Shimming of a Magnet for Calibration of NMR Probes

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UW PHYSICS REU 2013

Outline

- Background
  - The muon anomaly
  - The g-2 Experiment
  - NMR
- Design
  - Helmholtz coils producing a gradient
- Results
- Future work

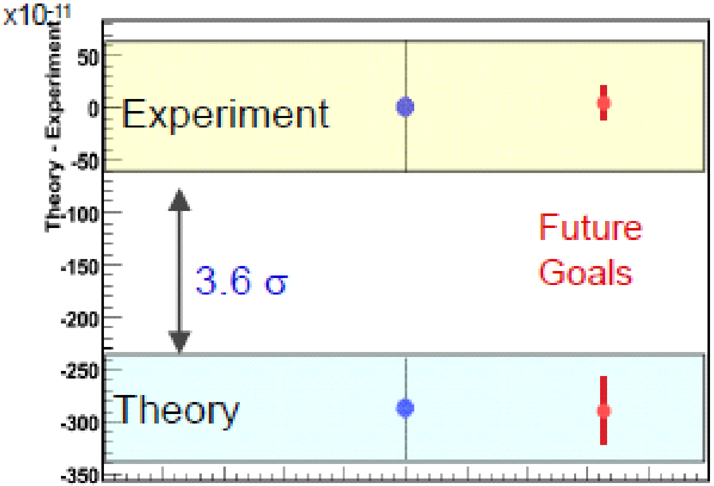
The Muon anomaly

- The magnetic moment of a particle:

$$\vec{\mu} = g\mu_B\vec{s} \qquad \mu_B = \frac{e\hbar}{2mc}$$

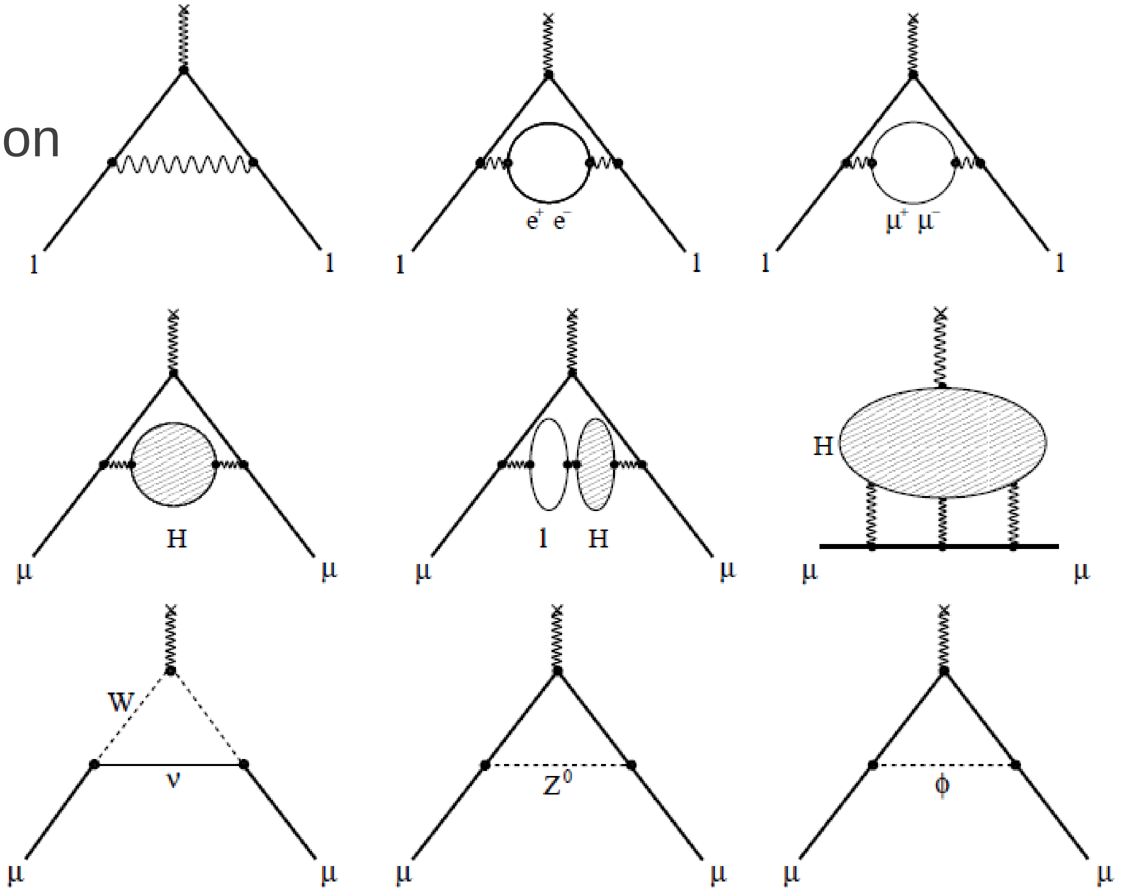
- A g factor of 2 is expected for point-like fermions
- There is a contribution to g from interactions with virtual fields

The Muon anomaly: $$a_\mu = \frac{g_\mu - 2}{2}$$



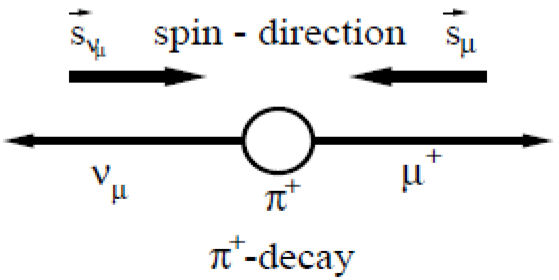
The Muon anomaly

1) Electromagnetic interaction
2) Strong interaction
3) Weak interaction
4) New physics – supersymmetry?



The Muon g-2 Experiment

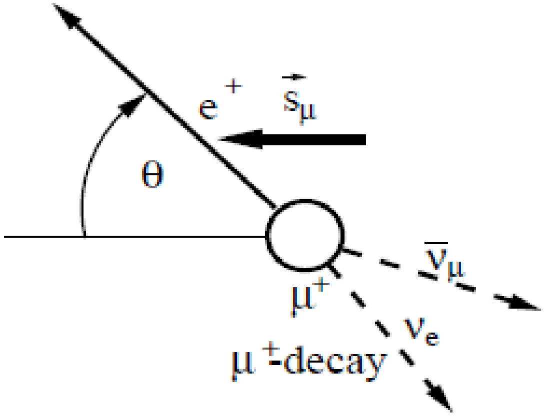
Goal: measure the anomalous magnetic moment of the muon to the precision of 0.14 ppm

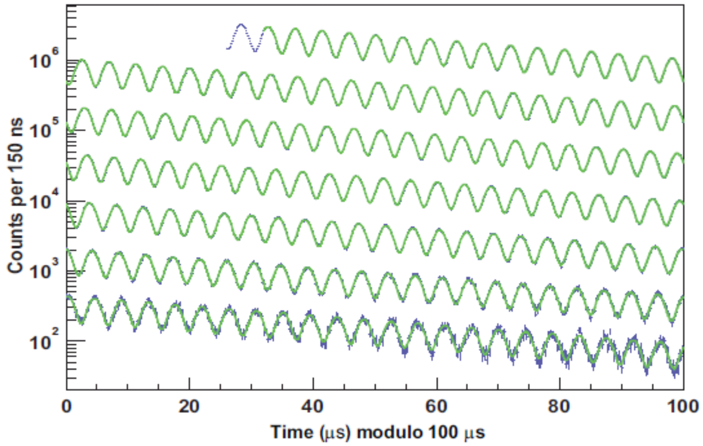


1) Collect polarized muons

2) Precession in $(g-2)$ storage ring

3) Measure arrival time and energy of positron from muon decay





The Muon g-2 Experiment

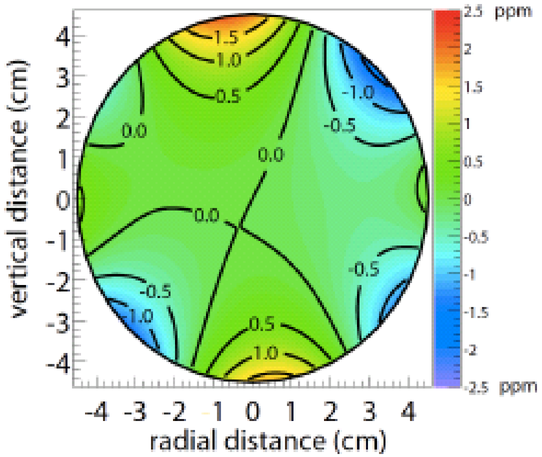
Determining the anomaly:

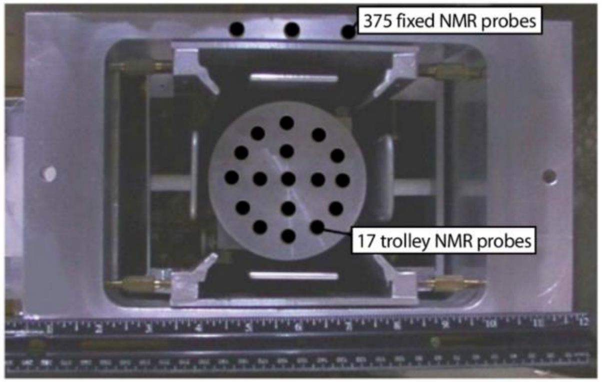
$$a_\mu = \frac{{}^{\omega_a}/_{\omega_p}}{{}^{\mu_\mu}/_{\mu_p} - {}^{\omega_a}/_{\omega_p}}$$

- ${}^{\mu_\mu}/_{\mu_p} = 3.183345137(85)$
  - From muonium hyperfine level measurements

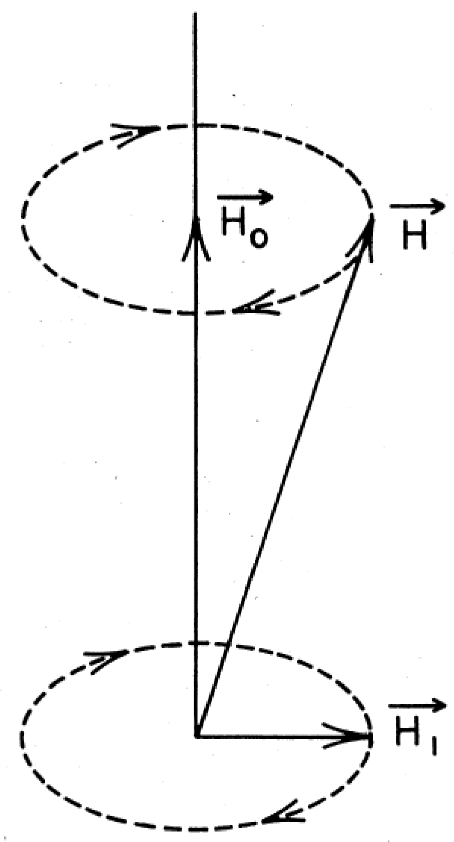
- ω_a: Difference frequency
  - $\omega_a = \omega_s - \omega_c = a_\mu \left(\frac{e}{m_\mu c}\right) B$
  - From detection of positrons

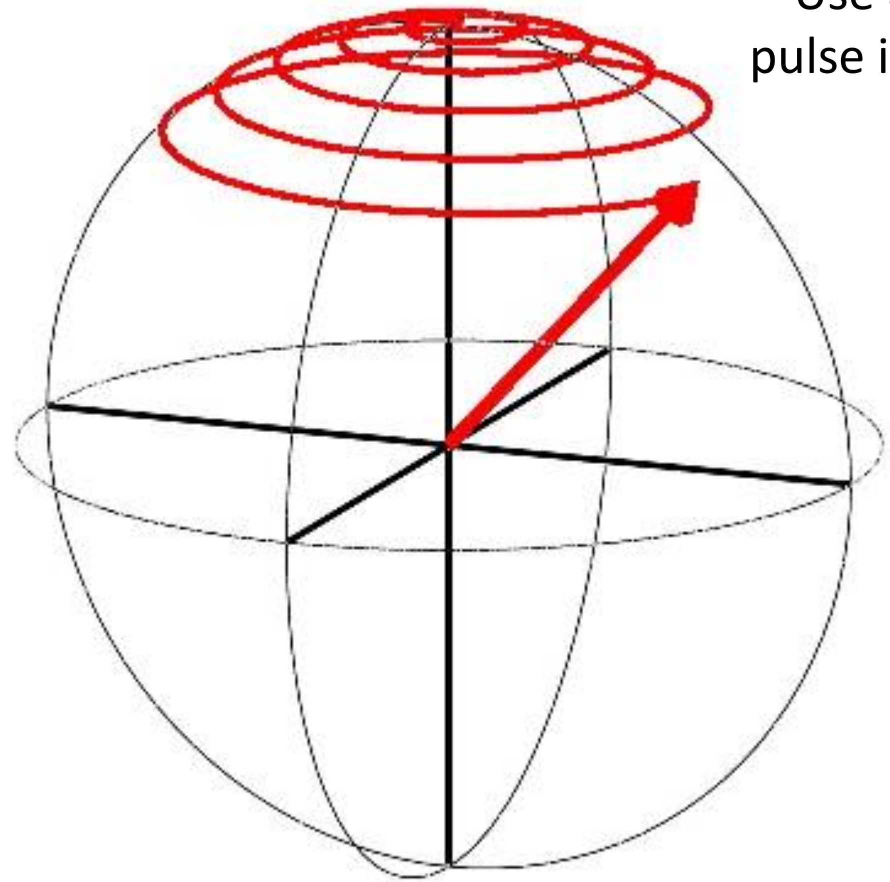
- ω_p: Larmor frequency of free protons
  - Measured with 400 NMR probes





NMR

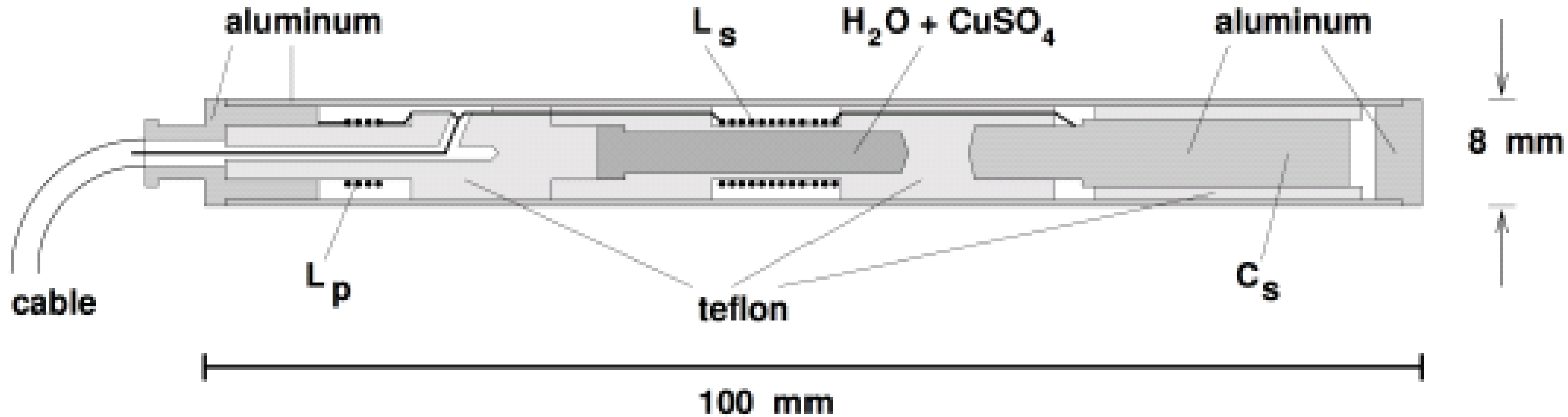


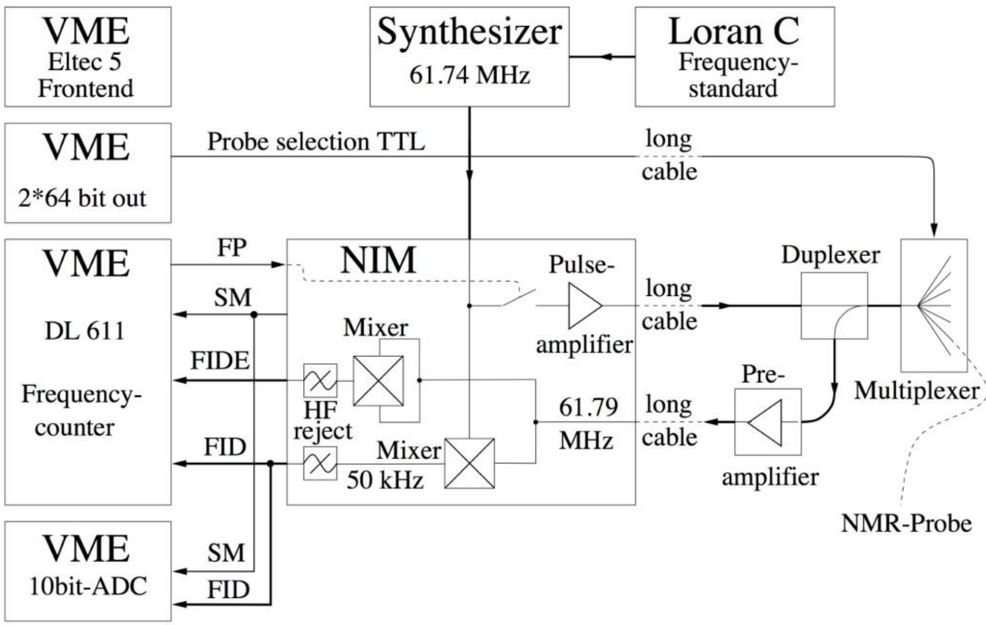


Use a $\pi/_2$ pulse in NMR

A moment precessing about a strong field $\overrightarrow{H_0}$ may be flipped with the addition of a weak field $\overrightarrow{H_1}$ rotating with a frequency close to resonance

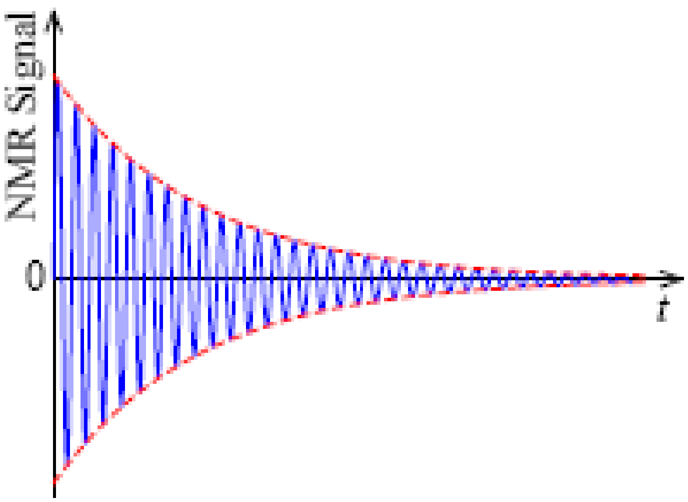
NMR

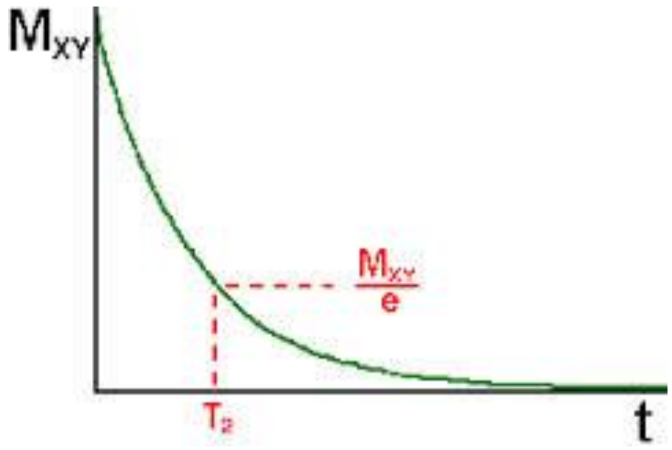

aluminum
L s
$H_2O + CuSO_4$
aluminum
8 mm
cable
L p
teflon
C s
100 mm


VME
Eltec 5
Frontend
Synthesizer
61.74 MHz
Loran C
Frequency-
standard
VME
2*64 bit out
Probe selection TTL
long
cable
VME
DL 611
Frequency-
counter
FP
SM
FIDE
FID
NIM
Pulse-
amplifier
long
cable
Duplexer
Multiplexer
Mixer
HF
reject
Mixer
50 kHz
61.79
MHz
long
cable
Pre-
amplifier
NMR-Probe
VME
10bit-ADC
SM
FID

NMR Time Constants

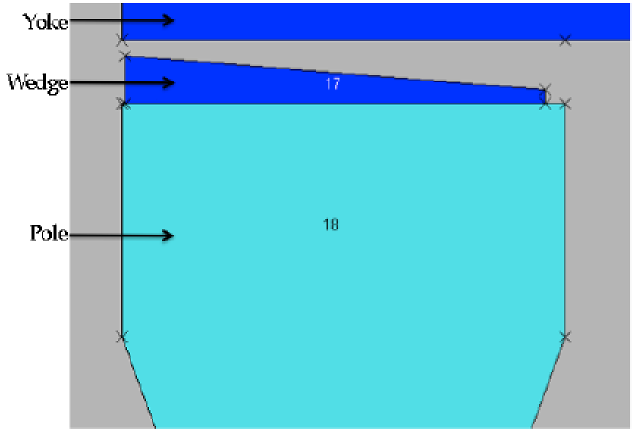
- T_1: spin-lattice relaxation
  - Time until magnetization reaches thermal equilibrium value (z-direction)
- T_2^*: spin-spin relaxation
  - due to magnetization parallel to rf field and field inhomogeneities

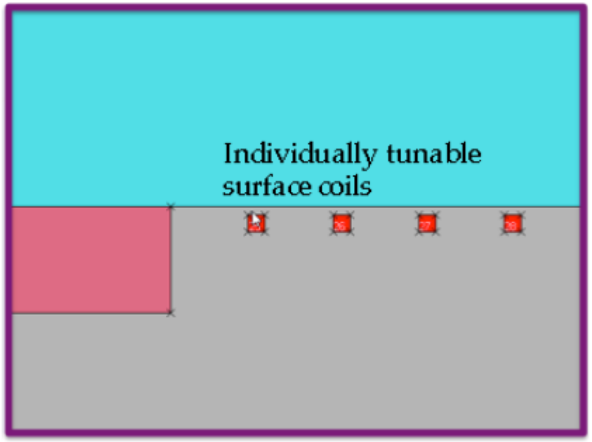




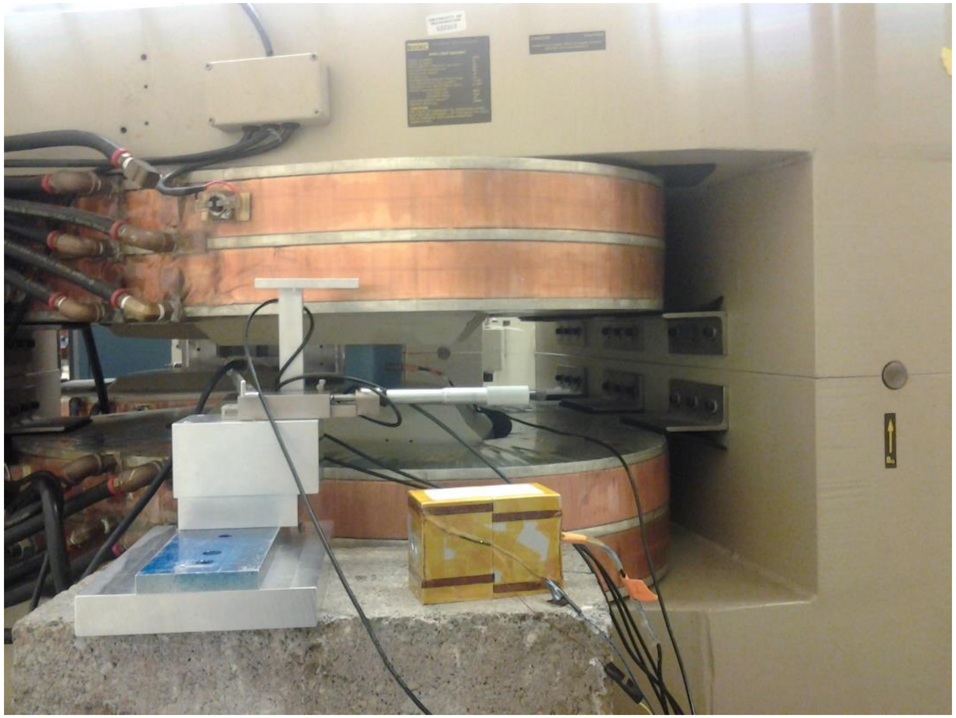
Shimming the magnet

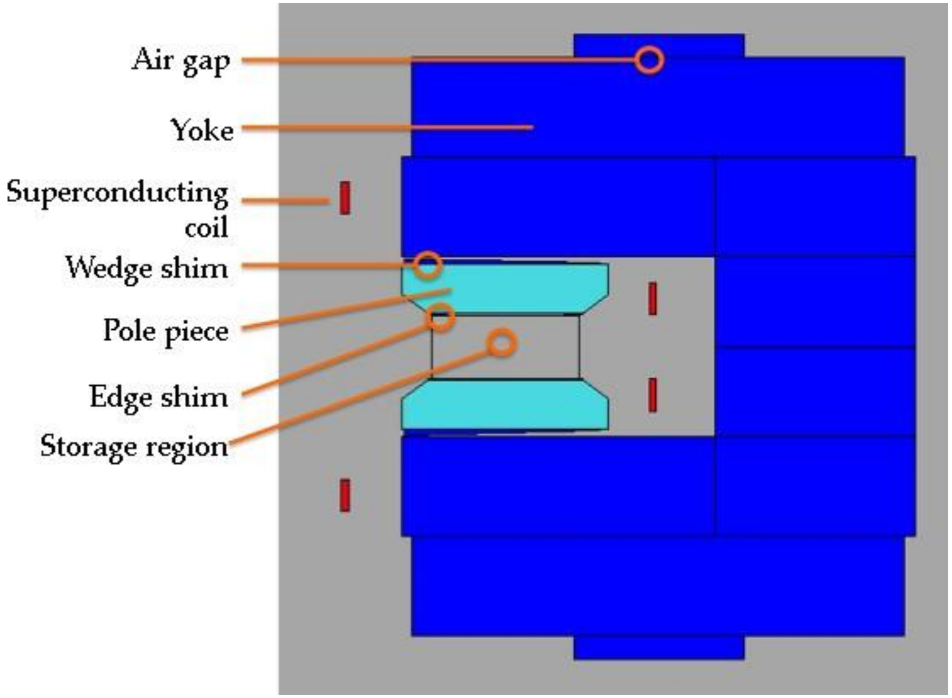
- Shimming: removing inhomogeneities in the field
- Passive shims
  - Iron pieces on the yoke and in gaps
  - Pole face alignment
  - Edge and wedge shims
- Active shims
  - Control of superconductor current
  - Surface correction coils
  - Dipole and gap correction loops



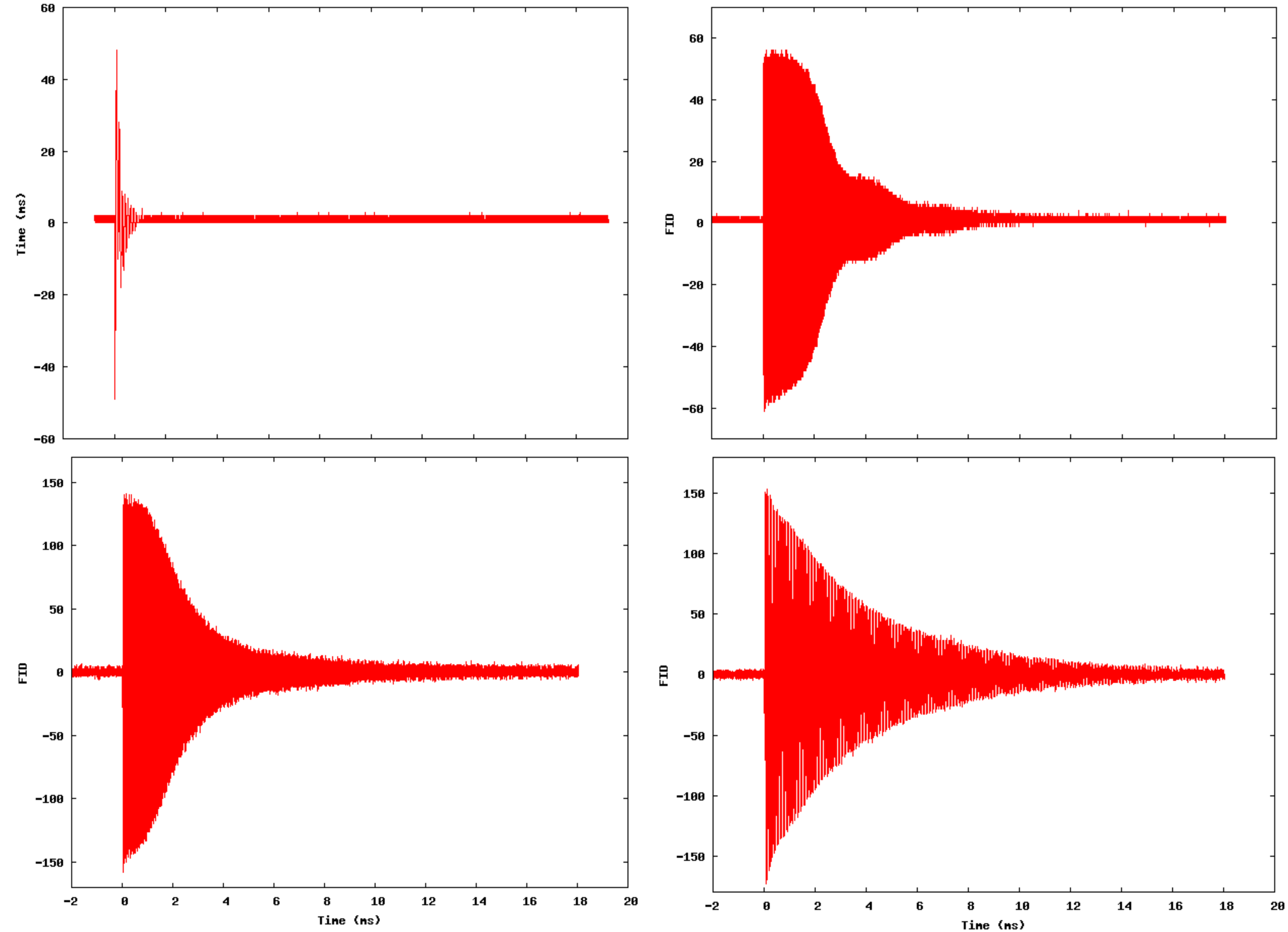


CENPA test magnet



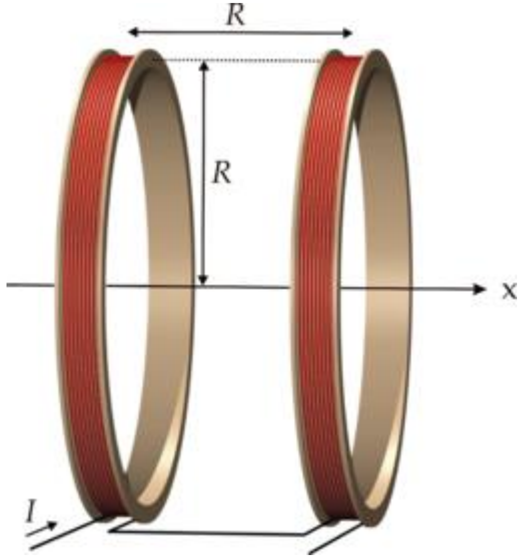


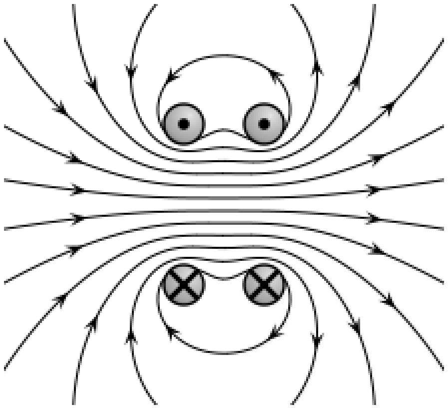
Original Signals



The best signals were around 3.8 ms (until amplitude reaches 1/e)

Helmholtz coils



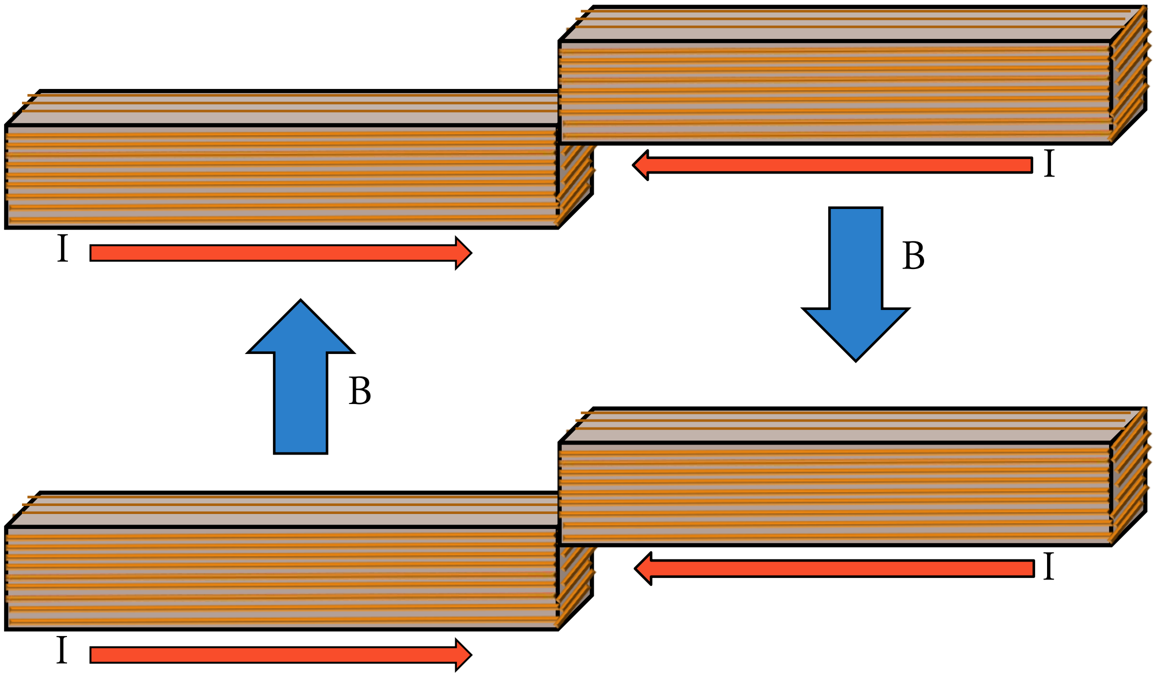


h (distance between the coils) should be equal to *R* (the radius of a coil) for maximum uniformity

Using the Biot-Savart Law, the field at midpoint between coils:

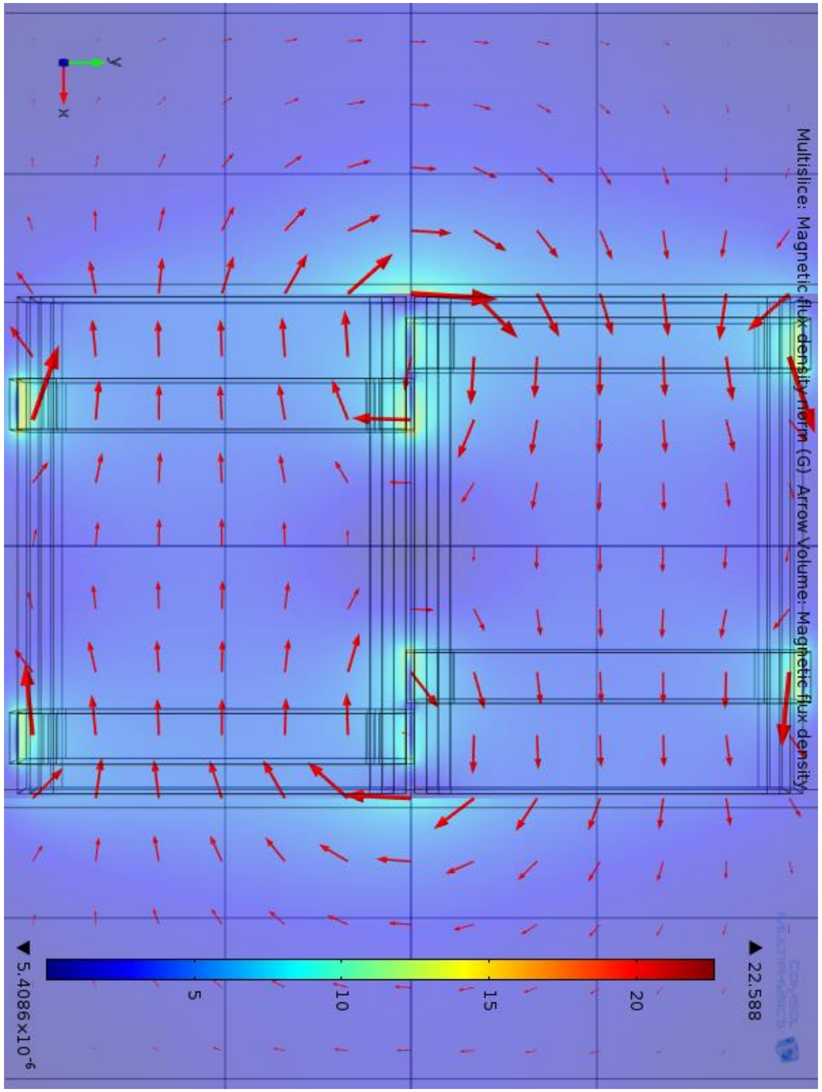
$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$

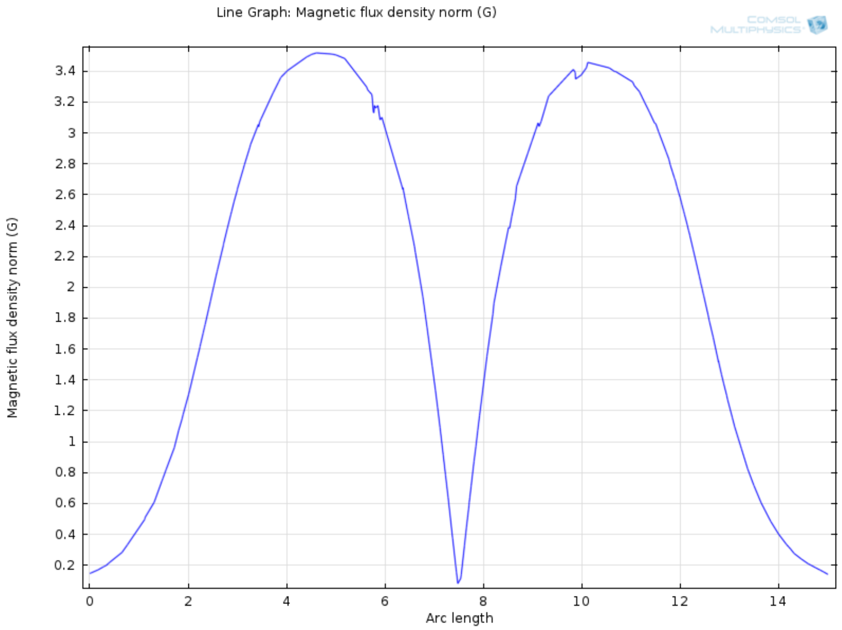
Helmholtz coils

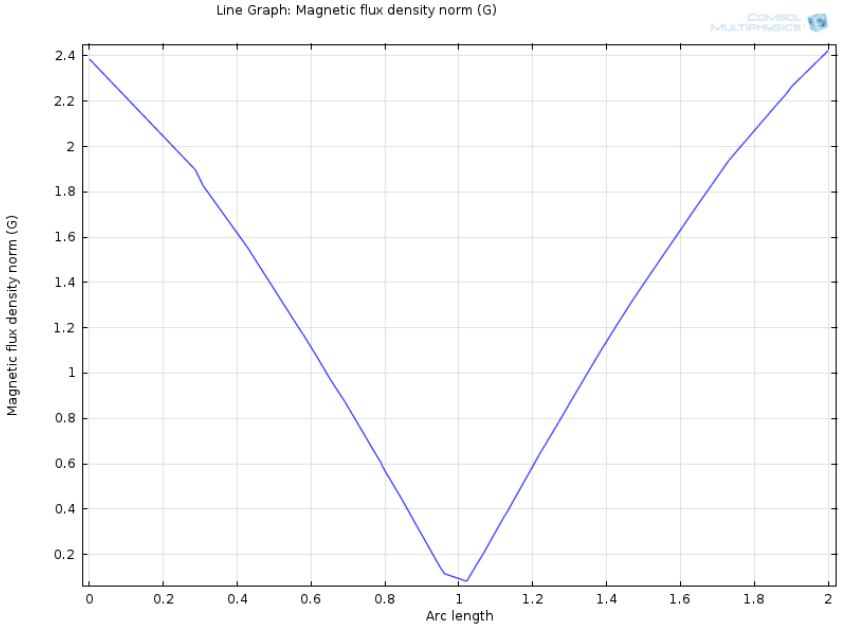


A linear gradient is created between the coils

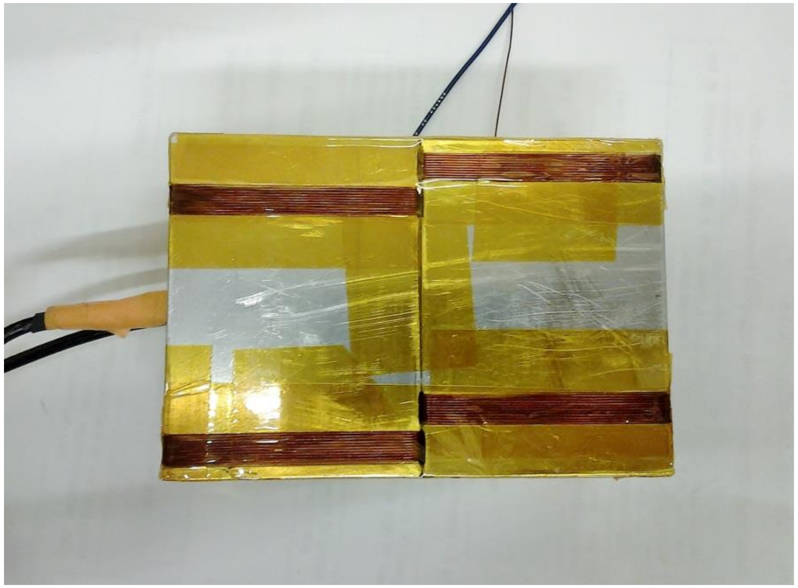
Helmholtz coils

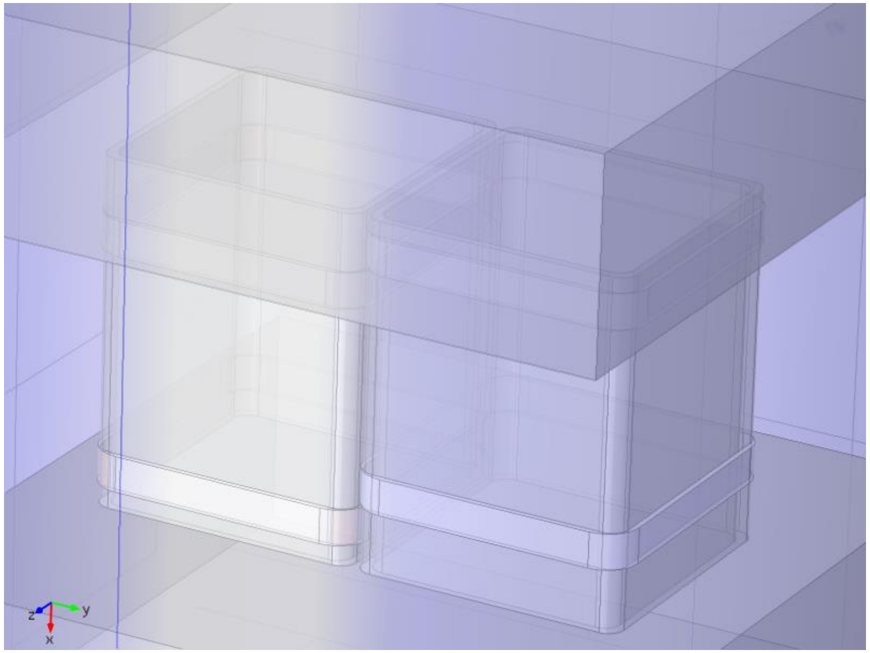


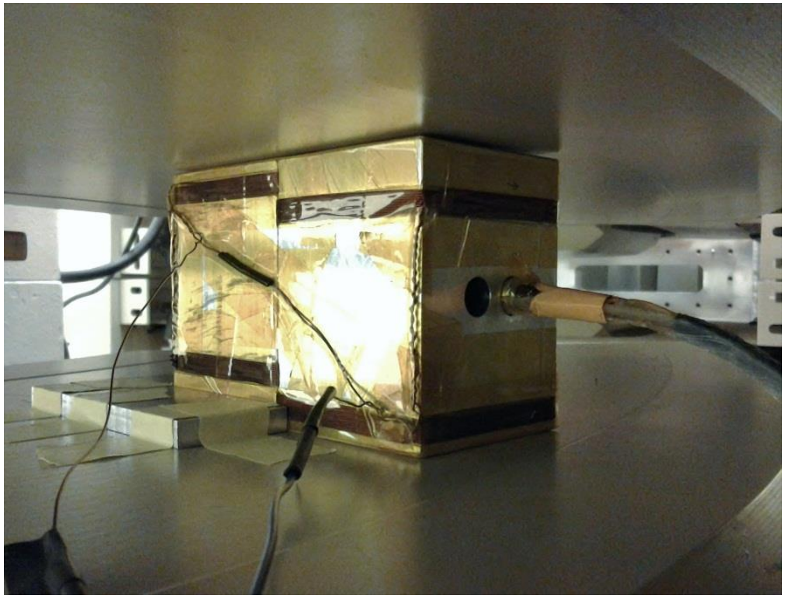




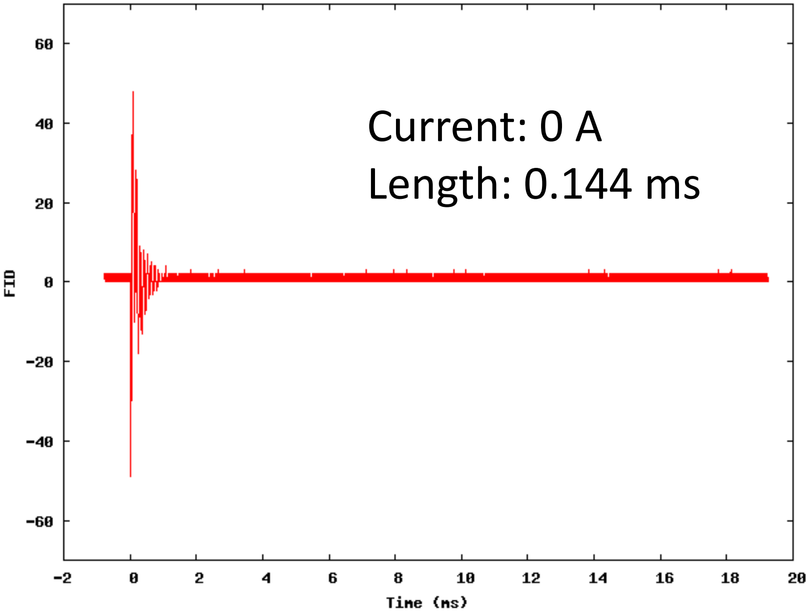
Helmholtz coils

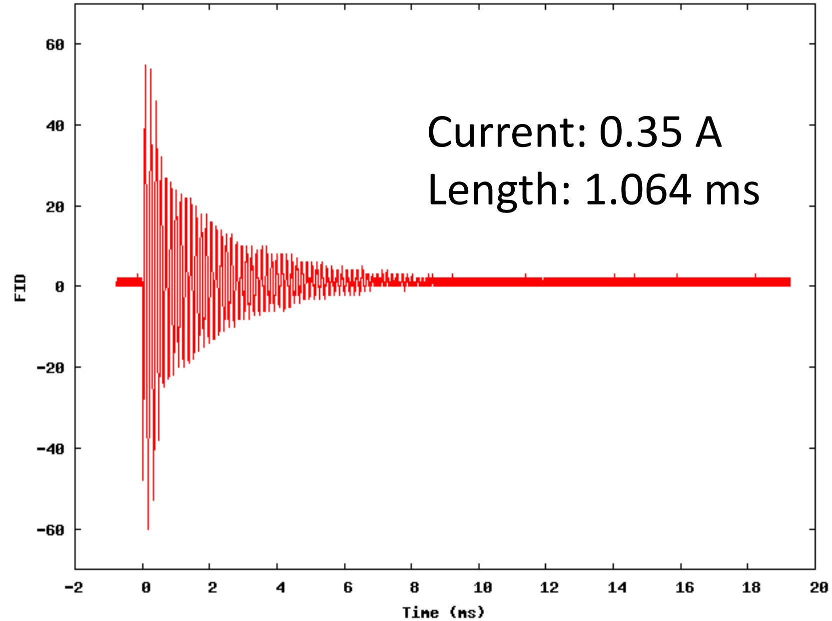


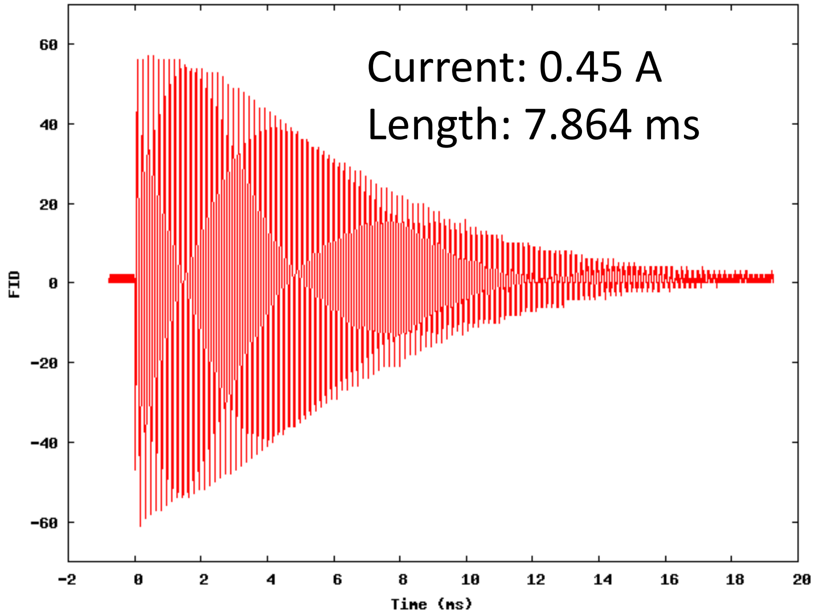


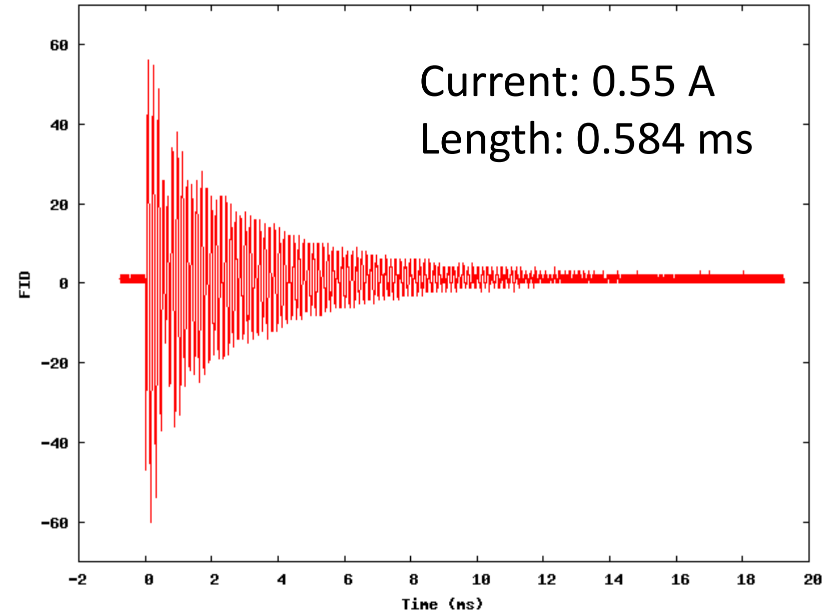


Results – varying the current



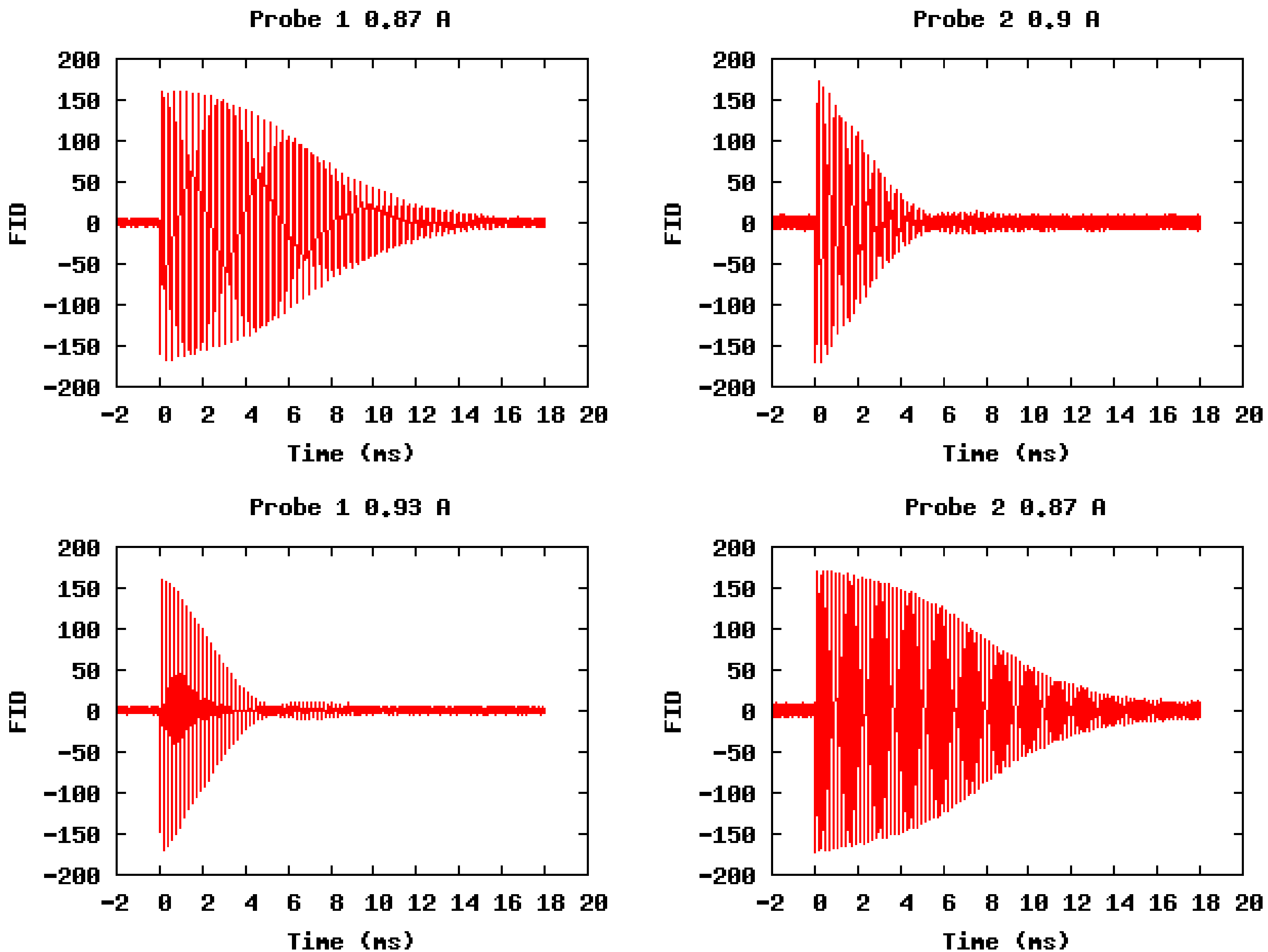






The best signals found were 9.14 ms at 0.87 A

Results – 2-probe coils

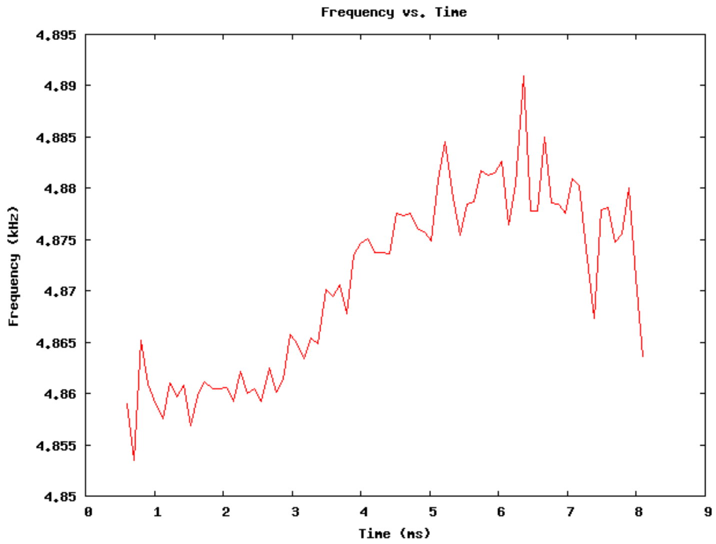


After switching the probe positions

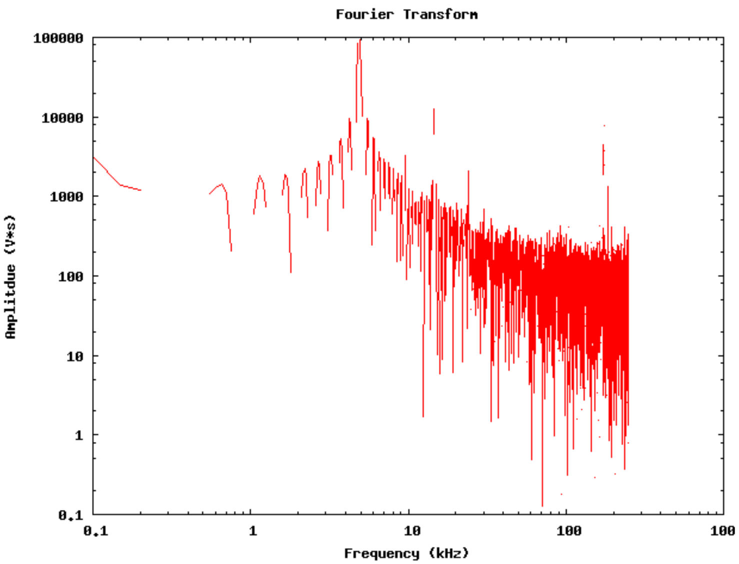
Maximizing both probes at the same current does not work, but one may be used for calibration. Long signals are repeatable.

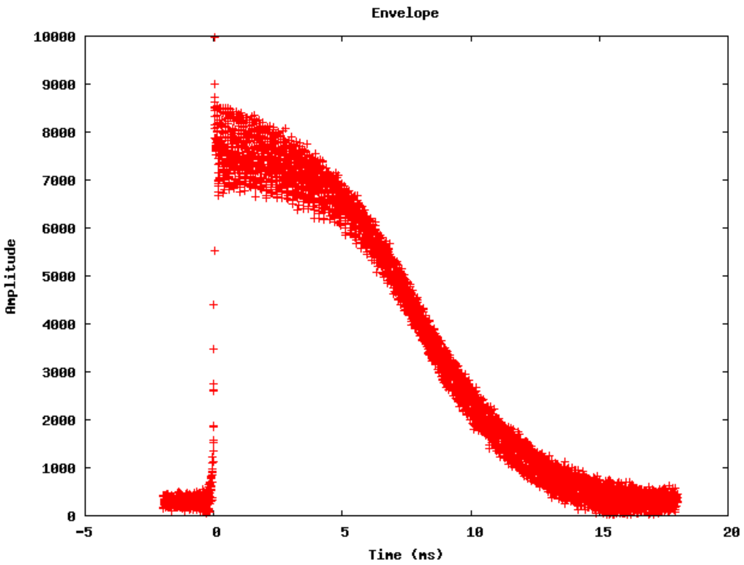
Extracting the Frequency

Zero-crossing method

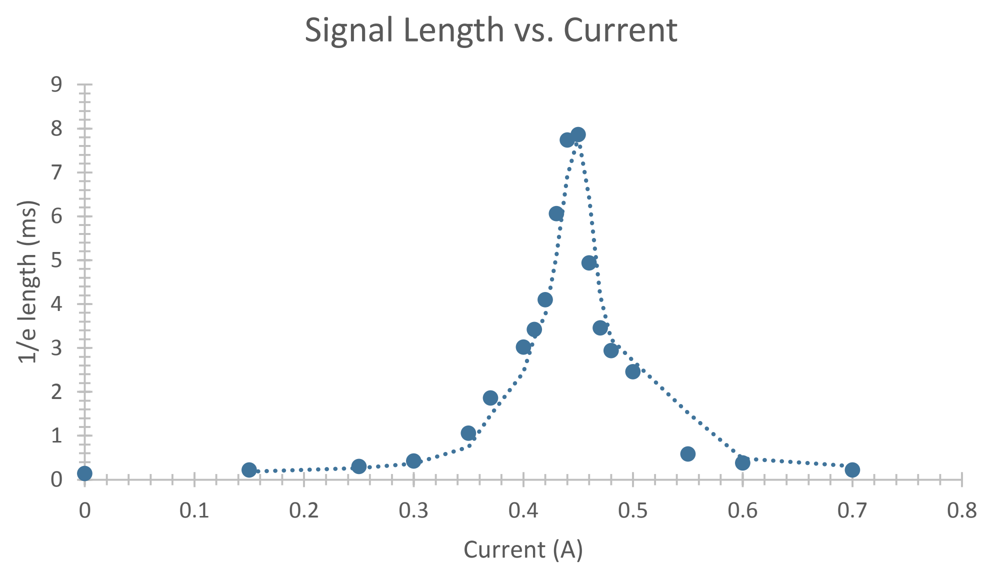


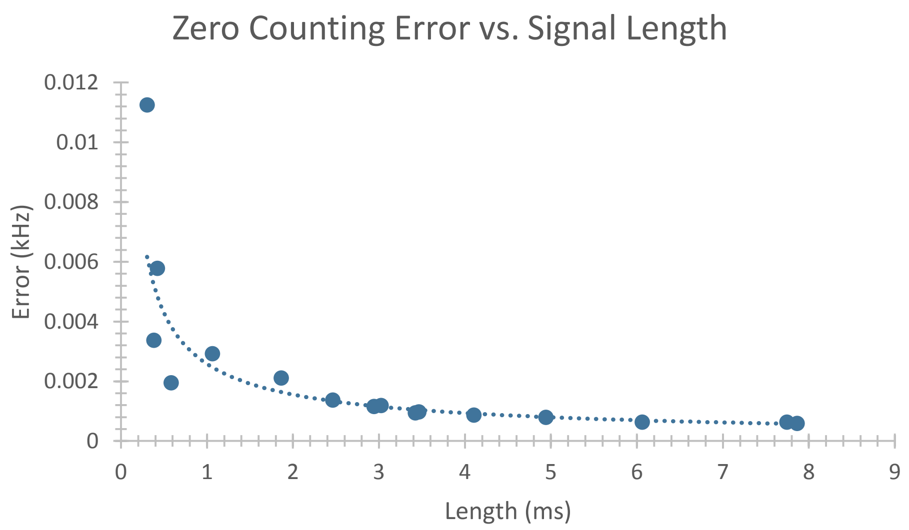
Fourier Transform method

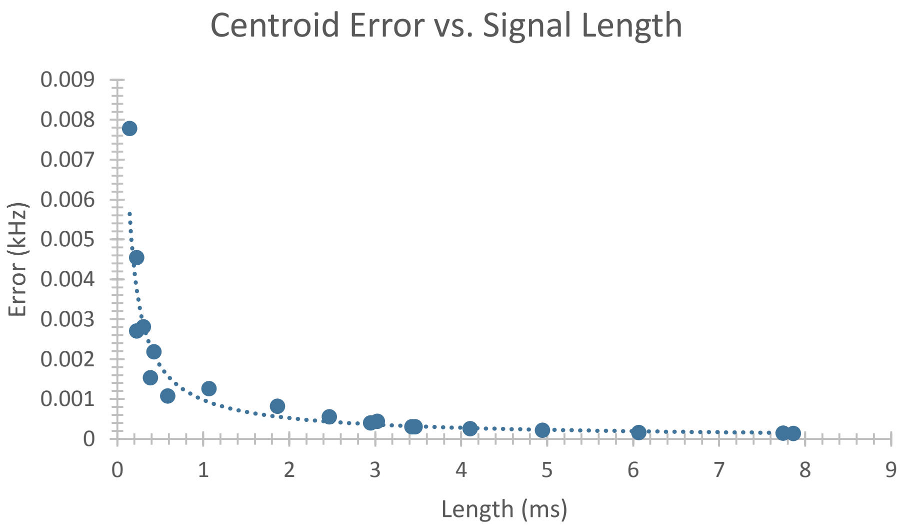




Error

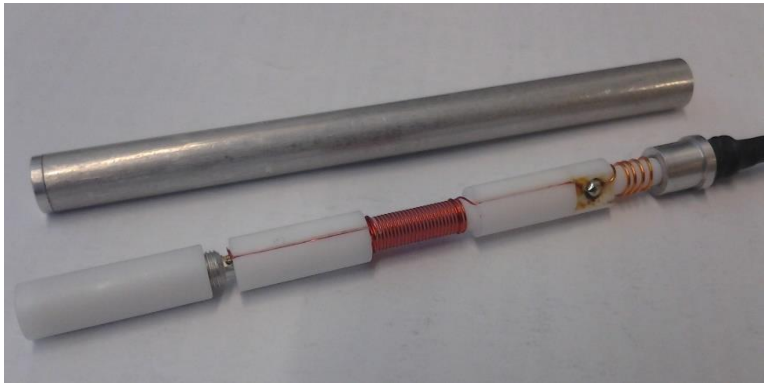

Signal Length vs. Current
1/e length (ms)
0
1
2
3
4
5
6
7
8
9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Current (A)

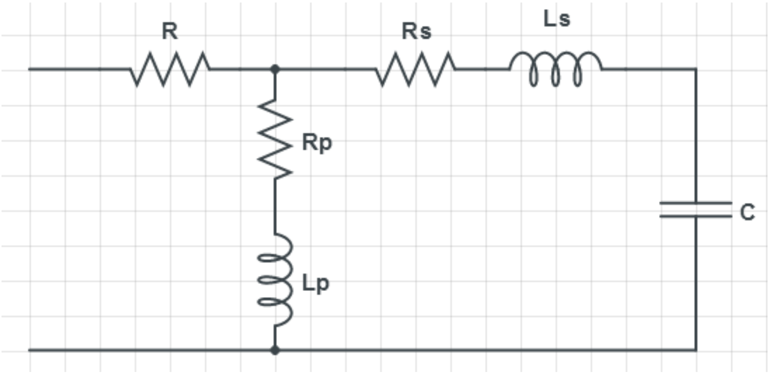

Zero Counting Error vs. Signal Length
Error (kHz)
0
0.002
0.004
0.006
0.008
0.01
0.012
0
1
2
3
4
5
6
7
8
9
Length (ms)

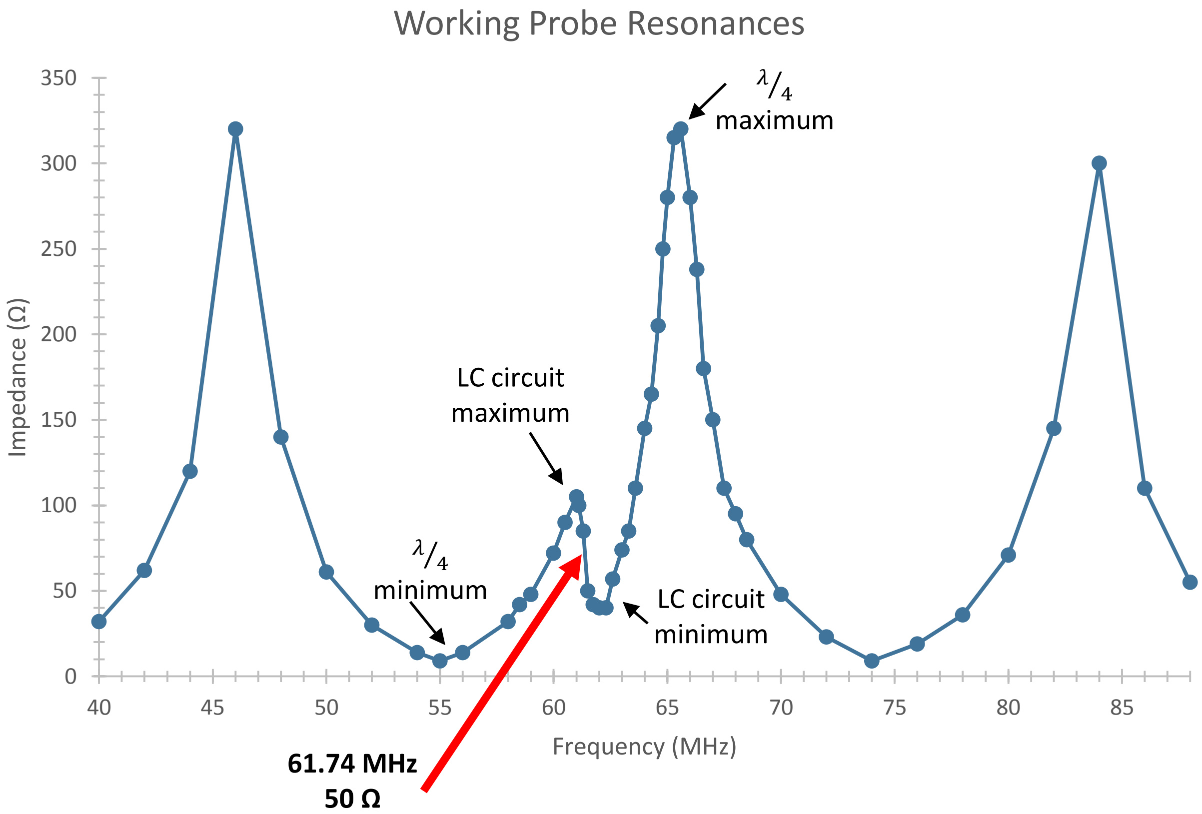

Centroid Error vs. Signal Length
Error (kHz)
0
0.001
0.002
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0
1
2
3
4
5
6
7
8
9
Length (ms)

Repairing Probes

- Diagnose problems with old probes using a vector impedance meter
- Repair broken circuitry or determine that the sample has leaked

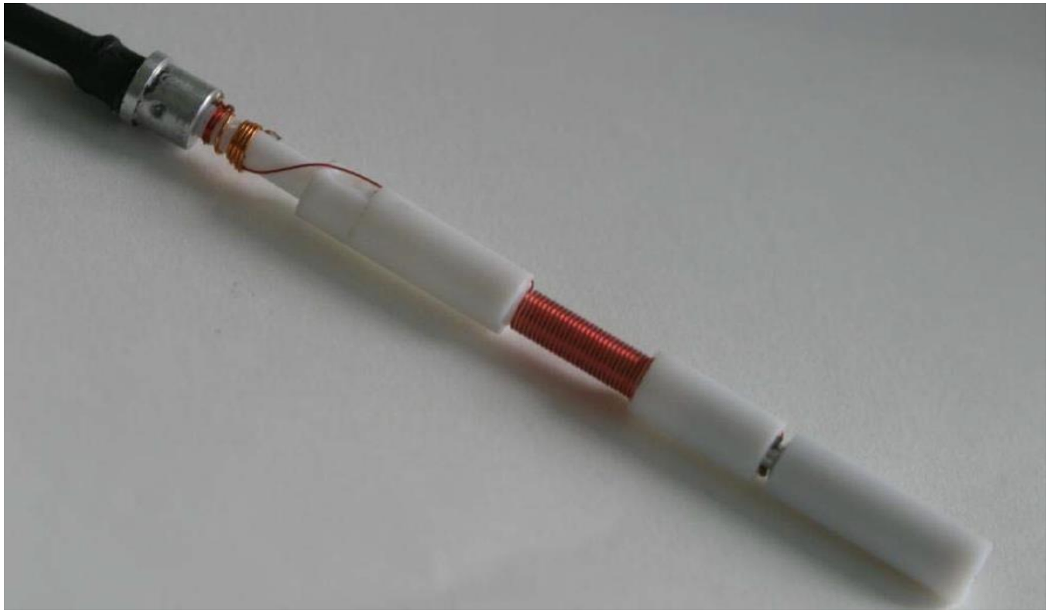






Future Work

- Test the temperature dependence of the NMR probes
- Diagnose problems with old probes
- Re-design and rebuild 400 probes
- Test and calibrate the probes using the coils



Acknowledgments

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NSF for funding

References

1) Bloembergen, N., Purcell, E., & Pound, R. (1948). Relaxation effects in nuclear magnetic resonance absorption.*Physical Review*, *73*(7), 679-715.
2) Grossman, A. (1998). *Magnetic field determination in a superferric storage ring for a precise measurement of the muon magnetic anomaly*. (Doctoral dissertation).
3) U.S. Department of Energy, (2013). *Muon g-2 conceptual design report* (DE-AC02-07-CH-11359). Batavia, Il: Fermi National Accelerator Laboratory.

Questions?

NMR

A system with angular momentum $\mathbf{j}$, magnetic moment $\mathbf{m}$, and gyromagnetic ratio γ:

$$\frac{\mathrm{d}\mathbf{j}}{\mathrm{d}t} = \mathbf{m} \times \mathbf{B_0} \qquad \mathbf{m} = \gamma \mathbf{j}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{m}(t) = \gamma \mathbf{m}(t) \times \mathbf{B_0}$$

Add a perpendicular field $\mathbf{B_1}(t)$ rotating about $\mathbf{B_0}$ with angular velocity ω

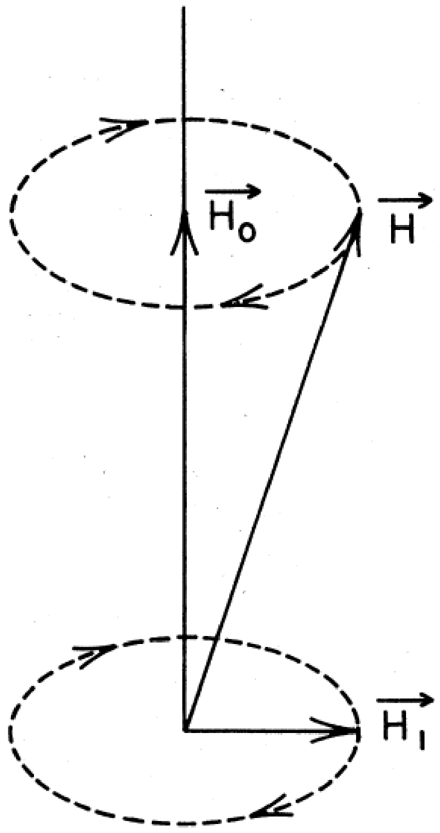
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{m}(t) = \gamma \mathbf{m}(t) \times [\mathbf{B_0} + \mathbf{B_1}(t)]$$

Set $\Delta\omega = \omega - \omega_0$ and move to the rotating frame

$$\left(\frac{\mathrm{d}\mathbf{m}}{\mathrm{d}t}\right)_{\mathrm{rel}} = \mathbf{m}(t) \times [\Delta\omega - \omega_1]$$

Resonance condition: $\Delta\omega \ll \omega_1$

The moment can be flipped with a small rotating field $\mathbf{B_1}$



How NMR works – Quantum Mechanics

The state vector of the spin system:

$$|\psi(t)\rangle = a_+(t)|+\rangle + a_-(t)|-\rangle$$

The Hamiltonian:

$$H(t) = -\mathbf{M}\,.\,\mathbf{B}(t) = -\gamma\mathbf{S}\,.\,[\mathbf{B_0} + \mathbf{B_1}(t)]$$

From the spin matrices:

$$H = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 \mathrm{e}^{-i\omega t} \\ \omega_1 \mathrm{e}^{i\omega t} & \omega_0 \end{pmatrix}$$

Define functions $b_+(t)$=$\mathrm{e}^{i\omega t/2}a_+(t)$ and $b_-(t)$=$\mathrm{e}^{-i\omega t/2}a_-(t)$ and set up set up the Schrodinger equation

$$\begin{cases} i\frac{\mathrm{d}}{\mathrm{d}t}b_+(t) = -\frac{\Delta\omega}{2}b_+(t) + \frac{\omega_1}{2}b_-(t) \\ i\frac{\mathrm{d}}{\mathrm{d}t}b_-(t) = \frac{\omega_1}{2}b_+(t) + \frac{\Delta\omega}{2}b_-(t) \end{cases}$$

The Hamiltonian is now time-independent (we are in the rotating frame)

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\tilde{\psi}(t)\rangle = \tilde{H}|\tilde{\psi}(t)\rangle$$

Find the transition probability

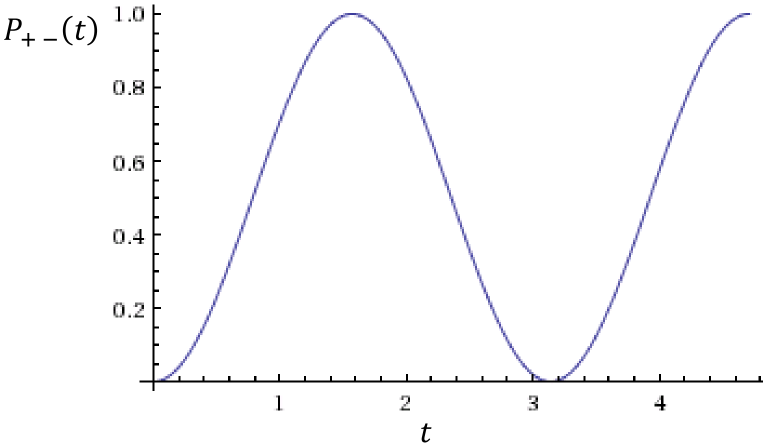
$$P_{+-}(t) = |\langle -|\psi(t)\rangle|^2 = \left|\langle -|\tilde{\psi}(t)\rangle\right|^2$$

Where the initial condition is $|\psi(0)\rangle = |+\rangle$

Rabi's formula:

$$P_{+-}(t) = \frac{\omega_1^2}{\omega_1^2 - (\Delta\omega)^2}\sin\left[\sqrt{\omega_1^2 + (\Delta\omega)^2}\,\frac{t}{2}\right]$$

Same resonance condition as classical mechanics: $\Delta\omega \ll \omega_1$



NMR Time Constants

- T_1: spin-lattice relaxation
  - Time until magnetization reaches thermal equilibrium value (z-direction)
  - ${}^{dn}/_{dt} = \left({}^{1}/_{T_1}\right)(n_0 - n)$
    - n: surplus population at lower levels
    - n_0: number at equilibrium
- T_2: spin-spin relaxation
  - due to magnetization parallel to rf field if the field is perfectly homogeneous
- T_2^*: spin-spin relaxation combined with field inhomogeneities
  - due to magnetization parallel to rf field and field inhomogeneities
  - $T_2^* = \frac{1}{2} g(\nu)$

    $g(\nu)$: shape factor of the absorption line of energy from the magnetic field

Circuit Resonances