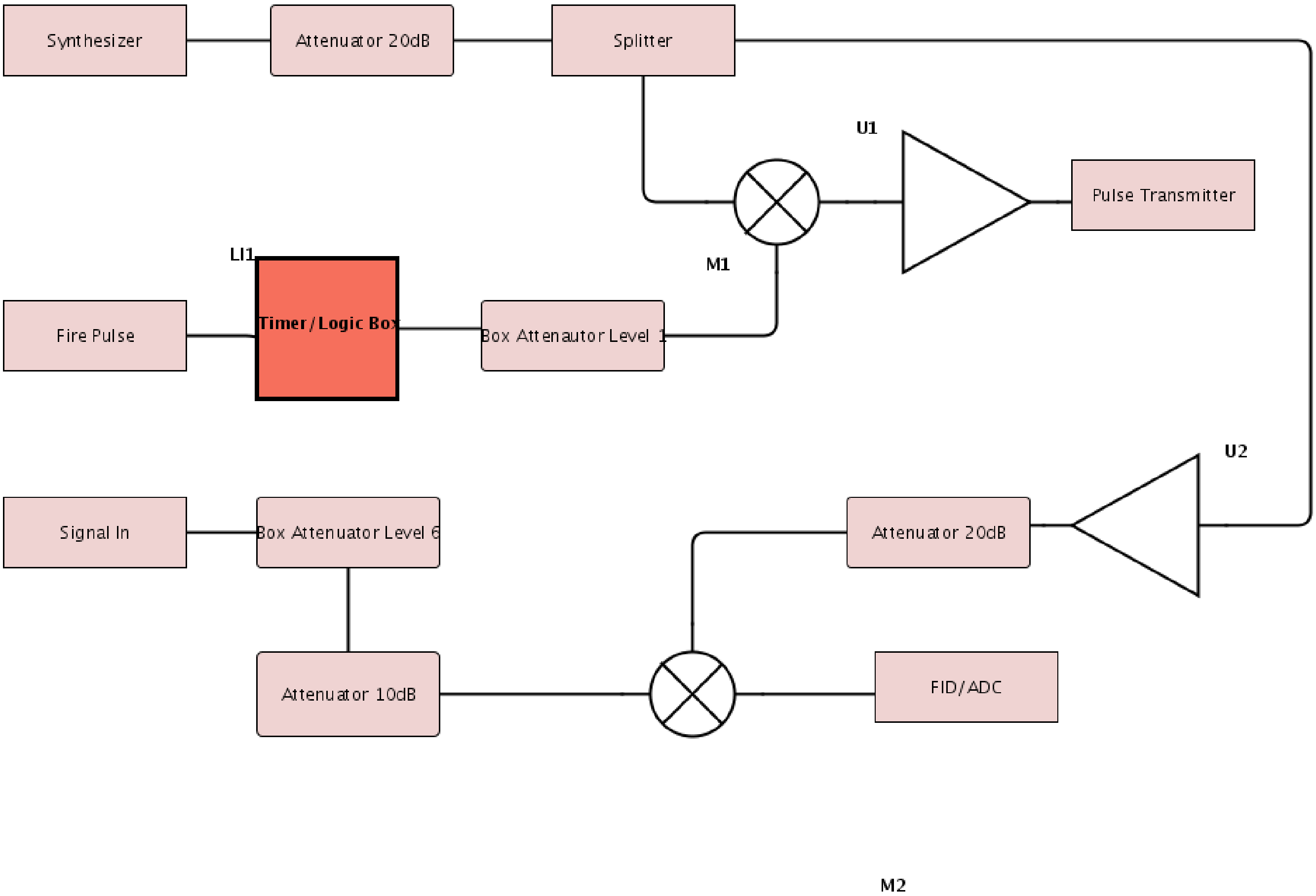

FIGURE 11: Block diagram of working, modified circuit with minimal components.