The Unitary Fermi Gas:
Universal Physics from Cold Atoms to Neutron Stars

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Outline

- From QCD to Cold Atoms: Universality in Physics

  BEC-BCS crossover, Unitary Fermi Gas,

  Benchmarks: Convergence of Theory and Experiment

  Novel Polarized Phases: p-wave superfluids, LOFF supersolid crystals

- The Many Body Problem

  Classical and Quantum: Bosons and Fermions

  Quantum Monte Carlo (QMC), Mean Field Theory,

  Density Functional Theory (DFT)

- From Cold Atoms to Nuclei and Neutron Stars

  DFT, Vortex pinning, Glitches

- From Cold Atoms to Cold Dark Matter

Universality

- One theory describes different systems
  - Cold Atoms

    Tune interactions to model other systems
  - Neutron Stars, Nuclei

    Interactions accidentally like cold atoms
  - Superconductors
  - Quark Matter
  - Dark Matter

Universality

Fermionic Superfluids

Neutron Matter

$k_F \sim \text{fm}^{-1}$

$a_{nn} = -19$ fm

$r_{nn} = 2$ fm

Nuclei

neutrons and protons

Unitary Fermi Gas

$a = \infty$

$r_e = 0$

Cold Atoms

$k_F \sim \mu\text{m}^{-1}$

Tuneable a

$r_{nn} \sim 0.1$ nm

Many systems

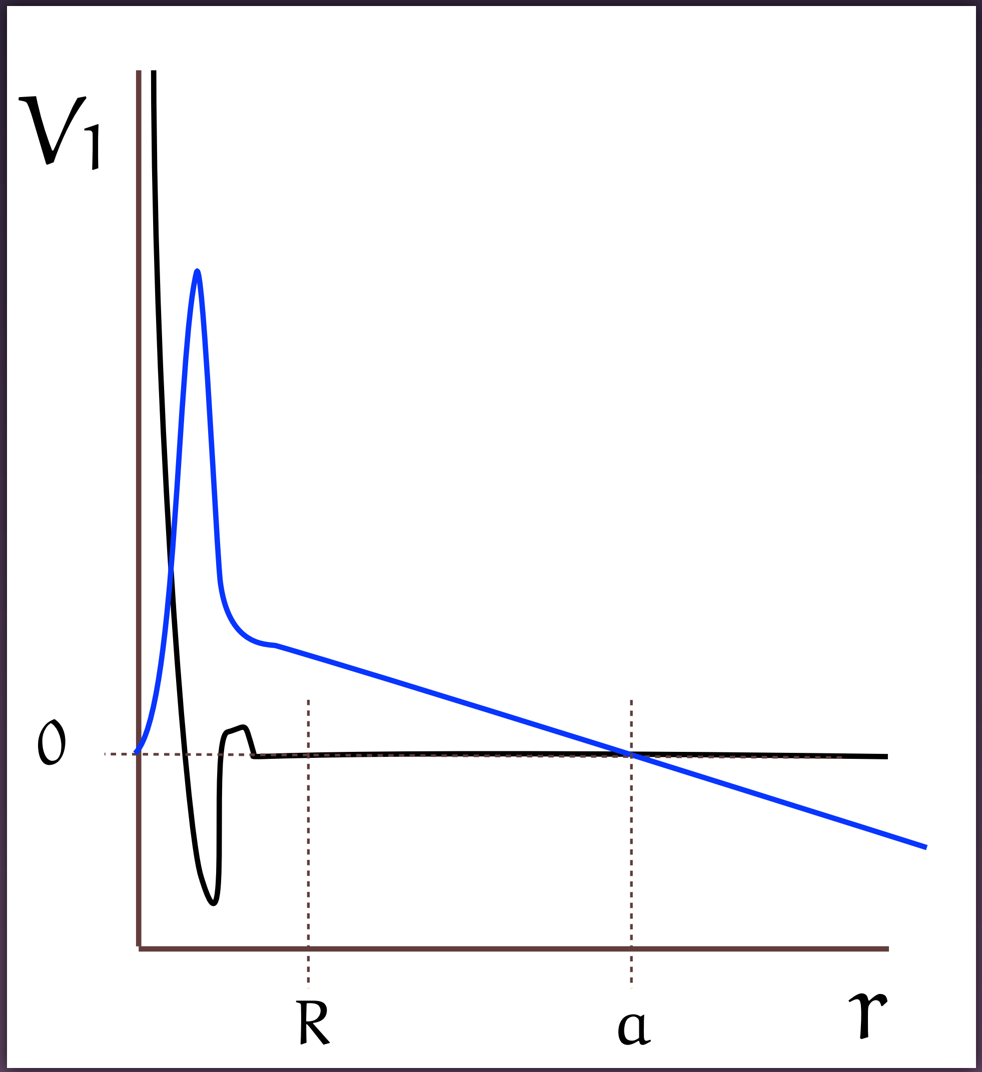
- different species
- dipole interactions
- optical lattices
- quantum simulators

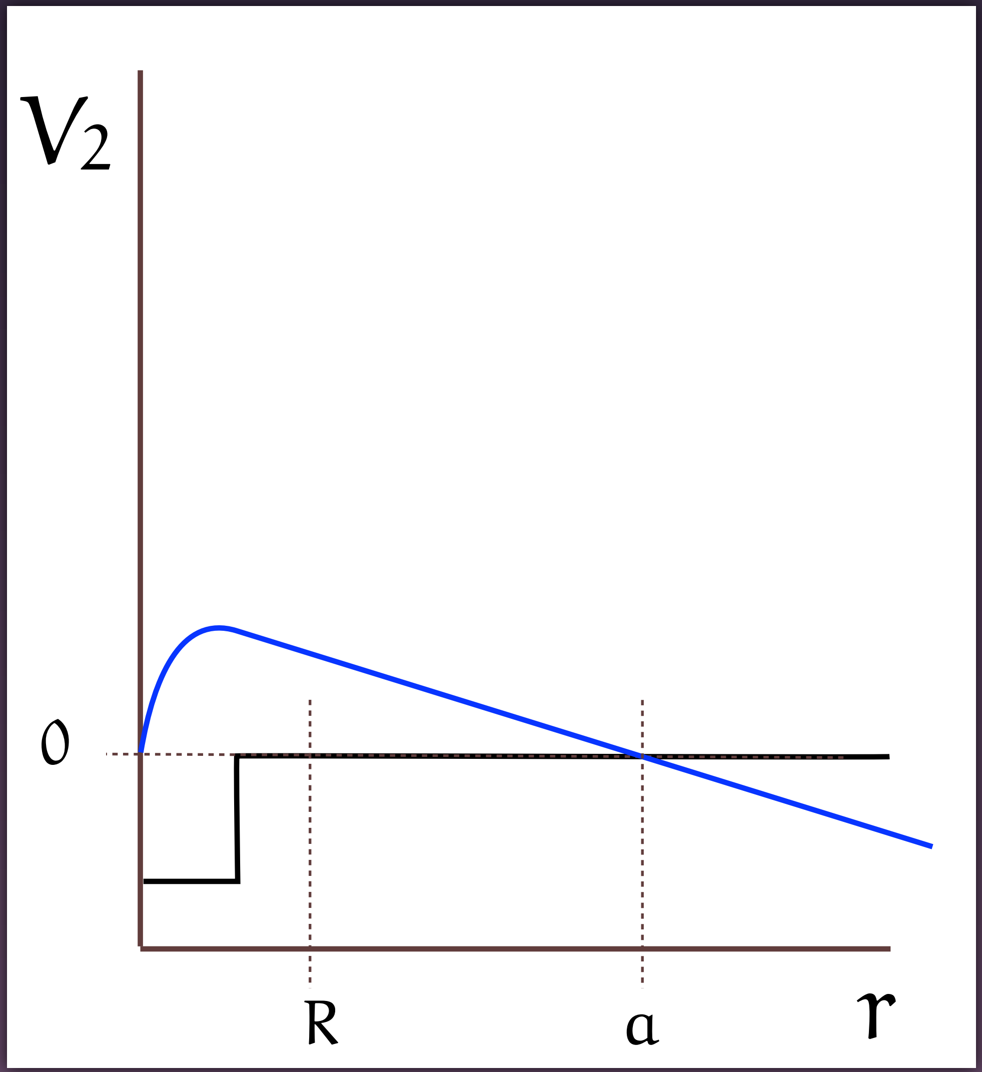
Other Superfluids

- Superconductors (charged + phonons)
- Quarks (gluon interactions, Dark Matter?)
- ^{3}He (p-wave)

Universality

- Short distance irrelevant:
  - At long distance $(r>R)$ potentials equivalent $V_1 \equiv V_2$
  - Characterized by scattering length a





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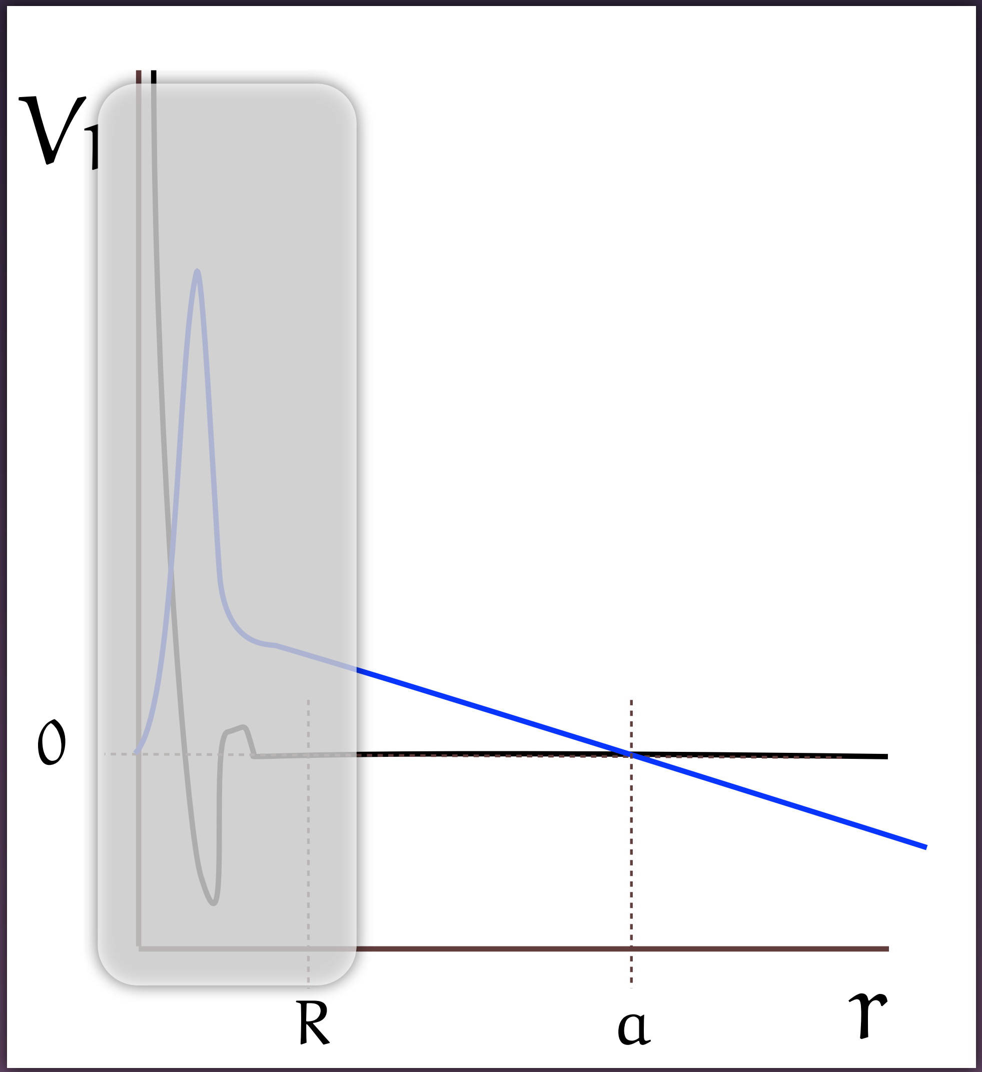
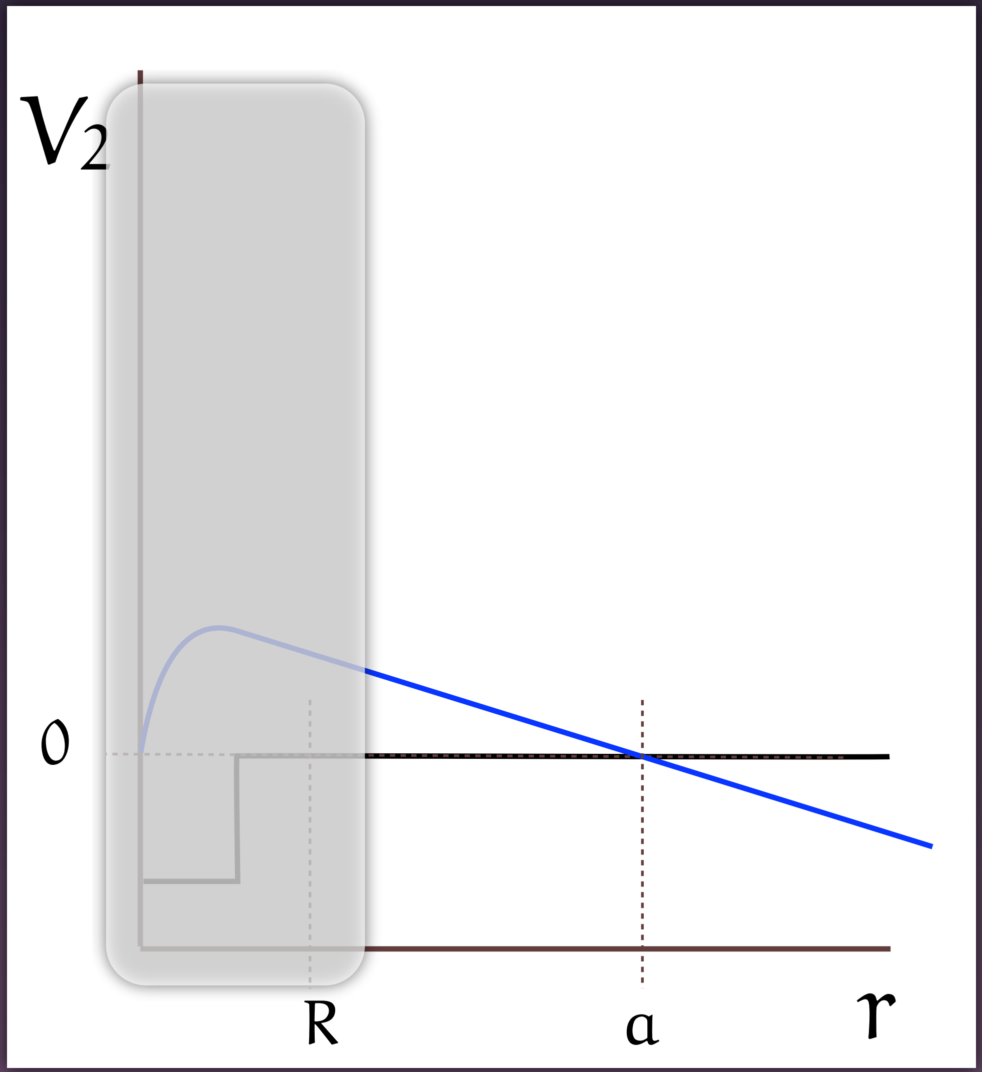
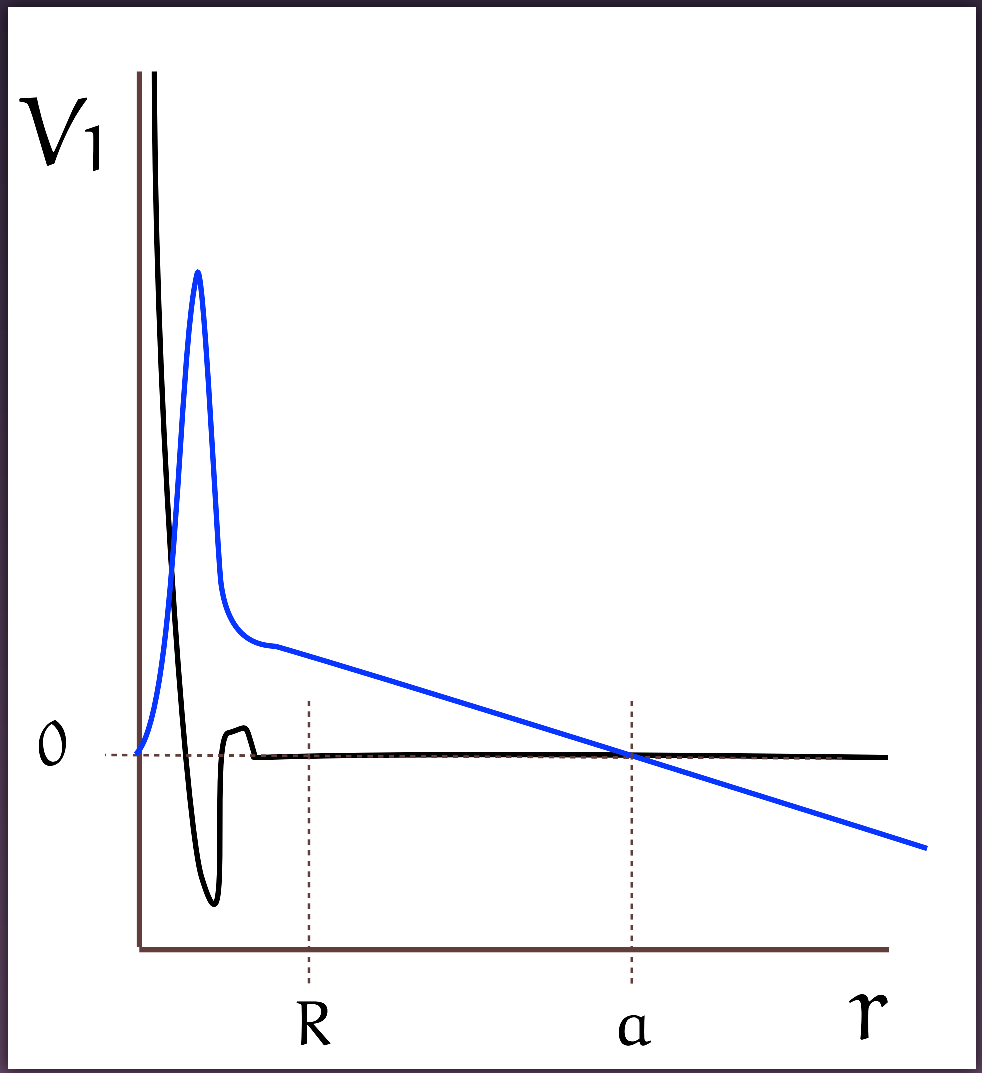
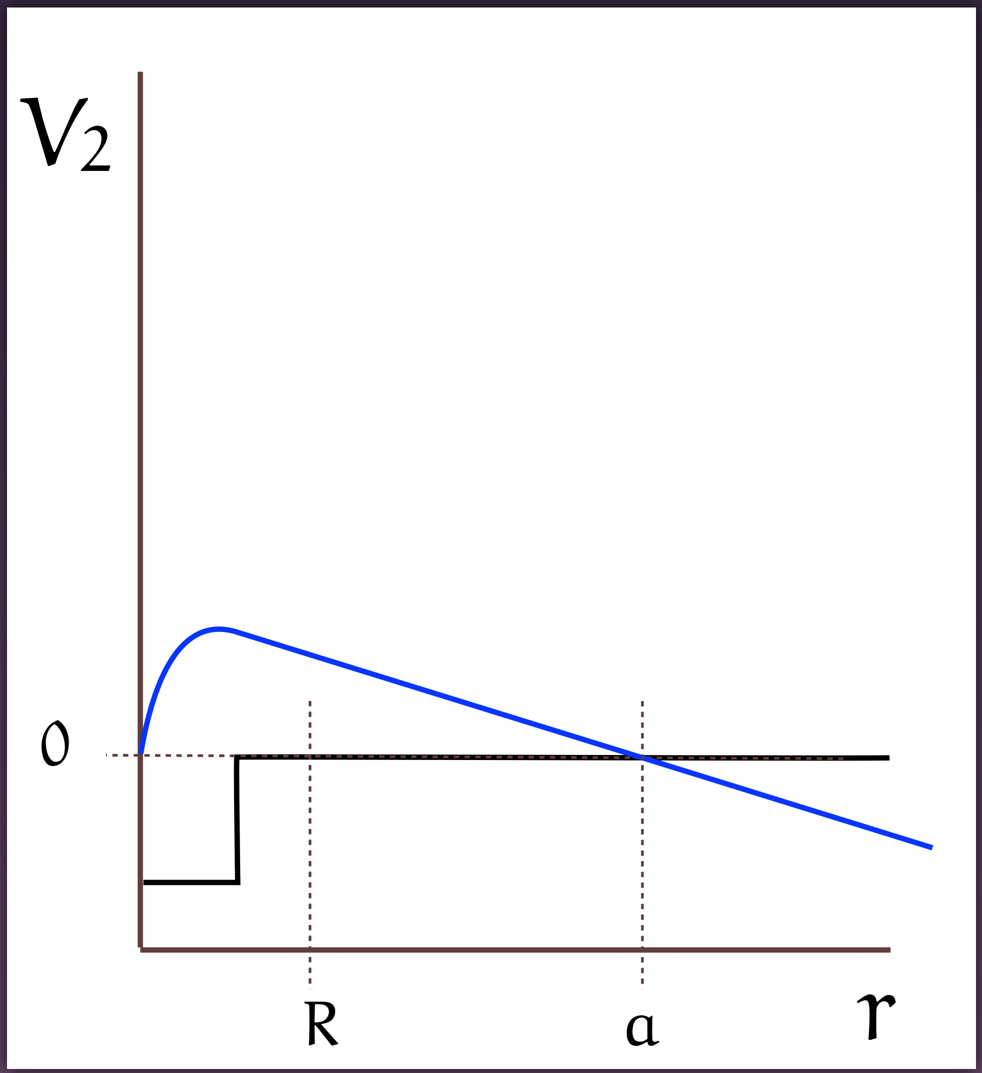




“Renormalization”

- Describe physics by low energy “effective” theory
- Replace complicated high-energy (short distance) with a few low energy parameters (a and R with cutoff)





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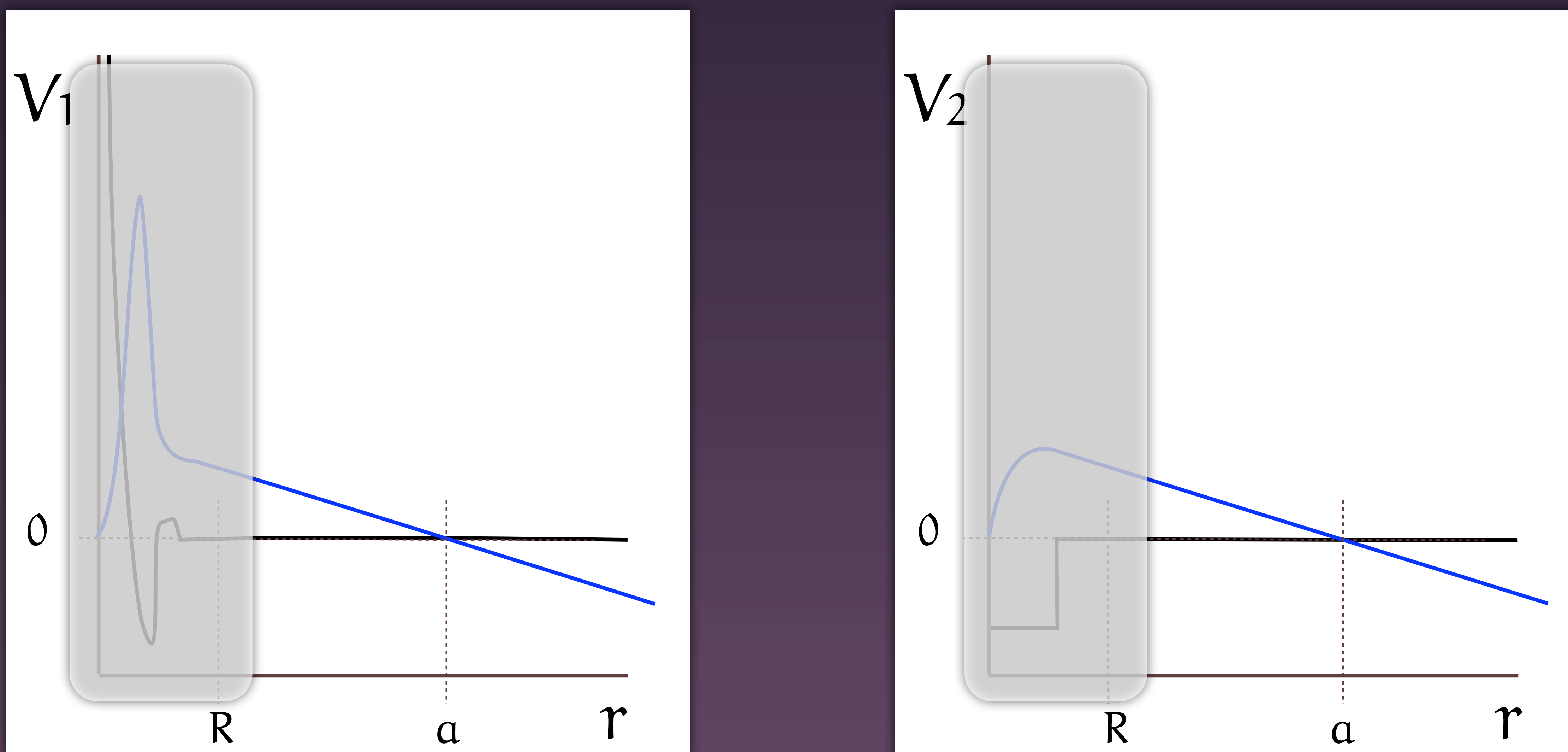
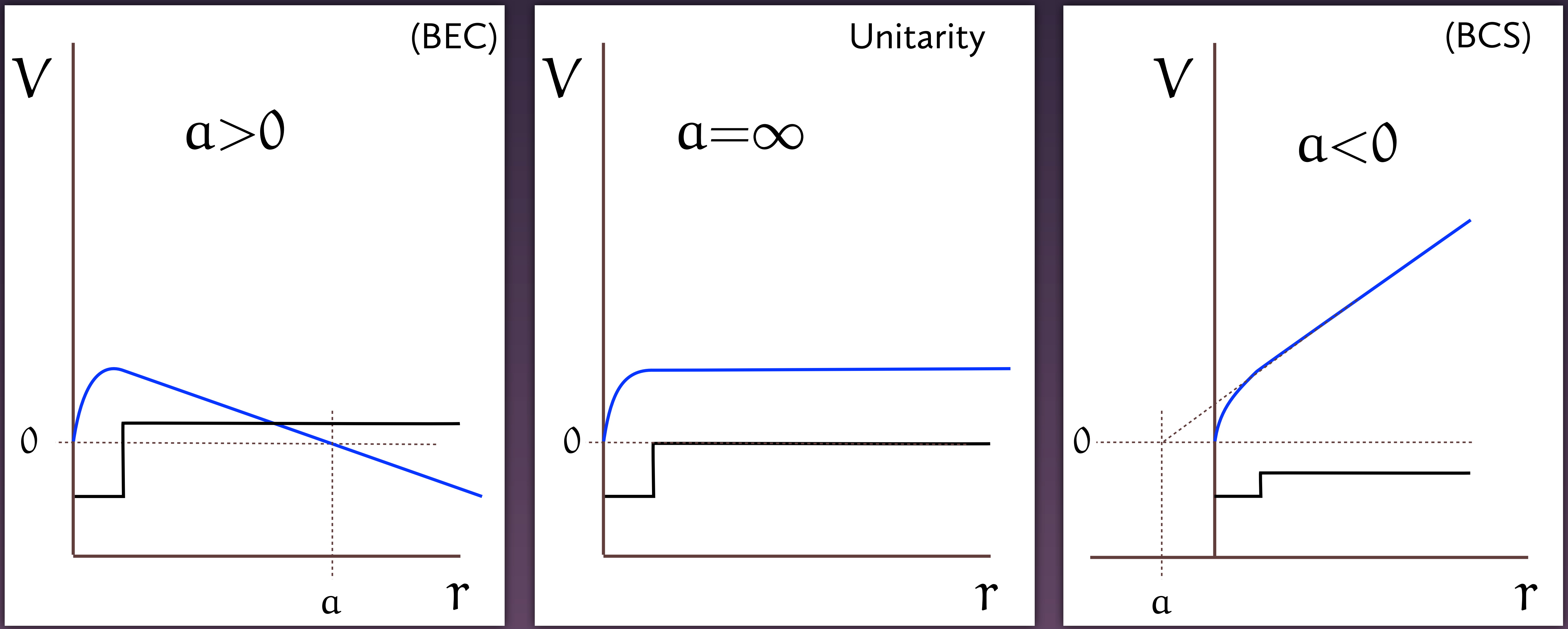


Unitary Fermi Gas

- S-wave scattering length
- BEC – Unitary – BCS crossover

  (use whatever interaction is convenient)



Unitary Fermi Gas

$$\widehat{\mathcal{H}} = \int \left(\widehat{a}^\dagger \widehat{a} E_a + \widehat{b}^\dagger \widehat{b} E_b \right) - \int V \widehat{a}^\dagger \widehat{b}^\dagger \widehat{b} \widehat{a}$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \quad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

- Dilute limit: interaction given by scattering length a
- Unitary limit $a = \infty$: No interaction length scale!
- Universal physics: Only scale is particle separation
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\xi = 0.376(5)$
- Lithium 6 and dilute neutron matter in neutron stars
  - $a_{nn} = -19$ fm

Unitary Fermi Gas

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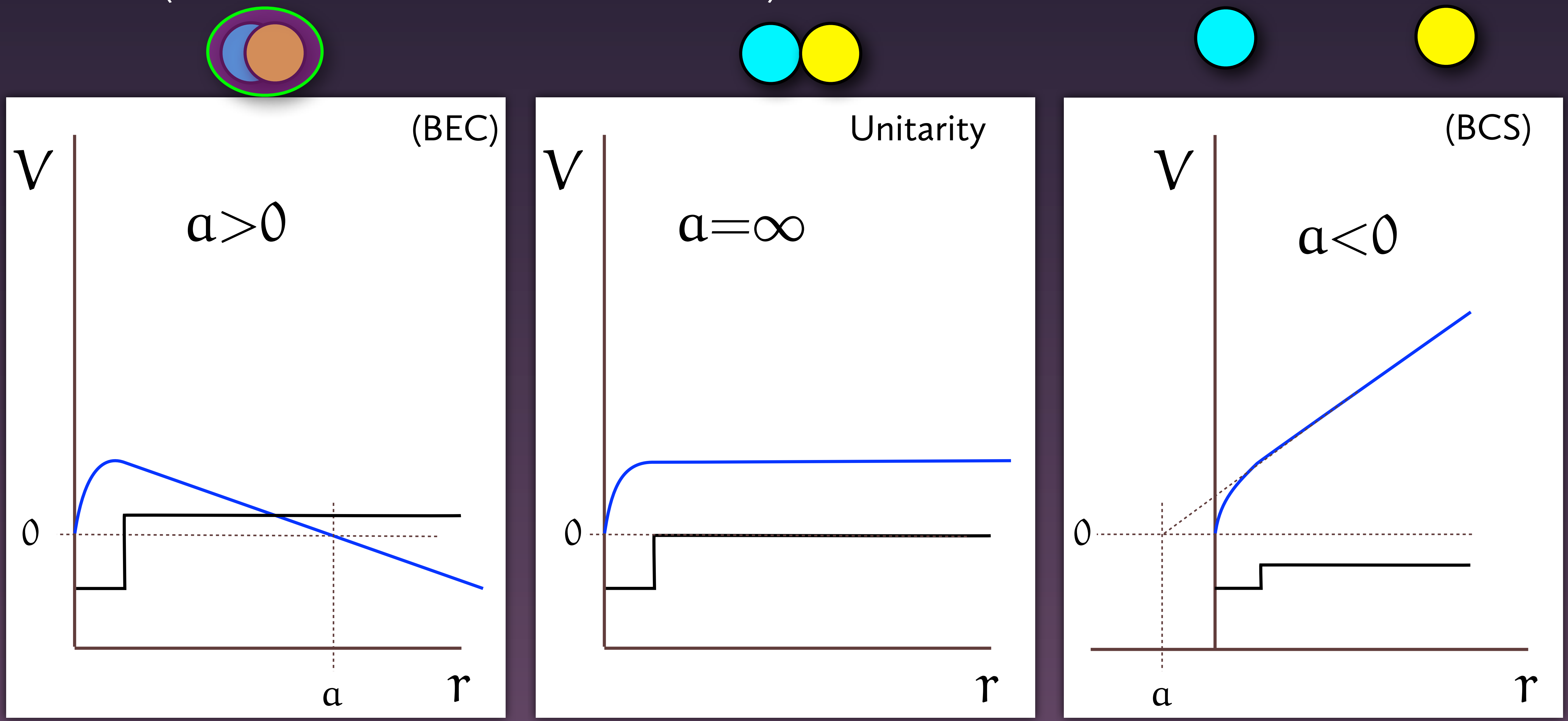
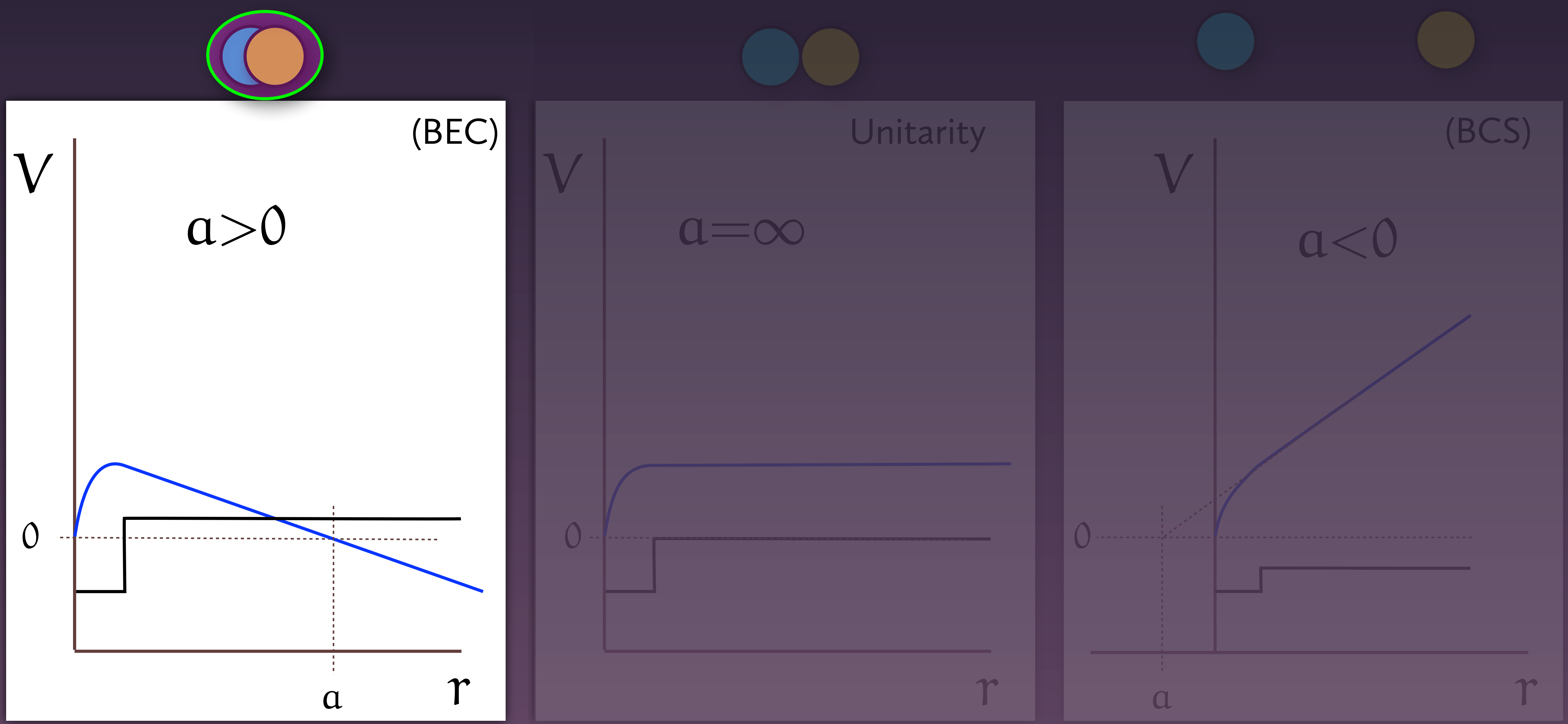


BEC Limit: Bosons

- Strong attraction: bosonic “dimers”
  - Tightly bound pairs of fermions act like bosons



Schrödinger Eq.

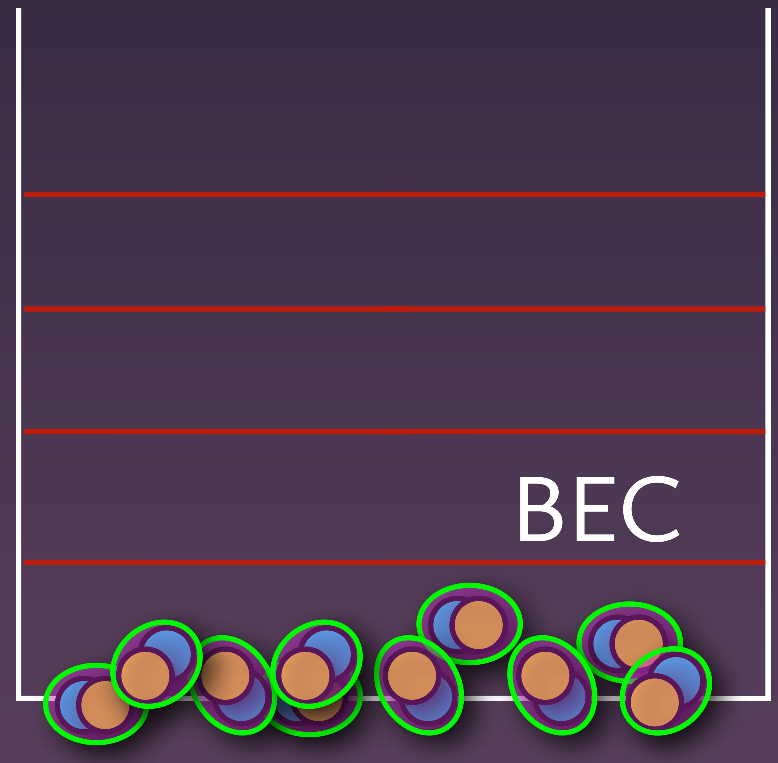
$$i\hbar\partial_t\Psi = \left(-\frac{\hbar^2\nabla^2}{2m_B} + V + V_{MF}(|\Psi|^2)\right)\Psi$$

- Wavefunction for a single particle
- Bose-Einstein Condensate (BEC)
  - Condensate wavefunction
  - All particles in same state
  - Density $\rho=|\Psi|^2$

Gross Pitaevskii Eq. (GPE)

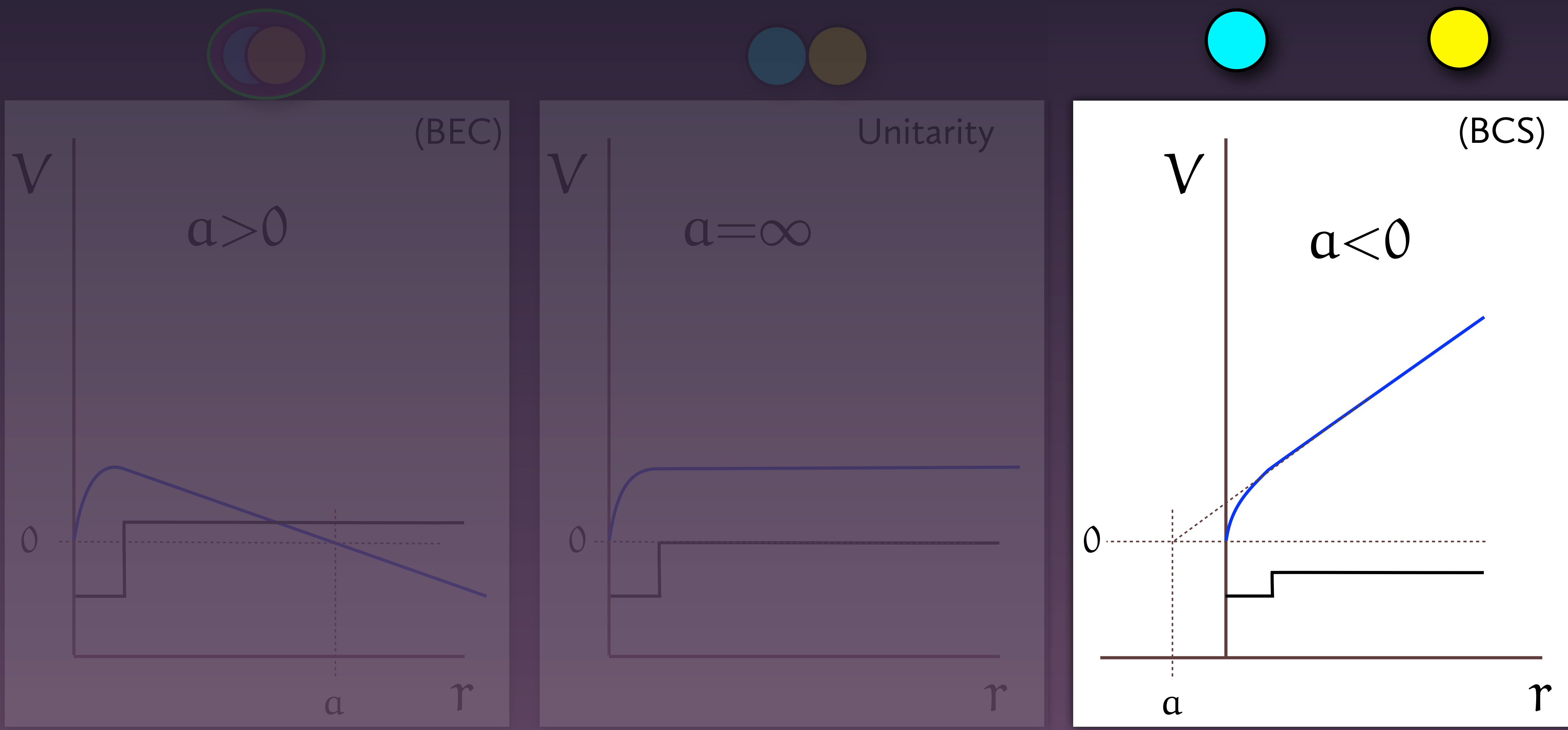
$$i\hbar\partial_t\Psi = \left(-\frac{\hbar^2\nabla^2}{2m_B} + V + V_{MF}(|\Psi|^2)\right)\Psi$$

- Interactions via "mean field" V_{MF}
  average (mean) of all particles
- Non-linear Schrödinger Equation
- Still evolve a single wavefunction



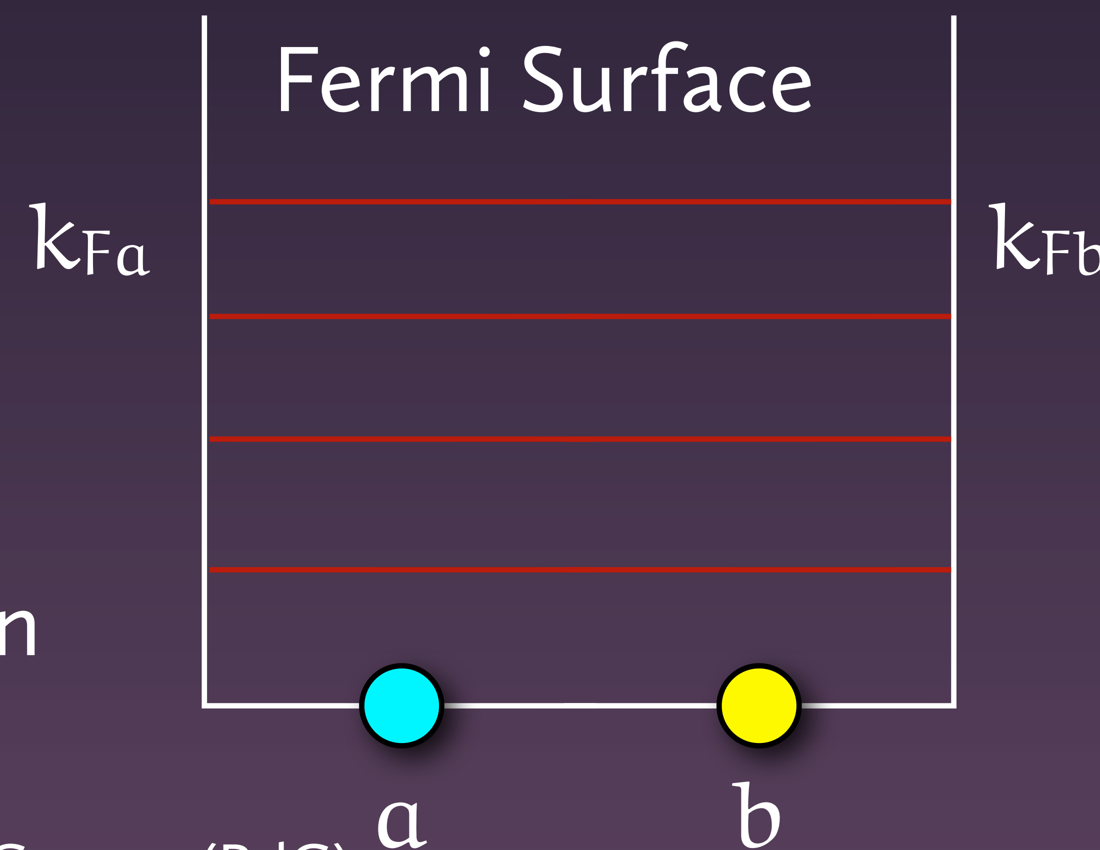
BCS Limit: Fermions

- Weak attraction: fermions almost free
  - Pauli exclusion principle dominates



Fermions are harder

$$i\partial_t \Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$



- Pauli Exclusion (blocking)
  - Particles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction

Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)

- Must use symmetries or supercomputers

$N \; 3N_x \; N_t$

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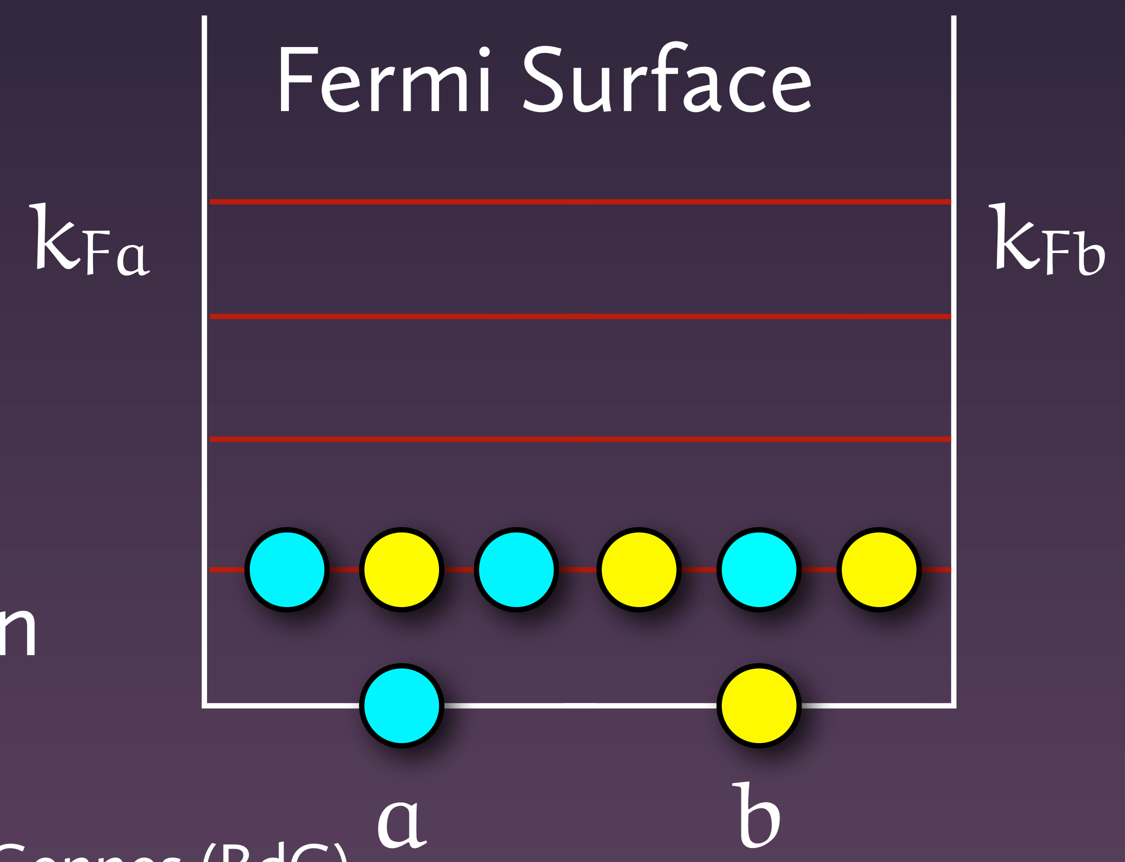

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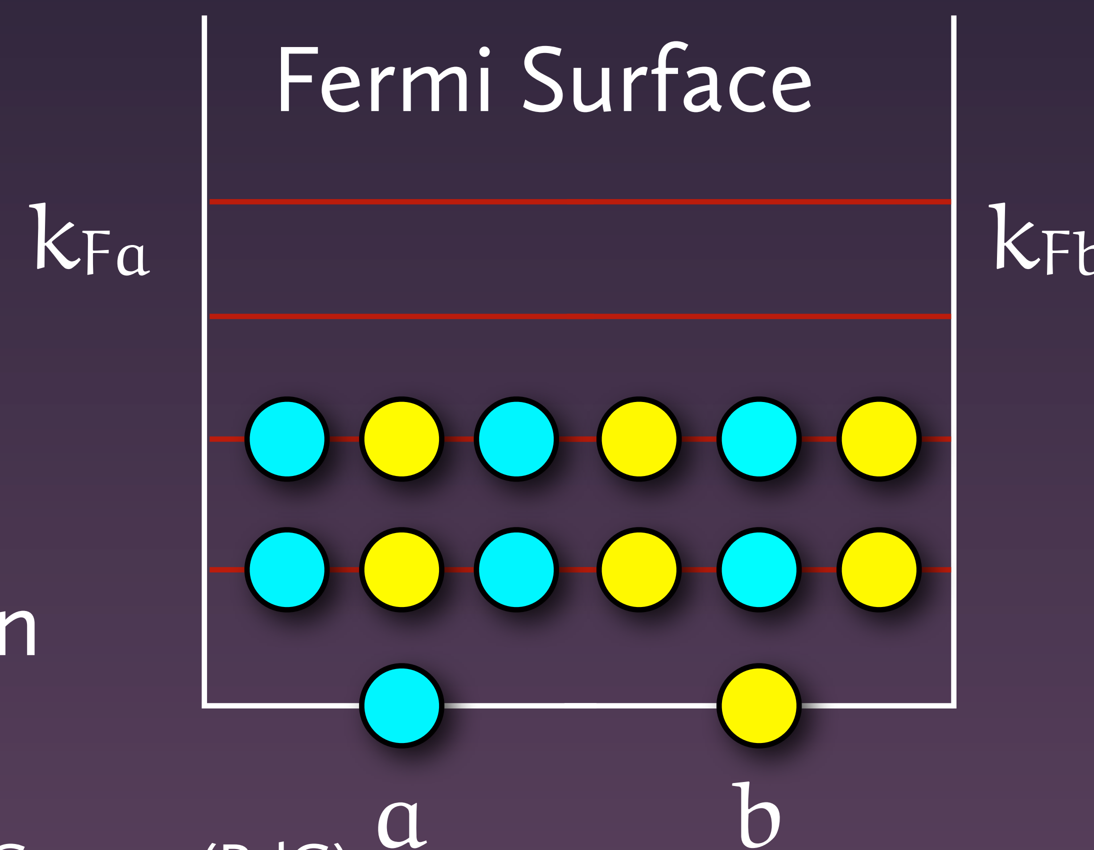

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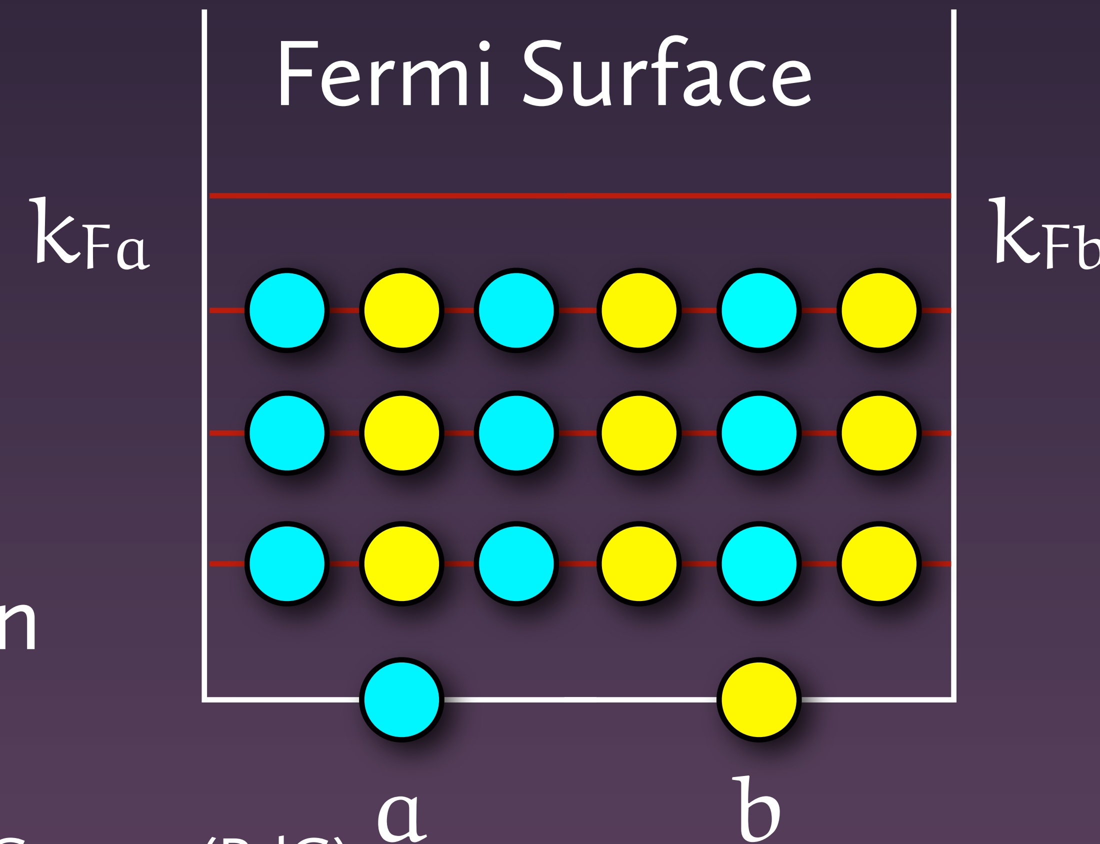

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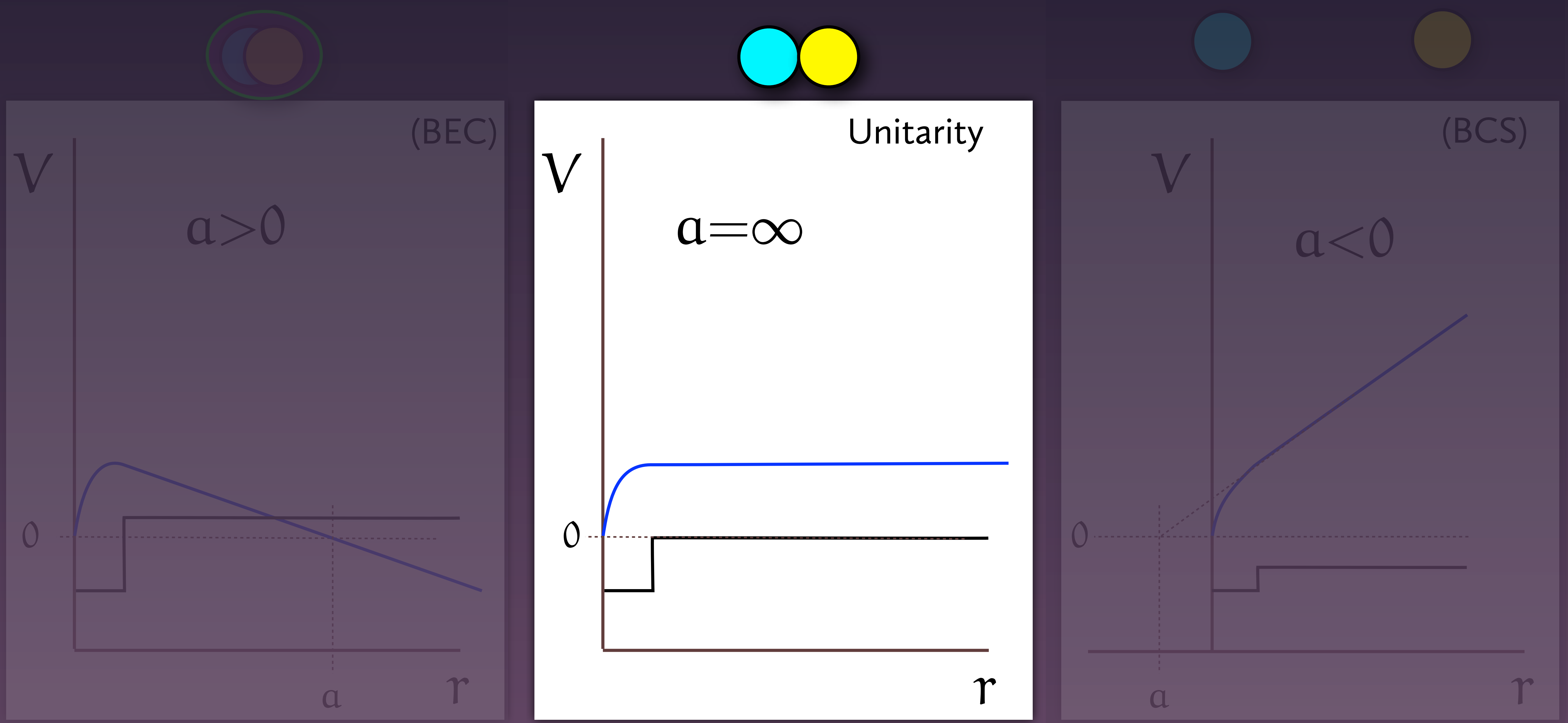
Unitary Fermi Gas

- Between: Properties of both BEC and BCS
- Strongly interacting: Non-perturbative



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- Unitary limit $a=\infty$: No interaction length scale!
- Universal physics:
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\xi=0.376(5)$
- Simplest non-trivial model (dimensional analysis)
- Non perturbative (no small parameters)
- Rich phase structure

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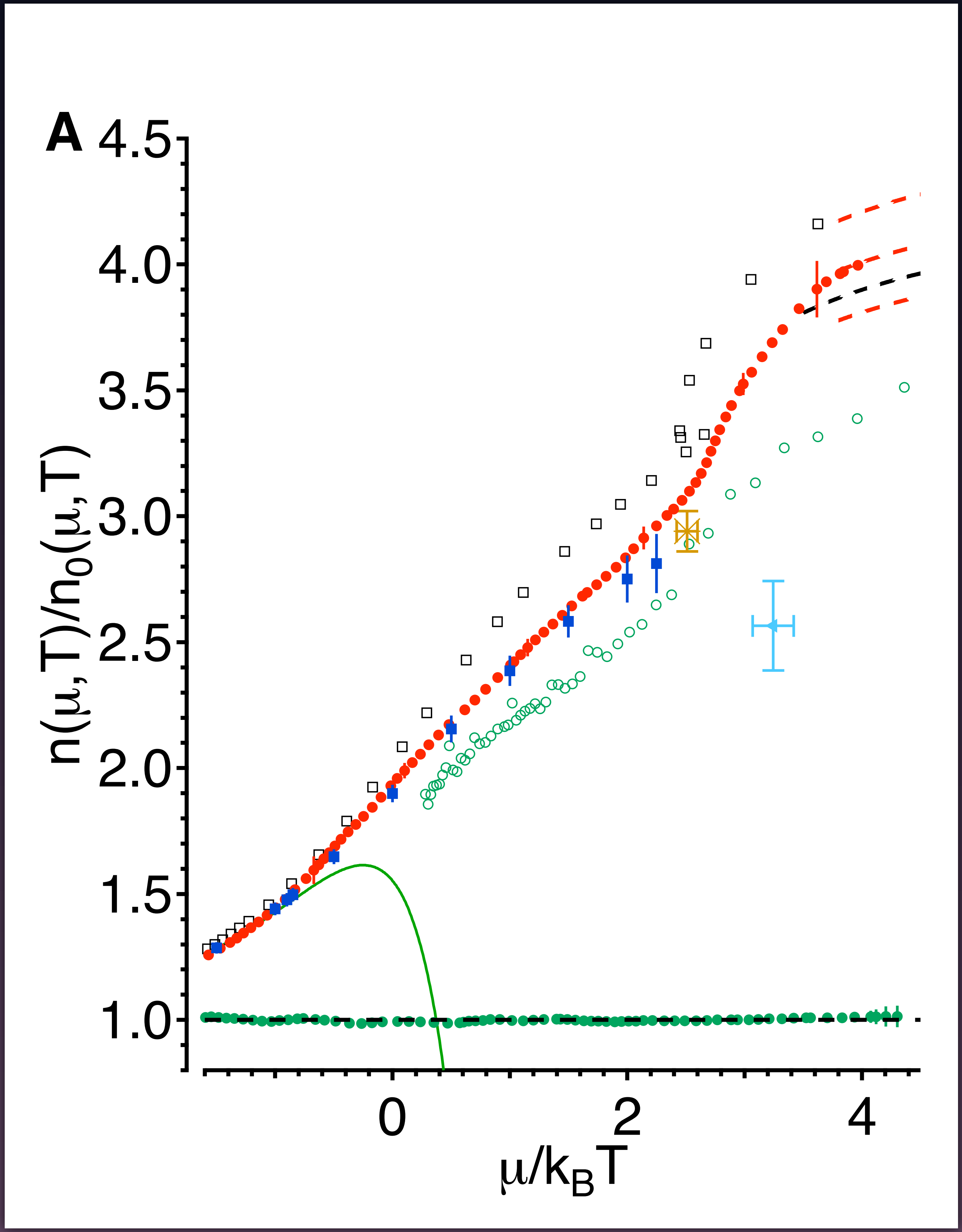
- Simple, but hard to calculate!

  Bertsch Many Body X-challenge

Unitary Equation of State



Ku, Sommer, Cheuk, and Zwierlein 2012

- Only scales: T and N
  - One convex dimensionless function $h_T(\mu/T)$

$$P = \left[T h_T\left(\frac{\mu}{T}\right)\right]^{5/2}$$

- Measured to percent level:
  - $\xi_{exp} = 0.376(5)$

Unitary Fermi Gas

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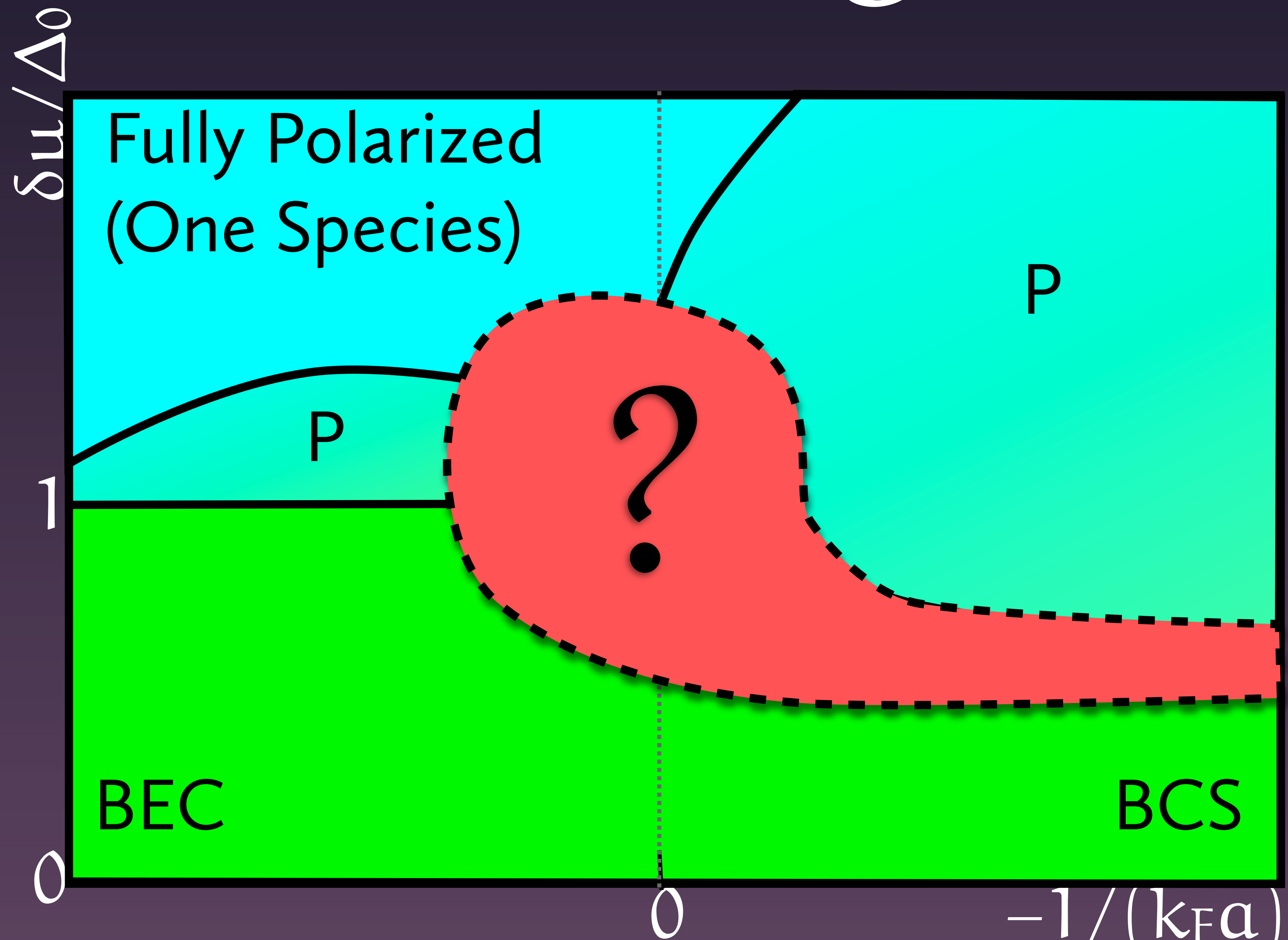
BEC-BCS Crossover Phase Diagram (T=0)



D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

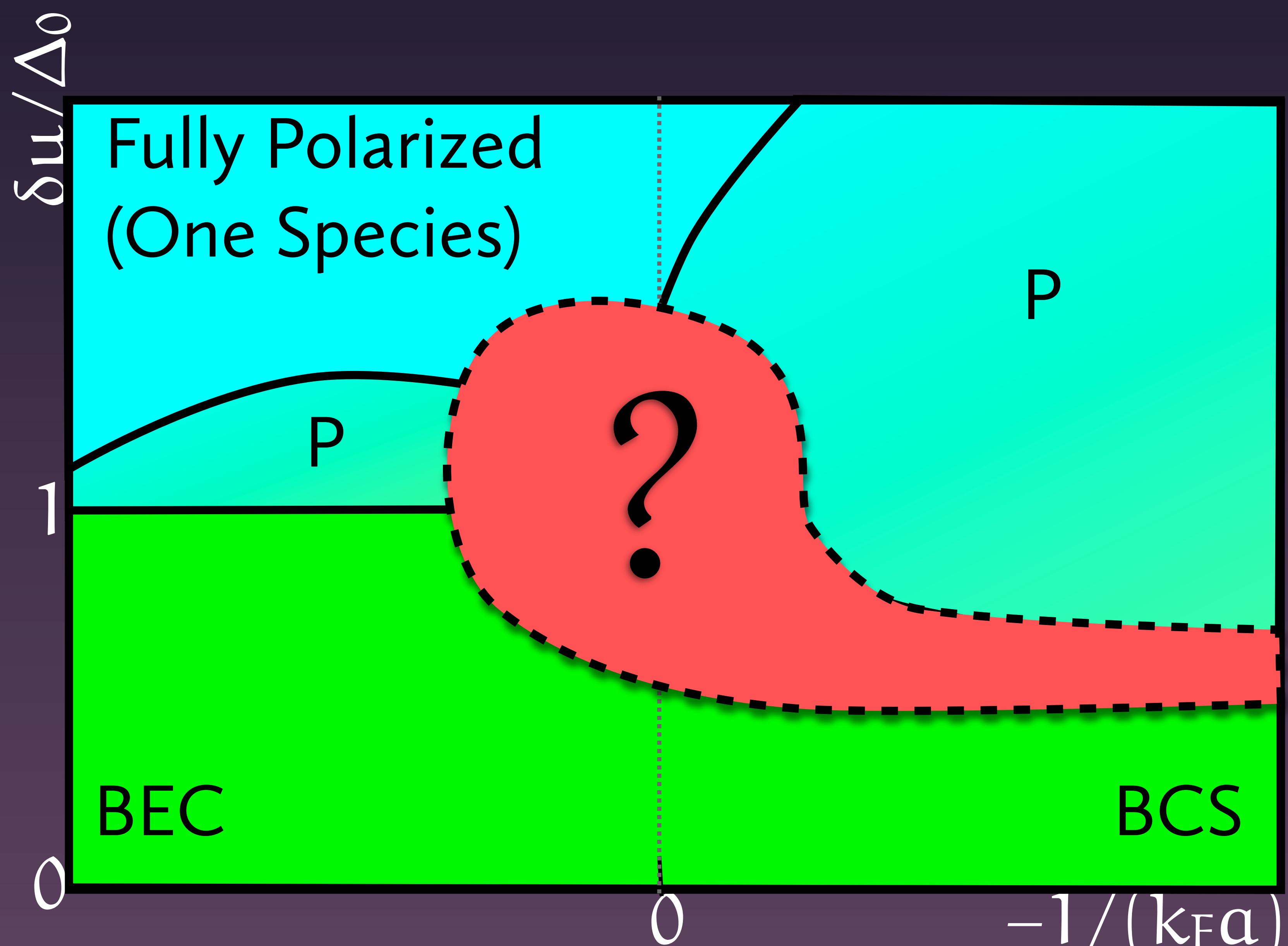
Grand canonical

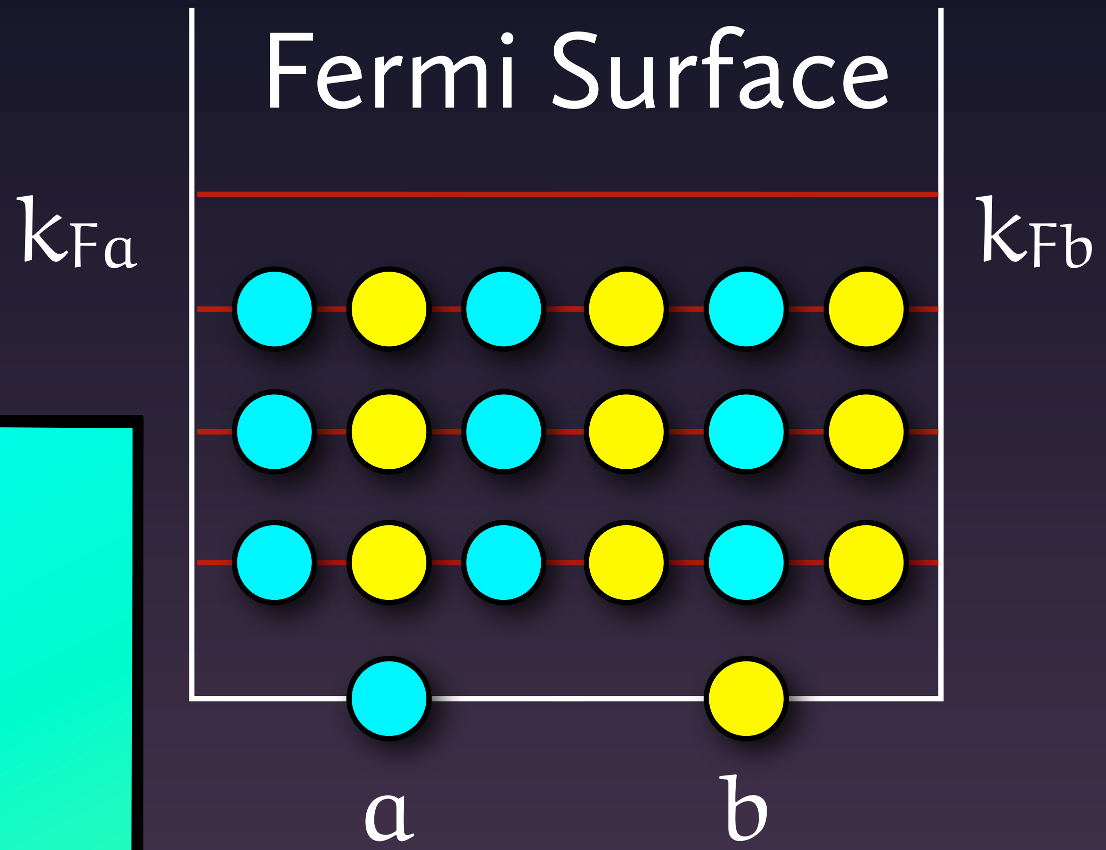
BCS-BEC Crossover

No solid evidence for what happens in the middle here

Need precision measurements

Symmetric Matter

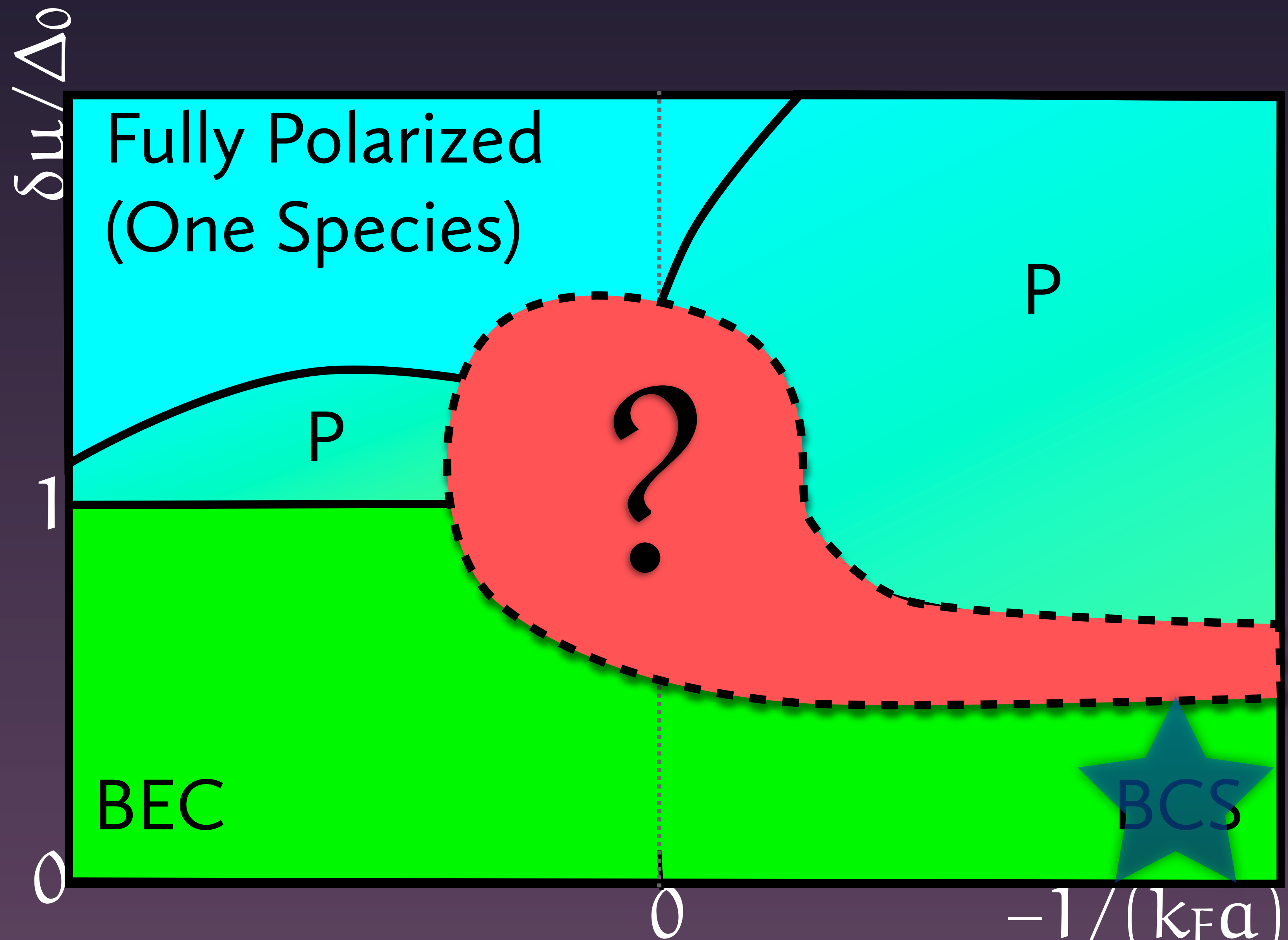


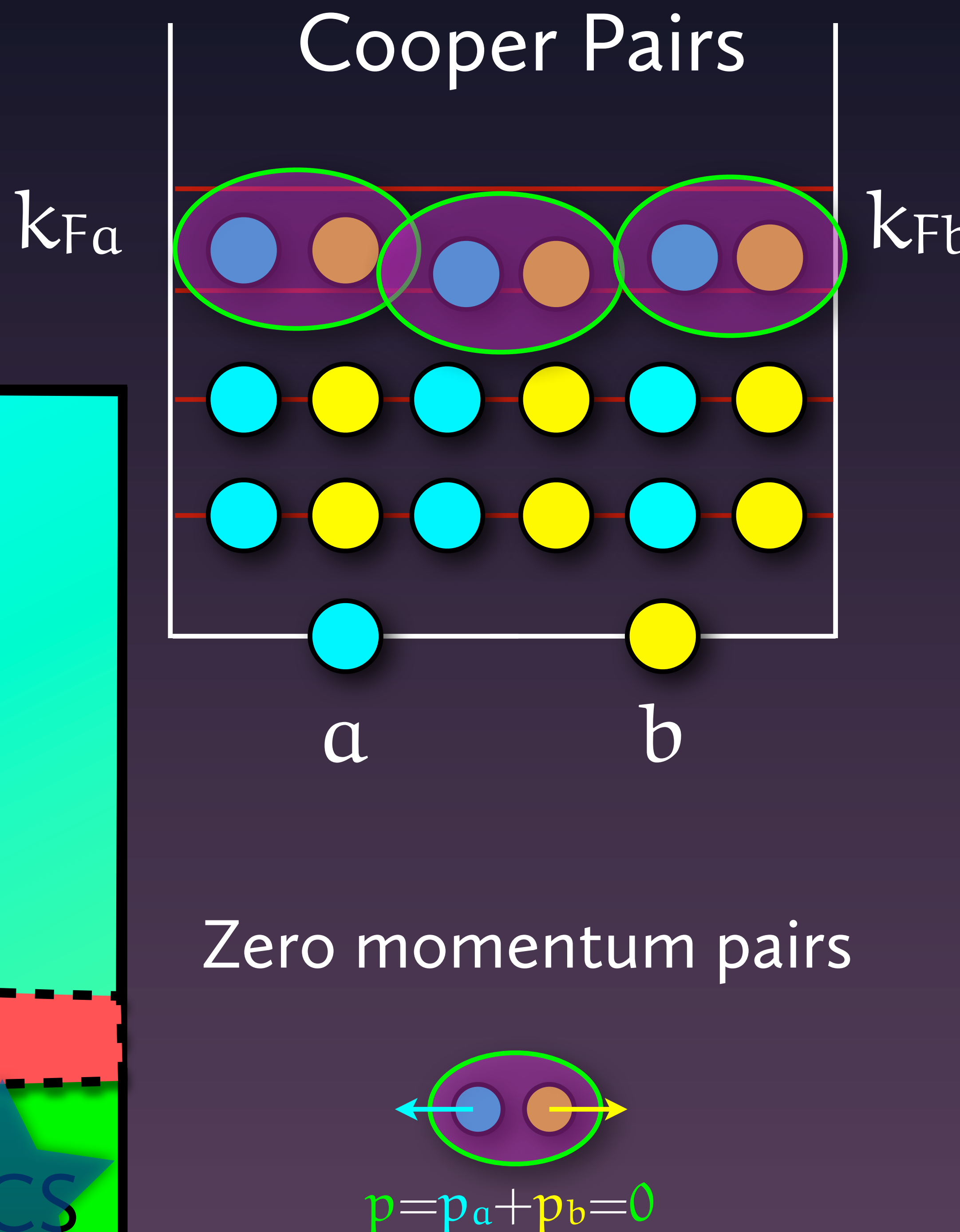


Equal Fermi surfaces

D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Symmetric BCS State





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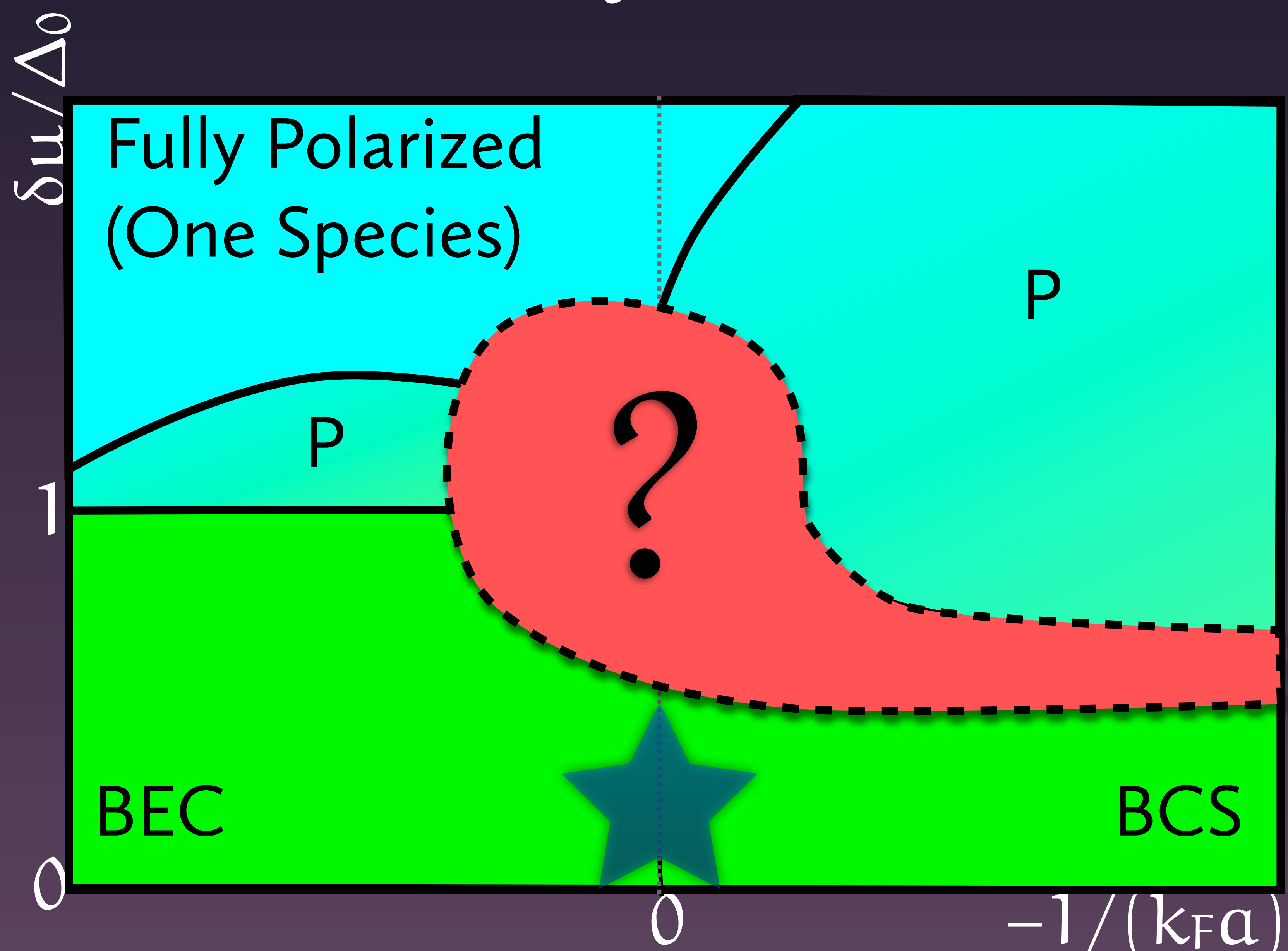
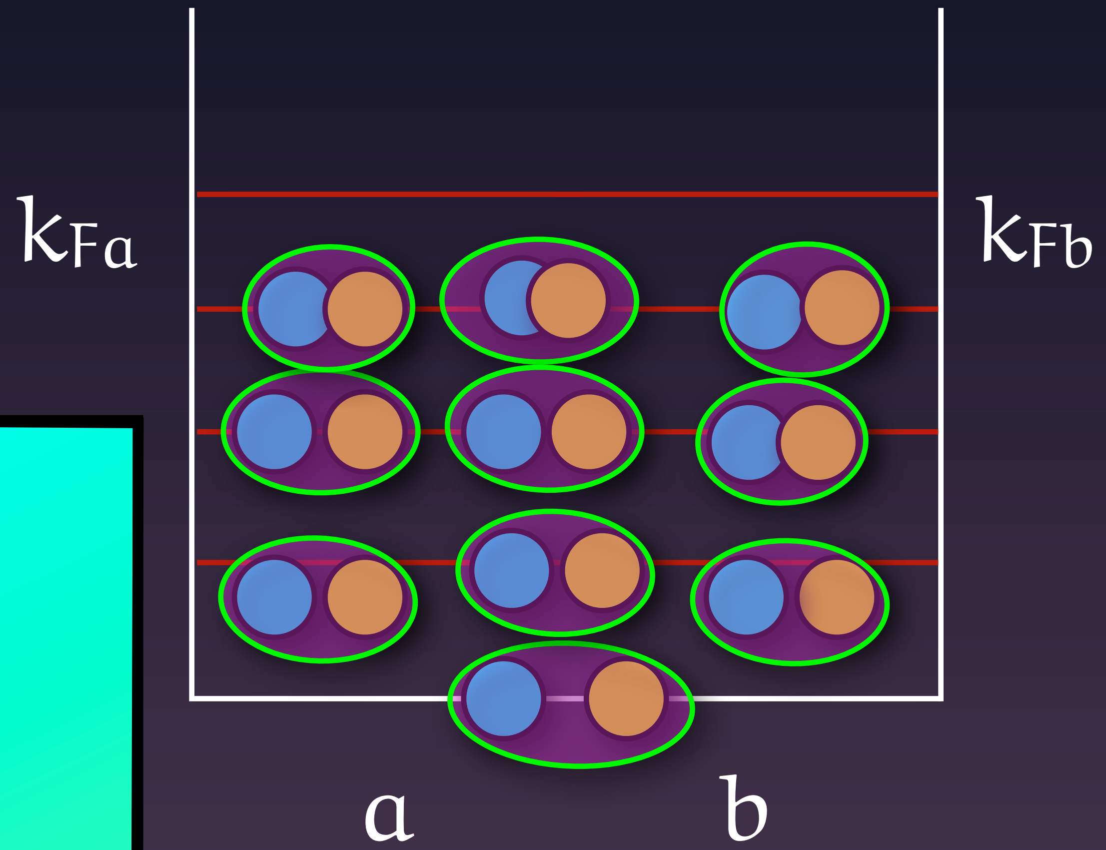
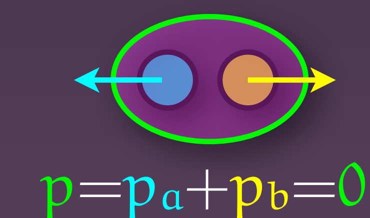
Symmetric Unitary Gas





Zero momentum pairs



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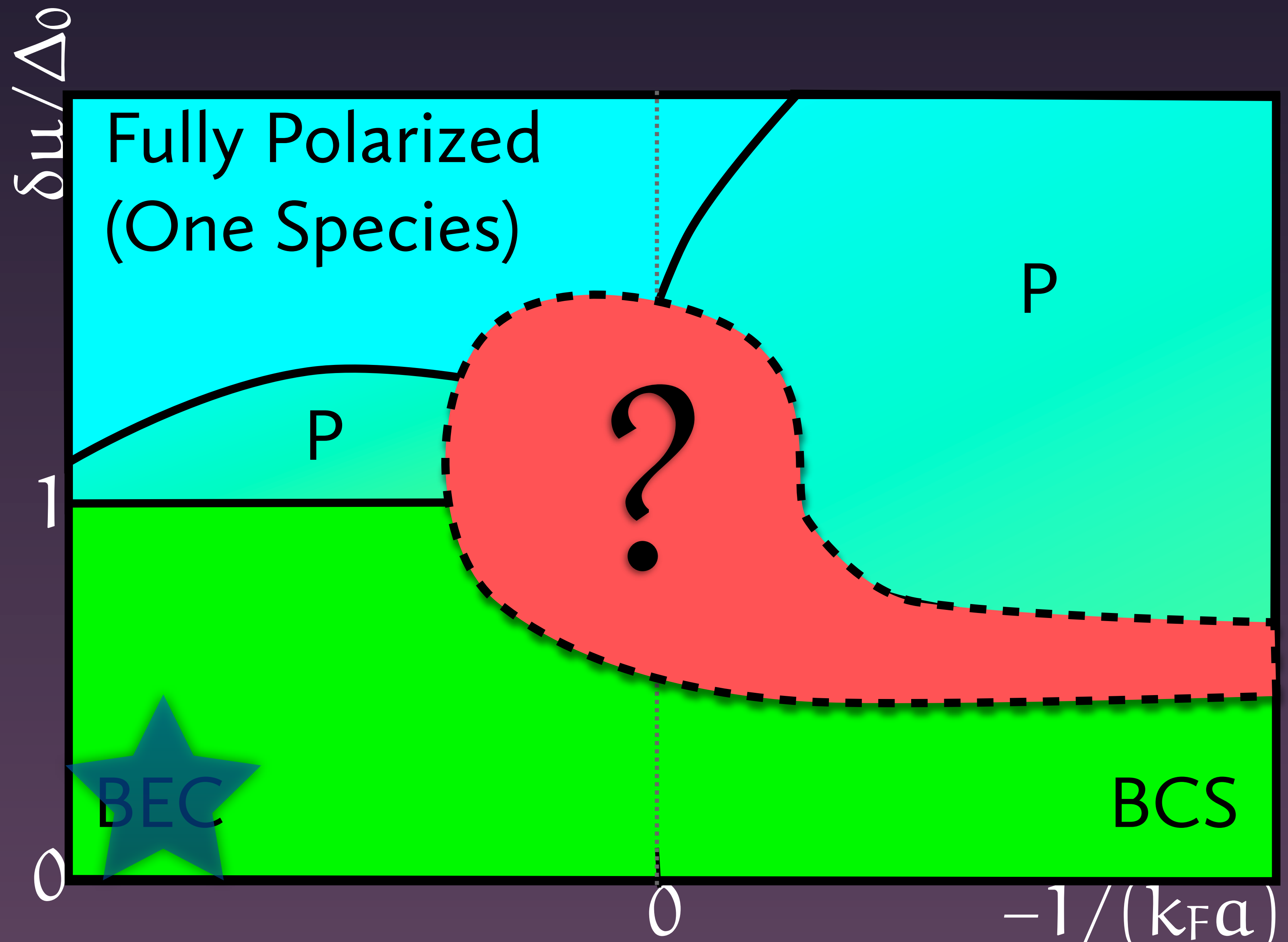
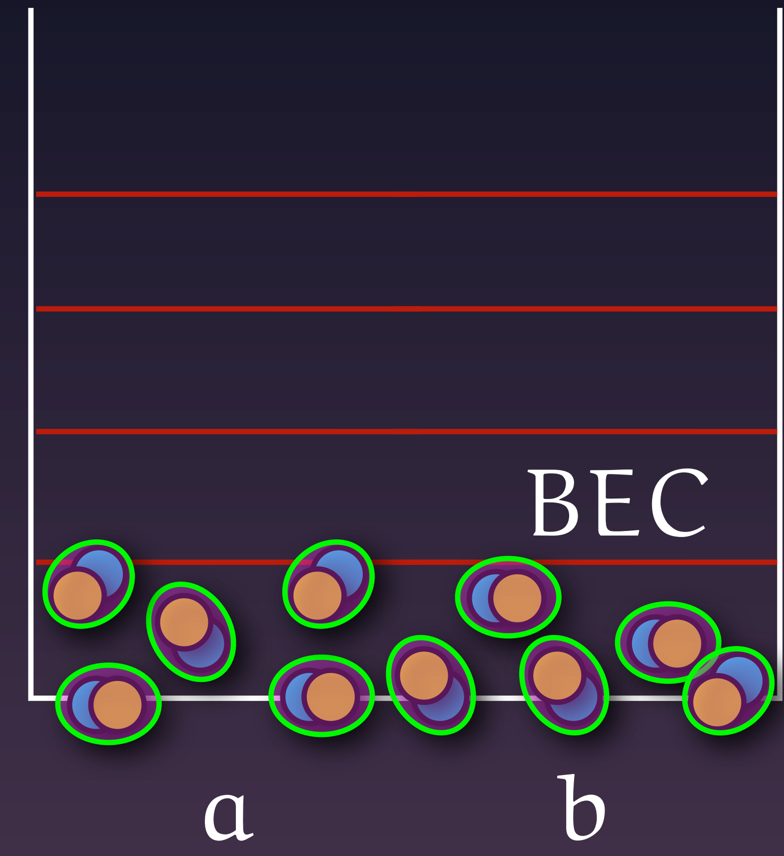
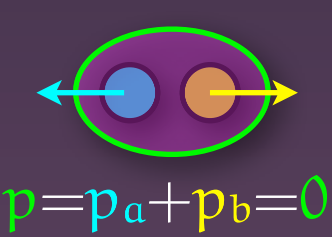
Symmetric BEC State





Tightly bound pairs



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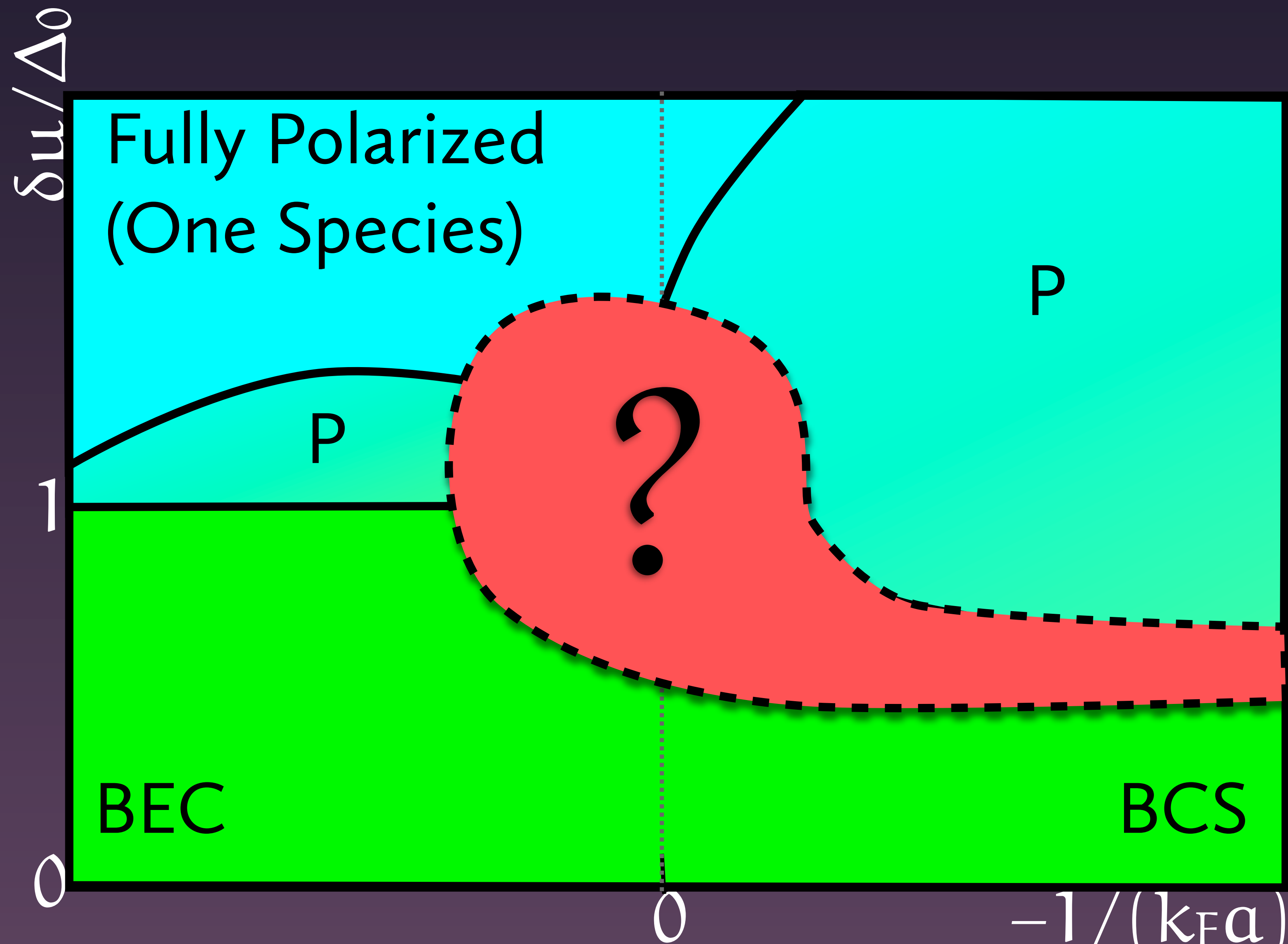
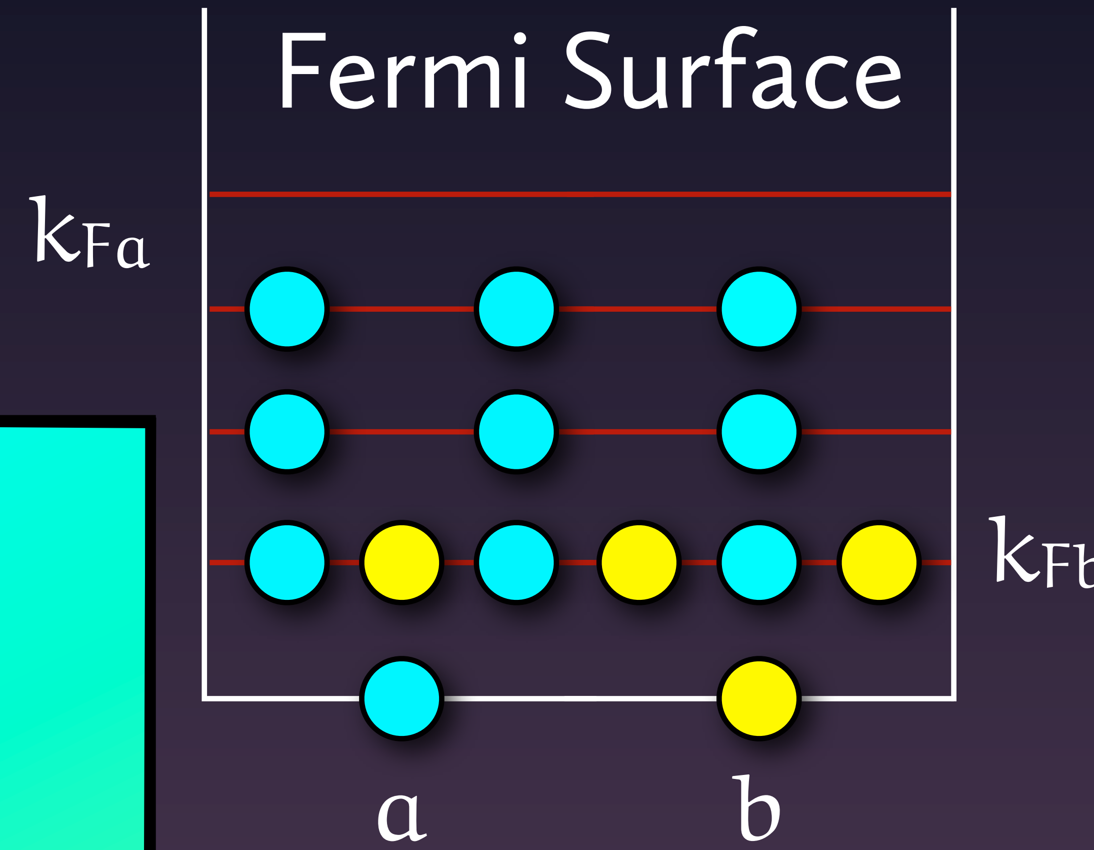
Asymmetric?





Unequal Fermi surfaces

- Frustrates pairing

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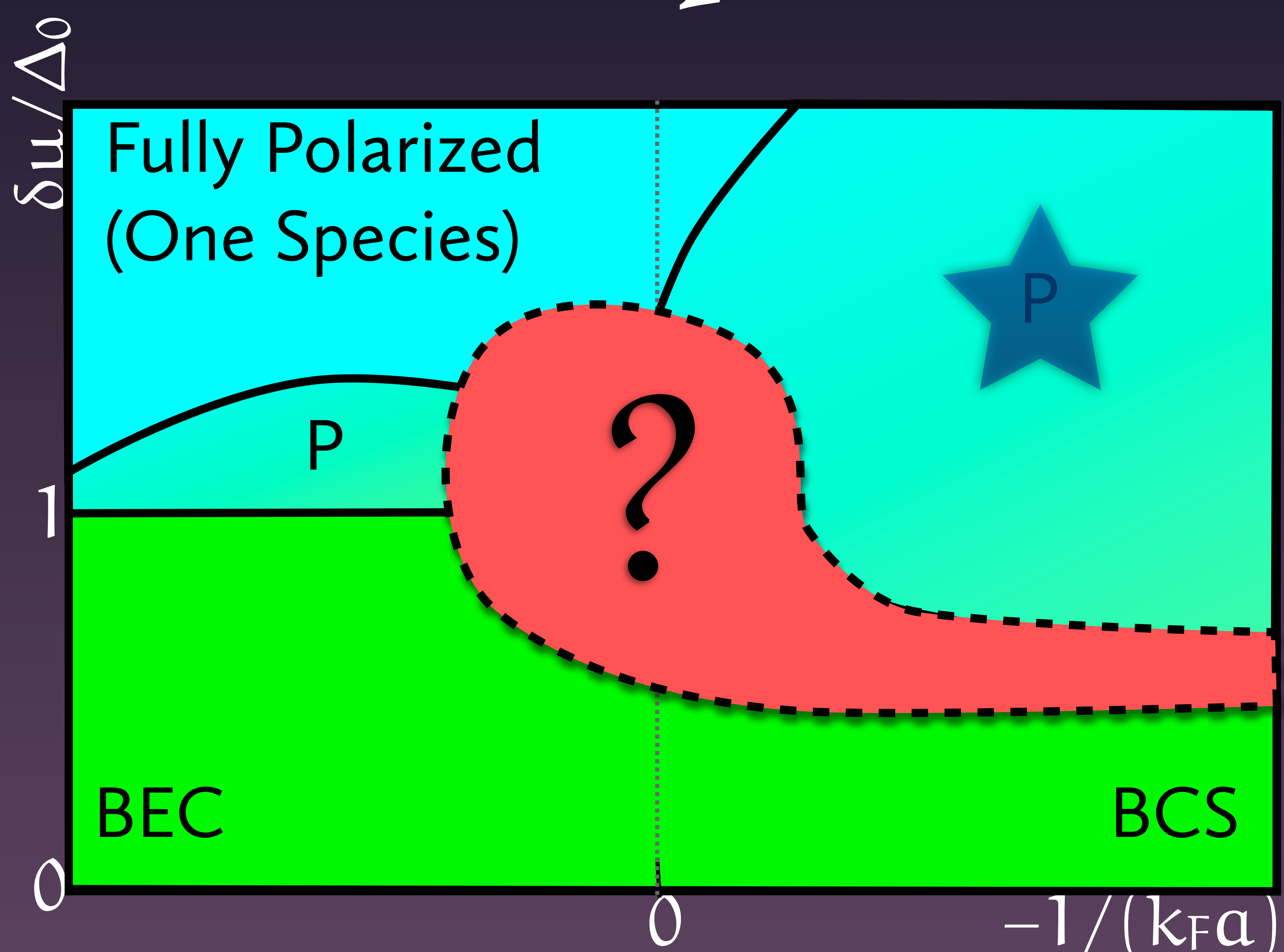
Asymmetric P-wave pairs



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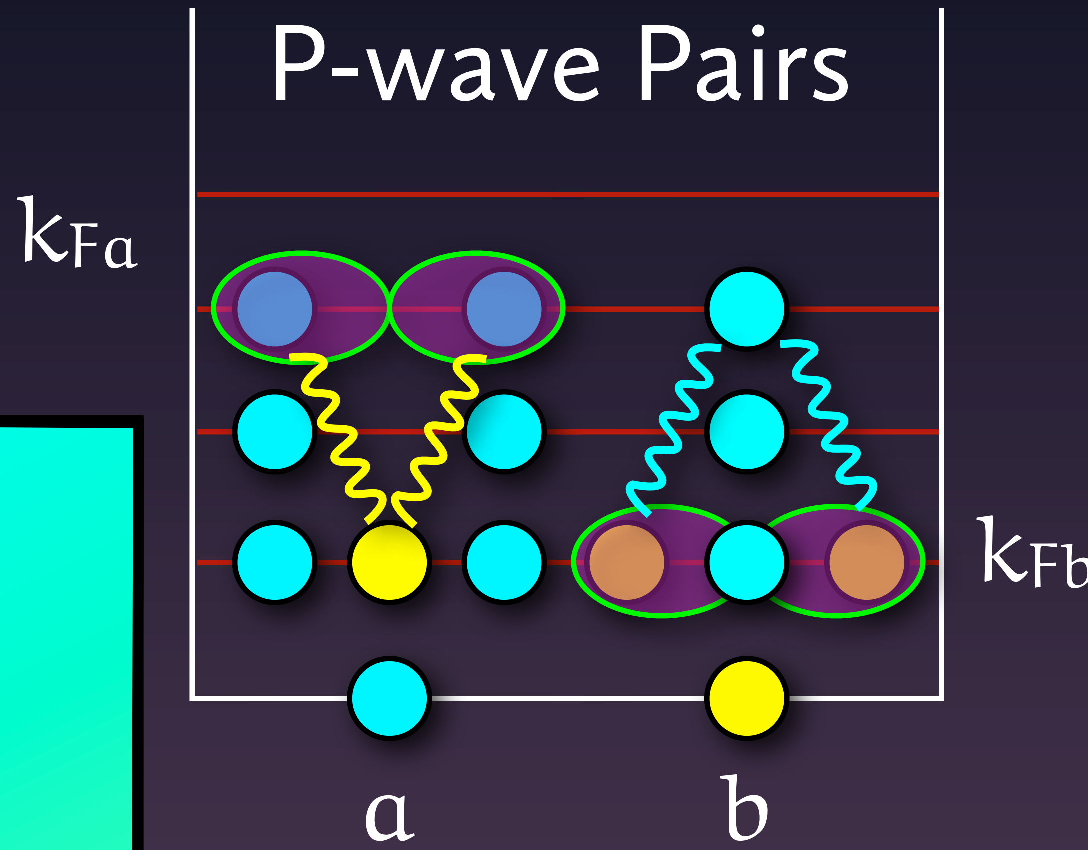
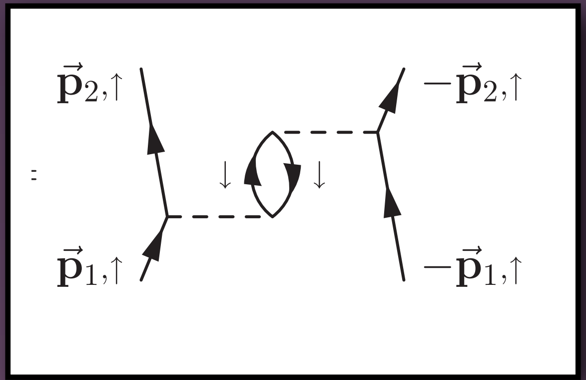
Intra-species P-wave Pairs

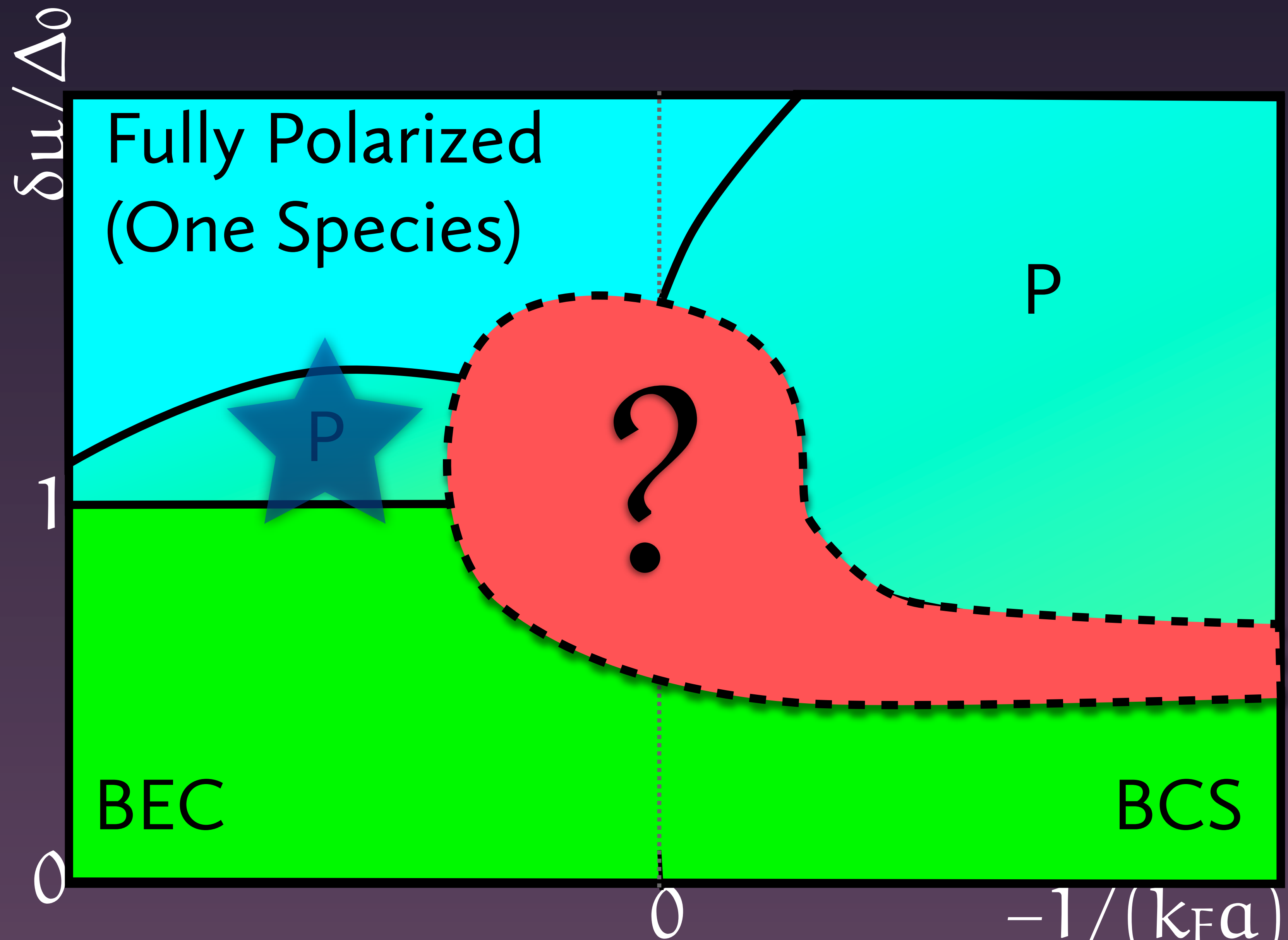


Kohn-Luttinger implies attractive at some l

Two coexisting superfluids



Asymmetric P-wave BEC



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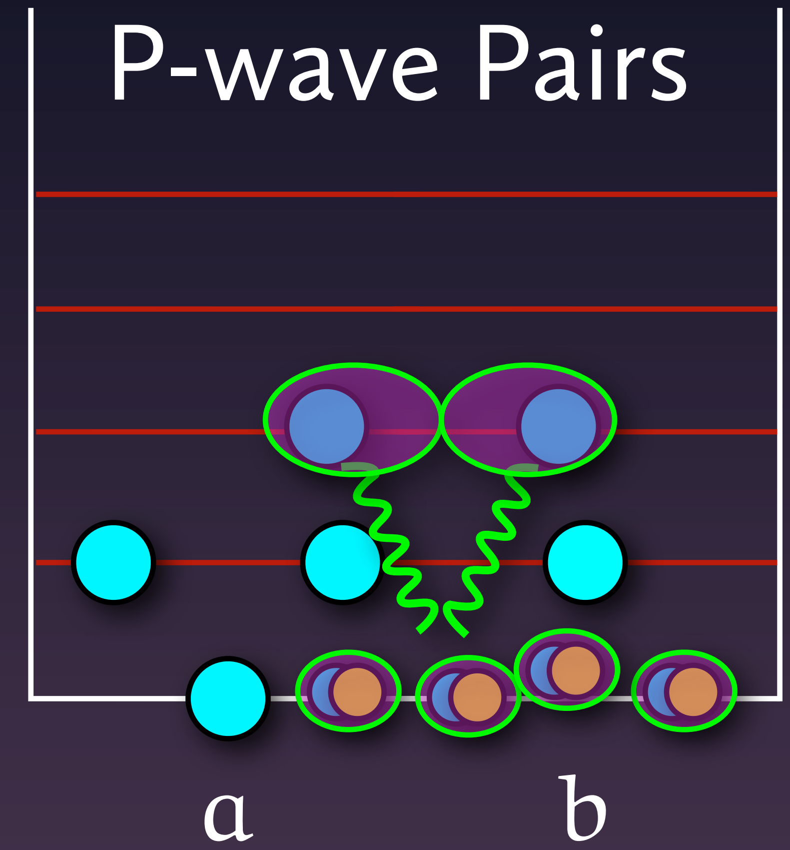
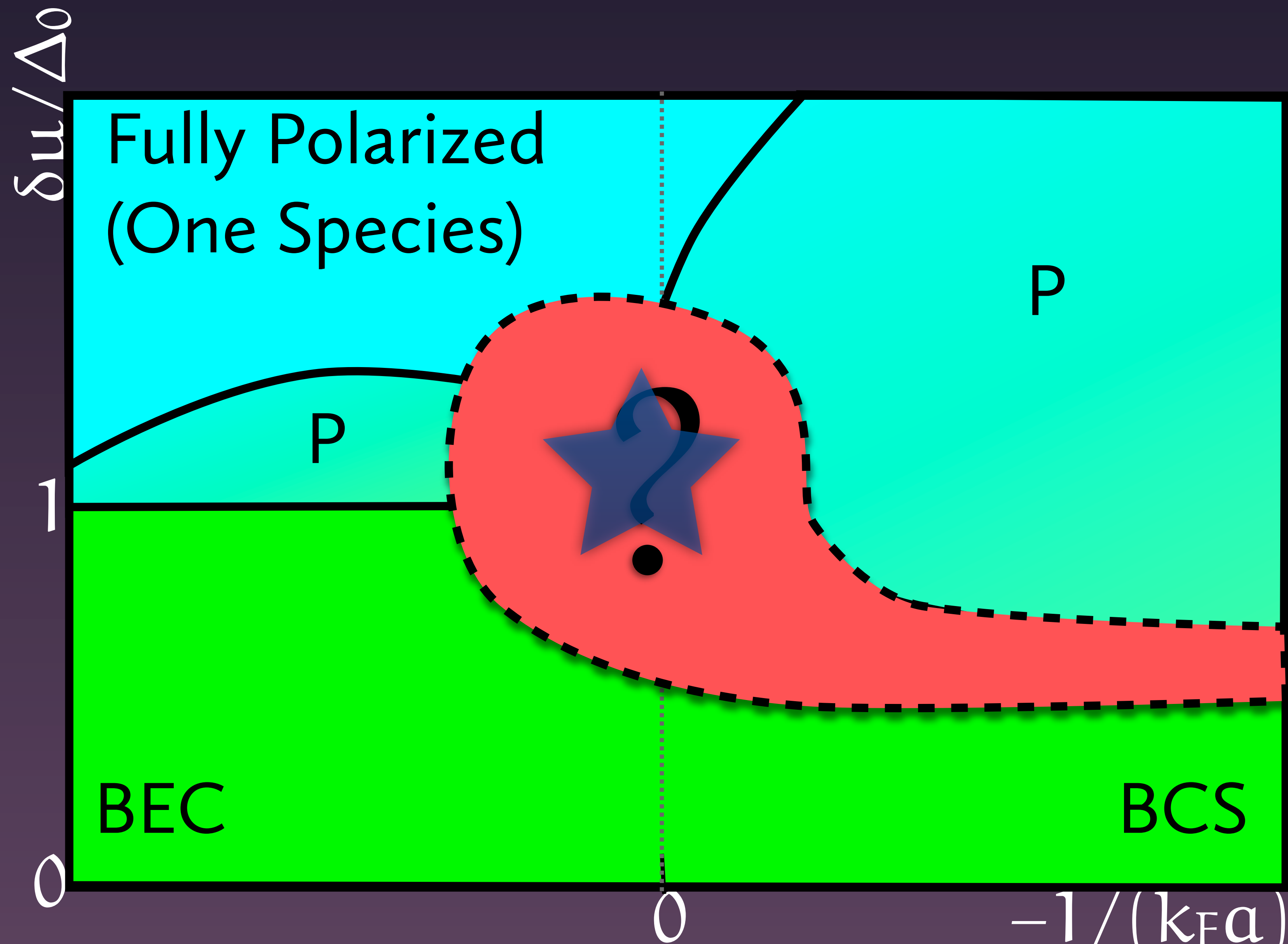


BEC and P-wave superfluids coexist homogeneously

Asymmetric Gapless SF



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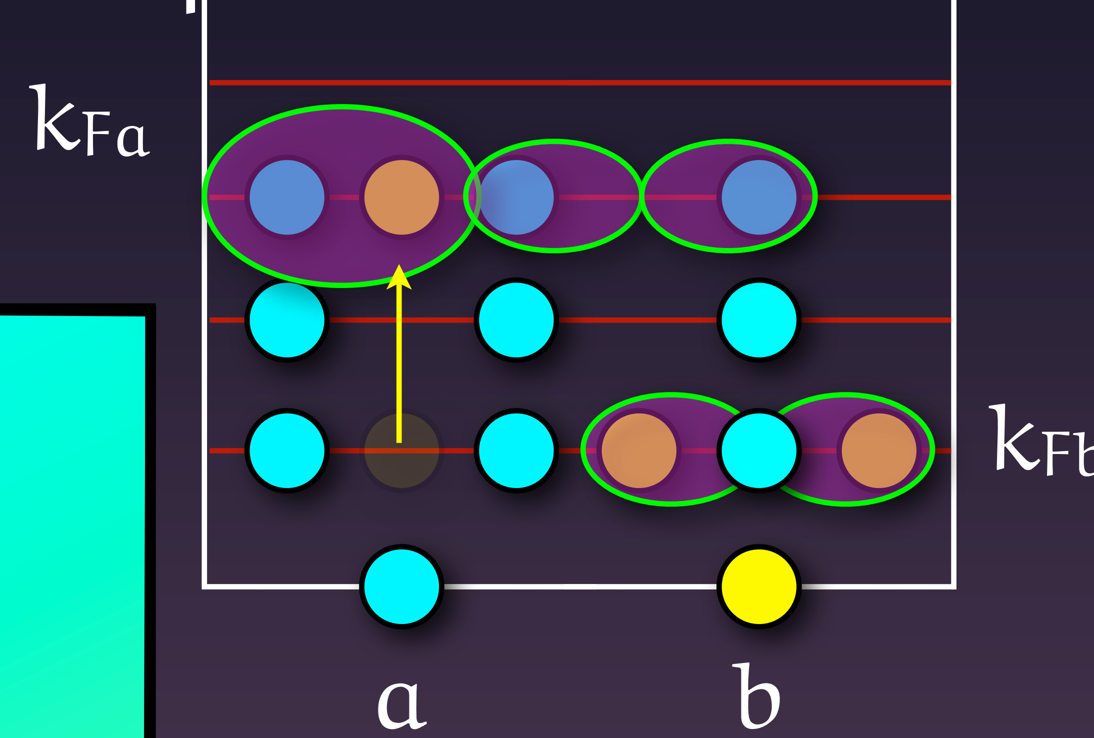
Pairing promotes particles?

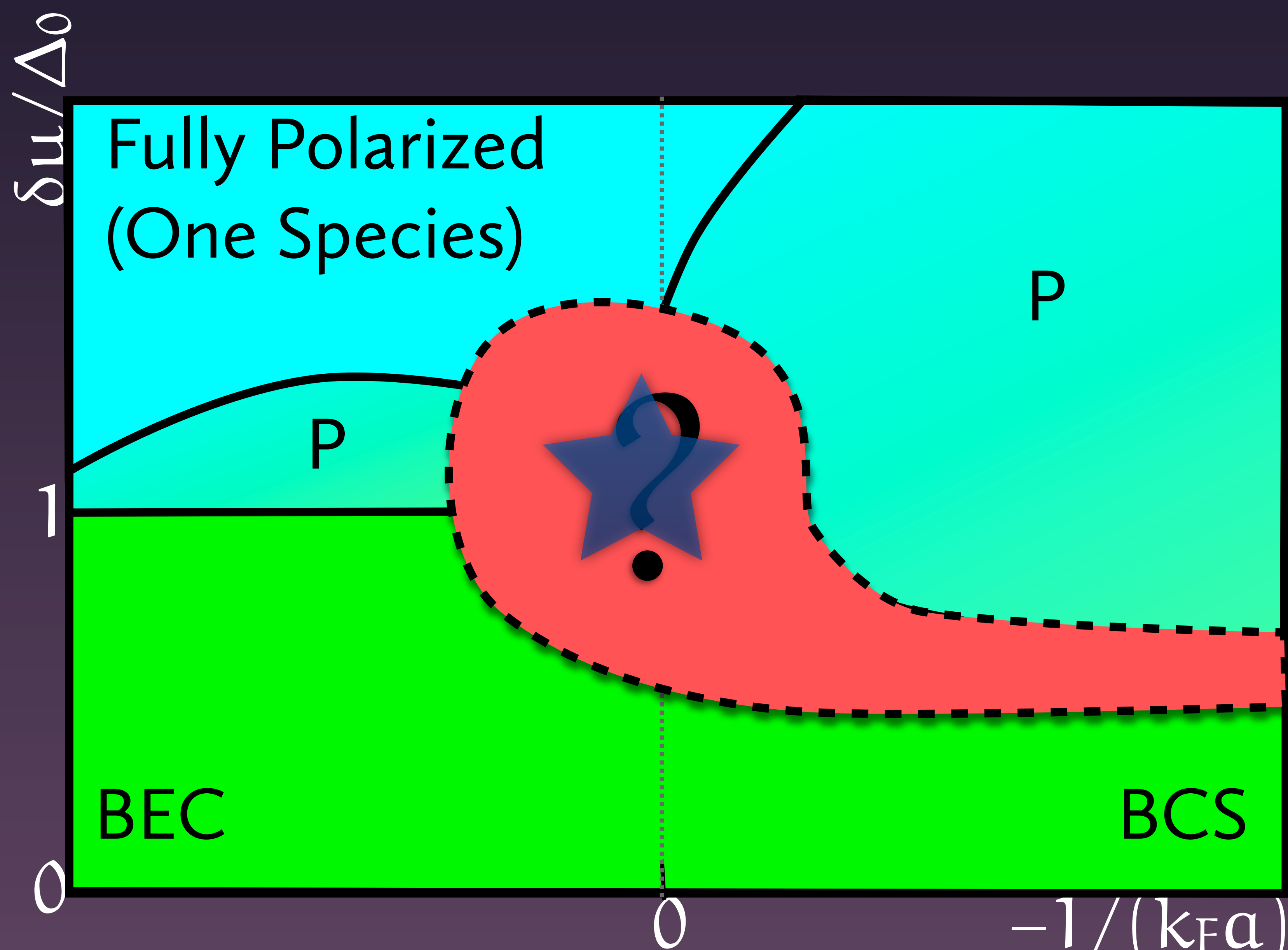


"Breach" in pairing

Still induced P-wave

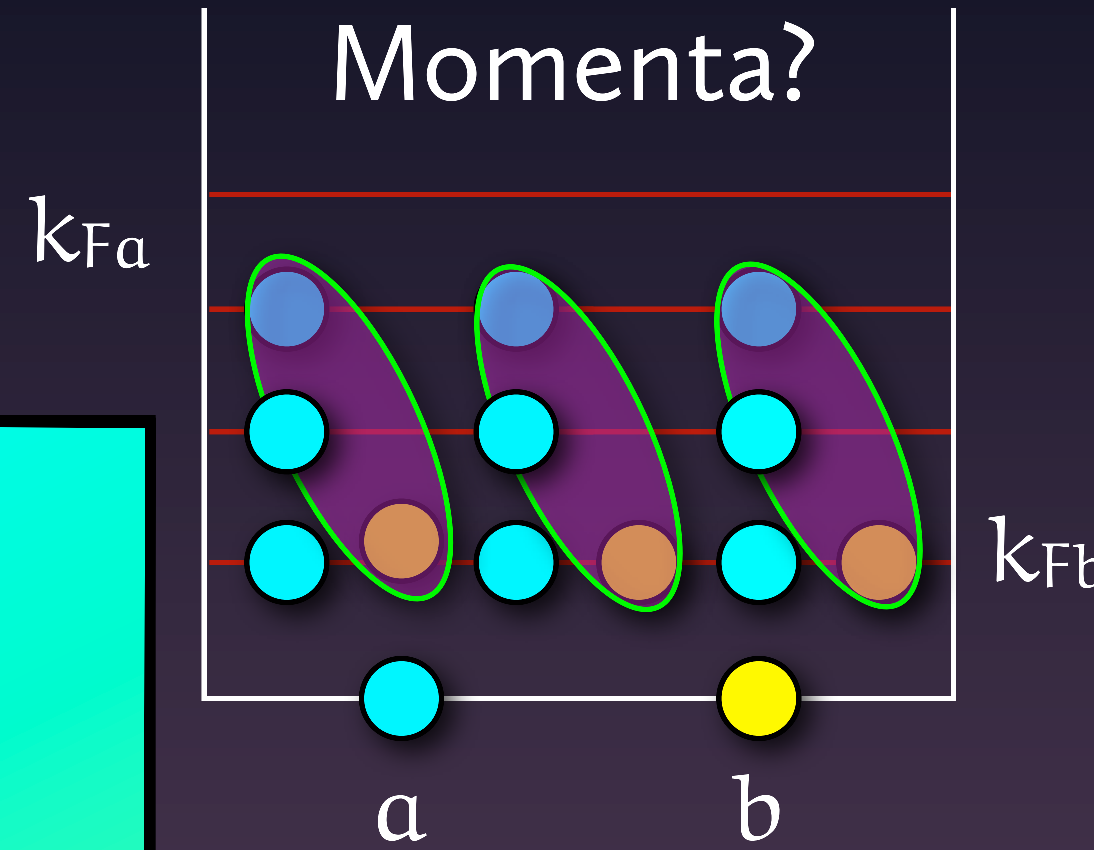
- May need large mass ratio or structured interactions (not likely at weak coupling in cold atoms)

Asymmetric FFLO



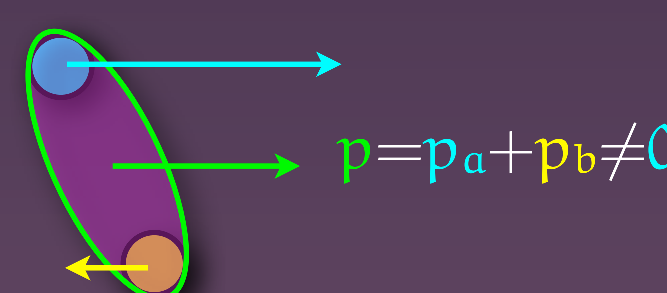
D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Pairs have Momenta?

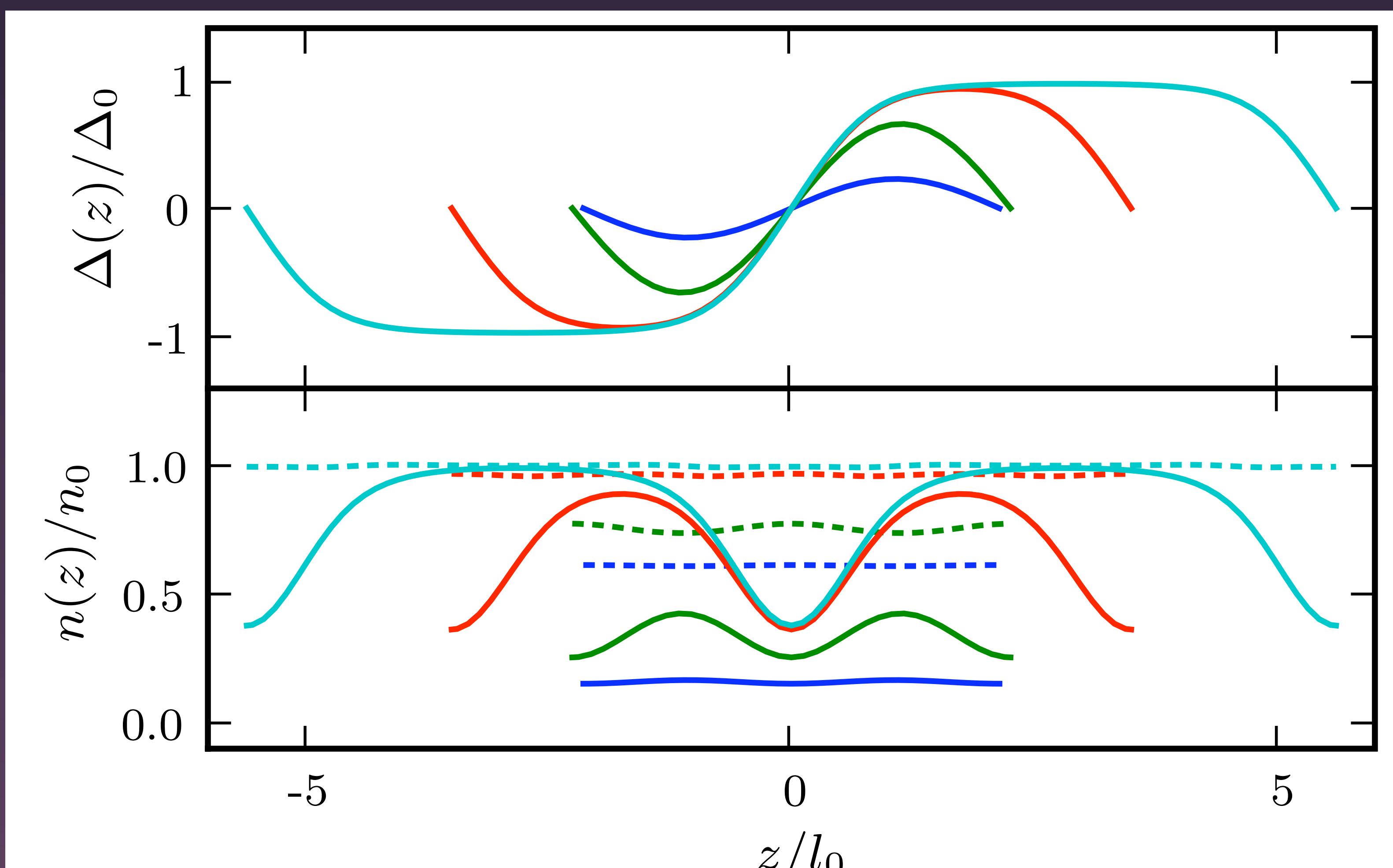


State (LO) is crystal (supersolid)

Pairs have momentum

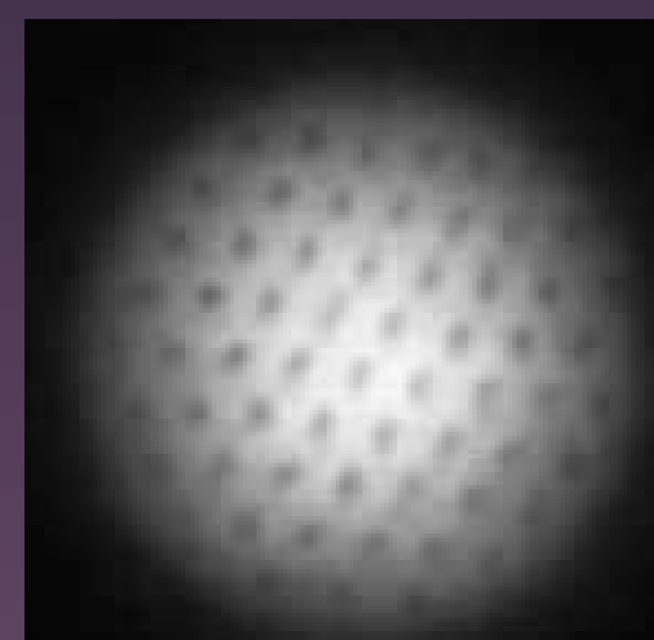


DFT predicts (FF)LO at Unitarity: Supersolid!



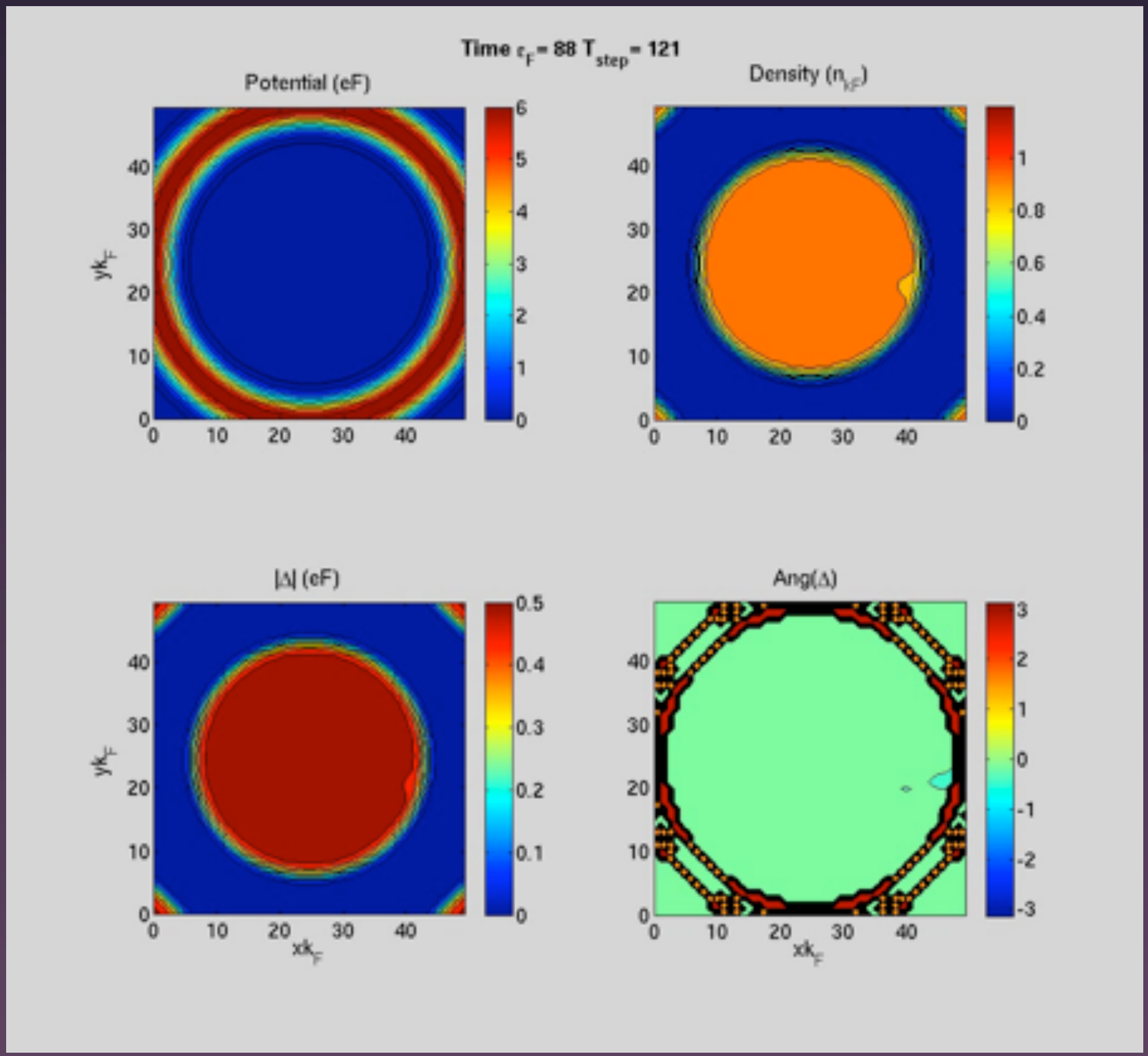
Large density contrast (factor of 2)

Similar to contrast of vortex core



Bulgac and Forbes PRL 101 (2008) 215301

Superfluid vortices have holes



Bulgac et al. (Science 2011)

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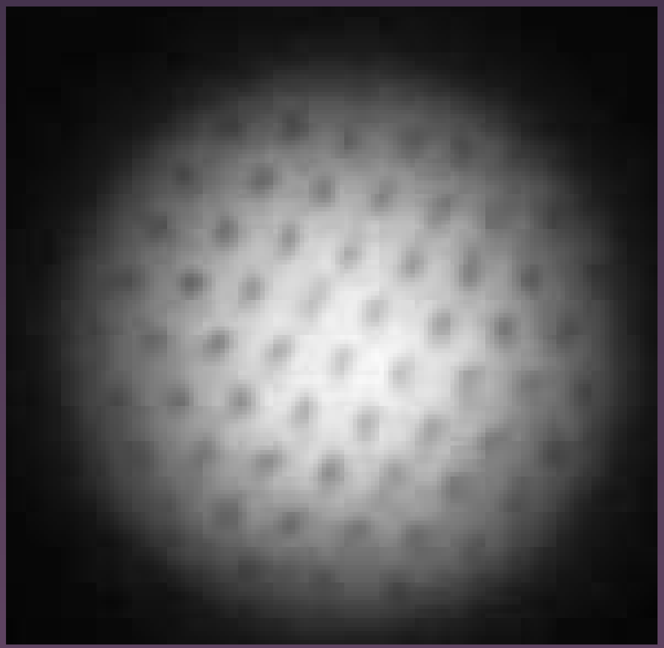
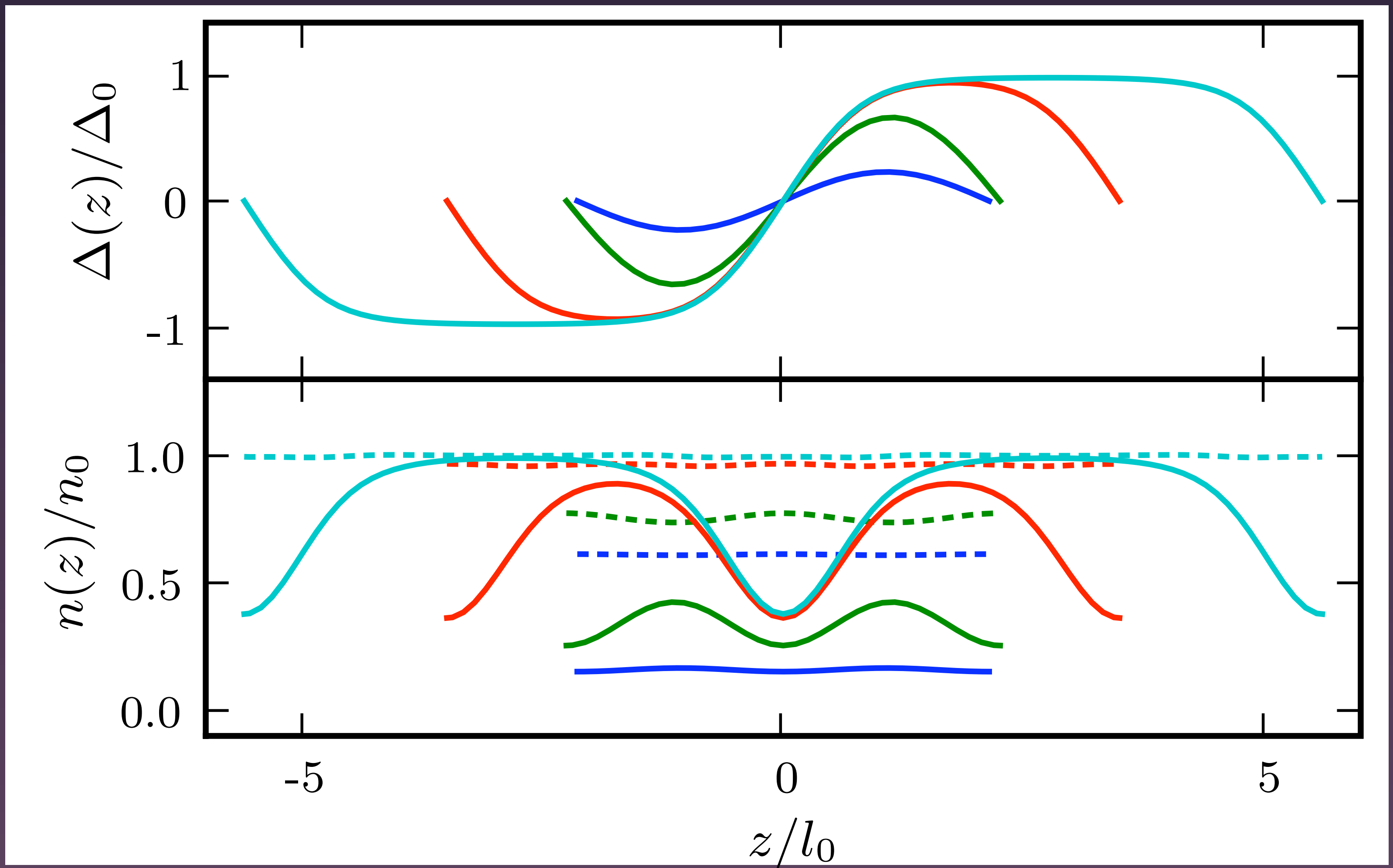
Similar to contrast of vortex core



DFT predicts (FF)LO at Unitarity: Supersolid!



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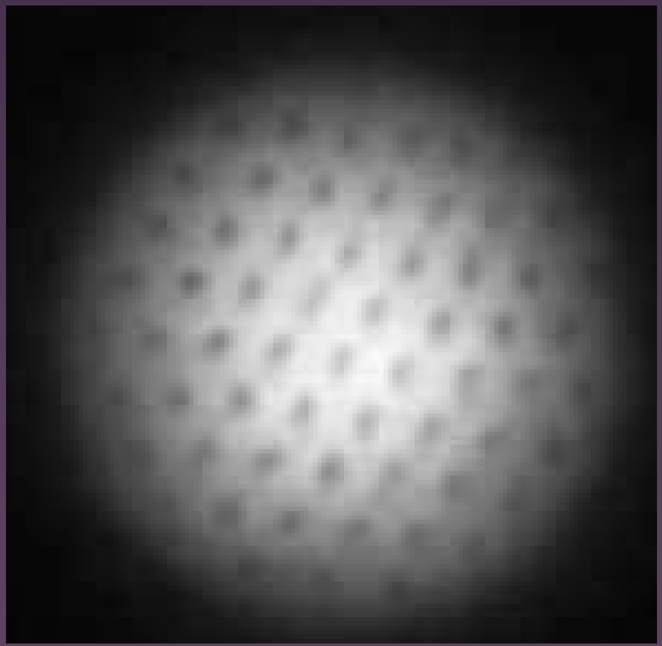
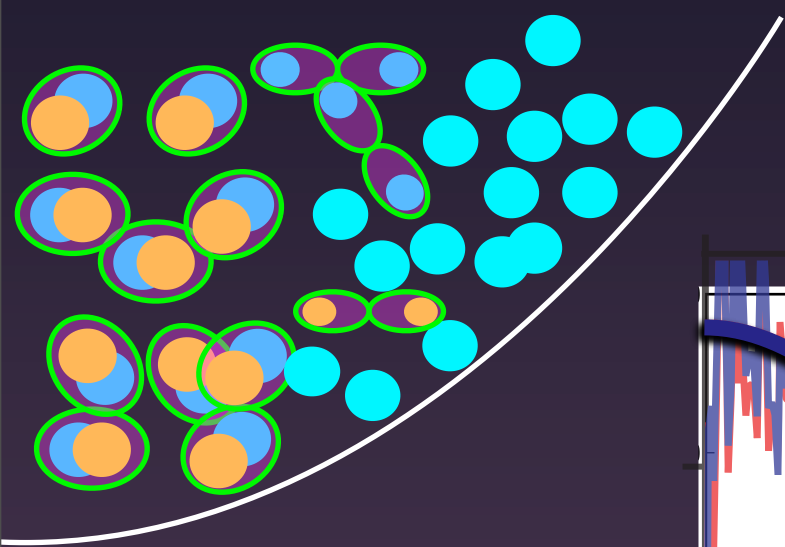
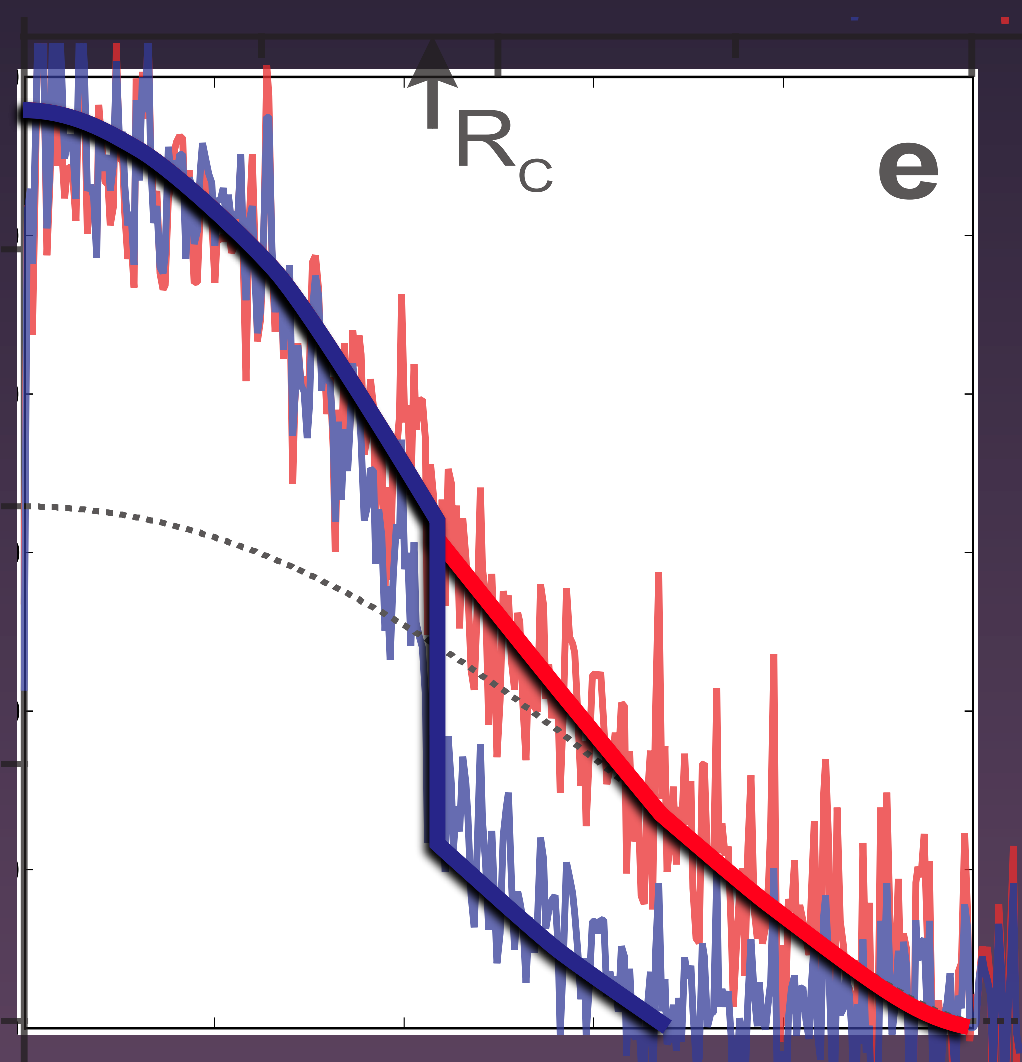


Bulgac and Forbes PRL 101 (2008) 215301

Observations: Nothing?





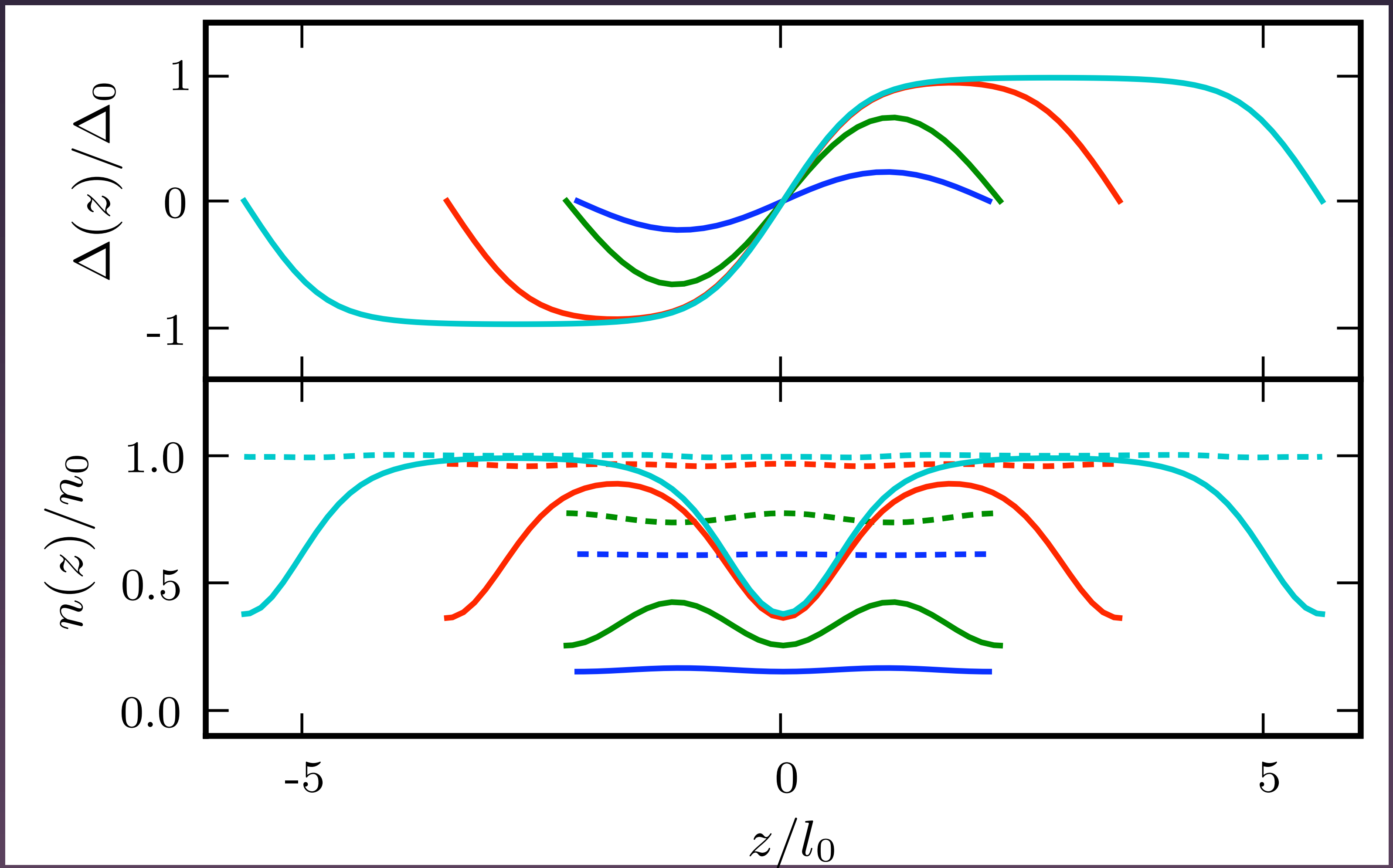
Paired core

Polarized wings

Maybe there are no interesting polarized superfluid phases?

MIT Experimental data from Shin et. al (2008)

DFT predicts (FF)LO at Unitarity: Supersolid!



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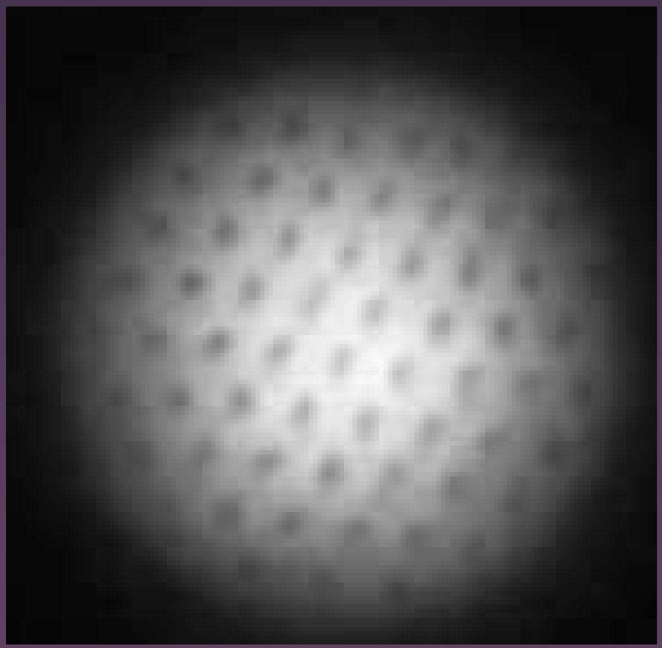
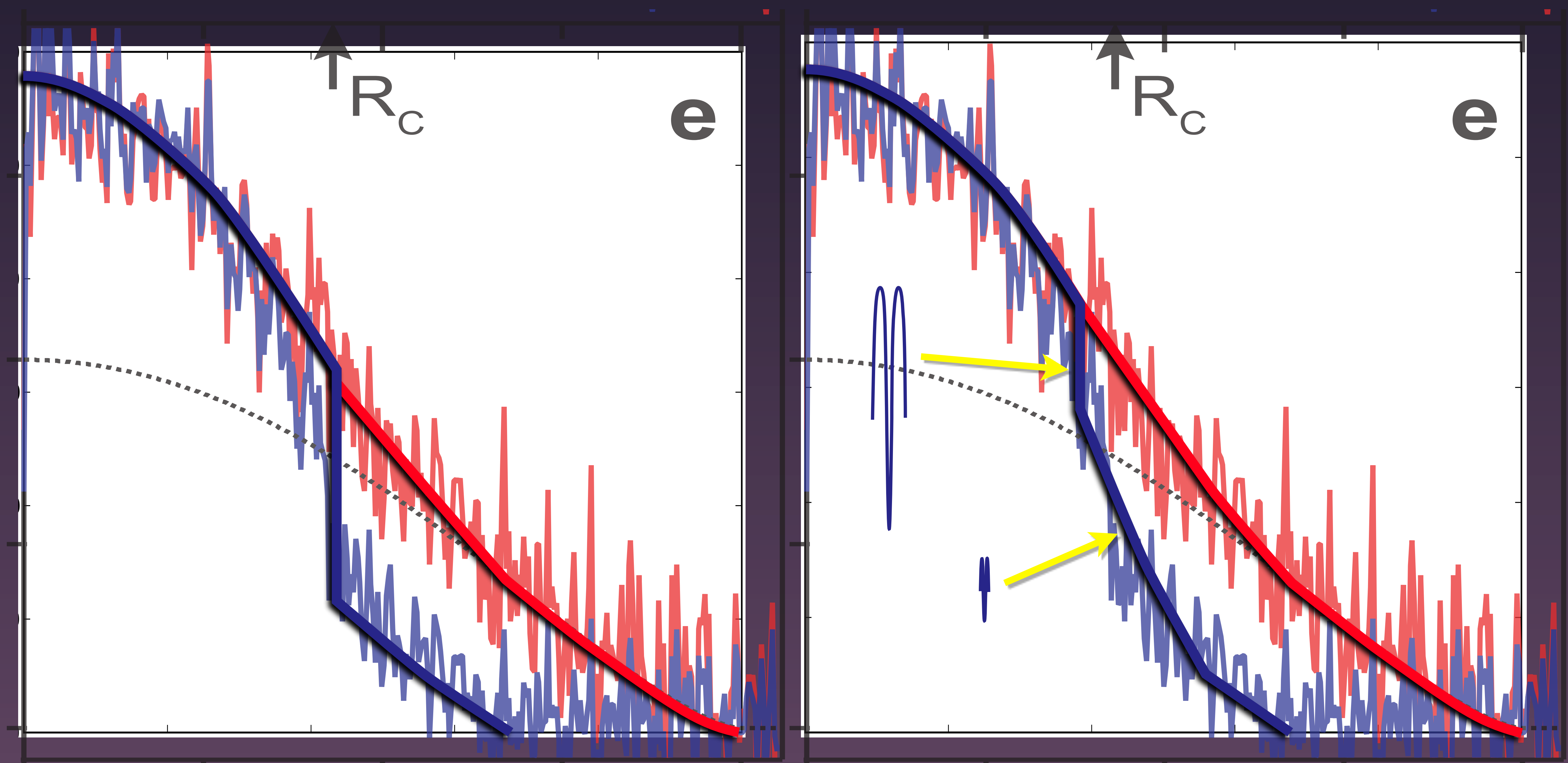


Bulgac and Forbes PRL 101 (2008) 215301

Observations: Inconclusive

- Need detailed structure or novel signature



MIT Experimental data from Shin et. al (2008)

Why FFLO not seen?

- It is not there:
  - Other homogenous phases might be better.
  - T might be too high (fluctuations kill 1D FFLO).
  - Trap frustrates formation (traps are not flat enough).
- It is not seen:
  - Noise washes out signature.
  - Small physical volume for FFLO.
- Need a nice flat trap: Large physical volume of FFLO

Cold Atoms

- Experimental Controls
  - Species, population, interactions, optical lattices
- Universal physics: quantum simulators?

  Can we simulate gauge theories? (Try to simulate lattice models)
- Benchmark Theory: Few to Many
- Unitary Fermi Gas models Neutron Matter

Universality

Fermionic Superfluids

Neutron Matter

$k_F \sim \text{fm}^{-1}$

$a_{nn} = -19$ fm

$r_{nn} = 2$ fm

Nuclei

neutrons and protons

Unitary Fermi Gas

$a = \infty$

$r_e = 0$

Cold Atoms

$k_F \sim \mu\text{m}^{-1}$

Tuneable a

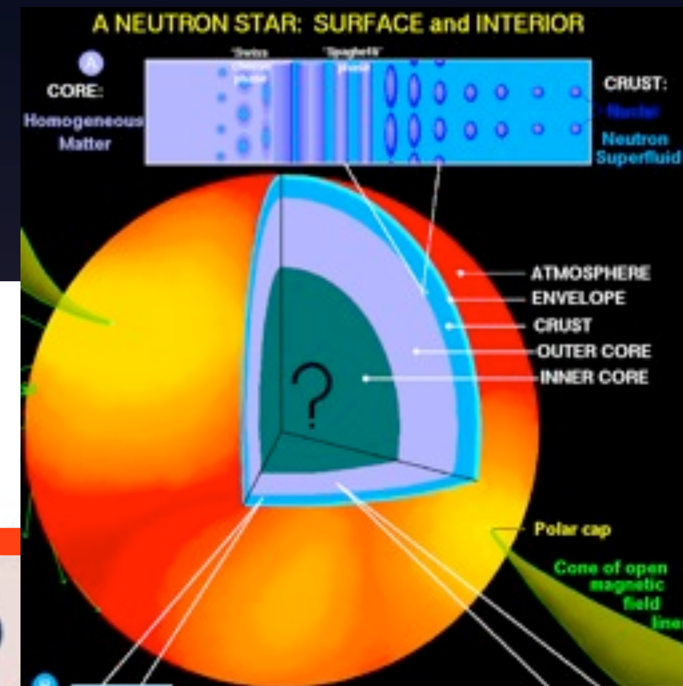
$r_{nn} \sim 0.1$ nm

Many systems

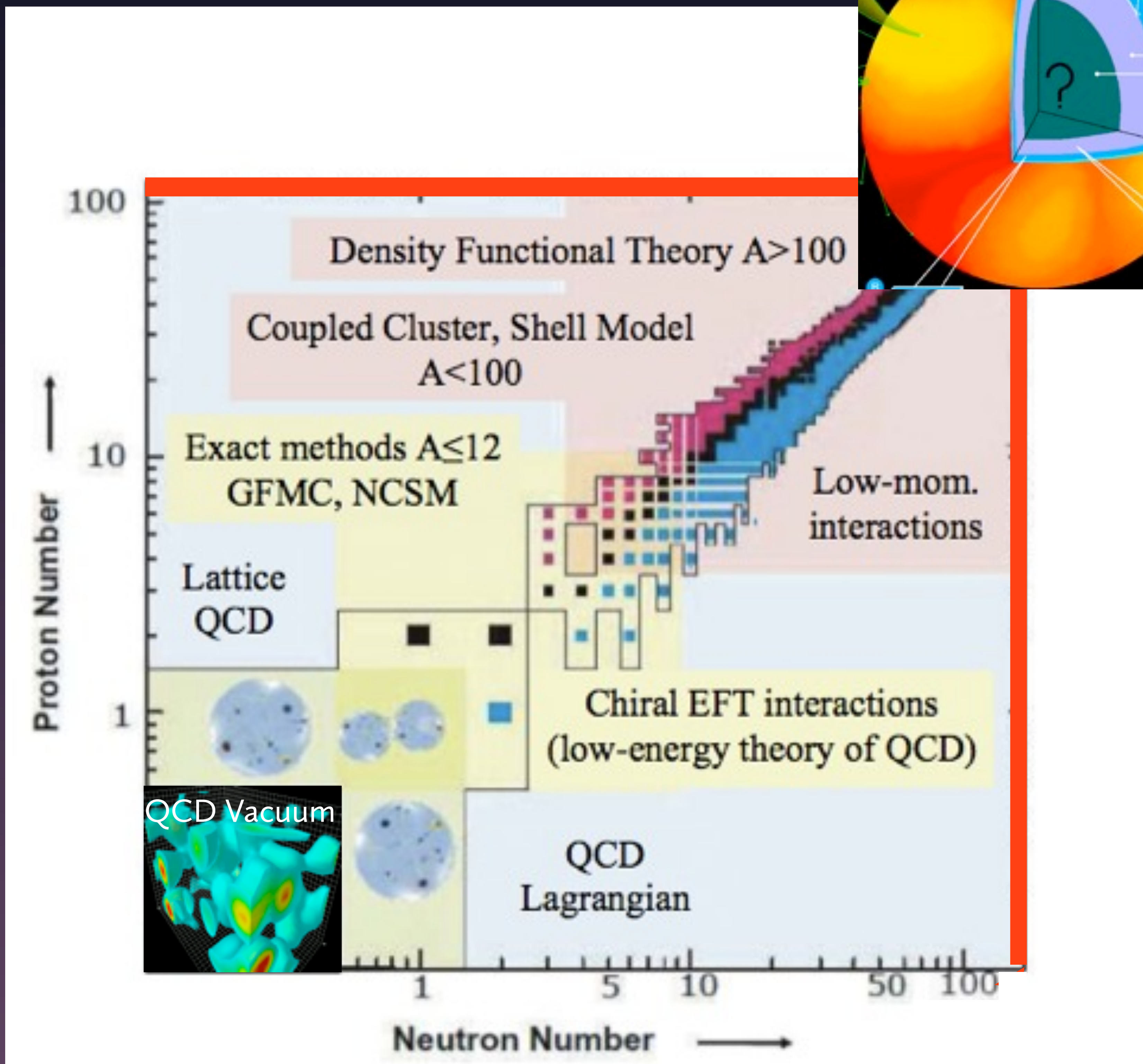
- different species
- dipole interactions
- optical lattices
- quantum simulators

Other Superfluids

- Superconductors (charged + phonons)
- Quarks (gluon interactions, Dark Matter?)
- ^{3}He (p-wave)



The Nuclear Landscape



- Lattice QCD,
  nucleons, interactions
- QMC, etc.
  small to medium nuclei
- DFT,
  medium to large nuclei
- Neutron stars?
  Molecular Dynamics
  Hydrodynamics

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

Many Body Problem

- From microscopic...

  quarks and gluons, electrons and photons, protons and neutrons, atoms

- ... to macroscopic

  nuclei, superconductors/superfluids, neutron stars, (dark matter)

- One, two, (three, four)... many.
- Exact method fail quickly
  - Approximate, or make models

Classical

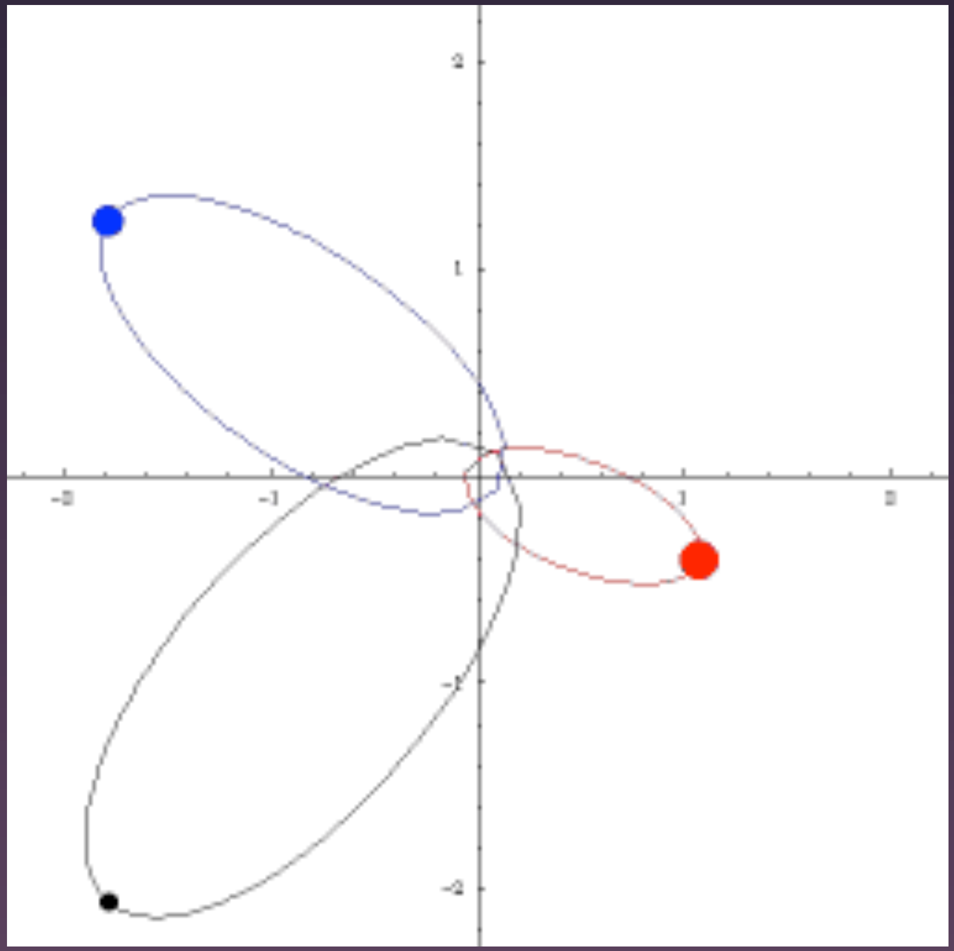
- Positions and velocities as functions of time: $(x, y, z; p_x, p_y, p_z)$
- One-body and two-body
  - Exact solutions
- Many two-body interactions

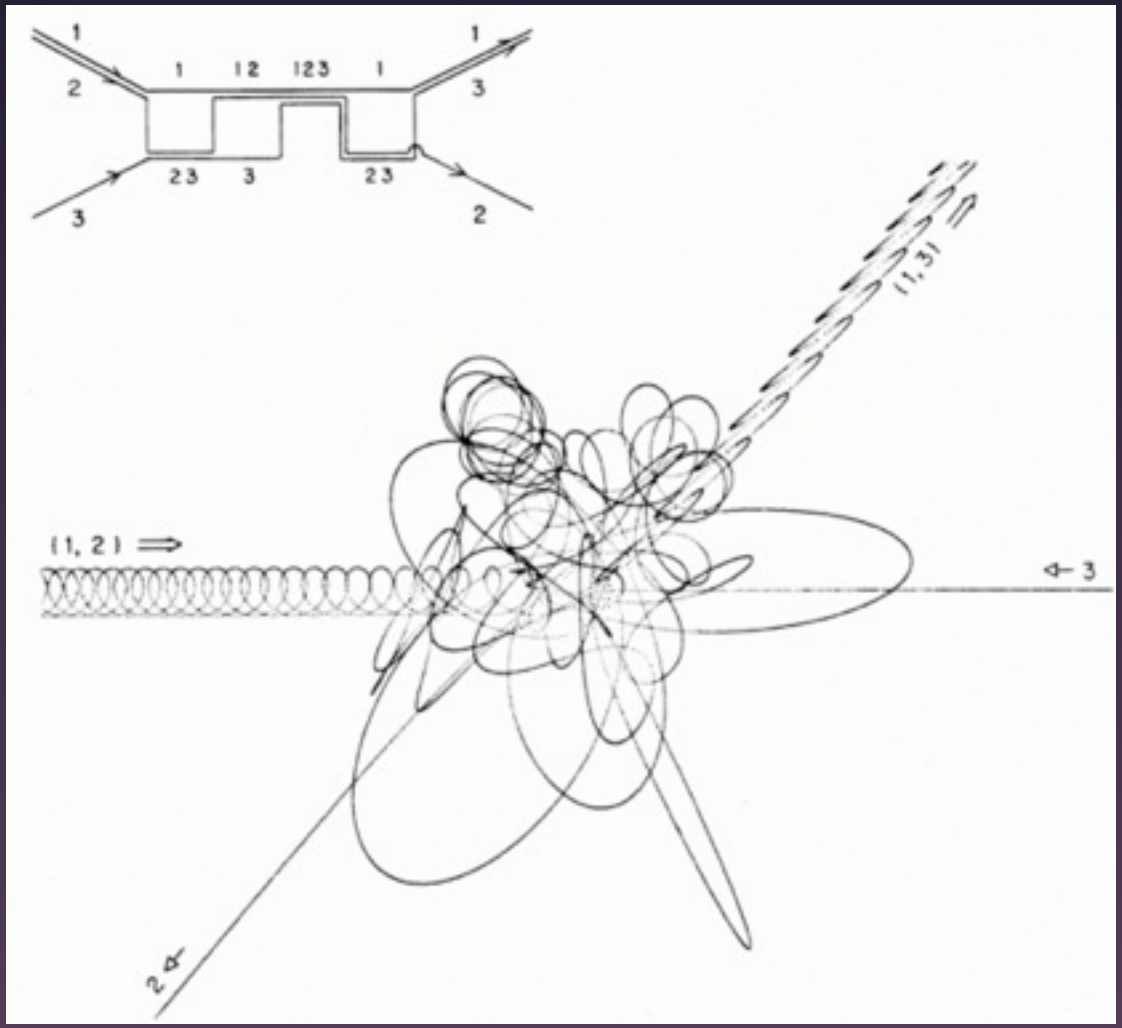
$$\vec{F} = \frac{GmM}{\|\vec{r}_1 - \vec{r}_2\|^2}, \qquad \vec{F} = -\vec{\nabla} V(\|\vec{r}_1 - \vec{r}_2\|)$$

$N \; N_x \; N_t$

Three Body

- Few exact solutons
- Enter chaos



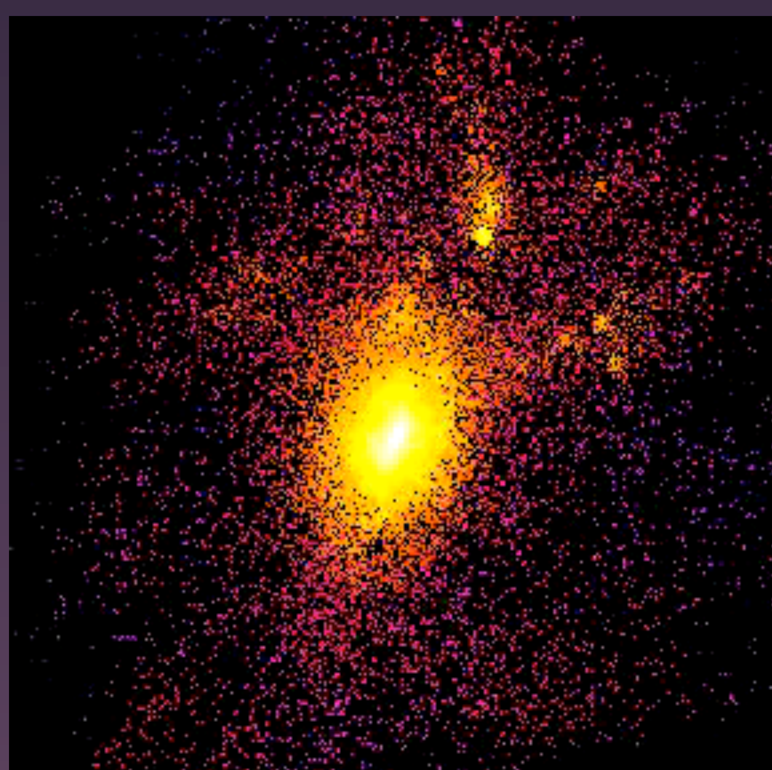


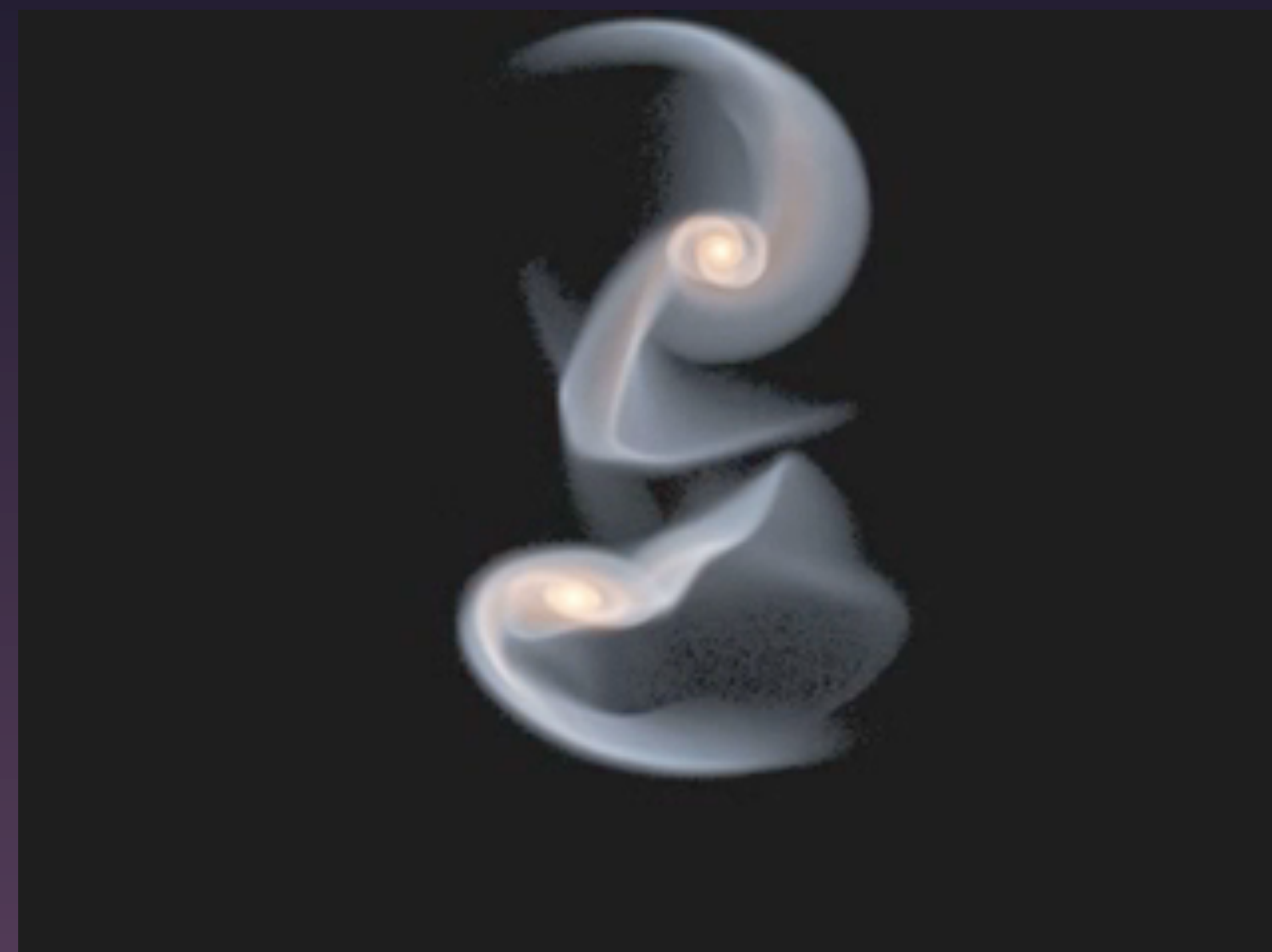
Hut and J.N. Bahcall ApJ 268, 319-41 (1983)

18 N_x N_t

Many Body

- Still tractable numerically
- Naïvely N^2
- Usually $N \log(N)$





John Dubinski (2008)
http://www.galaxydynamics.org/spiral_metamorphosis.html

Tom Quinn
http://www-hpcc.astro.washington.edu/picture/movies.html

$N \quad 6N_x N_t$

Quantum Systems

- Wavefunction $\Psi(x,t)$: $3N_x\ N_t$
- Exponentially hard: $(3N_x)^N\ N_t$
- Eg. Sn (Tin) $A \sim 120$: $(3N_x)^{120}\ N_t$
  - One state – more bytes than atoms in visible universe!
- Need to approximate

$(3N_x)^N\ N_t$

Harmonic Oscillator

$$\widehat{H} = \frac{\widehat{p}^2}{2m} + \frac{m\omega^2\widehat{x}^2}{2} = \hbar\omega\left(\widehat{a}^\dagger\widehat{a} + \frac{d}{2}\right), \qquad [\widehat{a}, \widehat{a}^\dagger] = 1, \qquad \langle\widehat{a}^\dagger\widehat{a}\rangle = n$$

- "Second Quantization"

  (Each point in space (or momentum) is like an HO)

- Two-particle Hamiltonian

$$\widehat{\mathcal{H}} = \int\left(\underbrace{\widehat{a}^\dagger\widehat{a}}_{n_a}E_a + \underbrace{\widehat{b}^\dagger\widehat{b}}_{n_b}E_b\right) - \int V\,\underbrace{\widehat{a}^\dagger\widehat{a}}_{n_a}\underbrace{\widehat{b}^\dagger\widehat{b}}_{n_b}$$

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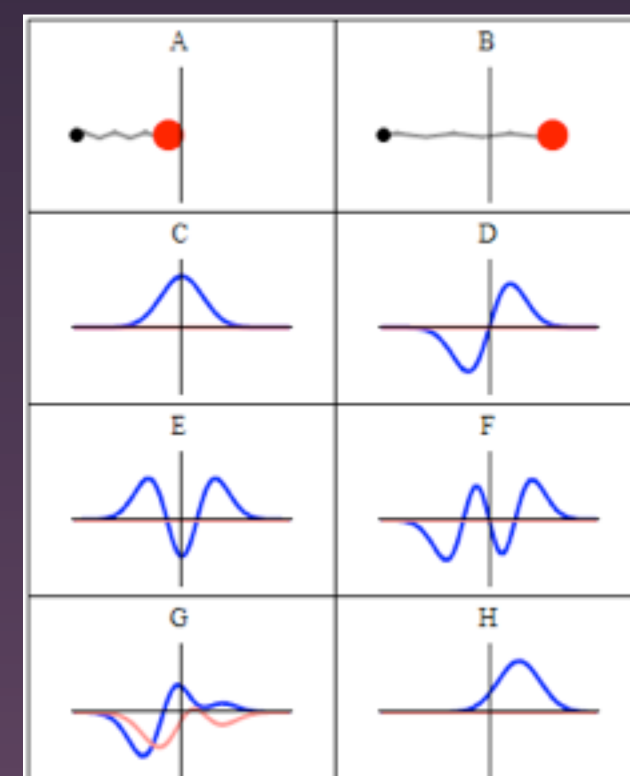

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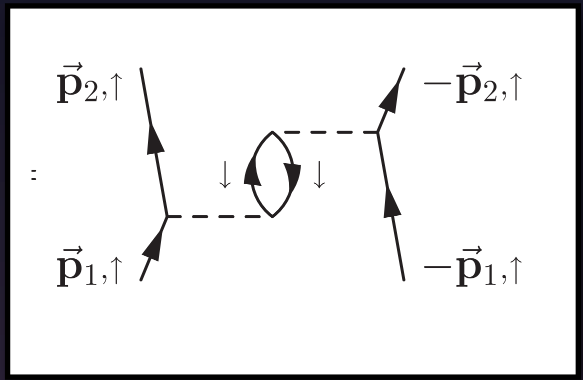
- Quadratic part easy to solve.
- Interactions make problem hard

$$\hat{\mathcal{H}} = \int \left(\underbrace{\hat{a}^\dagger\hat{a}}_{n_a} E_a + \underbrace{\hat{b}^\dagger\hat{b}}_{n_b} E_b\right) - \int V \underbrace{\hat{a}^\dagger\hat{a}}_{n_a}\underbrace{\hat{b}^\dagger\hat{b}}_{n_b}$$

$$(3N_x)^N N_t$$

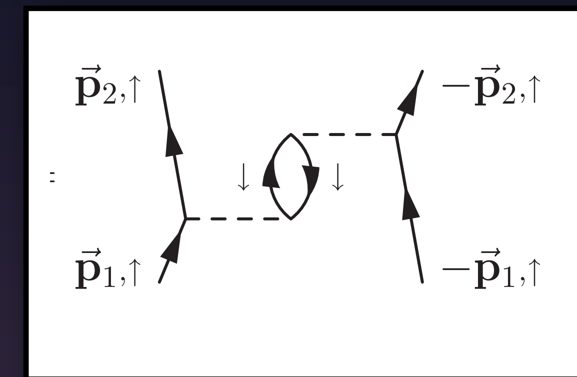
Methods



- Perturbation theory

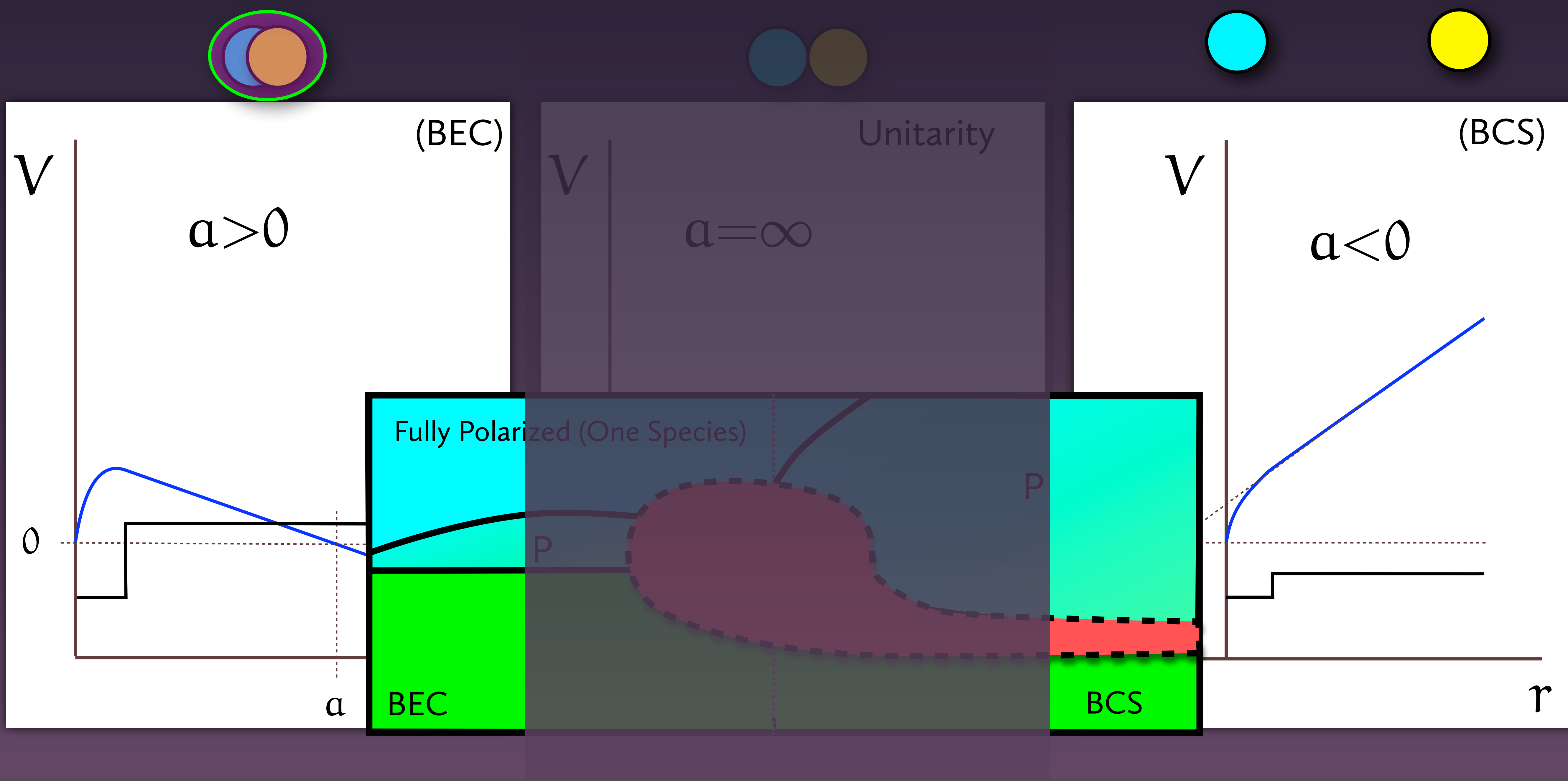
  Weak interactions, parameter expansion

Methods

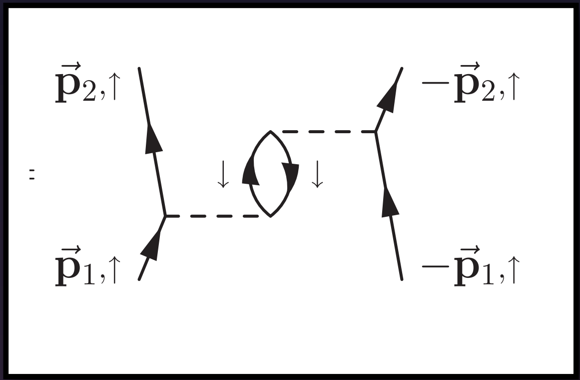


- Perturbation theory

  Weak interactions, parameter expansion



Methods



- Perturbation theory

  Weak interactions, parameter expansion

- Numerical methods

  DMRG, Quantum Monte Carlo, No-core Shell Model, Coupled Cluster

- Experiment

  Cold Atoms

- Models, Effective Theory

  Mean Field, Density Functional Theory, Hydrodynamics

Mean Field Theory

$$V\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b} \approx \hat{a}^\dagger\hat{a}\delta\mu_a + \hat{b}^\dagger\hat{b}\delta\mu_b + \hat{a}^\dagger\hat{b}^\dagger\Delta + \text{h.c.}$$

$$\delta\mu_a = V\langle\hat{b}^\dagger\hat{b}\rangle, \quad \delta\mu_b = V\langle\hat{a}^\dagger\hat{a}\rangle, \quad \Delta \sim V\langle\hat{b}\hat{a}\rangle$$

- Introduce "mean fields" to make Hamiltonian quadratic
- Single particles in an effective potential
- Self-consistent problem
- Equivalent to variational formulation:

$$\langle\Omega\rangle \leqslant \Omega_0 + \langle\hat{H} - \hat{H}_0\rangle_0$$

$N\ 3N_x N_t$

Density Functional Theory (DFT)

- The (exact) ground state density in any external potential V(x) minimizes a functional (Hohenberg Kohn):

$$\int d^3x \{\mathcal{E}[n(x)] + V(x)n(x)\}$$

- Functional may be complicated (non-local)
  - Need to find physically motivated approximations
- (think adjustable Mean Field Theory)

(N) $3N_x$ N_t

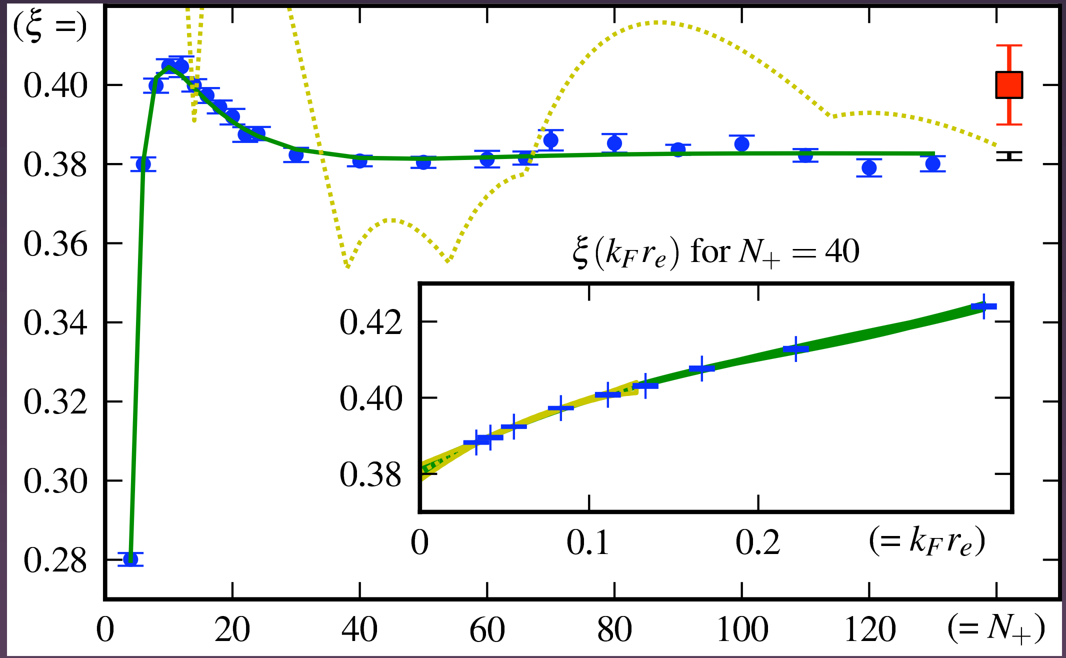
Density Functional Theory (DFT)

- Define functional with physically motivated model
- Fit parameters to experiment/QMC
- Functional extrapolates from small to large
- Seems very effective for the Unitary Fermi Gas

SLDA: Fit to QMC

$$\mathcal{E}(n,\tau,\nu) = \alpha\frac{\tau}{m} + \beta\frac{(3\pi^2 n)^{5/3}}{10m\pi^2} + g_{\text{eff}}\nu^\dagger\nu$$

- Three parameters
- Fit all boxes from 2 to 120 particles per box



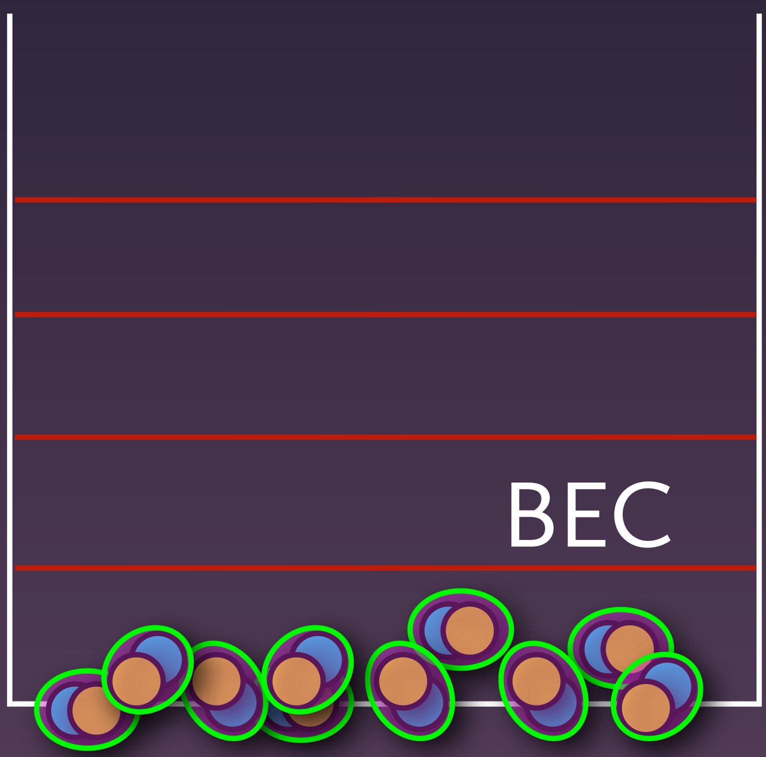
Forbes, Gandolfi, Gezerlis (2011, 2012)

Bosons are “easy”

$$E[\Psi] = \int d^3\vec{x} \left(\frac{\hbar^2 |\nabla\Psi(\vec{x})|^2}{2m_B} + V_F(\vec{x})\rho_F + g\frac{|\Psi|^4}{2} \right)$$

$$i\partial_t \Psi = \left(-\frac{\nabla^2}{2m_B} + [V + g|\Psi|^2] \right) \Psi$$

- Gross-Pitaevskii Equation (GPE)
- (all) bosons in single ground state
  - Include interactions through mean field
- Non-linear Schrödinger equation
- Only one wave function

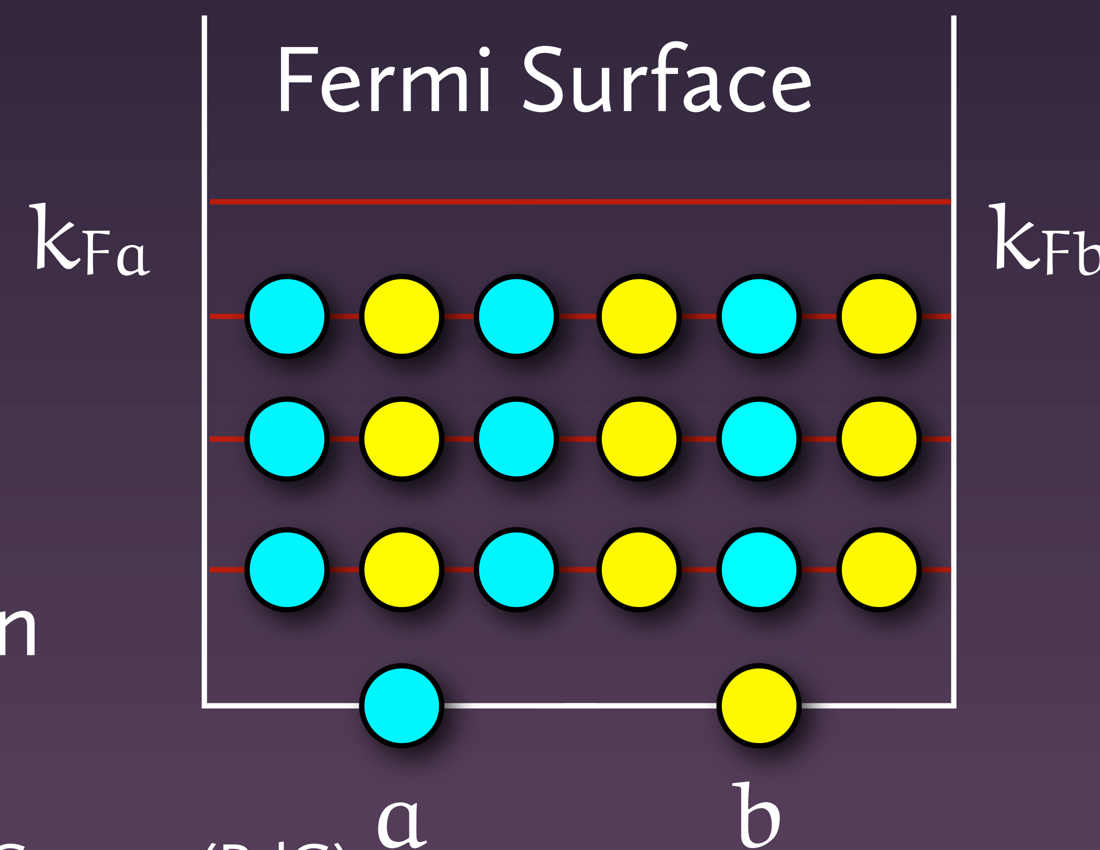


$3\ N_x\ N_t$

Fermions are harder

$$i\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Pauli Exclusion (blocking)
  - Particles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction



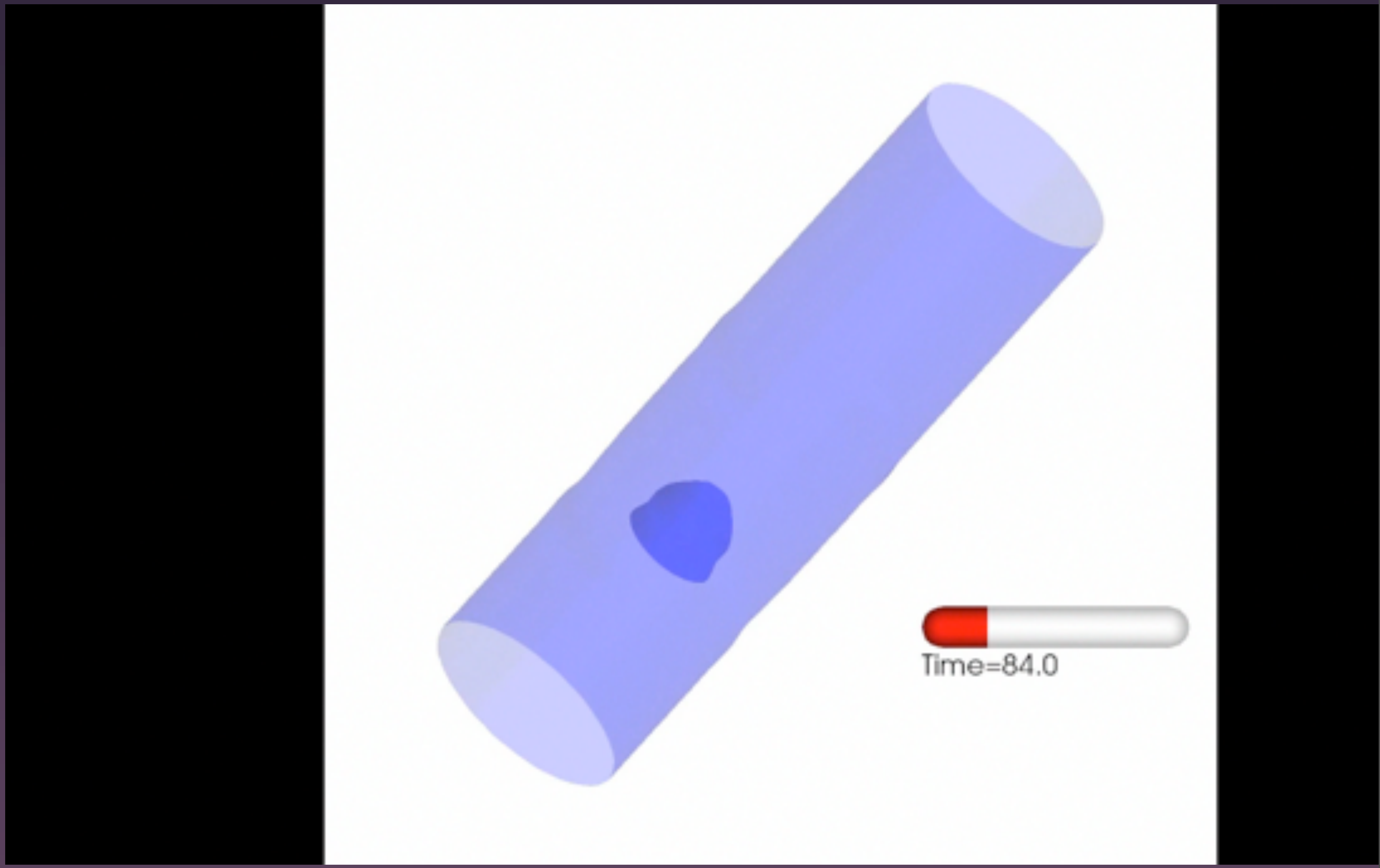
Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)

- Must use symmetries or supercomputers

$N\ 3N_x\ N_t$

DFT: Fermion still hard

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$



Bulgac, Luo, Magierski, Roche, Yu (2011)

$N \; 3N_x \; N_t$

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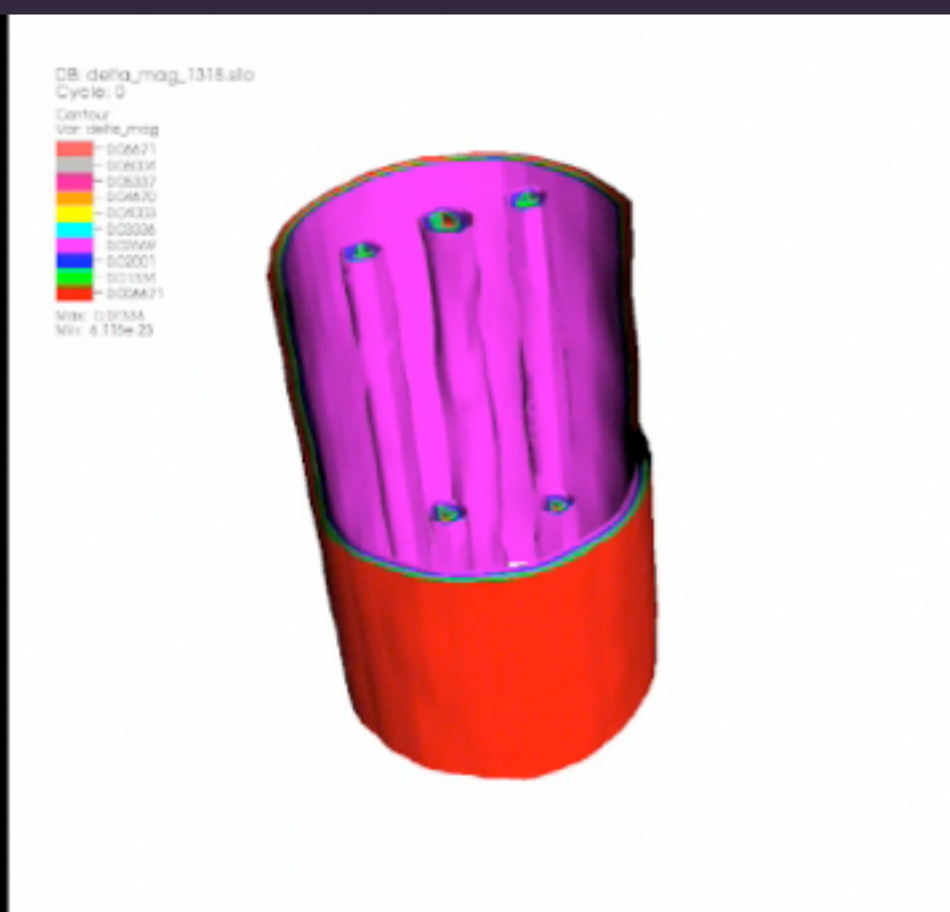
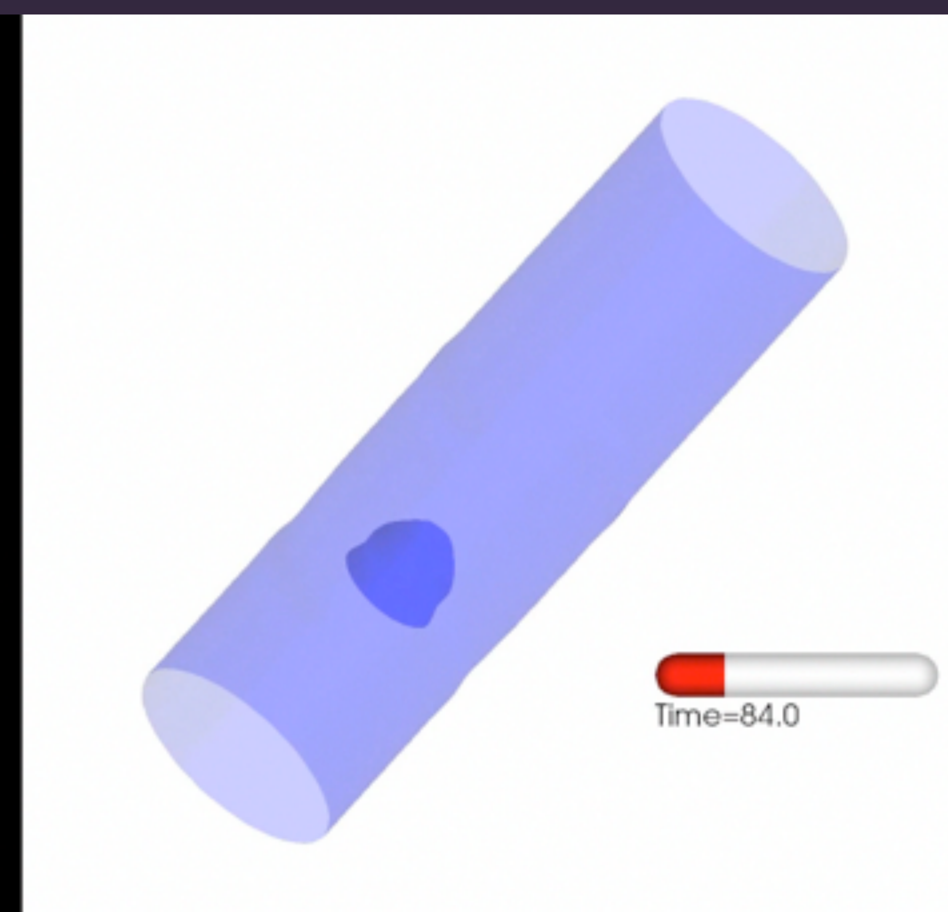




Bulgac, Luo, Magierski, Roche, Yu (2011)

$N \; 3N_x \; N_t$

GPE model for UFG?

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}(\rho_F, \{\nabla\rho_F\}) \right)$$

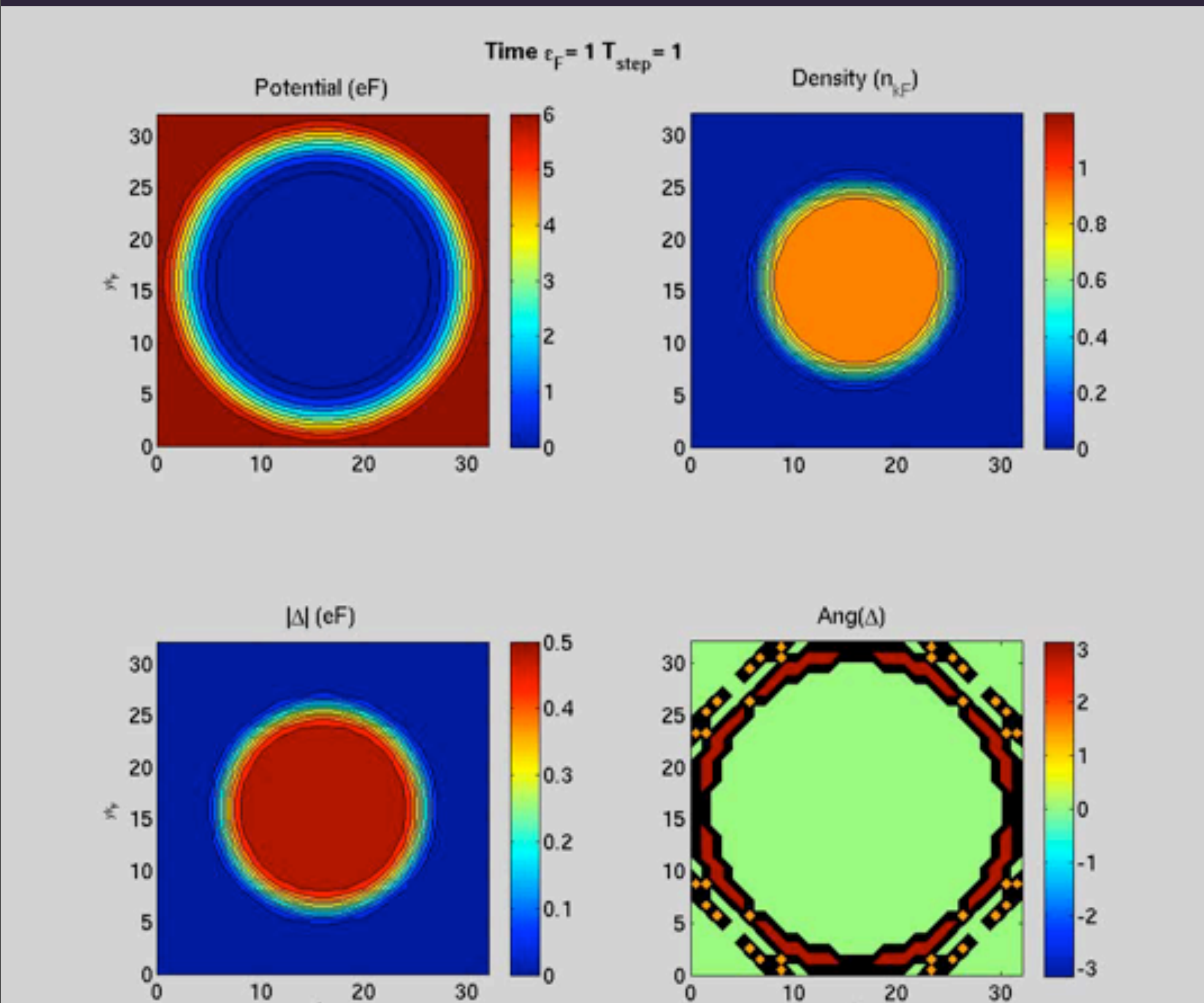
$$\imath\partial_t\Psi = \left(-\frac{\nabla^2}{4m_F} + 2[V_F + \xi\epsilon(\rho_F, \{\nabla\rho_F\})] \right)\Psi$$

- Think:
  - Boson = Fermion pair (dimer) $\quad \rho_F = 2|\Psi|^2$
- Galilean Covariant (fixes mass) $\quad \mathcal{E}_{FG} \propto \rho_F^{5/2}$
- Match Unitary Equation of State $\quad \epsilon_F = \mathcal{E}'_{FG}(\rho_F) \propto \rho_F^{3/2}$
- "Extended Thomas-Fermi" (ETF) model

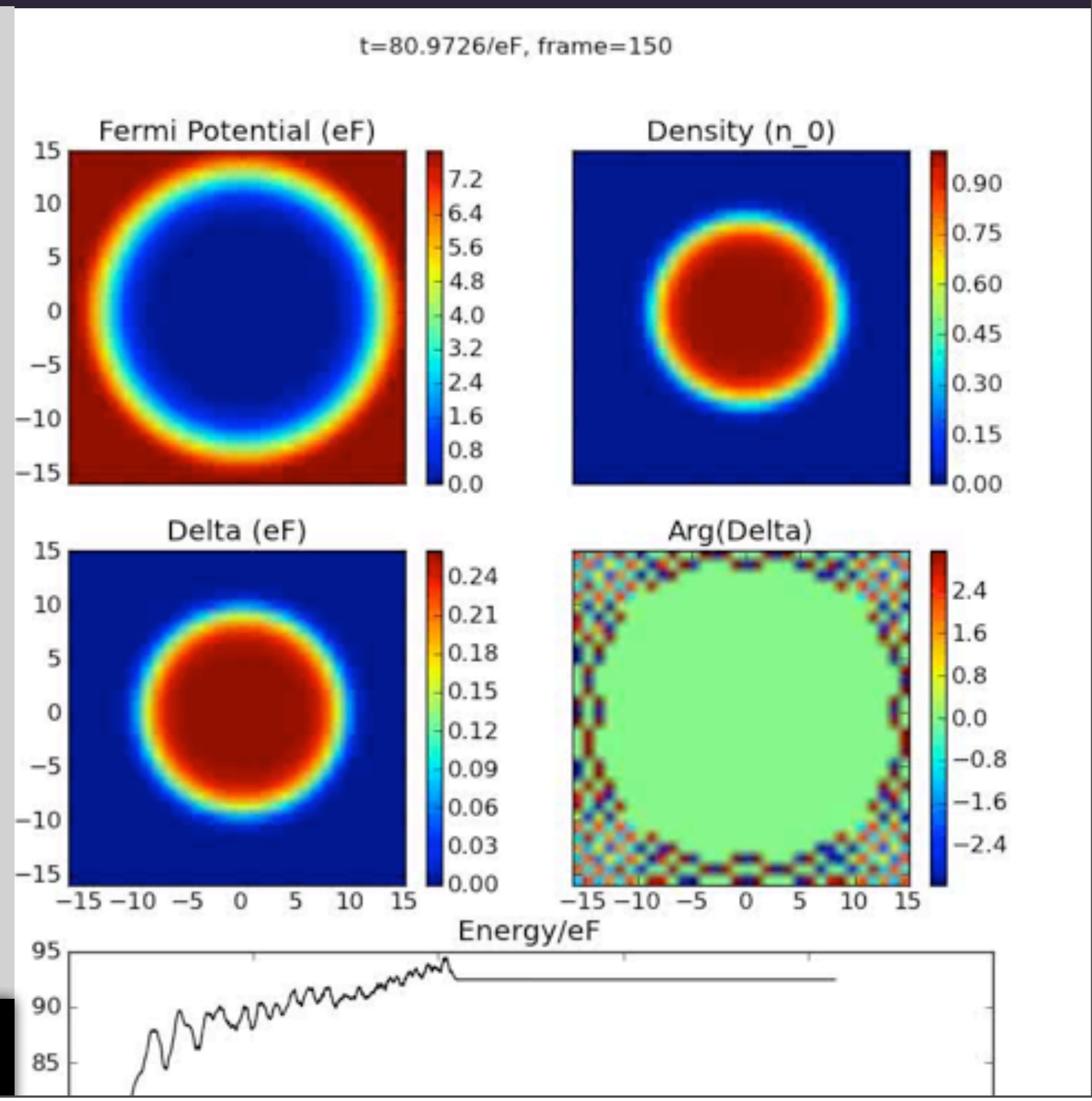
Comparison

Fermions
SLDA TDDFT

Gross Pitaevskii
model

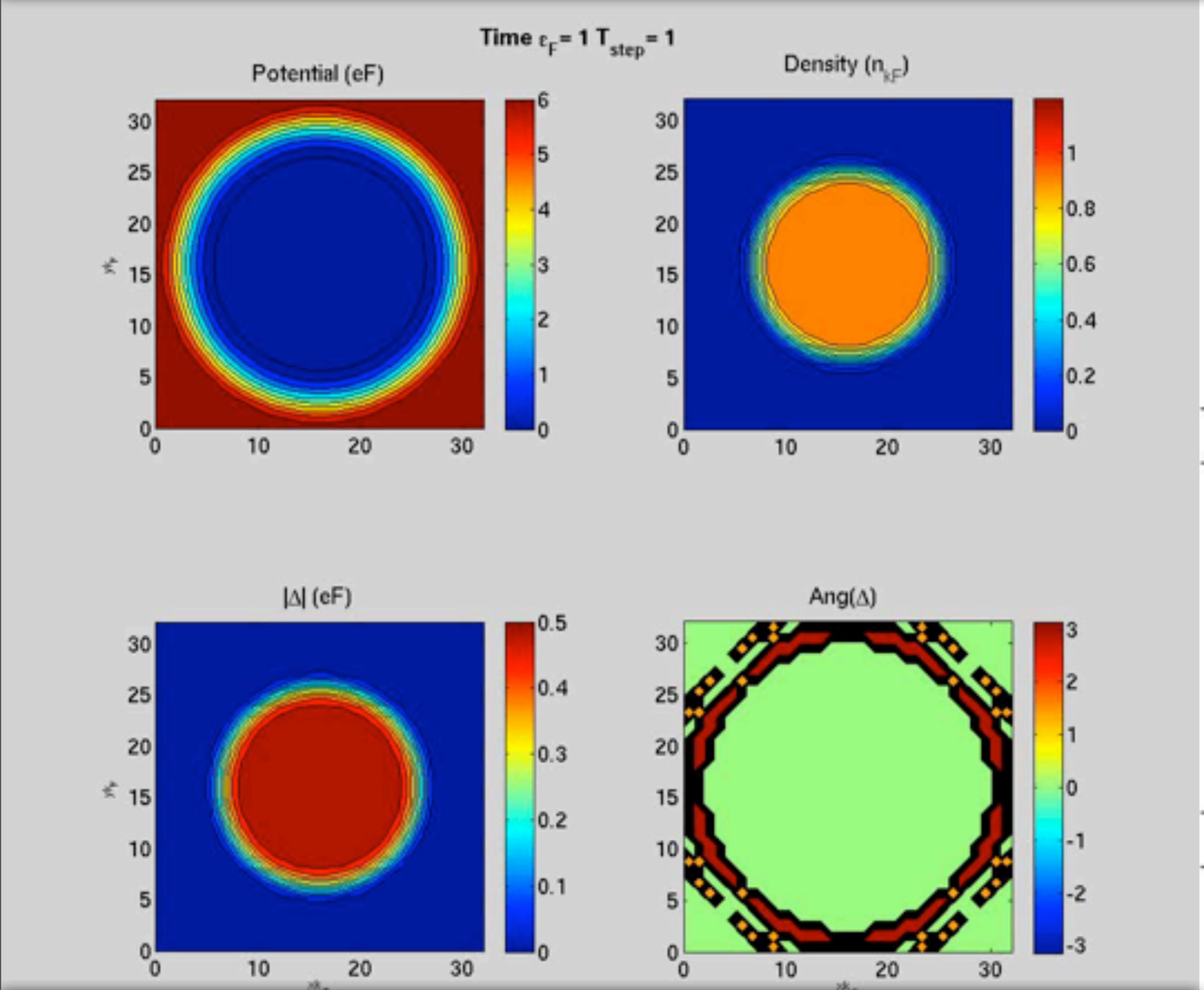


Bulgac et al. (Science 2011)



- Fermions:
- Simulation hard!
- Evolve 10^4–10^6 wavefunctions
- Requires supercomputers

- GPE:
- Simulation much easier!
- Evolve 1 wavefunction
- Use supercomputers to study large volumes



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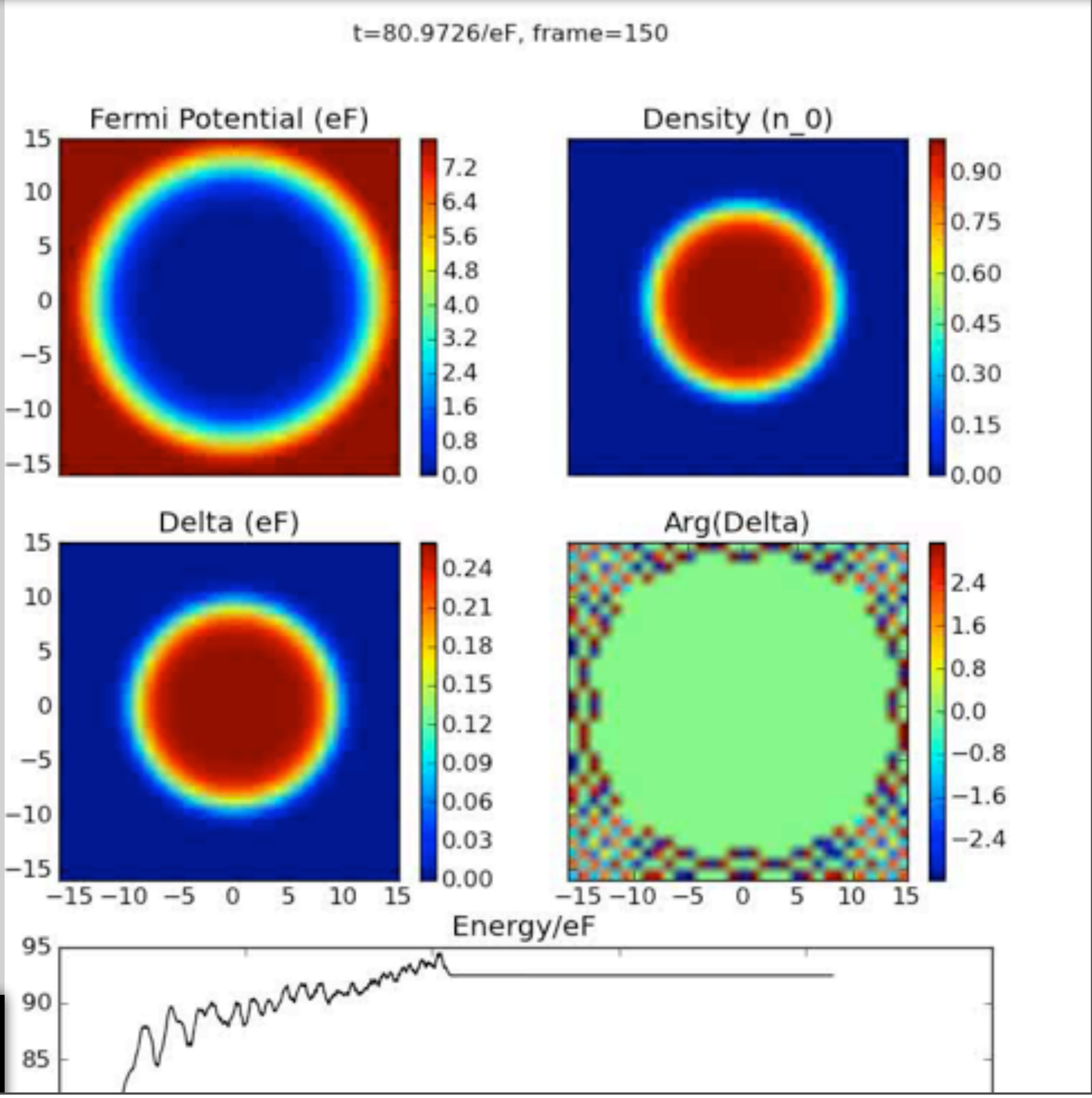
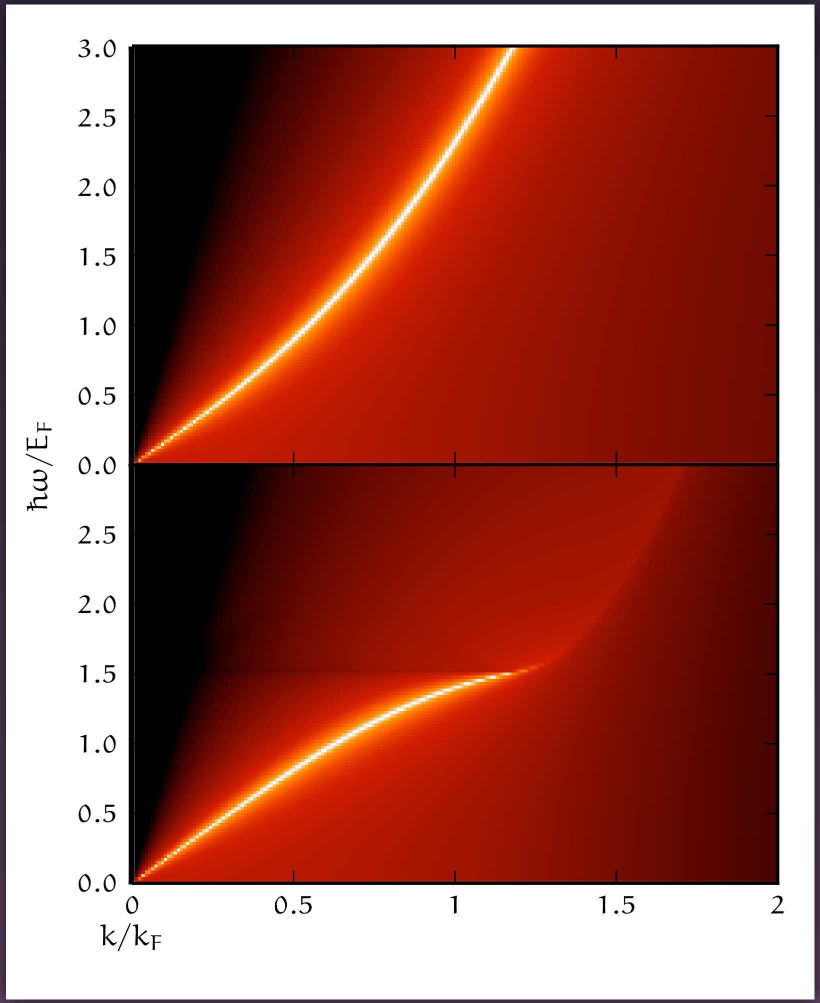


Matching Theories: The Good

- Galilean Covariance (fixes mass/density relationship)
- Equation of State
- Hydrodynamics
  - speed of sound (exact)
  - phonon dispersion (to order q^3)
  - static response (to order q^2)

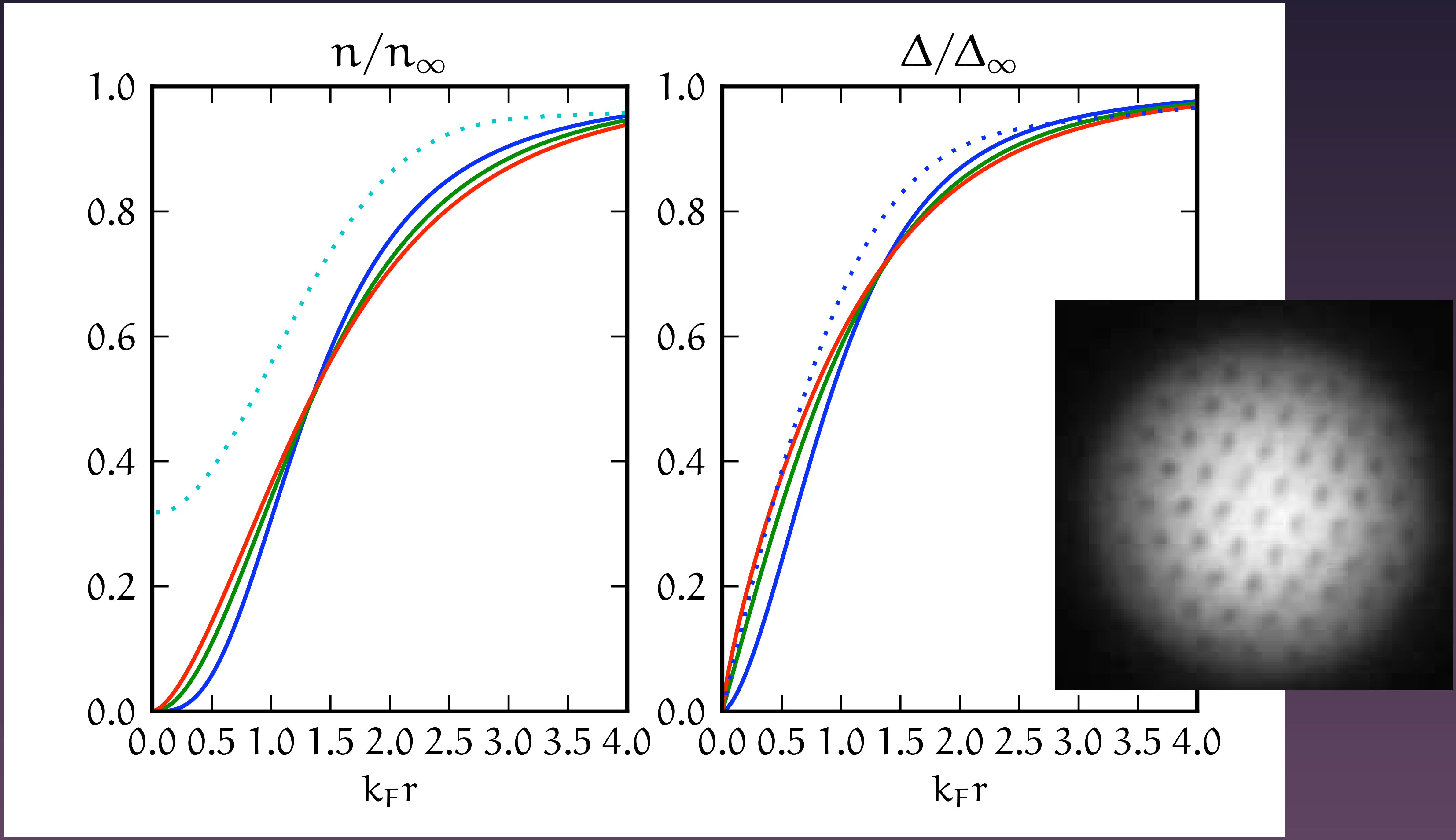


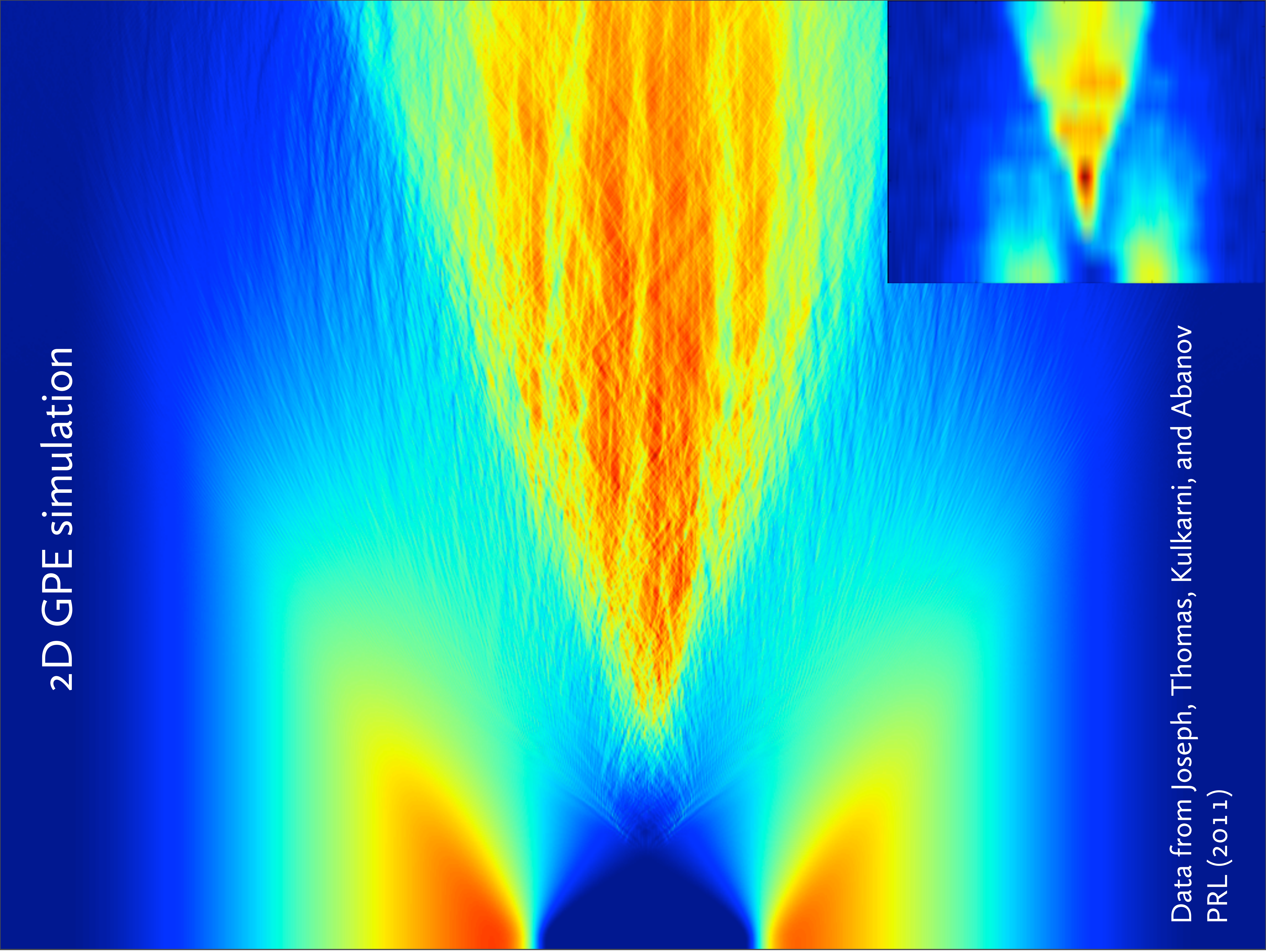
Forbes and Sharma (in prep)

Matching Theories: The Bad

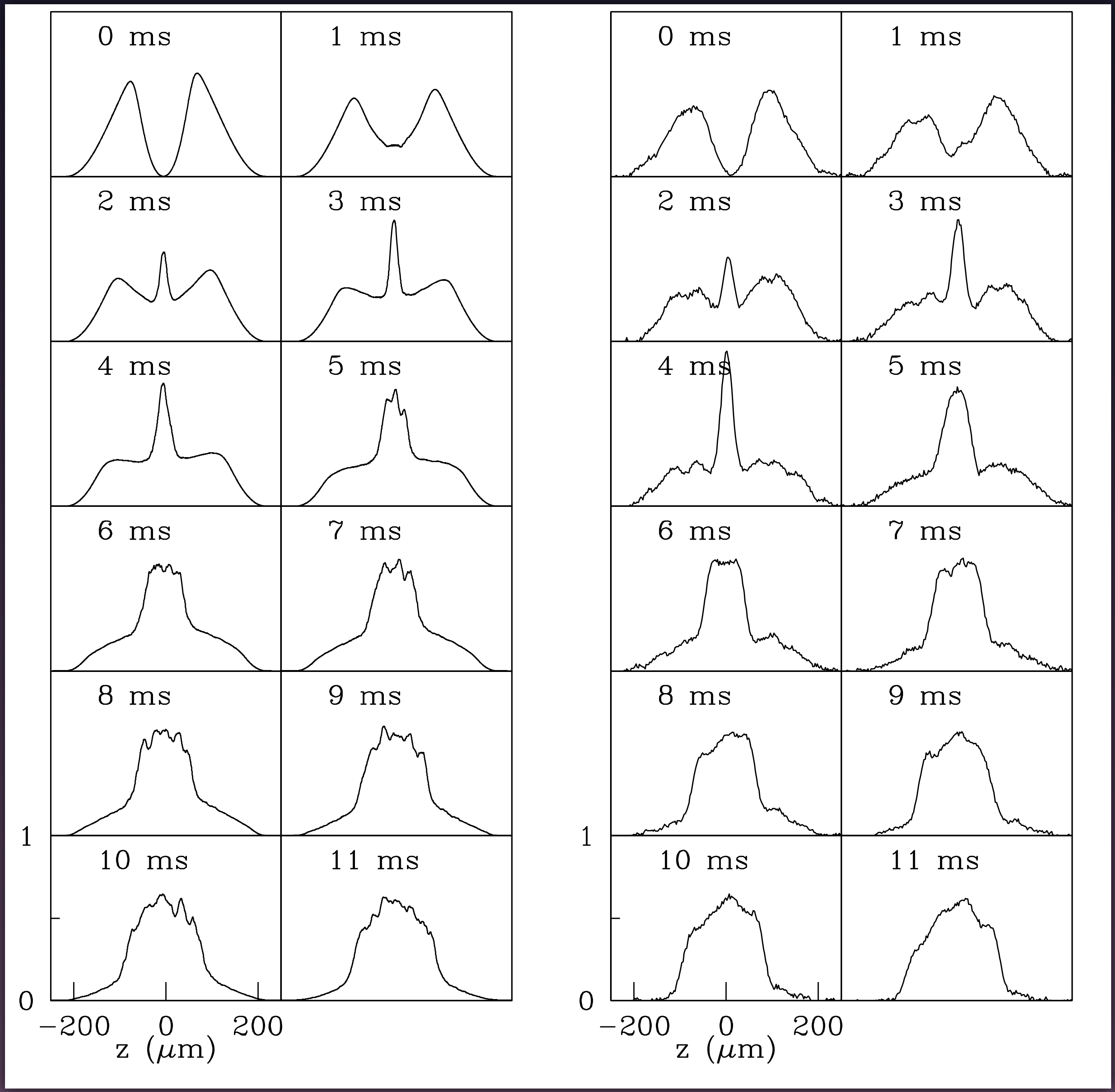
- GPE has $\rho = 2|\Psi|^2$
  - Density vanishes in core of vortex
  - Implies $\int|\Psi|^2$ conserved
    - (Approximate conservation $\int|\Psi|^2$ in Fermi simulations provides measure of applicability)
- No “normal state”
  - Two fluid model needed?
  - Coarse graining (transfer to “normal” component)

Vortex Structure



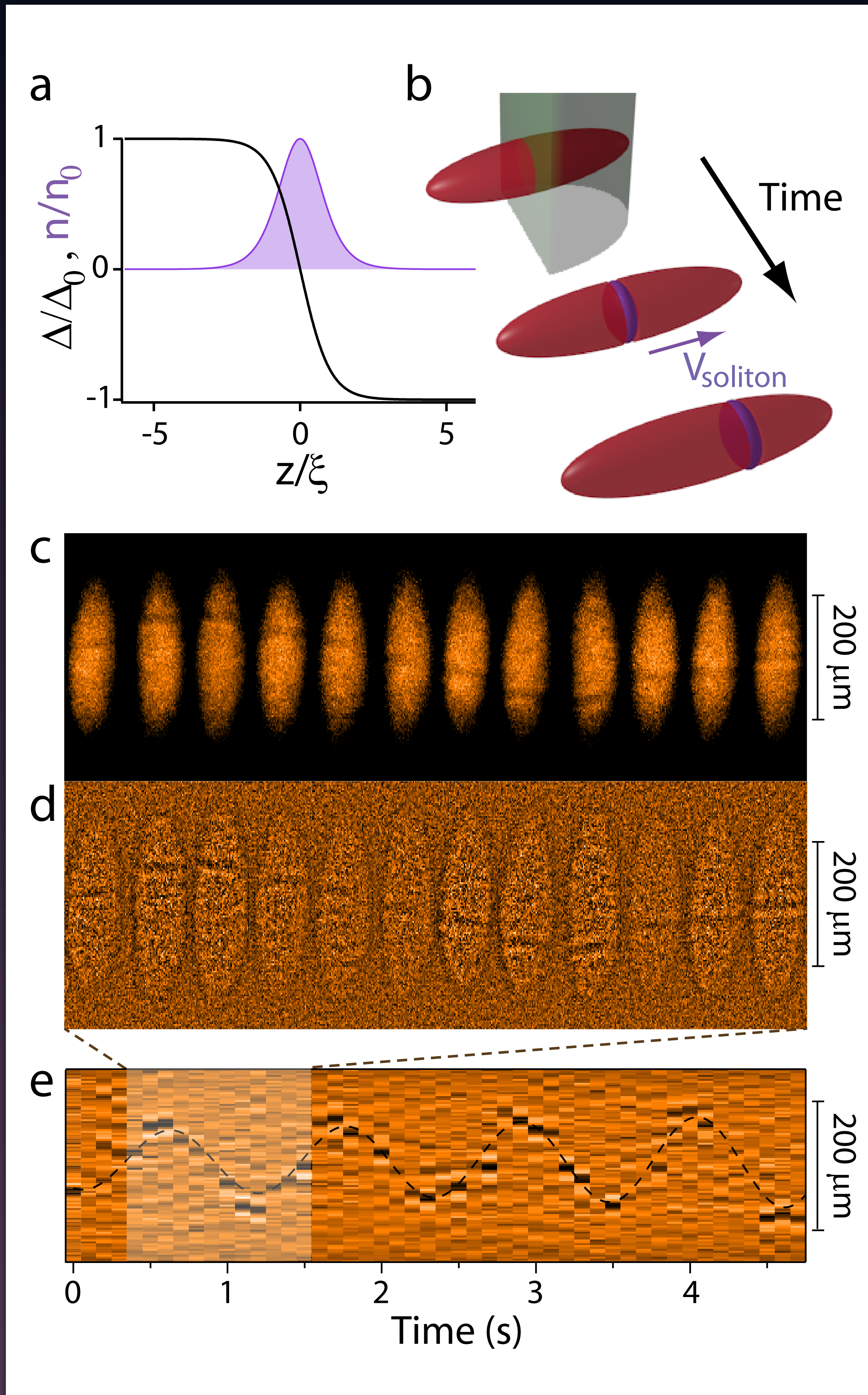

2D GPE simulation
Data from Joseph, Thomas, Kulkarni, and Abanov
PRL (2011)

GPE vs. Experiment



Ancilotto, L. Salasnich, and F. Toigo (2012)

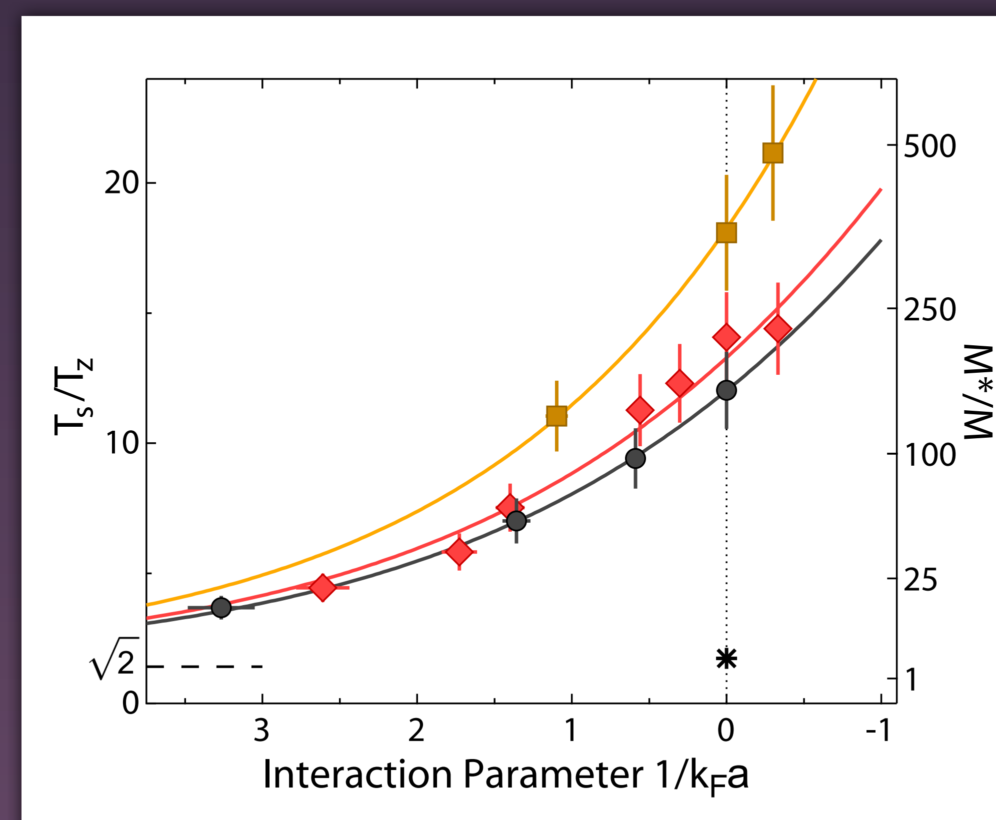
Soliton Motion



Yefsah et al. (MIT Experiment) arXiv:1302.4736

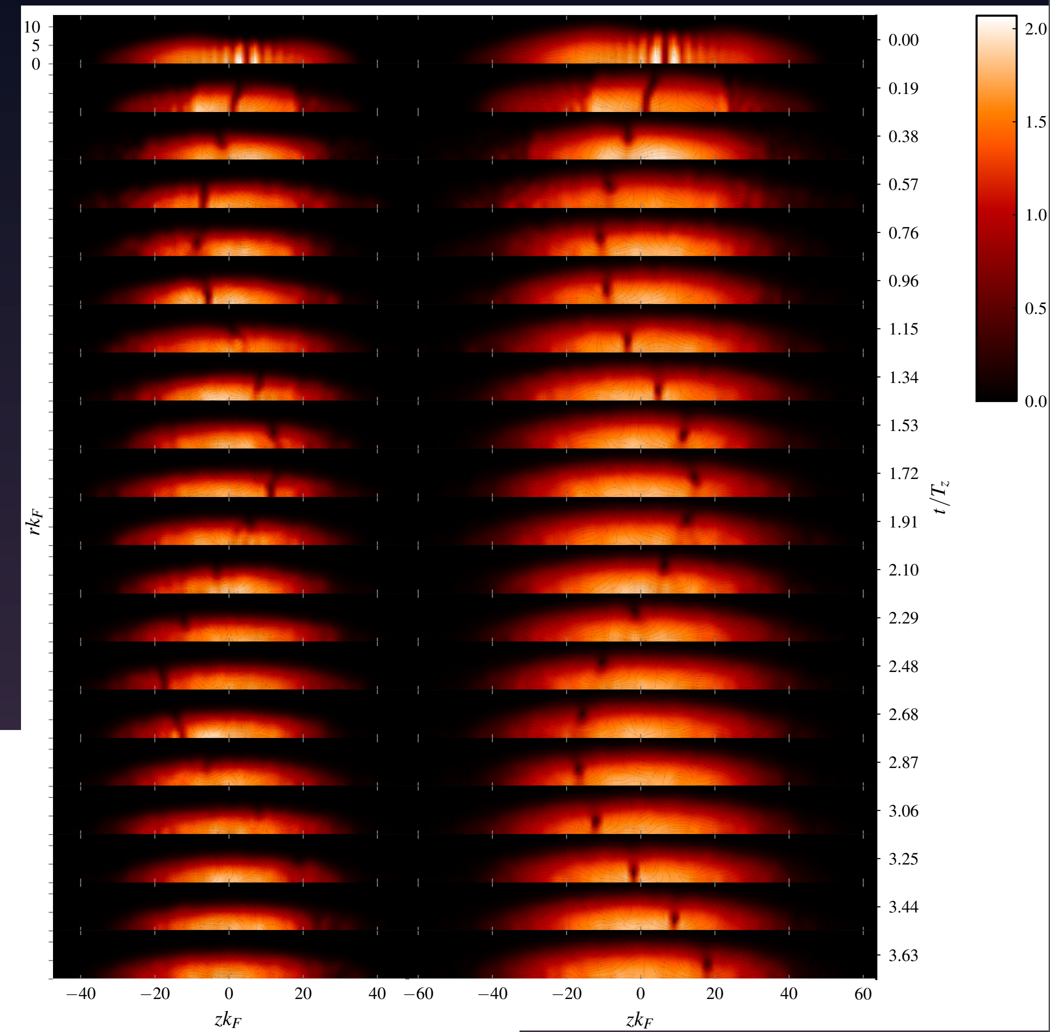
Soliton?

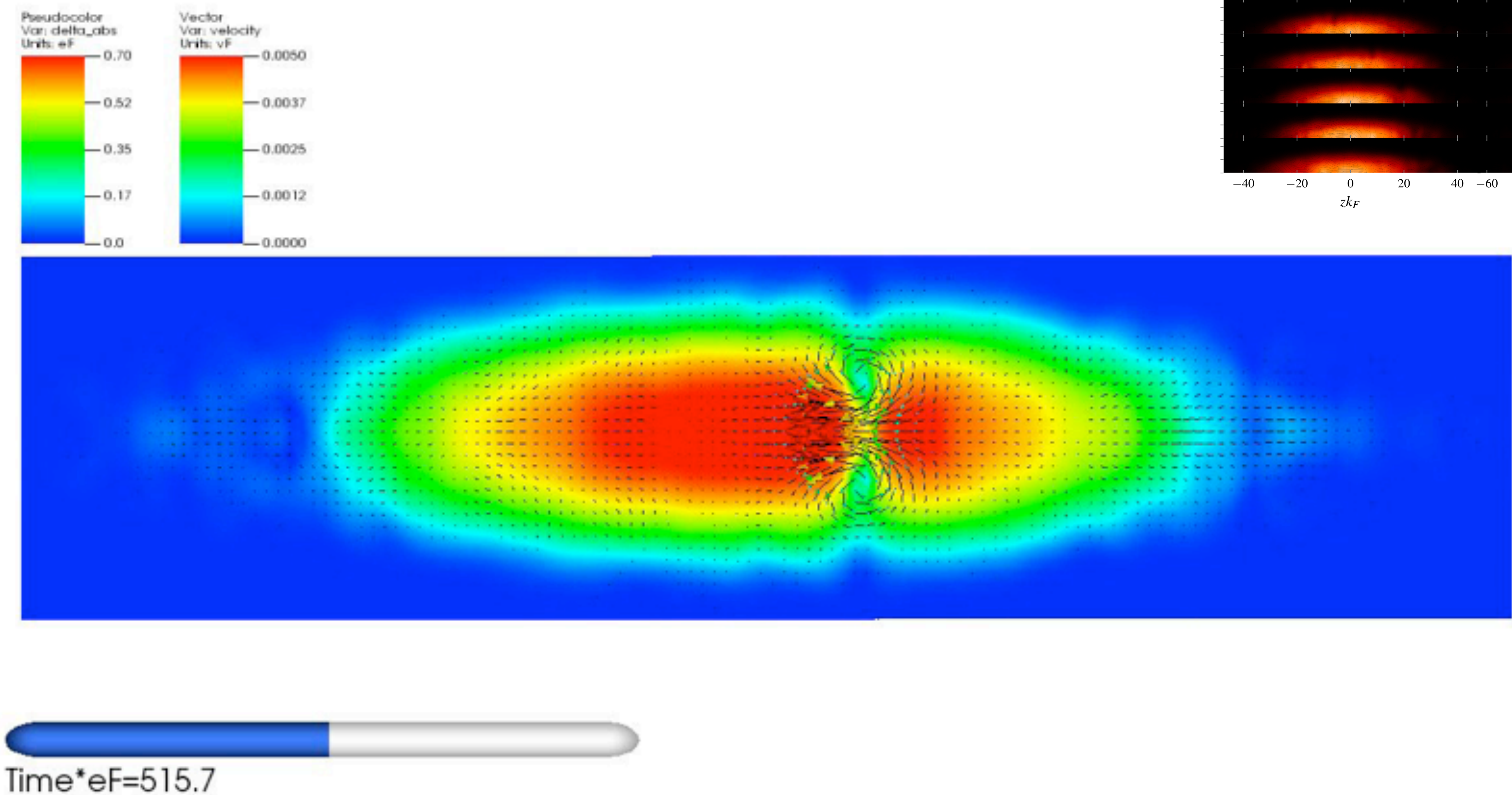
Moves much slower than expected



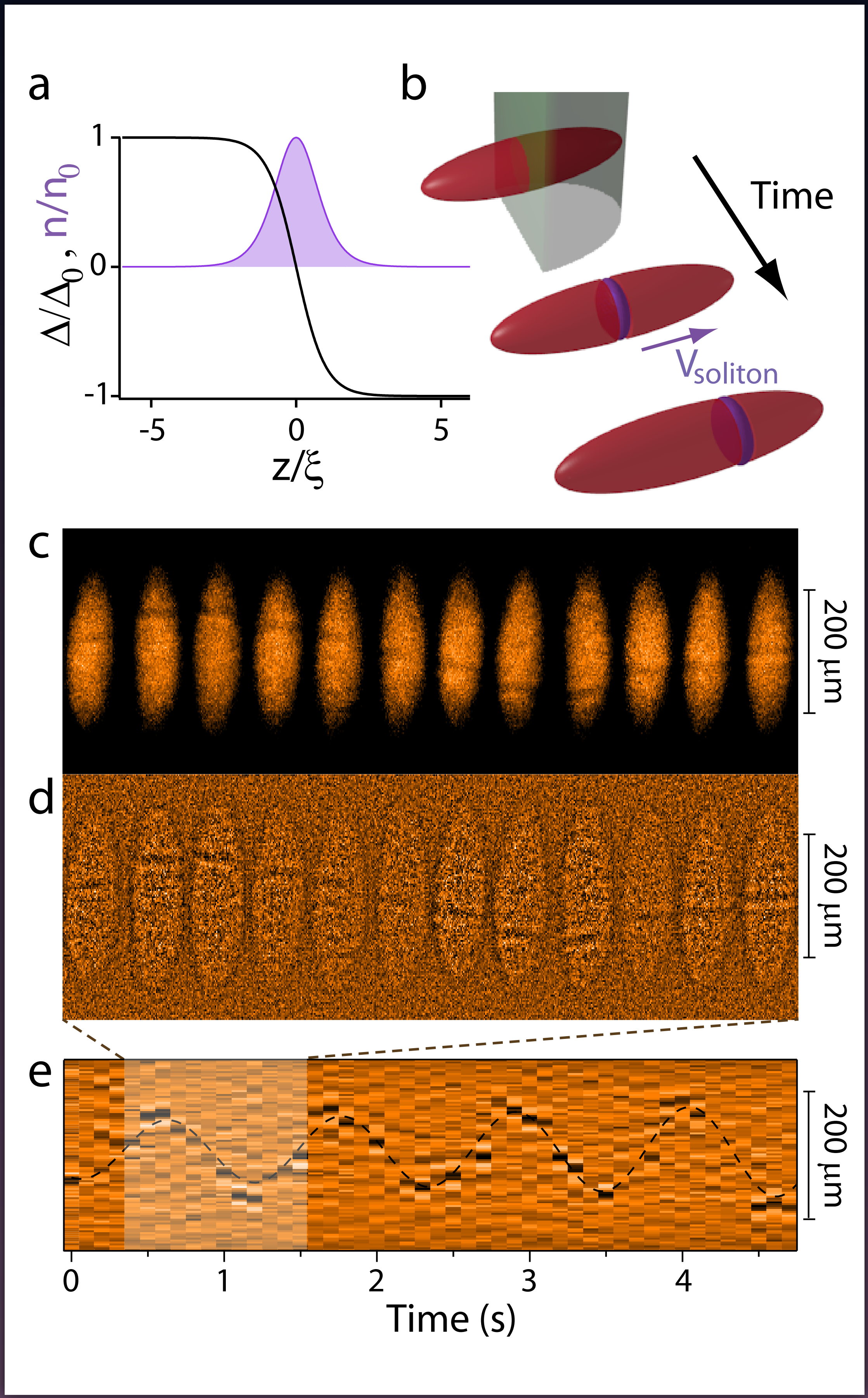
Vortex Rings?

- Result of expected snake instability
- Naturally move slowly



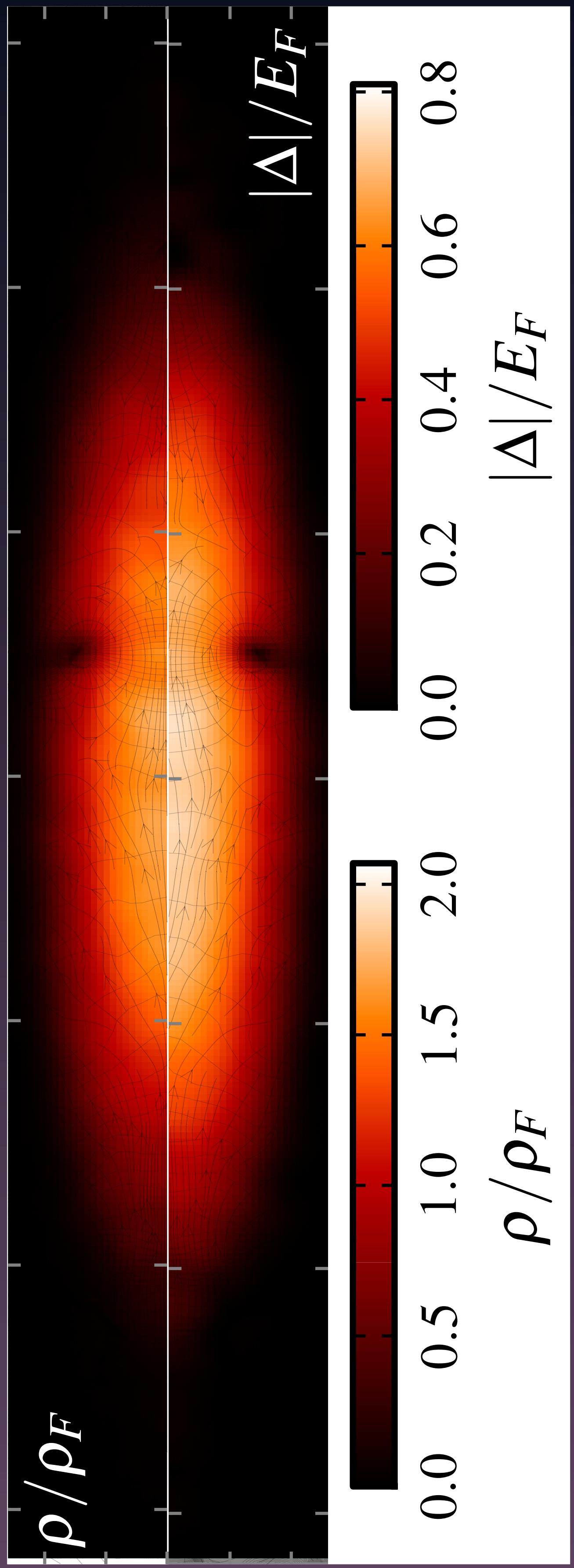


Bulgac et al.
arXiv:1306.4266



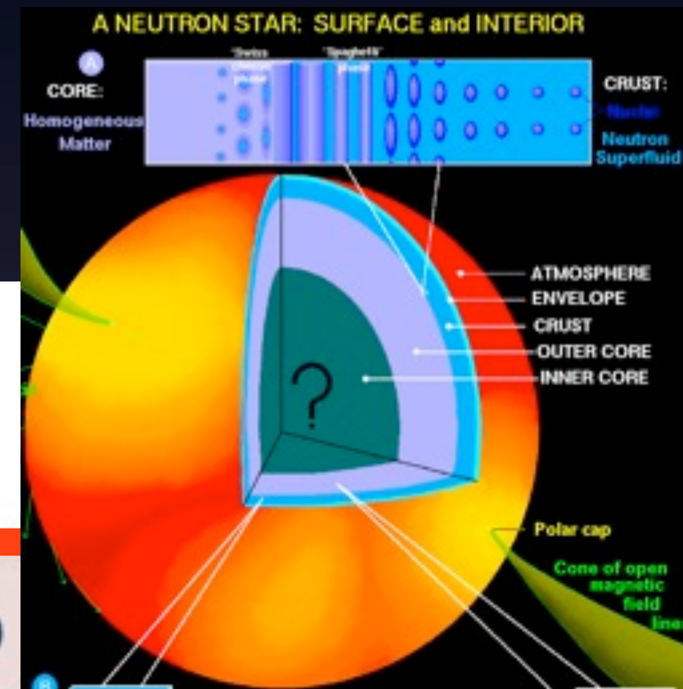
The same?

expansion?
vortex tangle?

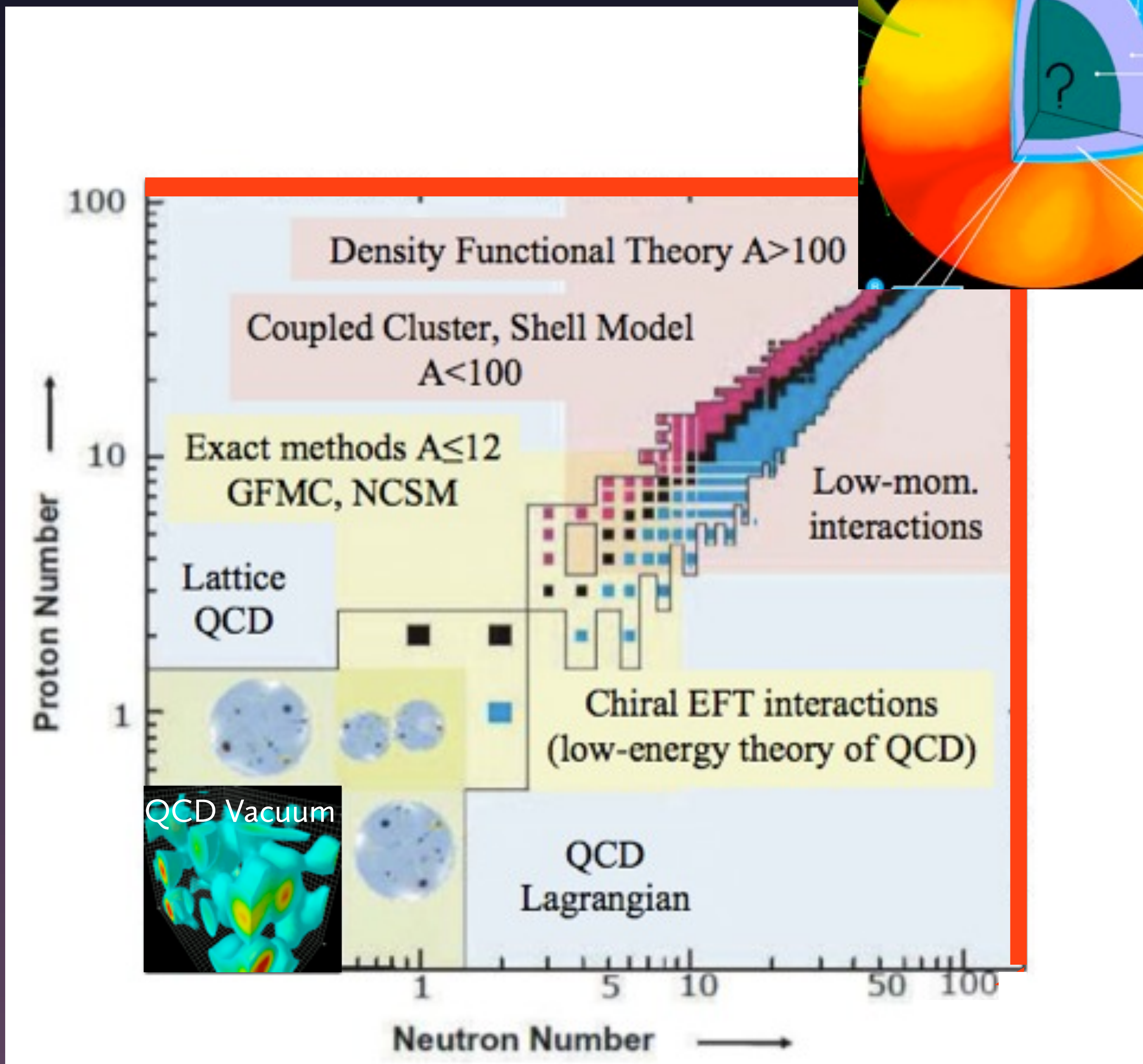


Yefsah et al. (MIT Experiment) arXiv:1302.4736

Monday, July 29, 13



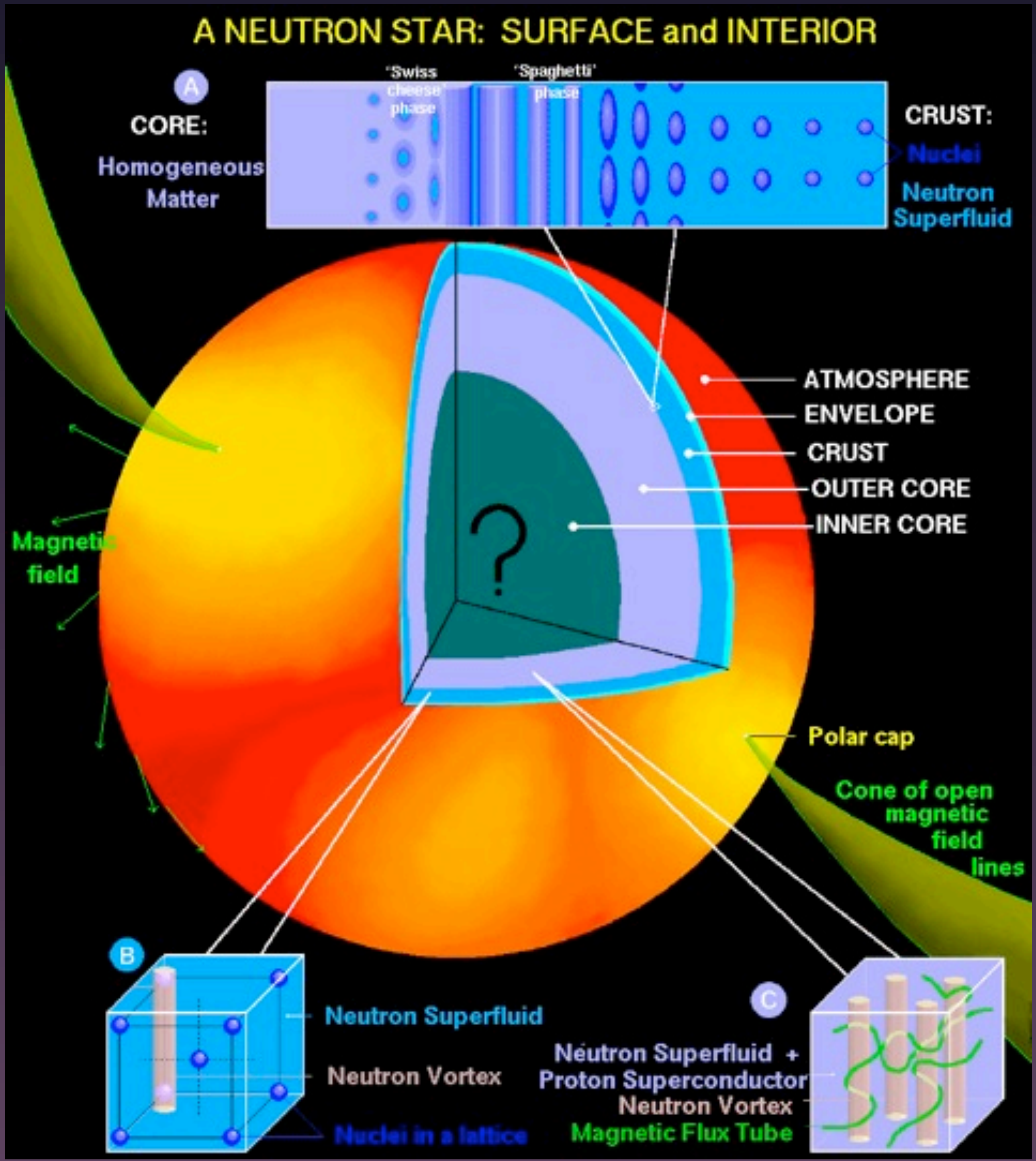
The Nuclear Landscape



- Lattice QCD,
  nucleons, interactions
- QMC, etc.
  small to medium nuclei
- DFT,
  medium to large nuclei
- Neutron stars?
  Molecular Dynamics
  Hydrodynamics

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

Neutron Stars



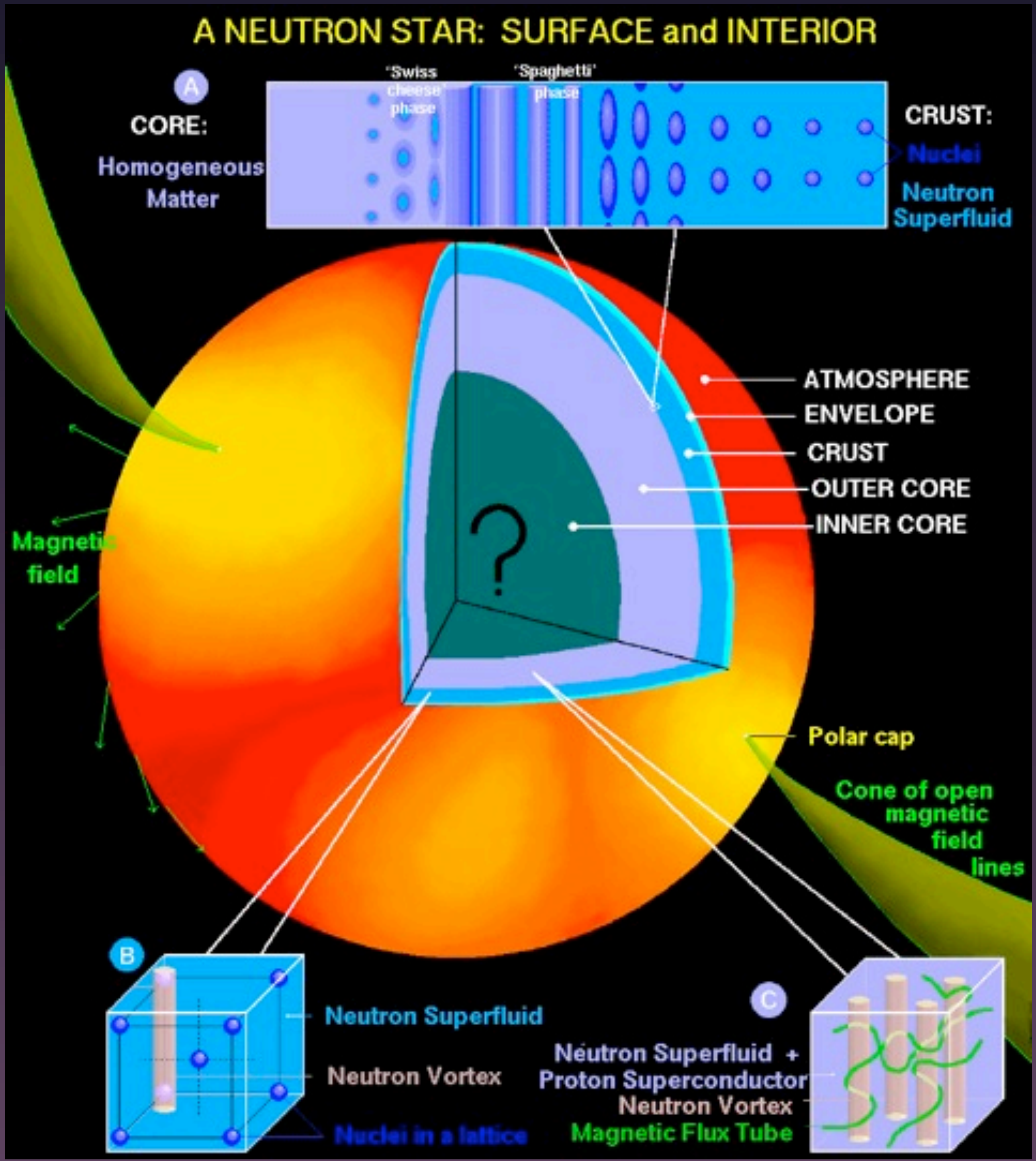
Dany Page: http://www.astroscu.unam.mx/neutrones/NS-Picture/NS-Picture.html

Neutron superfluid in Crust is almost a Unitary Fermi Gas
($a_s \sim -7r_e$, $k_F a_s \sim -10$)

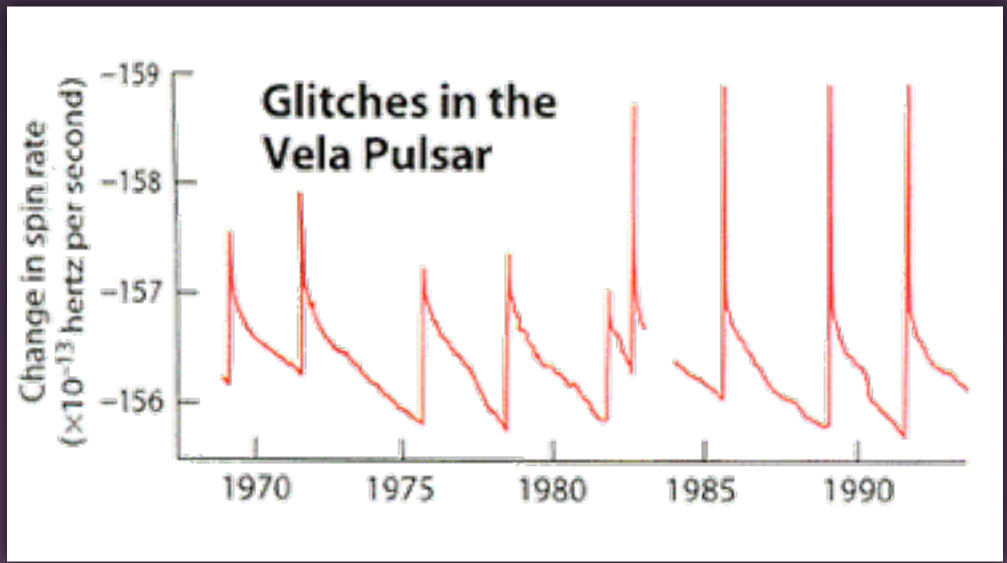
Many relevant phenomena
- Vortex pinning (glitches)
- Heat transport
- Equation of State

Can we use cold-atoms to model nuclear matter?
- More complicated interactions
  - Three-body, tensor forces etc.

Glitches



- Rapid increase in pulsation rate
- Anderson and Itoh (1975) suggested pinned superfluid vortices



Pulsar Astronomy by Andrew G. Lyne and Francis Graham-Smith

Dany Page: http://www.astroscu.unam.mx/neutrones/NS-Picture/NS-Picture.html

From Cold Atoms to Neutron Stars

- Use (expensive) Fermi calculations to determine parameters (vortex nucleus interaction)
  - Validate with cold atoms
  - Time-dependent method scales well: Bulgac, Forbes and Sharma (2013)
- Fit a GPE-like theory
  - Use this to model macroscopic dynamics

Conclusion Femion Superfluids

- QCD
- Cold Atoms
- Nuclei
- Neutron Stars
- Dark Matter

- Universal aspects of many body physics
- Similar techniques, different physical problems
- Use one field to test and understand others