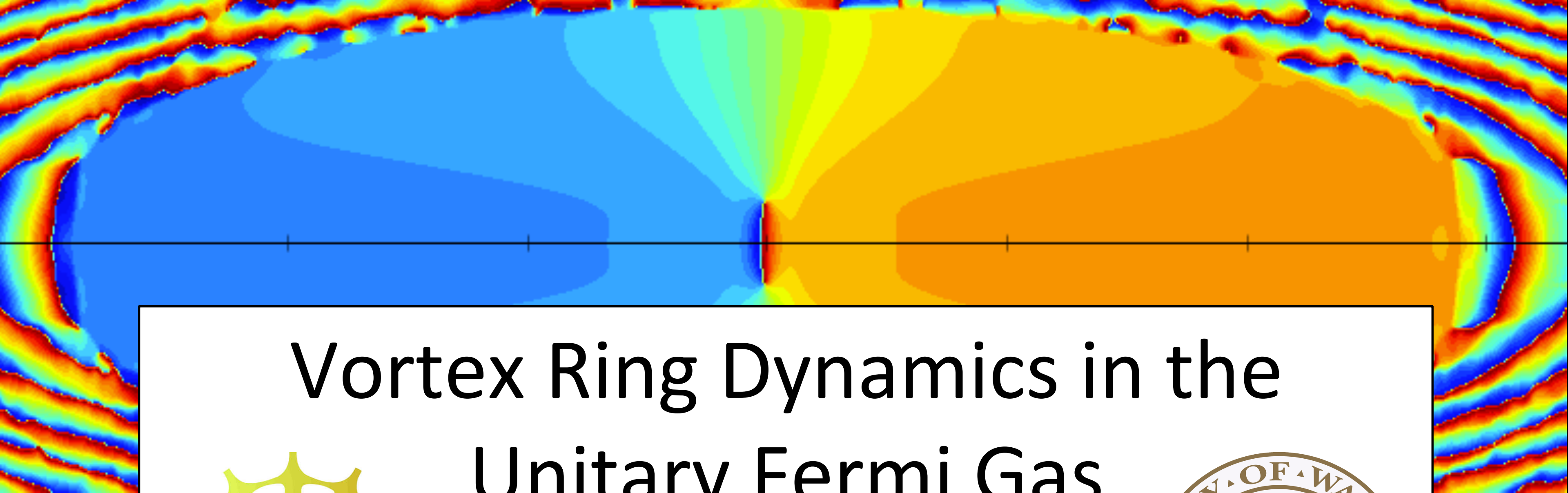

Vortex Ring Dynamics in the Unitary Fermi Gas

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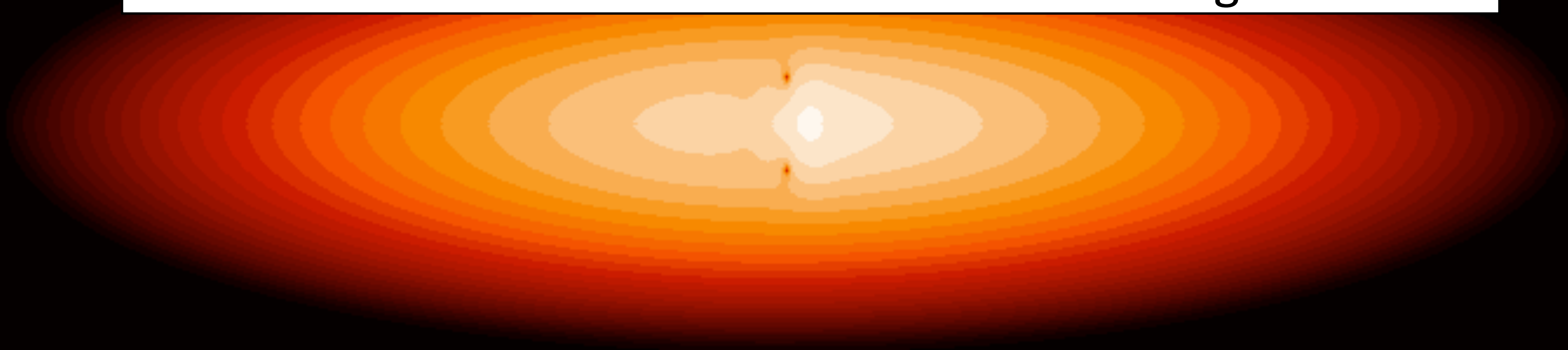
Advisors: Michael Forbes and Aurel Bulgac



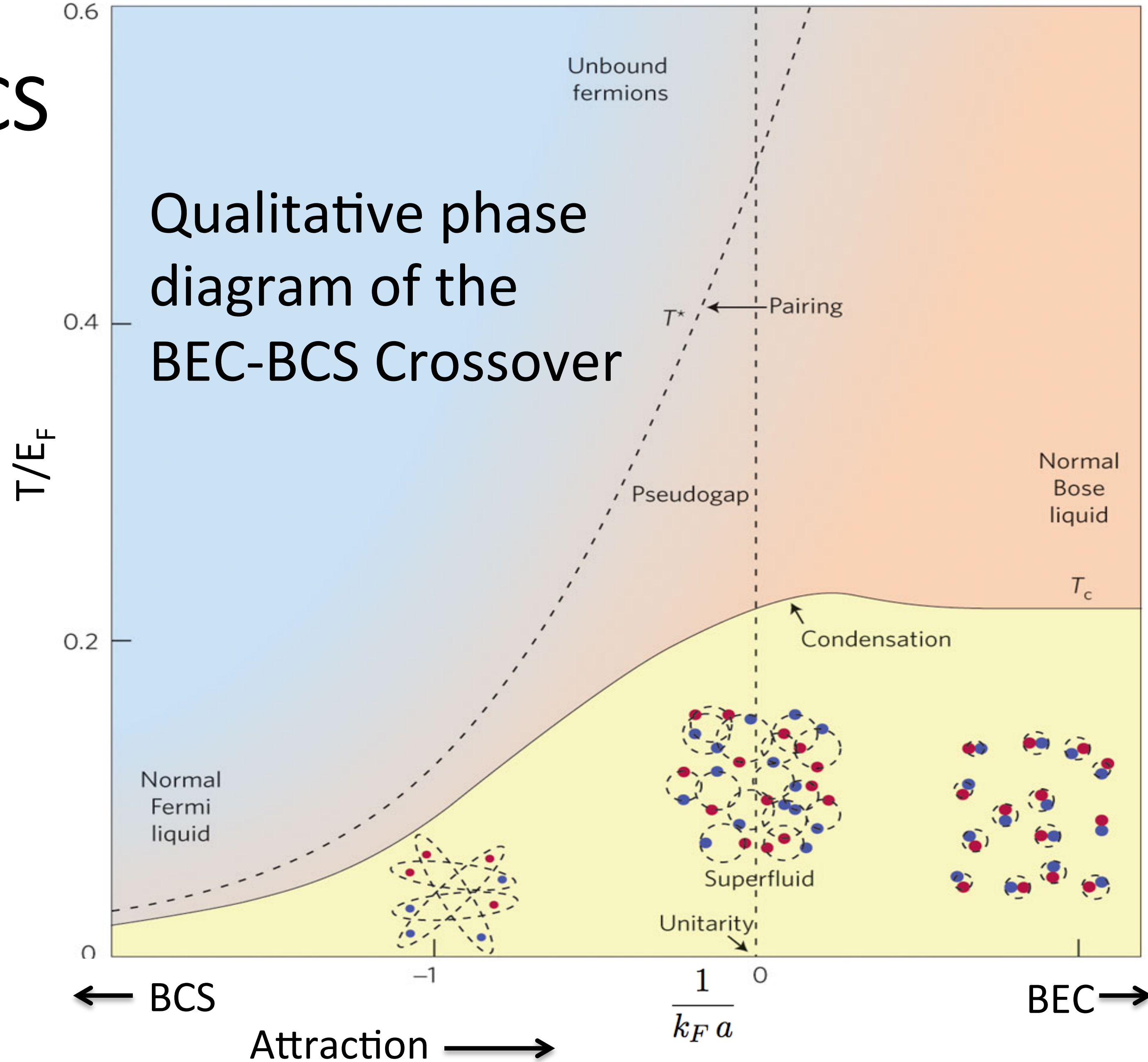
The BEC-BCS Crossover

Theoretically predicted, realized experimentally in 2002

Dilute Limit:

$$r_0 << \frac{1}{k_F}$$

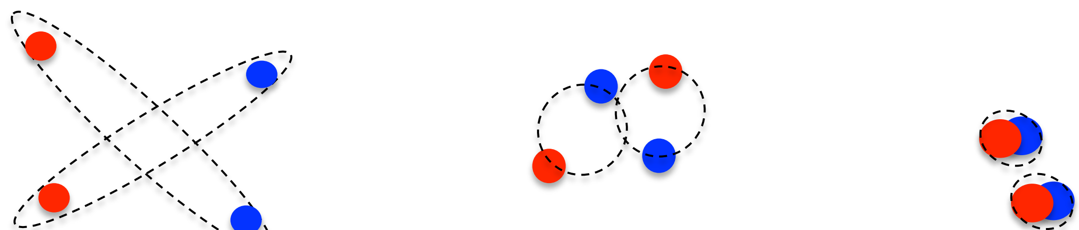
Qualitative phase diagram of the BEC-BCS Crossover



Randeria et al. DOI: 10.1038/nphys1748

Ultracold atom interactions can be tuned with the external magnetic field

Feshbach resonance: $a(B) \sim \frac{C}{B - B_{\mathrm{res}}}$



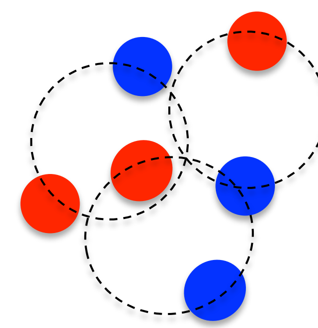
BCS ← Unitary → BEC

Long range cooper pairs	Overlapping cooper pairs	Tightly bound dimers
$\frac{1}{k_F\, a} \to -\infty$	$\frac{1}{k_F\, a} = 0$	$\frac{1}{k_F\, a} \to \infty$

The Unitary Fermi Gas: $\frac{1}{k_F a} = 0$

Length scales drop out to obtain a theory with a dimensional scale set by particle density

Measurable quantities take a universal form
and dimensional analysis can be used
(e.g. energy density)



$$\mathcal{E}(\rho) = \xi\, \mathcal{E}_{FG}(\rho) \qquad \xi = 0.376(5)$$

Agreement between theory and experiment

The Many-Body Problem

Wavefunctions become *exponentially hard*

One particle: $\Psi(x,t) = N_x^3 \; N_t$

N particles: $\Psi(x,t) = (N_x^3)^N \, N_t$

Numerical methods, models, effective theories are needed (QMC, DFT, MFT, ...)

BEC-BCS crossover provide new avenues to check the many-body theory

Further implications (Nuclei, Neutron matter, ...)

Density Functional Theory

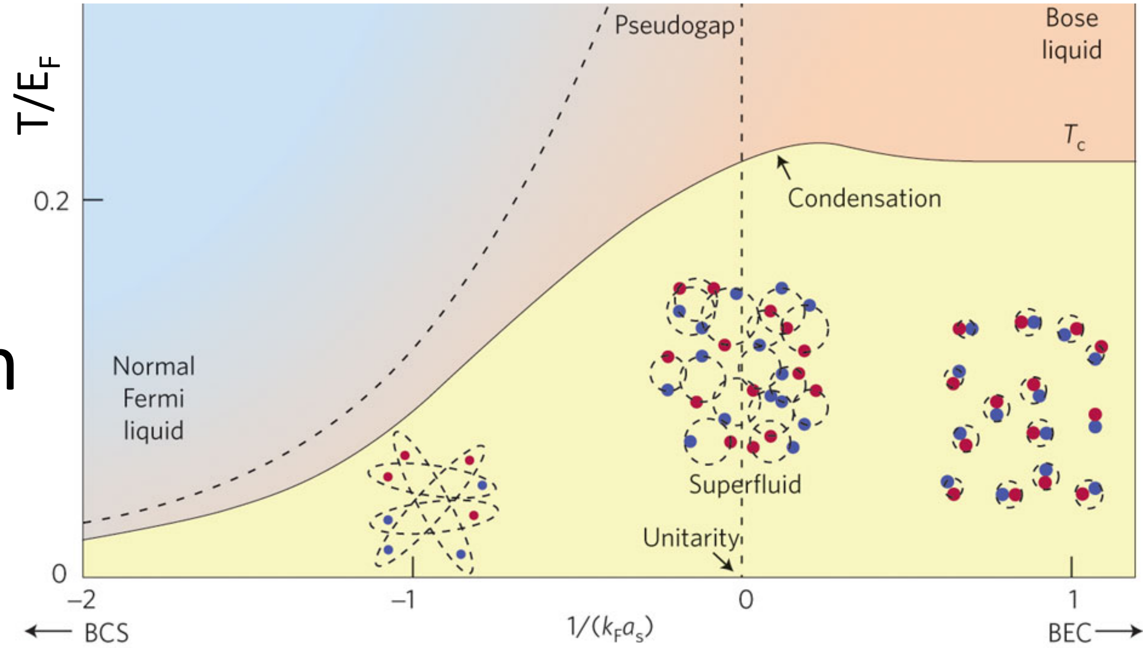
Hohenberg-Kohn theorem:

There exists a functional when minimized that exactly describes the ground state of a system

$$E[n(\mathbf{r})] + \int d^3\mathbf{r}\, V_{\text{ext}}(\mathbf{r})\, n(\mathbf{r})$$

Time-Dependent Superfluid Local Density Approximation (TDSLDA), very effective but requires world's largest supercomputers

Less expensive computationally to describe system with the Gross Pitaevskii Model



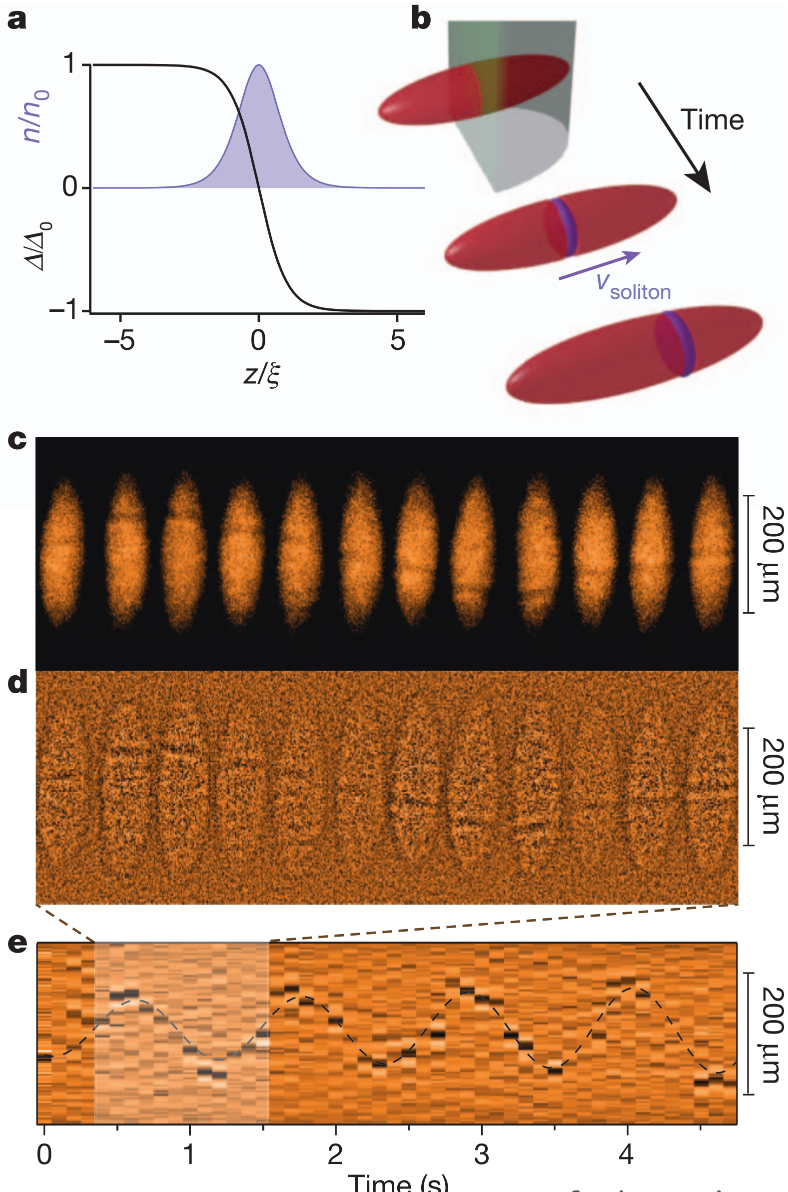
Attraction ⟶

The extended Thomas-Fermi model well describes the Unitary Fermi Gas in superfluid regime

$$i\hbar\partial_t\Psi = \left(\frac{-\hbar^2\nabla^2}{4m_F} + 2V + 2\xi E_F(\rho)\right)\Psi$$

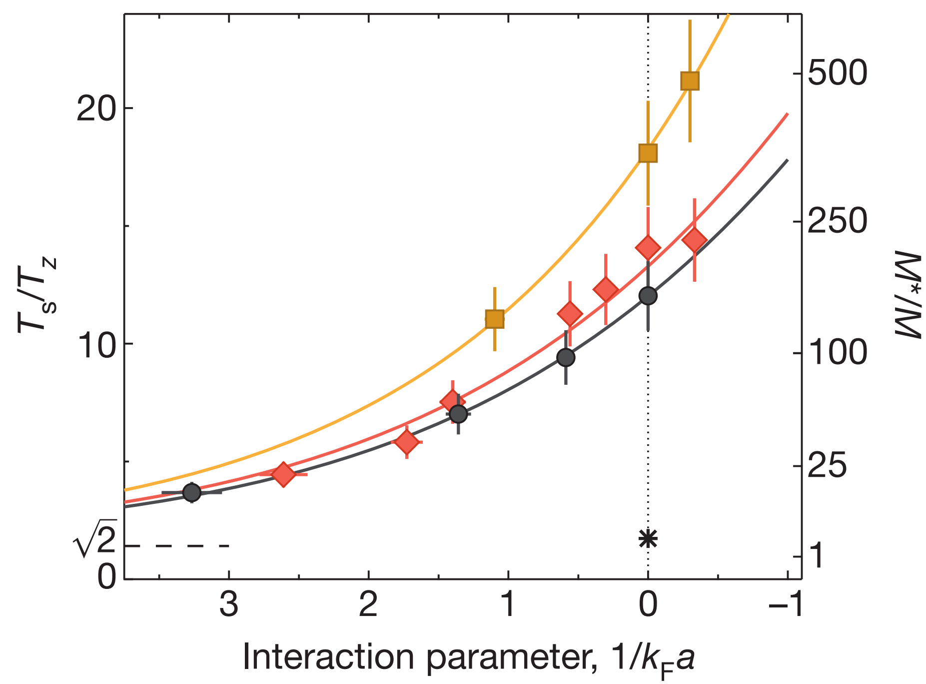
Randeria et al. DOI: 10.1038/nphys1748

MIT-Harvard Center for Ultracold Atoms



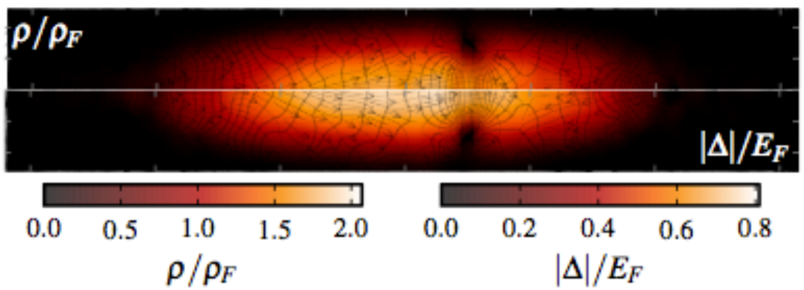
Oscillations observed longer than predicted

Heavy soliton?



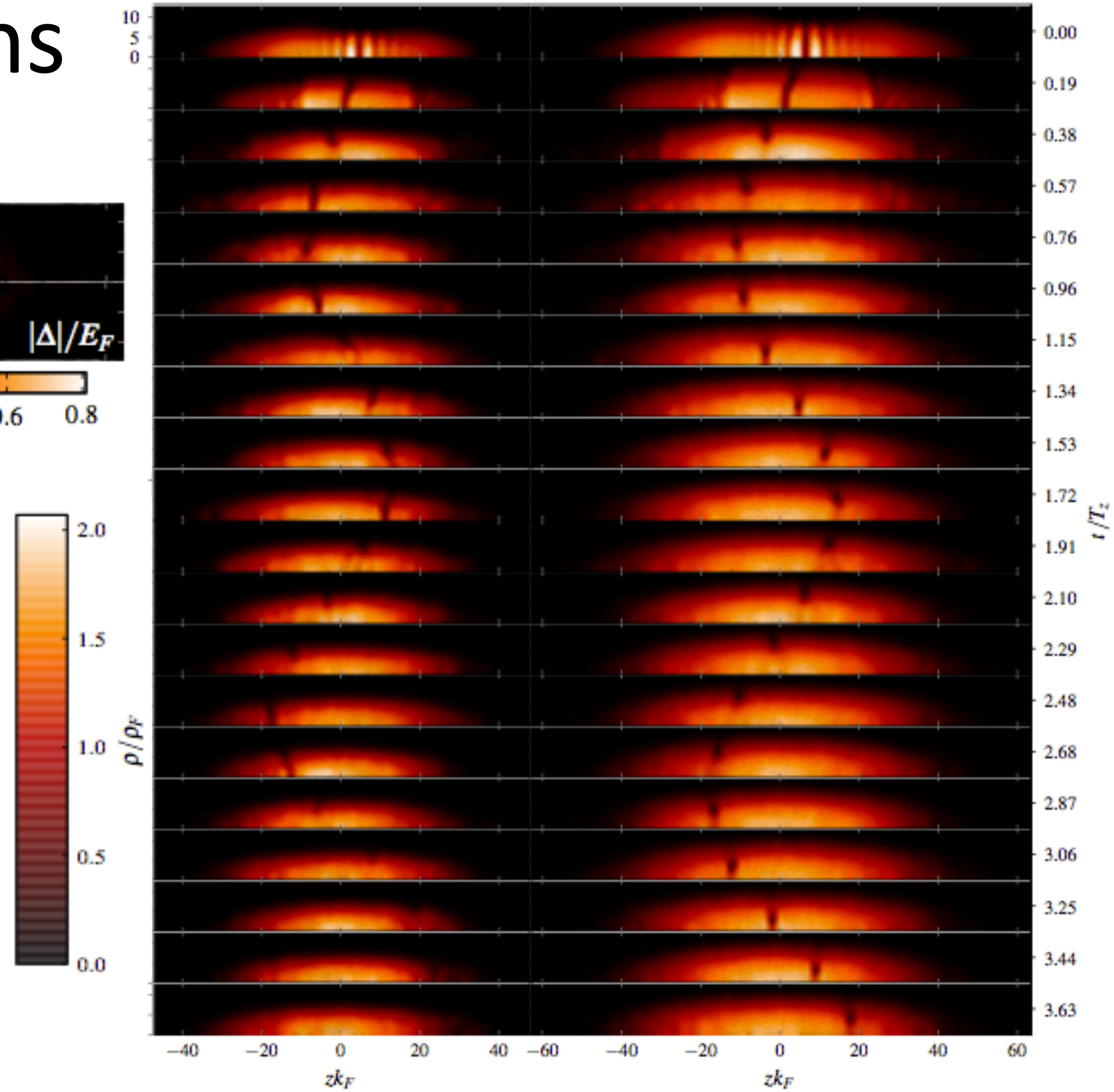
Yefsah et al. DOI: 10.1038/nature12338

Vortex Rings naturally describe the long oscillations



Vortex rings product of Snake Instability

Period as expected in domain wall limit

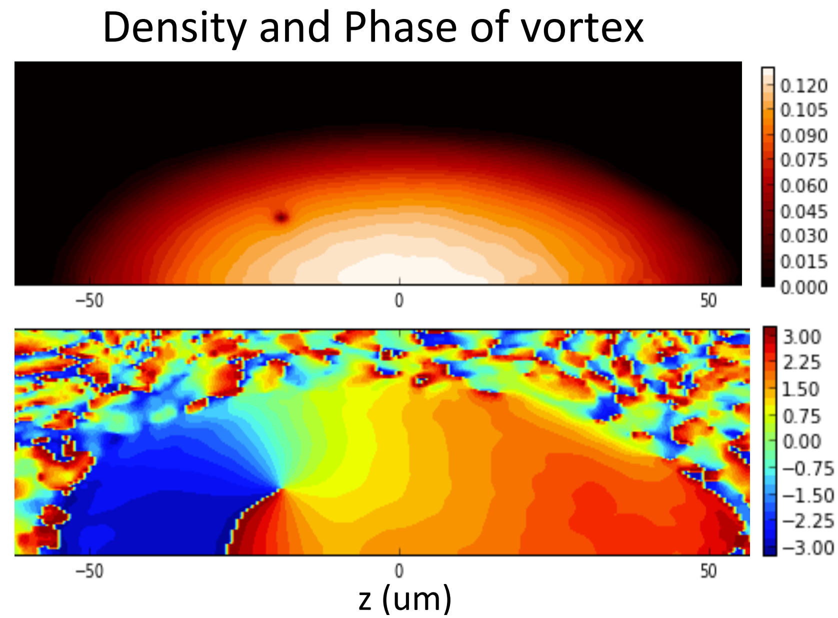


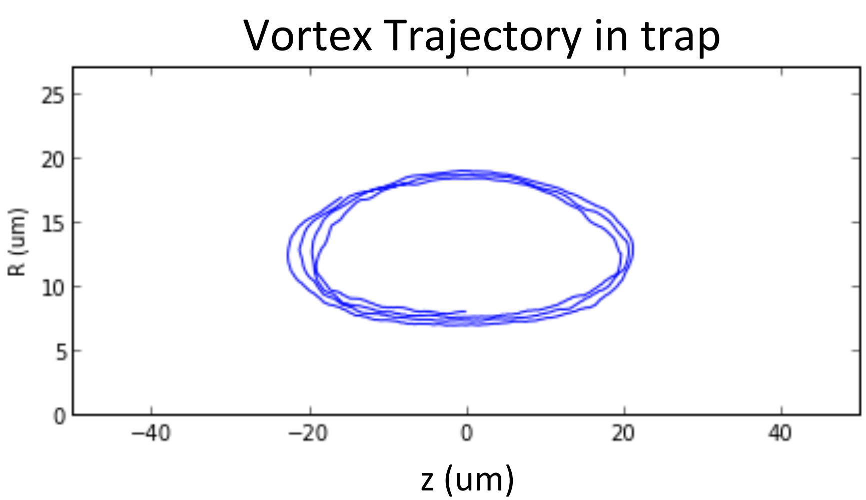
Bulgac et al. arXiv:1306:4266

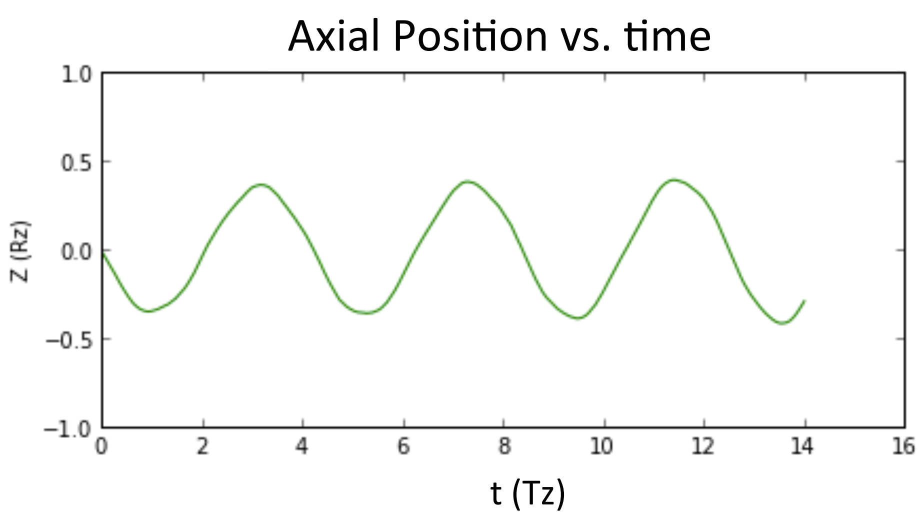
First Task: One vortex ring in a small trap

Fewer particles, small trap size for quicker simulations

Created a vortex tracking function







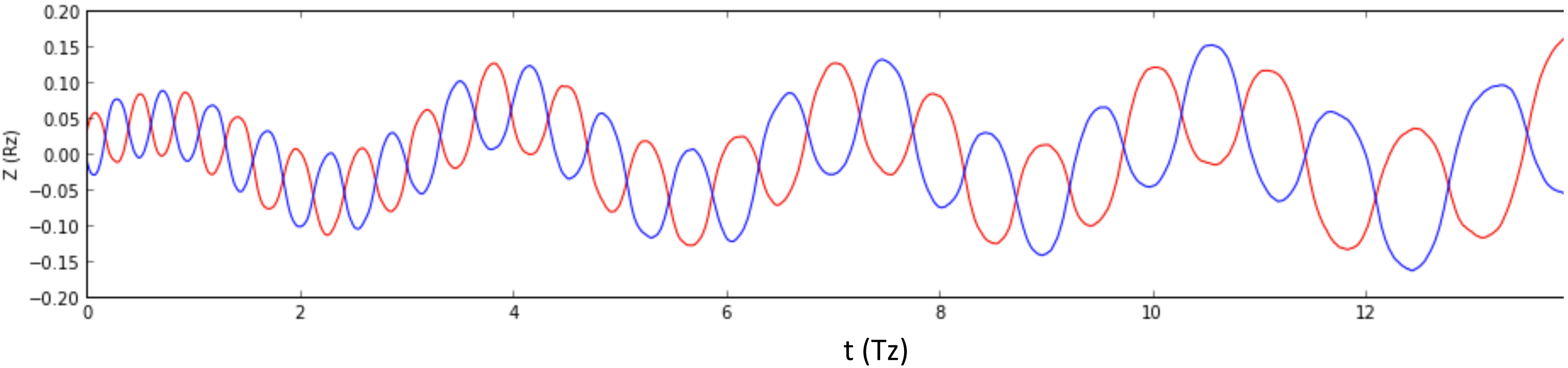
Second Task: Two orbiting vortex rings in a small trap

Source: http://i.minus.com/iUQsj2aoNRlQI.gif

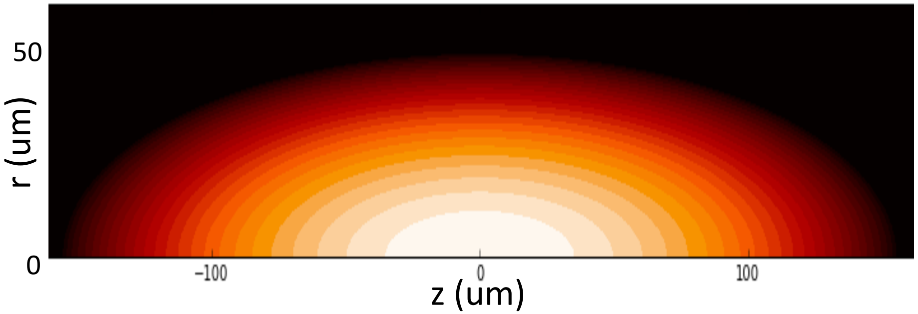
Shorter period than one vortex at similar radius

Next: Vortex tangle?

Axial Position vs. time for both vortices

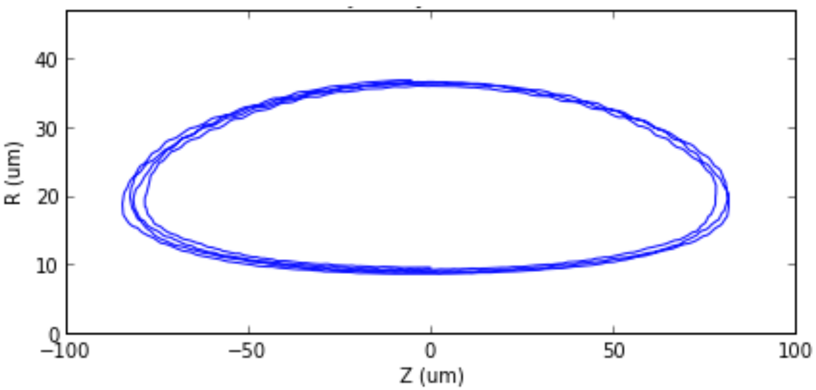


One vortex ring in a realistic trap

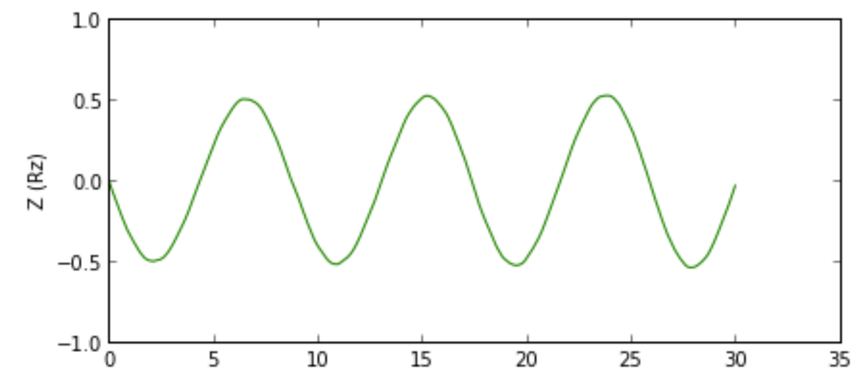


Imprint Location	Period
$R_0 = 0.20\ R_\perp$	$T = 8.6\ T_z$
$R_0 = 0.30\ R_\perp$	$T = 9.9\ T_z$
$R_0 = 0.40\ R_\perp$	$T = 10.7\ T_z$
$R_0 = 0.50\ R_\perp$	$T = \ 11\ T_z$

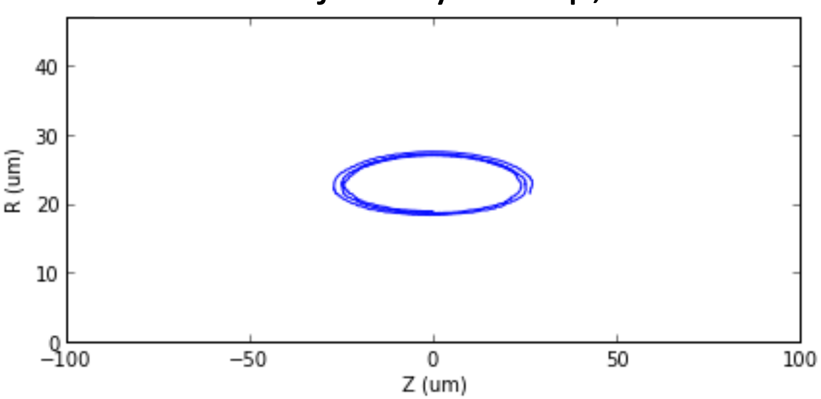
Vortex Trajectory in trap, R0=0.2



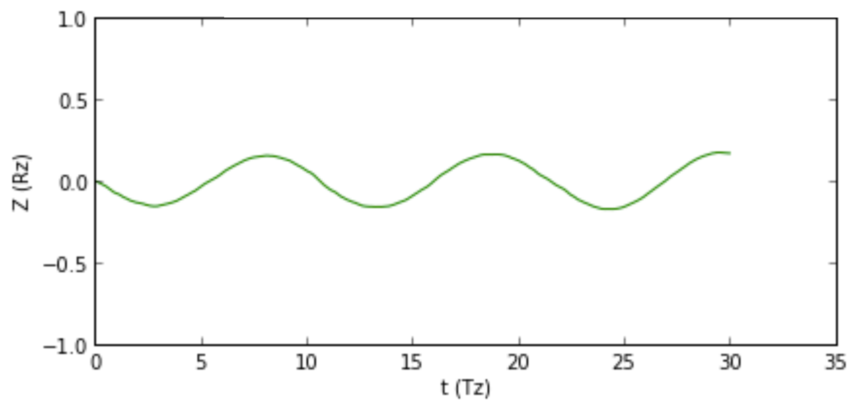
Axial Position vs. time



Vortex Trajectory in trap, R0=0.4

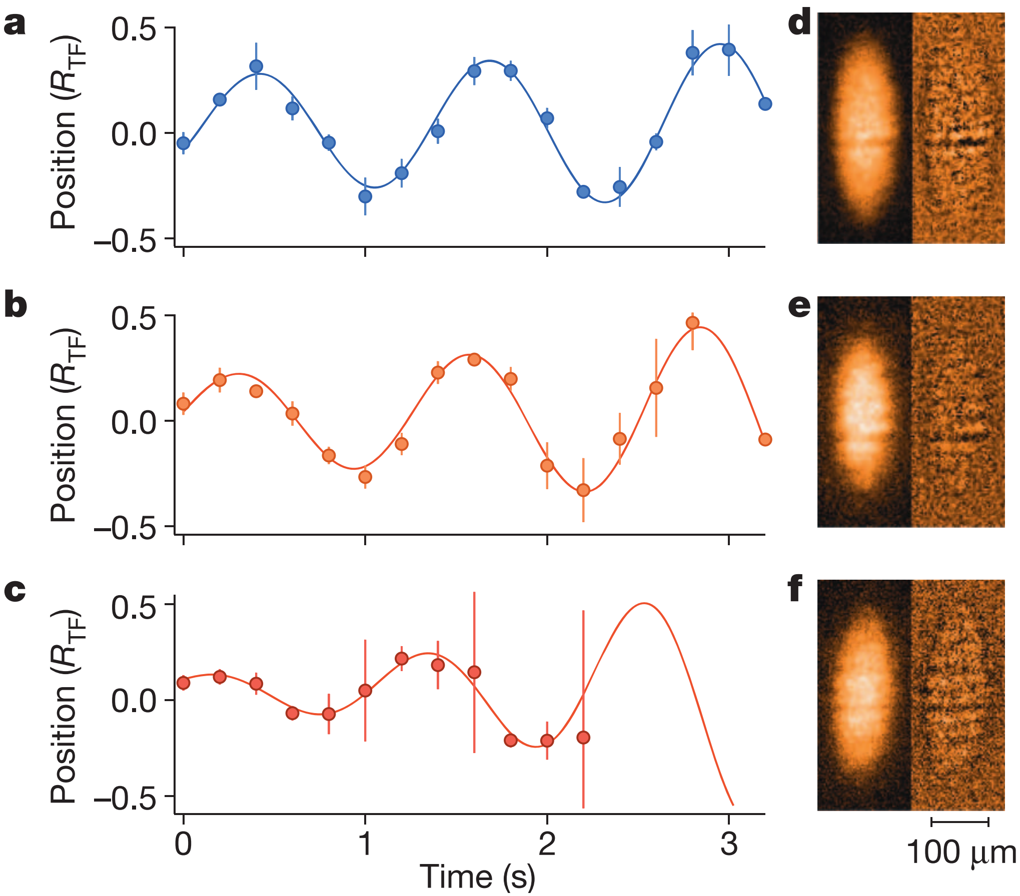


Axial Position vs. time



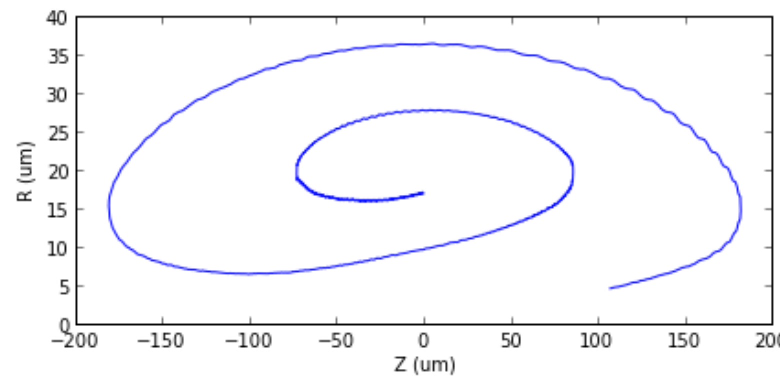
Anti-damping in oscillations result from systems cooled insufficiently

Experiment:

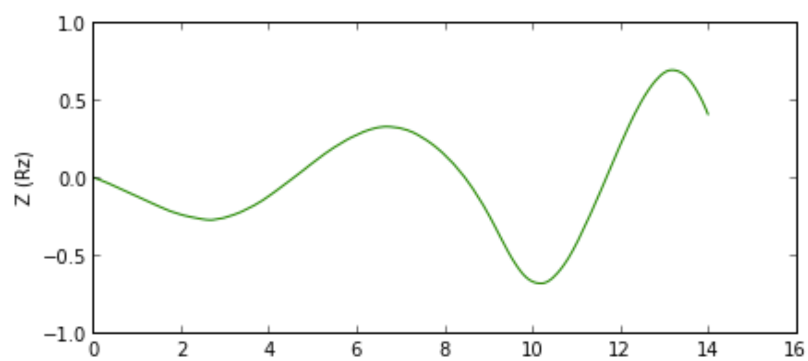


Yefsah et al. DOI: 10.1038/nature12338

Vortex Trajectory, insufficiently cooled



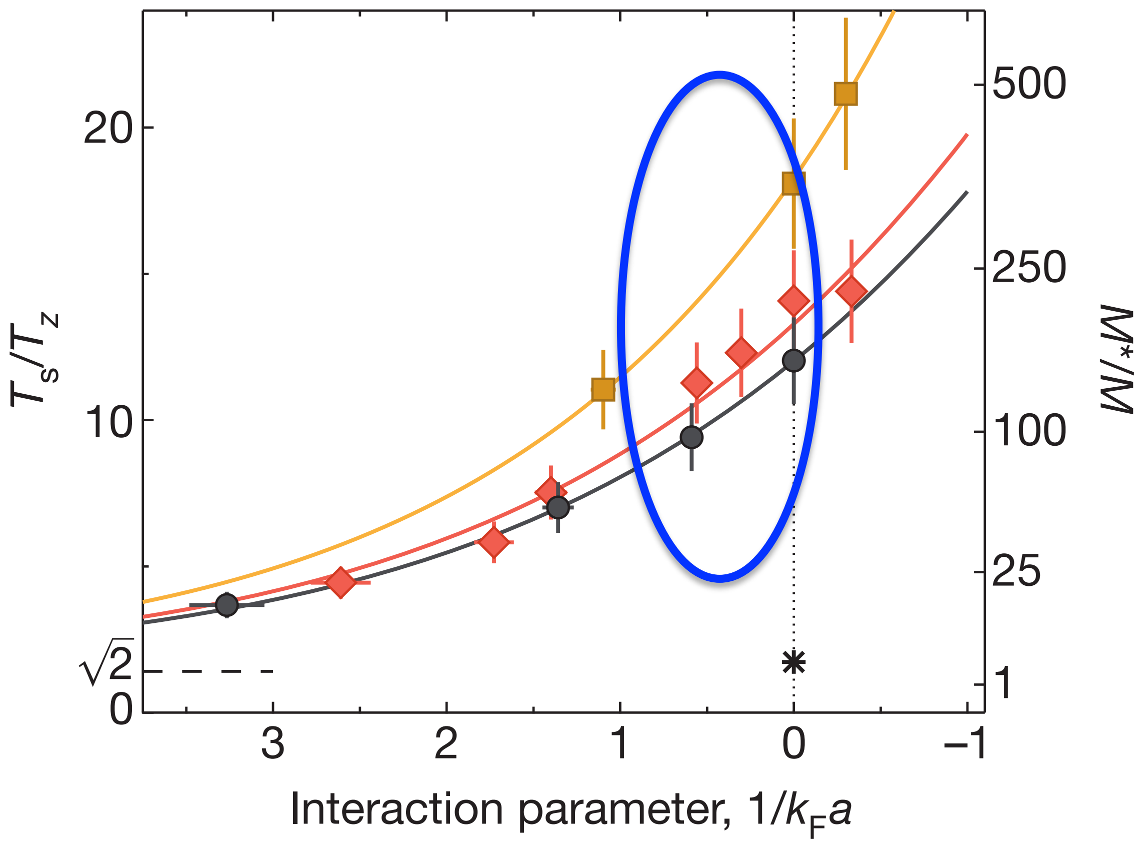
Axial Position vs. time



Tasks still in process…

Extracting period dependence on:

- Different aspect ratios
- Magnetic fields in BEC regime



Yefsah et al. DOI: 10.1038/nature12338

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- All the REU students

Systems evolved with Split Operator method

$$\Psi(T) = \lim_{N\to\infty} \left(e^{iK\delta_t/2} e^{i\int_t^{\delta_t} V\,dt} e^{iK\delta_t/2} \right)^N \Psi(0)$$

$$\delta_t = T/N$$

Splitting the Kinetic energy makes error scale to 3rd order, as opposed to 2nd order.

Superfluid Local Density Approximation (SLDA)

Energy density functional takes the form

$$\mathcal{E}(\mathbf{r}) = \alpha\frac{\tau(\mathbf{r})}{2} + \beta\frac{3(3\pi^2)^{2/3}n^{5/3}(\mathbf{r})}{10} + \gamma\frac{|\nu(\mathbf{r})|^2}{n^{1/3}(\mathbf{r})}$$

Particle density: $n(\mathbf{r}) = 2\sum_k |v_k(\mathbf{r})|^2$

Kinetic density: $\tau(\mathbf{r}) = 2\sum_k |\nabla v_k(\mathbf{r})|^2$

Anomalous density: $\nu(\mathbf{r}) = \sum_k v_k^*(\mathbf{r})u_k(\mathbf{r})$

Parameters α, β, and γ fitted with QMC