

Shimming of a Magnet for Calibration of NMR Probes for the Muon g-2 Experiment

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Abstract

The Muon g-2 Experiment at Fermilab aims to measure the anomalous magnetic moment $a_\mu \equiv \frac{(g-2)}{2}$ of the muon to a precision of 0.14 parts per million. This experimental value of a_μ can then be compared to the similarly precise theoretical predictions of the Standard Model in order to test the completeness of the model. The value of a_μ is extracted from muons precessing in a magnetic field. The magnetic field will be measured with a set of approximately 400 Nuclear Magnetic Resonance (NMR) probes, which have the ability to measure the magnetic field to tens of parts per billion. Some of the probes will be set in grooves at the top and bottom of the muon beam chamber, and some will be placed in a trolley which will be moved around the storage ring. Before the Muon g-2 Experiment can take place, new NMR probes must be designed, built, and tested using a 1.45 Tesla test magnet at the University of Washington Center for Experimental Nuclear Physics and Astrophysics (CENPA). In order to achieve a significant signal from NMR probes, the magnetic field in which the probes are immersed must be extremely uniform. The existing magnet at CENPA has an approximately linear gradient in magnetic field of about 1 Gauss per centimeter in the smoothest direction. A pair of adjacent square Helmholtz coils was designed and built to create a linear gradient in order to cancel the existing gradient. The length of the NMR signals improved with the implementation of the coils. Problems with the old NMR probes were also diagnosed using resonances from the probe circuit.

1 Introduction

Precision measurements are one method of testing the theoretical model of modern physics. The electron's magnetic anomaly is one of the best measured universal constants in physics, and the value agrees very well with QED predictions. The measurement of the electron anomaly demonstrates the accuracy to which QED processes may be predicted. The muon is 207 times more massive than the electron, so the

muon has a smaller Compton wavelength and the muon anomaly therefore probes the vacuum more deeply for the effects of virtual particles.

1.1 The Muon Anomaly

The magnetic moment of a particle, $\vec{\mu}$, is related to spin, \vec{s} , by:

$$\vec{\mu} = g\mu_B\vec{s}$$

where μ_B is the Bohr magneton of the particle, related to the charge e and mass m by:

$$\mu_B = \frac{e\hbar}{2mc}$$

The proportionality constant g , called the g-factor, is equal to 2 for point-like fermions according to relativistic quantum mechanics. The g-factor of non-point-like particles, such as protons and neutrons, may differ largely from 2 due to their inner structure, and the g-factor of point-like leptons, such as muons, may differ slightly from 2 due to interactions between the particle and virtual fields. The magnetic anomaly is the deviation from the g-factor of a point-like fermion, defined as $a_\mu = \frac{g-2}{2}$. Over over 99.99% of the virtual field contributions to the muon anomaly are due to the electromagnetic interaction. The next largest contribution is from the strong interaction, and theoretical work on interactions and scattering into hadrons is ongoing. The weak interaction is another contribution to the muon anomaly, predicted from the lowest order single loop diagrams. The final contribution to the muon anomaly is from physics not yet contained in the standard model, such as vacuum polarization loops with the lightest particles in supersymmetric models [5].

1.2 The Muon g-2 Experiment

The Muon g-2 Experiment at Fermilab aims to measure the muon anomaly to 0.14 ppm. The measurement of a_μ will require precise measurements of the muon spin frequency within a magnetic field, as well as the average value of the magnetic field over the muon distribution. The equation used in the g-2 Experiment to determine the muon anomaly is given by:

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

where μ_μ/μ_p is the ratio between the proton and muon magnetic moments which was determined in muonium hyperfine splitting experiments [?], ω_a is the difference frequency between the muon spin and cyclotron frequencies, and ω_p is the Lar-

mor precession frequency of free protons.

The g-2 experiment will measure the anomalous magnetic moment of the muon from muons precessing in a magnetic storage ring, called the g-2 ring. A pulsed proton beam will be accelerated and collide with a target to create pions. The pions undergo the decay:

$$\pi^\mp \rightarrow \mu^\mp + \bar{\nu}_\mu(\nu_\mu)$$

The pion spin is zero and the neutrino is left-handed (the anti-neutrino is right-handed). The muon spin must be antiparallel to the neutrino spin and therefore a polarized beam of muons may be obtained by selecting by momentum either the forward beam or backward beam of muons.

This polarized beam of muons will be injected into the g-2 storage ring, where the muons will precess about the axis of the magnetic field with a spin frequency:

$$\omega_s = -g\frac{Qe}{2B} - (1 - \gamma)\frac{Qe}{\gamma m}B$$

The muons also travel around the storage ring with a cyclotron frequency:

$$\omega_c = -\frac{Qe}{\gamma m}B$$

The frequency taken in the g-2 Experiment is the difference frequency given by:

$$\omega_a = \omega_s - \omega_c = -a_\mu \frac{Qe}{m}B$$

The value of ω_a may be directly measured by the decay of muons in the g-2 storage ring. Muons undergo the decay:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

Due to parity violation, there is a preferred angle for the emission of the positron. Because of this preferred direction, detectors placed around the g-2 storage ring detect oscillations in positron count rates [1]. The frequency of oscillation in the positron count rate is related to ω_a . These oscillations are shown in Figure 1.

The average magnetic field over the muon distribution is related to ω_p , the Larmor preces-

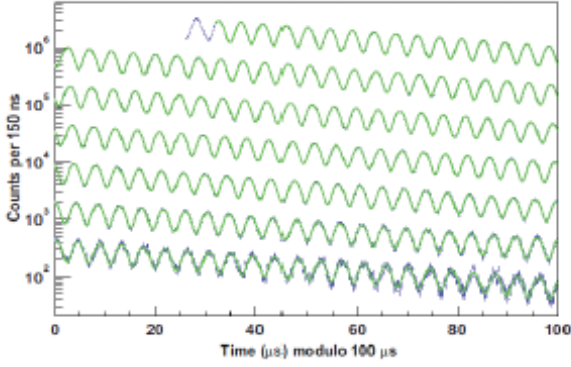


Figure 1: Positron Detection Frequency

sion of free protons. This value will be measured in the g-2 Experiment using approximately 400 NMR probes, some of which will be set at the bottom and top of the storage ring and some of which will be placed in a trolley which will move around the storage ring when no muons are present. The NMR probes will be built and calibrated at the University of Washington. Careful determination of the magnetic field in the g-2 storage ring is important for the measurement of a_μ .

1.3 Nuclear Magnetic Resonance

NMR probes will be used in the g-2 Experiment to determine ω_p . The evolution of a spin 1/2 particle in a magnetic field may be described both classically and quantum mechanically. To understand the classical treatment, consider a system with angular momentum \mathbf{j} and magnetic moment $\mathbf{m} = \gamma\mathbf{j}$, where γ is the gyromagnetic ratio. If this system is placed in a magnetic field \mathbf{B}_0 , the system will precess around the \mathbf{B}_0 field with a frequency ω_0 , which is called the Larmor precession frequency. The field exerts a torque on the

system, given by $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}_0$. The motion of the system is described by:

$$\frac{d\mathbf{j}}{dt} = \mathbf{m} \times \mathbf{B}_0$$

or, using the torque equation:

$$\frac{d}{dt}\mathbf{m}(t) = \gamma\mathbf{m}(t) \times \mathbf{B}_0$$

Adding a perpendicular linearly oscillating field $\mathbf{B}_1(t)$ is equivalent to adding a rotating field with an angular velocity ω . The total field is changed to $\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t)$ and the equation describing the motion of the moment becomes:

$$\frac{d}{dt}\mathbf{m}(t) = \gamma\mathbf{m}(t) \times [\mathbf{B}_0 + \mathbf{B}_1(t)]$$

Moving into the frame rotating with angular velocity ω , the relative motion of $\mathbf{m}(t)$ is:

$$\left(\frac{d\mathbf{m}}{dt}\right)_{rel} = \frac{d\mathbf{m}}{dt} - \omega\mathbf{e}_z \times \mathbf{m}(t)$$

Then, given the relations:

$$\omega_0 = -\gamma B_0$$

$$\omega_1 = -\gamma B_1$$

and substituting $\frac{d}{dt}\mathbf{m}(t)$ from the non-rotating frame into the rotating frame equation, motion in the rotating frame becomes:

$$\left(\frac{d\mathbf{m}}{dt}\right)_{rel} = \mathbf{m}(t) \times [(\omega - \omega_0)\mathbf{e}_z - \omega_1\mathbf{e}_x]$$

Setting $\Delta\omega = \omega - \omega_0$, the effective field in the rotating frame, \mathbf{B}_{eff} , becomes:

$$\mathbf{B}_{eff} = \frac{1}{\gamma}[\Delta\omega\mathbf{e}_z - \omega_1\mathbf{e}_x]$$

The resonance condition for this effective field is approached when $\Delta\omega$ approaches 0, or when $\omega \simeq \omega_0$. At the resonance condition, \mathbf{B}_{eff} has a component only in the x-direction, therefore, the moment will begin to precess around the field in the x-direction. Combining the effect of the precession in the rotating frame with that in the non-rotating frame, the effect of adding a linearly oscillating field is to change the precession of the moment from one pointing in the +z-direction and traveling around the z-axis, to one that passes through the x-y plane and eventually points in the -z-direction [2]. In NMR measurements, it is not desirable to completely flip the spins of the protons in the sample, but rather bring them down into the x-y plane and then let

them relax back to the $+z$ -direction with a " $\frac{\pi}{2}$ " pulse. These pulses are used because precession in the x-y plane will give the largest signal in an NMR probe, shown in Figure 2.

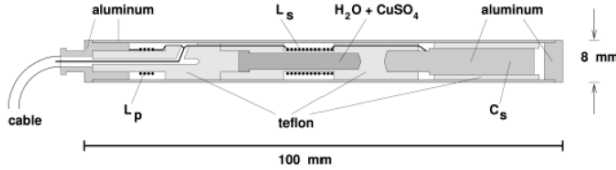


Figure 2: From Ref.[5]: NMR Probe used in the last g-2 experiment

The probe in Figure 2 represents a possible design for the trolley probes in the g-2 Experiment. The trolley probes will be placed in a trolley that will travel around the g-2 storage ring to take measurements of the magnetic field at different positions. In Figure 2, the protons are contained within a sample of water mixed with Copper sulfate. The sample is housed in the L_s inductor. This inductor will apply the linearly oscillating field to the sample in order to generate a signal. The same inductor will collect the signal from the protons when their magnetic moments are flipped into the x-y plane. The other components which complete the circuit of the probe are the L_p inductor, located at the left end of the probe, and the capacitor formed by inner aluminum of the probe, the aluminum cover, and the Teflon dielectric between the two aluminum pieces. All of these components have associated intrinsic resistances, but these resistances are small and will be ignored in this description of the resonant circuit. The probe is also connected to a coaxial cable which brings the signal to the rest of the NMR electronics. The circuit diagram of the probe is shown in Fig-

ure 3.

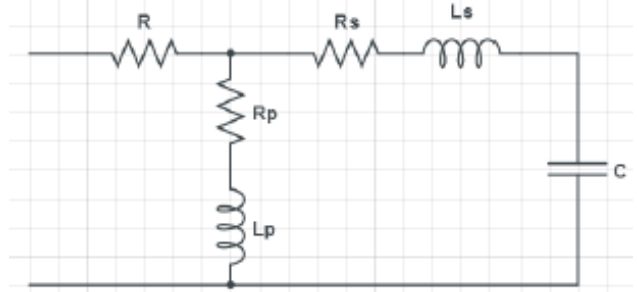


Figure 3: Probe Circuit

The LC circuit in the NMR probe and the attached coaxial cable lead to resonances at various signal frequencies. The series LC circuit is expected to reach minimum impedance at its resonant frequency because impedance in this circuit is given by $X = \omega L - \frac{1}{\omega C}$, where L is the inductance and C is the capacitance, and resonance occurs when the capacitor and inductor impedances cancel. The parallel LC circuit is expected to reach a maximum at resonance because the impedance of this circuit is given by $X = \frac{1}{\frac{1}{\omega L} - \omega C}$, so when the inductor and capacitor resonances match, the impedance should approach infinity. Resonances from the coaxial cable are added to the resonant features from the LC circuit in the probe. The impedance equations from transmission line theory state that when the cable length is a multiple of a quarter of the wavelength of the signal, the impedance in the cable will reach either a maximum or a minimum [7].

The resonances from the LC circuit are close together (over the range of a few MHz), and the $\frac{\lambda}{4}$ resonances from the coaxial cable are periodic and spaced with about 10 MHz between each pair of maxima or minima. In an ideal NMR probe, the minimum and maximum from the LC

circuits would be found on either side of the Larmor precession frequency, which is 61.79 MHz in a magnetic field of 1.45 T, the field to be used in the g-2 Experiment. Between these two resonances, the circuit of the probe would ideally have an impedance of 50 Ω because the characteristic impedance of the coaxial cable is 50 Ω . In order to avoid signal reflections at the end of coaxial cables, the impedance must be matched at either end, so when the impedance of the probe circuit and the coaxial cable match, the maximum signal will be transmitted through the cable. This maximum signal is desired to be at the Larmor precession frequency.

The electronic system used to obtain a signal from the NMR probe and coaxial cable is shown in Figure 4.

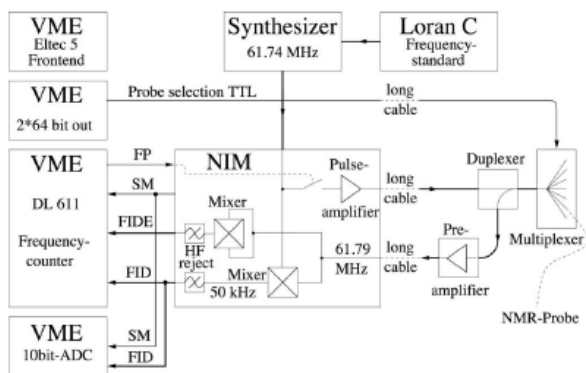


Figure 4: From Ref. [3] and [4]: Sketch of electronics used in the last g-2 experiment

The process for obtaining an NMR signal shown in Figure 4 begins with the fire pulse (labeled FP from the VME). The fire pulse is a $\frac{\pi}{2}$ pulse and carries the frequency of 61.74 MHz from the synthesizer. This reference frequency is about 50 kHz different from the Larmor precession frequency of the protons in the sample. The pulse

is amplified and sent through the duplexer and multiplexer. The duplexer allows for a strong RF pulse to transmit to the probe but prevents this pulse from entering the sensitive pre-amplifier. The multiplexer selects one of twenty probes to receive the RF pulse. The pulse causes the spin flip in the protons of the water sample in the probe, and the inductor L_s in the probe picks up the frequency of their precession. This signal is sent back through the multiplexer and duplexer. The signal is then mixed with the reference frequency from the synthesizer, creating a signal with high frequency components and a low-frequency beat pattern, which is the difference in frequency between the signal frequency and the reference frequency. After filtering out the high frequencies, only the difference frequency remains, and this is what is taken to be the signal from an NMR probe [1].

An ideal NMR signal is exponential and will last until the protons relax back into their equilibrium state. Two time constants govern the decay of the NMR signal. The first is T_1 and called the spin-lattice relaxation time. The length of T_1 is related to the longitudinal fields between molecules in the sample. The second time constant is T_2^* is called the spin-spin relaxation time. The length of T_2^* is related to the transverse components of the fields in the sample, as well as magnetic field inhomogeneities [6]. Field inhomogeneities cause poor signals because if the field is not completely uniform, protons may precess at one frequency at one location in the sample and with a different frequency at a different location. Because of these different frequencies, as time passes, the signal will decohere and decay quickly. The length of T_2^* is always less than that of T_1 , but unlike T_1 , T_2^* may be improved by adjusting the magnetic field in which the sample is immersed.

1.4 Shimming the Magnet

Shimming a magnet is the process of removing inhomogeneities in the magnetic field. This is an important process in NMR measurements because an extremely homogeneous field is required to obtain signals of significant length. Passive shimming is a process in which pieces of high magnetic permeability material are built into a magnet in order to increase field homogeneity. Examples of passive shimming include appropriately aligning the poles of the magnet, placing wedge shims on the poles of the magnet, and inserting iron pieces into gaps in pieces of the magnet in order to direct magnetic field lines. Active shimming uses coil loops and control of the superconductor current to change the magnetic field and increase uniformity [1].

The 1.45 T magnet at CENPA that will be used to calibrate NMR probes for the g-2 experiment is shown in Figure 5.

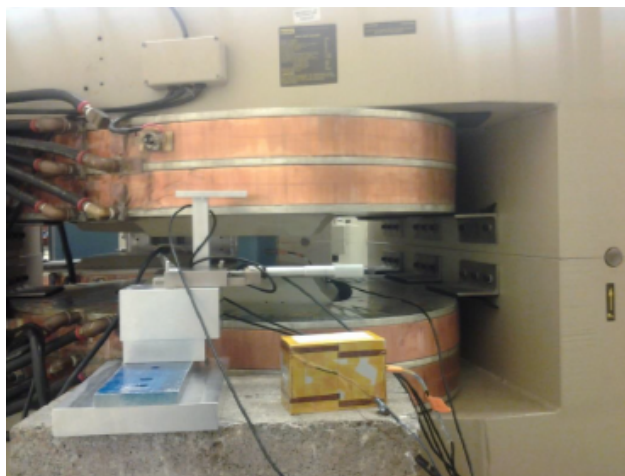


Figure 5: CENPA Test Magnet

The CENPA test magnet was measured to have a non-uniform magnetic field. The field in the z-direction of the magnet changes linearly by

about 1 Gauss per centimeter moving from one side of the magnet to the other (from the left to right side of the magnet in Figure 5). The result of the inhomogeneity of the field in the CENPA magnet was poor signals, such as those shown in Figure 6.

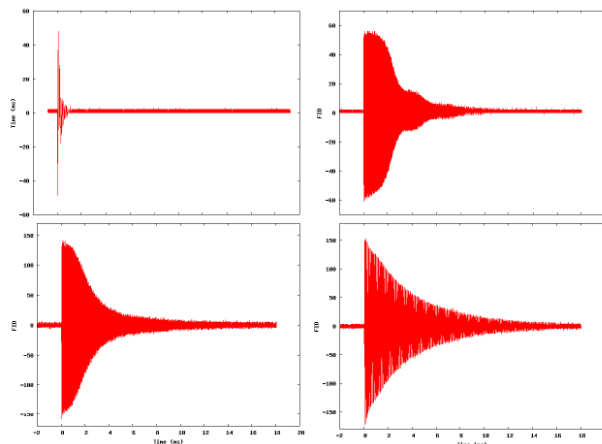


Figure 6: Poor Signals

Figure 6 shows some of the original signals from the CENPA test magnet. The two left signals are very short, upper right signal displays bumps as a result of T_2^* , and the bottom right signal is among the best original signals from the CENPA test magnet, but it still lasts only 3.8 ms until the signal amplitude reaches $\frac{1}{e}$ of the original amplitude. The poor signals produced by the magnetic field in the CENPA test magnet suggest that the magnet is in need of shimming.

2 Design

The device built to shim the magnetic field in the CENPA test magnet by canceling the linear gradient was a pair of adjacent, square Helmholtz coils, with the current traveling in different di-

rections in the two coils. A traditional set of Helmholtz coils is shown in Figure 7.

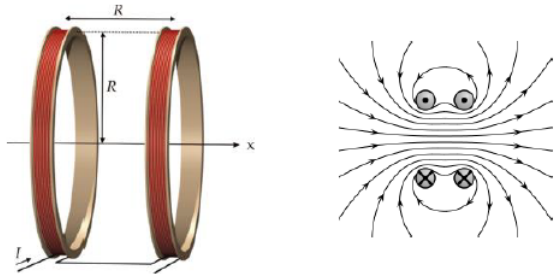


Figure 7: Traditional Helmholtz Coils

Helmholtz coils are typically used for the extremely uniform magnetic field generated in the center region of the coils, if the current in both coils is the same in magnitude and direction. The magnetic field in the center region of a set of Helmholtz coils is given by:

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 n I}{R}$$

The coils used to shim the CENPA magnet were made by two sets of Helmholtz coils, shown in Figure 8.

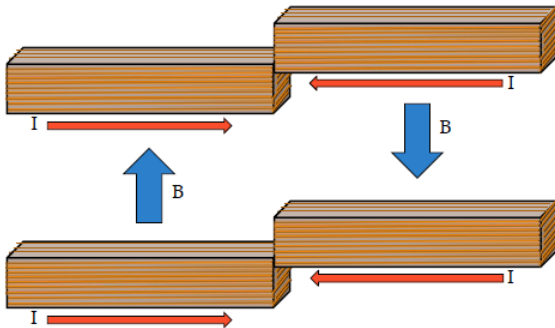


Figure 8: Square Helmholtz Coils

The coils were square so they could sit completely adjacent to each other. In order to form a linear gradient, the current was directed in

one direction in one of the sets of coils and the other direction in the other set. The coils were wrapped around an aluminum base. Aluminum is not strongly affected by magnetic fields. The circuit generating current through the copper coils was in series and directed with carefully placed wires.

The gradient from the coil design is shown in a COMSOL representation in Figure 9.

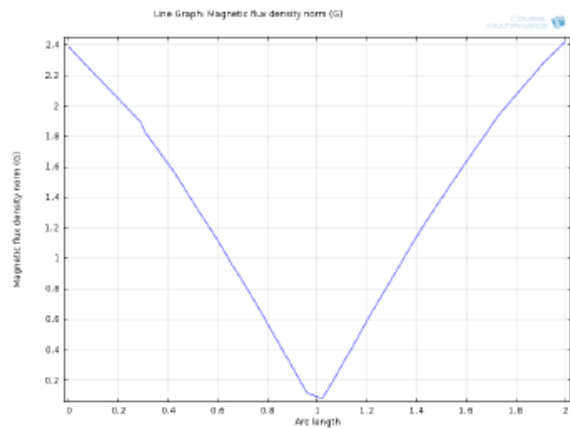
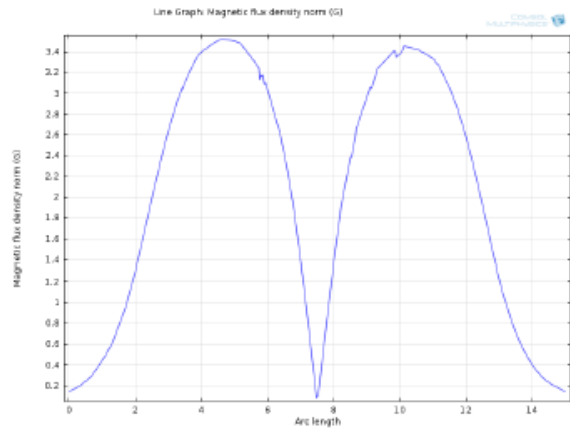


Figure 9: Helmholtz Coils Gradient

The top image in Figure 9. shows the magnitude of the magnetic field over the entire region of the

coils. The bottom image in Figure 9 is a representation of the absolute value of the magnetic field spanning the area directly around the center of the coils, where the two pairs meet. As Figure 9 suggests, the coils are expected to generate a very linear gradient.

3 Results

The coils were tested by introducing them to the magnet, inserting a probe, and slowly changing the current until desirable signals were reached.

3.1 Current Variation

The first test of the coils was to determine the relationship between current and signal length. The current was varied at a number of locations within the magnet, and different optimal currents were found at each spot. Figure 10 shows signals at a specific location within the magnet at a variety of currents.

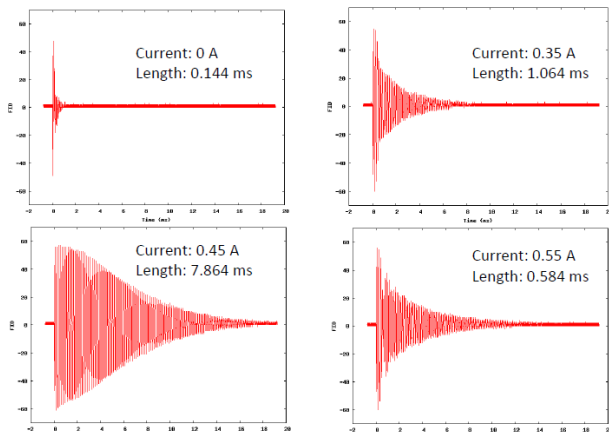


Figure 10: Current Variation

As Figure 10 suggests, there is a specific current for optimization of signal length. This opti-

mization is shown in Figure 11. The best signals found with use of the coils were over 9 ms until the amplitude was reduced by $\frac{1}{e}$.

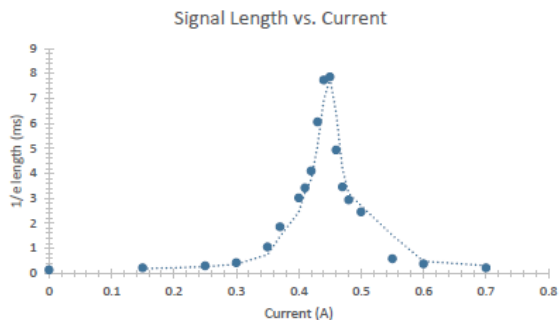


Figure 11: Optimizing Signal Length

3.2 The Two-Probe System

A second probe system was constructed which could fit two NMR probes at the same time. The magnetic field was different enough over the space between the two probes that the signals did not look the same in the two probes at any current. Nor could both signals be optimized at the same current. However, a corner of aluminum was placed within the magnet so that the coils and probes could be returned to the same location repeatedly, and this method was successful. The probes could be taken out, switched, and placed back in the magnet and expected to generate the same signal. A signal of around 8 ms may be expected in one of the probes at around 0.87 A through the Helmholtz coils. Figure 12 displays a successful switch of the probes.

3.3 Signal Frequency

The benefit of increasing signal length is a reduction of error when determining the frequency

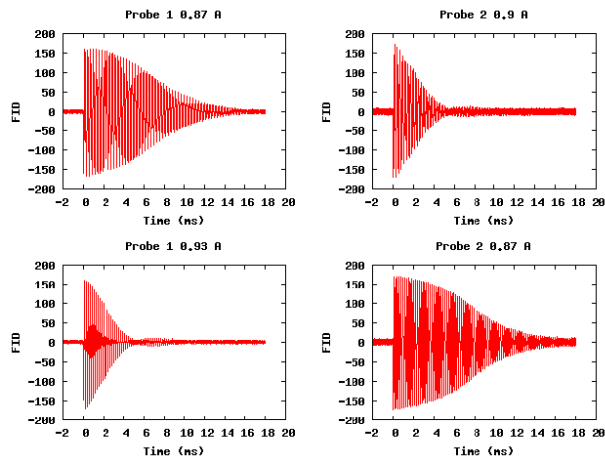


Figure 12: Two-Probe Switch

from the NMR pulse. The method of determining the frequency used in previous g-2 experiments was to count the zero-crossings of the signal and divide by the time between the first and last zero-crossing. Improved frequency extraction methods are being considered for use in the new g-2 Experiment.

One method of frequency extraction is an improved zero-counting method. This method locates the position of the zeros of the signal by using linear regressions on the points around the zero-crossing and hysteresis to ensure the crossings are not false. The frequency is taken between every two points so that the frequency may be tracked over the progression of an entire signal, as shown in Figure 13. Figure 13 shows changing frequency over the course of a signal. The frequency over the entire signal is averaged to give an accurate representation of the frequency.

A second method of determining the frequency is to take the Fourier transform of the signal. The Fourier transform of a particular signal is

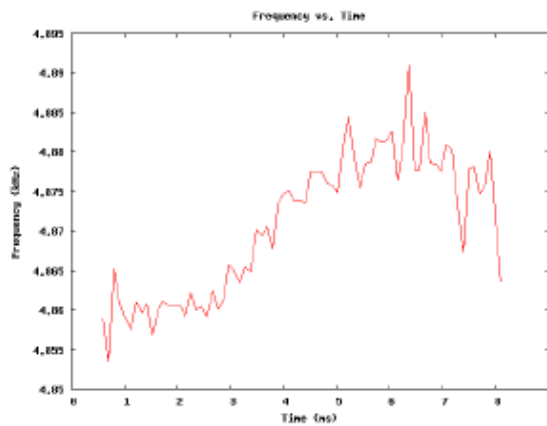


Figure 13: Zero Crossing Frequency vs. Time

shown in Figure 14. This Fourier transform displays the large noise levels that come from an average NMR signal. In order to determine the frequency of the signal, the centroid of the Fourier transform is found, ignoring the contributions from noise. A useful feature of the Fourier transform method is the envelope functions, which are easily returned. Figure 15 shows the envelope of a signal. The envelope functions are helpful in determining the signal length, which is defined in this paper to be the time until the amplitude of the signal is $\frac{1}{e}$ of the original.

An important feature of both of the aforementioned frequency determination methods is that both use error propagation and noise levels to give an error in frequency on the pulse. Errors below 1 Hz are needed to determination of the muon anomaly to the precision goal. The decrease in error with increase in signal length is shown in Figure 16. In typical error propagation techniques, error scales as $\frac{1}{\sqrt{N}}$, where N is the number of data points. As signals get longer, more usable data points are taken on any given oscilloscope capture of the waveform.

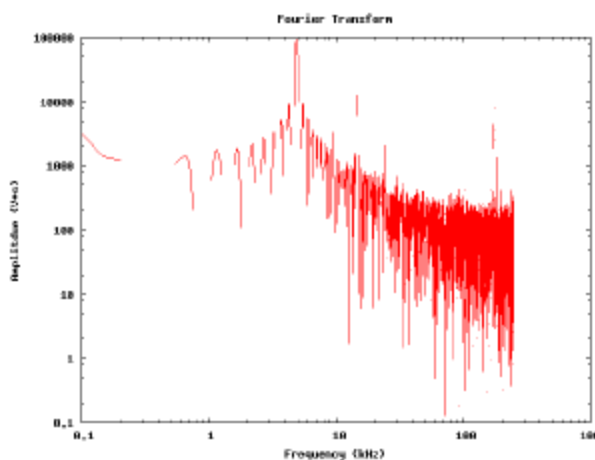


Figure 14: Fourier Transform of NMR Signal

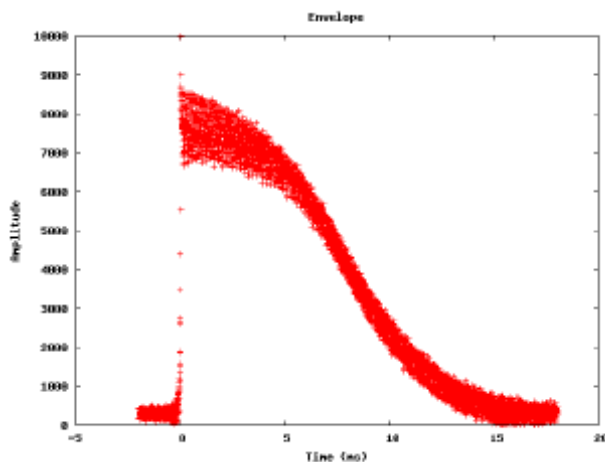


Figure 15: NMR Signal Envelope

4 Broken Probe Diagnosis

The process of broken probe diagnosis is another step in the process of improving NMR measurements for the g-2 Experiment. The University of Washington has received all of the probes from the old g-2 Experiment, and many of the old probes do not produce NMR signals. One possibility for probe damage is that the water sample has leaked out of the probe. Another possible source of problems with old probes is the fact that the soldered pieces in the LC circuit may have broken. A possibility for diagnosing probe problems is to check for the expected resonances from the LC circuits and from the coaxial cable described in section 1.3. A measurement of these resonances was taken on a working probe by varying the frequency of signal through the probe and measuring the impedance using a vector impedance meter. The result is shown in Figure 17.

Figure 17 clearly shows the coaxial cable resonances, and the LC circuit resonances on a smaller

scale. Because the resonances of a working probe are known, resonances may now be taken with probes found to be broken, and the placement or lack of these resonances may point to the problem with the probe.

5 Summary

A device for shimming the 1.45 T magnet at CENPA was made with two pairs of square, adjacent Helmholtz coils. The coils produced a linear gradient to cancel that in the test magnet. Implementation of the coils in the magnet increased signal duration from 3.8 ms at best to over 9 ms at best. A corner of aluminum was placed in the magnet so that long signals can now be achieved repeatably. Signals close to 8 ms may be expected at around 0.87 A using the aluminum corner and Helmholtz coils.

A working probe was tested with a vector impedance meter and resonances from the coaxial cable as well as the series and parallel LC circuit were found. Broken probes may now be

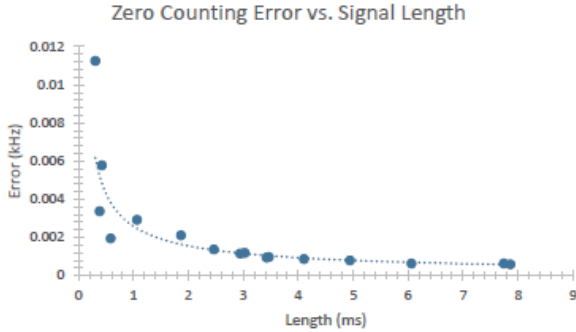


Figure 16: Zero Counting Error

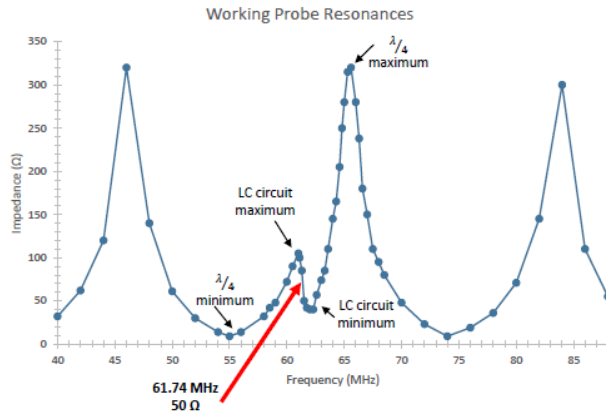


Figure 17: NMR Probe Resonances

diagnosed by searching for these resonances at the same frequencies as they were found in the working probe.

The future work of the g-2 NMR project includes continued diagnosis of problems with the NMR probes and redesigning the probes so that the problems do not occur in the future g-2 Experiment. Once the probes are designed, 400 must be built. They will then be tested and calibrated using the Helmholtz coils to achieve increased signal lengths.

6 Acknowledgements

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