Tuning RF Cavities Via End Plate Geometry

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Abstract

Adding ridges to the endplates of a right cylindrical resonant cavity can lower the frequencies of the cavity's modes substantially. This offers an effective means of tuning the Axion Dark Matter eXperiment (ADMX) detector cavity to sweep lower axion mass ranges than otherwise possible.

1 Background

Axions are hypothetical particles which may play an important role in both high energy physics and cosmology. The particles originate in a theory[1] which was proposed by Roberto Peccei and Helen Quinn to resolve the "Strong CP Problem" of the Standard Model[2]. The theory does not predict the mass of the axion, but if the mass should turn out to fall within a certain range (approximately 1 μeV to 1 meV), axions might also serve as constituents of dark matter. Hence, this particle stands to resolve two major open problems in physics at once.

The properties of axions, as predicted by Peccei-Quinn theory, make them very difficult to detect; indeed, if this were not the case, axions would be ineligable as dark matter particles. Axions are predicted to decay into photons with an expected half life of about 10^{42} years, which renders the efficacy of detecting this decay mode in, say, particle accelerators quite minimal. However, if axions are indeed dark matter particles, then their natural abundance makes detection much more feasible. This is the basis for the Axion Dark Matter eXperiment (ADMX): assuming that axions are dark matter particles, then in a carefully chosen situation the prevalence of axions should enable the detection of these otherwise rare decay modes.

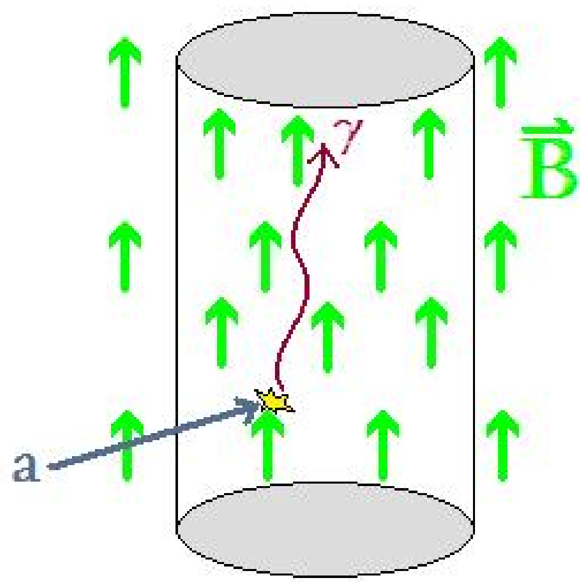

Figure 1: **Sikivie's axion detector setup.**

The particular "carefully chosen situation" used by ADMX was proposed in 1983 by Pierre Sikivie[3], and amounts to a very strong magnetic field (of sev-

[1] See Peccei, Roberto D.; Quinn, Helen R. (1977). "CP Conservation in the Presence of Pseudoparticles". *Physical Review Letters* 38 (25): 1440.

[2] Briefly, the Strong CP Problem is concerned with why certain parameters of the Standard Model are zero when there is no reason *a priori* that they must be.

[3] See Sikivie, P. (1983). "Experimental Tests of the 'Invisible' Axion". *Physical Review Letters* 51 (16): 1415.

eral Teslas) to stimulate axion decays located within a resonant cavity to capture the photons resulting from the decays. For a cavity of the appropriate shape and sufficiently high quality factor, the photons from many axion decays can accumulate and excite a standing wave, which can then be detected directly (by an antenna, for example). For this program to succeed, a first requirement for the cavity is that it be able to sustain a standing electromagnetic wave with the frequency of the photons produced by the axion decays, i.e. the cavity has a normal mode of that frequency. This poses a challenge, since the axion mass is unknown, and thus the frequency of the photons produced in axion decays is also unknown. To compensate, the cavity used in ADMX is equipped with a tuning mechanism, so that a range of frequencies (viz axion masses) can be probed.

In its present design, the ADMX cavity is a right cylinder, 1 meter tall and .5 meters in diameter, with a copper interior. It is tuned using two copper rods running the length of the cavity which can be rotated towards or away from the cavity's center axis; when the rods are towards the cavity axis the frequency of the cavity modes increases, and when the rods are away from the cavity axis the mode frequencies are lower, but in either case the frequencies cannot go below the corresponding frequencies for a bare cavity with no tuning rods. To achieve lower frequencies, dielectric rods can be substituted for the copper rods (although this can have other undesirable effects, described below).

The existing tuning mechanism for ADMX permits access to the entire range of possible masses for dark matter axions, but there is another consideration in choosing the design of the cavity which depends more delicately on the properties of axions. Axion coupling strength is proportional to $\vec{E} \cdot \vec{B}$, where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field (this dependence on $\vec{B}$ explains why a strong magnetic field is present in the cavity). As such, the normal modes which are most easily excited by axion decays are those for which this coupling is large over the entire cavity, which effectively means that the modes' electric fields are generally aligned with the static magnetic field in the cavity. In the ADMX design, the static magnetic

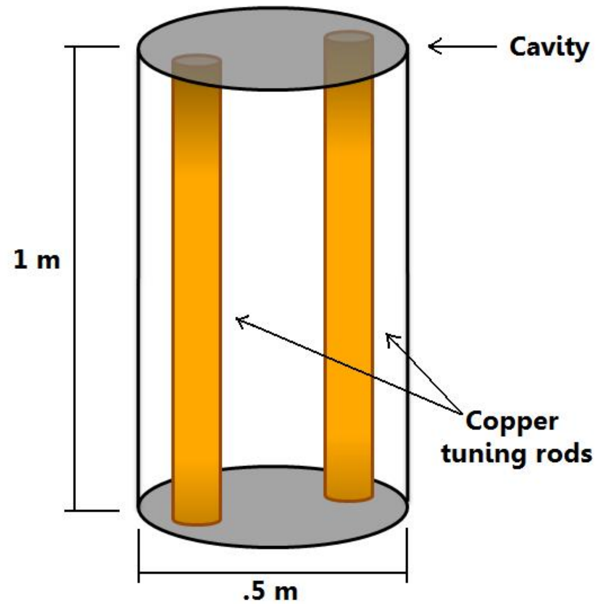

Figure 2: **Tuning rod placement within the cavity.**

field is parallel to the axis of the cavity, and thus axion coupling to a given mode may be summarized by a parameter dubbed the "form factor" of the mode, defined as[4]

$$f = \frac{\left(\int_V E_z dV\right)^2}{V \cdot \int_V E^2 dV}$$

where V is the cavity volume and the z-axis is aligned with the cavity axis. Efficacy of axion detection by a given mode depends on the square of the form factor, so a large form factor is imperitive for an viable axion search. An immediate consequence of this is that tranverse electric (TE) modes are not usable for axion detection, since $\vec{E}$ is everywhere perpendicular to the cavity axis, and thus the form factor vanishes. Among transverse magnetic (TM) modes, the TM_{010} mode has a form factor of about .7, and other modes' form factors are generally somewhat lower than this. Form factors down to about .1 are probably workable for ADMX purposes.

Placing dielectric tuning rods in the cavity can

[4]See Daw, E. J. *A Search for Halo Axions.* Diss. M.I.T., Cambridge, MA, 1998.

substantially reduce the form factors of the cavity modes, and thus this mechanism for tuning the cavity to low frequencies is somewhat undesirable (necessitating longer periods of data taking). The dielectric rods are also lossy and fragile, and on account of these problems an alternative means of lowering the cavity modes' frequencies would be appealing. One such alternative (proposed by Leslie Rosenberg, the ADMX group leader) was inspired by a technique used to lower mode frequencies in waveguides: so-called "slow wave structures". The idea is to perturb the side walls of a waveguide with a wire or a ridge in the shape of a helix along the length of the waveguide. Waves travelling along the waveguide are constrained to follow this perturbation, and as the wave winds along the helix its wavelength is effectively compressed in the direction of propagation down the guide. Hence a longer wavelength (lower frequency) can be accomodated into the guide. This method only works for TE modes, but in the setting of a resonant cavity an analogous modification may be made to the end plates of the cavity (see Figure 3), which hopefully should have a comparable effect on the TM modes. This was the motivation for my REU project with ADMX: model and test the effect of adding ridges to the end plates of the ADMX cavity. This paper describes the results of that project.

2 Results

2.1 Modelling

Various end plate geometries were modeled using the rf cavity solver program "Poisson/Superfish", distributed by the Los Alamos code group[5]. This program only accomodates cylindrically symmetric geometries, but since the ADMX cavity is itself cylindrically symmetric, this is not a major impediment. In general, the geometries that were explored consisted of one or more right-angled ridges protruding into the cavity from the end plates, as shown in Figure 3. Alternatively, the "ridges" might also extend into the endplates, away from the main cavity[6].

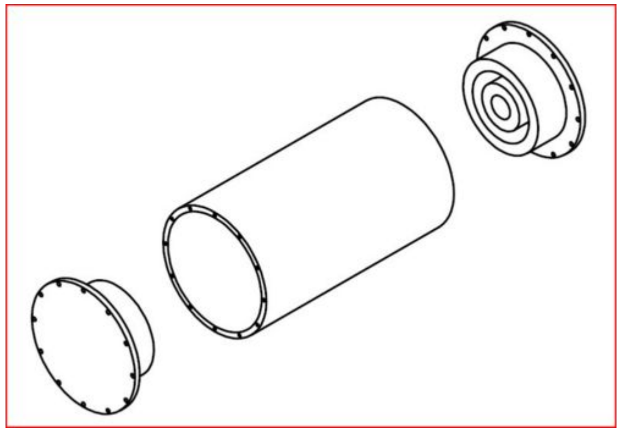

Figure 3: **A typical cavity with ridged end plates.**

The motivation for studying such geometries, as opposed to more complicated curved surfaces, was twofold: firstly, such right-angle surfaces are much easier to realize physically (i.e. in a machine shop) than curved surfaces; and secondly, these right-angled surfaces would seem to be more amenable to tuning in an actual ADMX setup (by, for instance, attaching a plunger to the ridges so that the length may be adjusted).

The first remarkable finding of the Poisson/Superfish simulations was that the frequencies of the lowest TM modes for ridged end plate geometries could be made substantially lower than the corresponding frequencies of a flat end plate geometry. An ADMX sized cavity (1 meter tall with a 25 centimeter radius) with flat end plates has a TM_{010} mode frequency of 459 MHz, whereas an identical cavity with a single ridge in the end plate of inner radius 10 cm, outer radius 15 cm, and depth 15 cm has a lowest TM mode frequency of 312 MHz. By extending the depth of the ridge to 25 cm the lowest TM mode frequency falls to 224 MHz. The lowest TM mode frequency of various end plate geometries is shown in Figure 5.

[5]See laacg1.lanl.gov/laacg/services/download_sf.phtml for more details and a download link.

[6]This might, however, create complications for the ADMX setup by protruding out of the cavity into other components of the experiment.

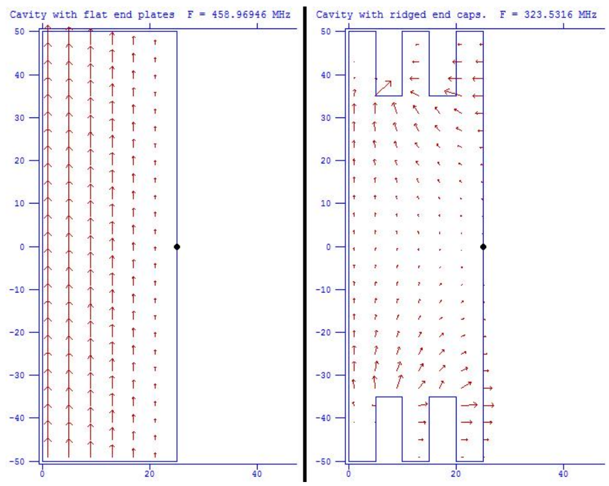

Figure 4: **Poisson/Superfish solutions for two cavities with different end plate geometries. The shape outlined by the blue line represents a cross section of the cavity extending from the center axis outwards. That is to say, revolving the shape about the the left edge yields the 3-d cavity shape.**

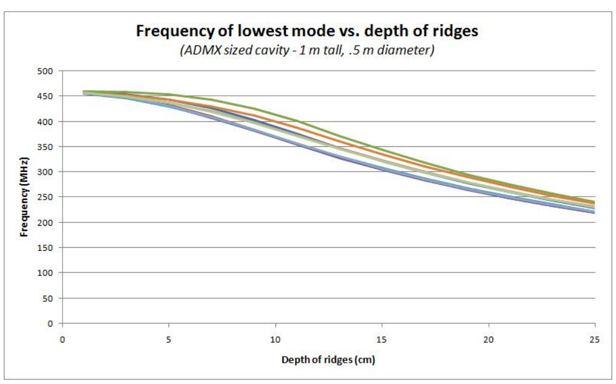

Figure 5: **A graph of lowest TM mode frequency vs. depth of the end plate ridges for various ridge configurations. Each line represents a different configuration of ridges (e.g. one line represents a configuration consisting of a single ridge with inner radius 15 cm and outer radius 20 cm, and the depth of the ridge is the x-axis parameter).**

Loosely speaking, the results of the Poisson/Superfish simulations showed that the lowest TM mode frequency depended strongly on the depth of the ridges and weakly on the number and widths of the ridges; this can be seen in Figure 5, where the curves representing various configurations of numbers and sizes of ridges lie close to one another, but all show a definite dependence on the depth of the ridge (which is the x-axis parameter). It is also turns out that the effect of letting the ridges protrude either into or away from the main cavity was much the same. It therefore appears that most any geometry should provide the desired frequency shift, provided that the depth of the ridges is sufficiently great.

The variation of form factor among end plate geometries is more appreciable than the variation of normal mode frequency (ranging an order of magnitude in some cases). As a point of comparison, among configurations of ridges for which the lowest TM mode frequency was half that of the TM_{010} mode for a flat end plate geometry, the form factor could be as low as .05 or as high as .4. As such, the form factors of the various end plate configurations played an important role in determining which configurations deserved further investigation. Eventually, a geometry consisting of two ridges protruding into the cavity (shown in Figure 6) was selected for further testing. Poisson/Superfish was used to find higher mode frequencies for this geometry, so as to assemble a partial spectrum for eventual comparison with physical data from a real cavity with this shape of end plates.

2.2 Testing a Real Cavity

To produce the cavity with the desired geometry, we started with an aluminum pipe approximately 5 inches in diameter and 10 inches long. Two pairs of end plates were then machined from a solid aluminum bar, one pair flat and the other with ridges as in Figure 6. The pipe and end plates were fitted with bolt holes to hold the entire unit together. The cavity and endcaps produced are shown in Figure 7.

This new cavity was probed by two weakly coupled antennas[7] inserted through small (3/16 inch) holes

[7]The antennas were inserted by hand and adjusted to the

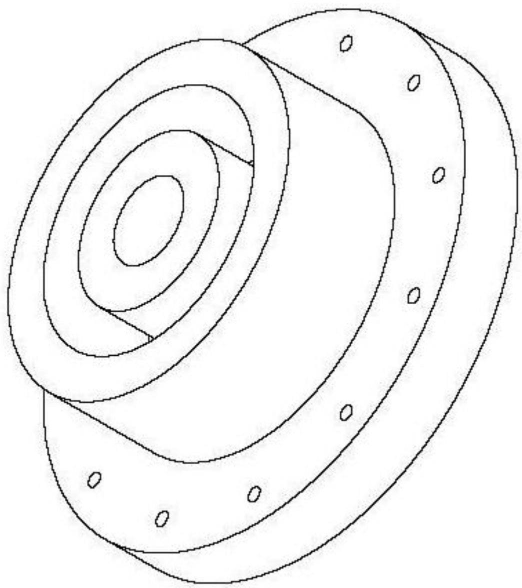

Figure 6: **End plate design with ridges.**

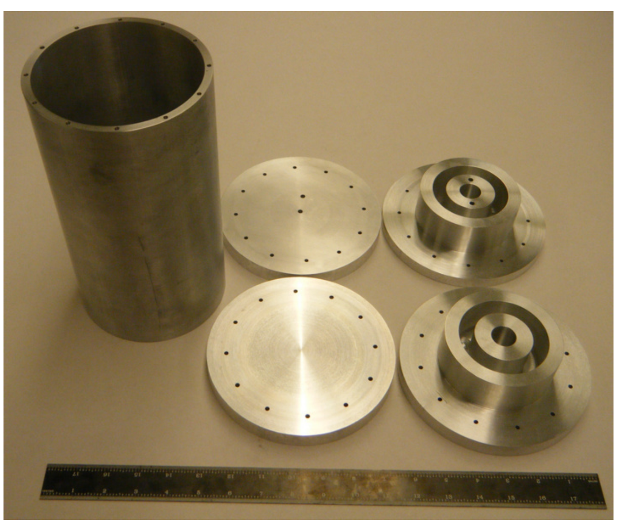

Figure 7: **Aluminum test cavity with two pairs of end plates.**

in one of the end plates and connected to an Agilent E5071C network analyzer. The resulting traces for both the flat end plates and those with ridges are shown in Figure 8. The important observation is that the frequency of the modes in the physical cavity corroborate well the frequencies predicted by Poisson/Superfish, as seen in Table 1. The Q factor measured by the network analyzer for the lowest mode was around 18000 for the flat end plates and 3700 for the ridged end plates.

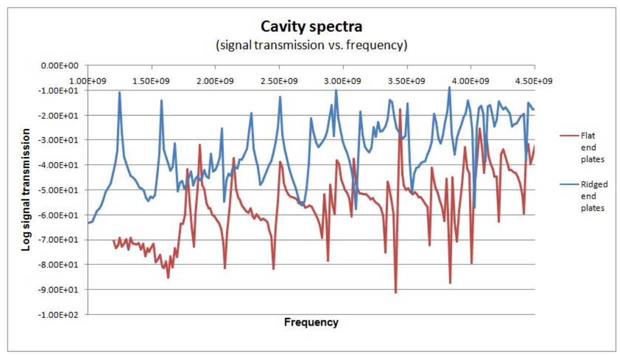

Figure 8: **Traces for each pair of end plates on the aluminum test cavity showing the normal mode frequencies of the cavity.**

3 Summary

This result justifies the notion that ridging end plates can provide an efficient mechanism for tuning the mode frequencies of a resonant cavity. The form factor of the relevant modes also remains reasonably high throughout this tuning. This has practical applications to the axion detection cavity of the ADMX setup, providing a direct means of probing otherwise difficult to access axion mass ranges.

Looking ahead, there are many geometric configurations, other than those tested already, which would

point that the cavity spectrum was just visable. Antenna port locations were chosen to be near high field points of the TM modes predicted by Poisson/Superfish; hence, some other cavity modes (including most TE modes) may not show up in the resulting spectrum, as the corresponding fields nearly vanish in the vecinity of the antennas.

Cavity mode frequencies (GHz)			
Ridged end plates		Flat end plates	
Predicted	Observed	Predicted	Observed
1.26	1.25	1.78	1.78
1.60	1.58	1.88	1.88
	1.68	2.14	2.14
1.97	1.96	2.50	2.51
	2.05	2.85	2.84
2.33	2.28	2.90	2.90
	2.51	2.95	2.96
2.76	2.75	3.08	3.08
	2.91	3.35	3.35
	2.94	3.44	3.45
	3.14	3.69	3.70
	3.24	3.81	3.81
3.28	3.28	3.86	3.86
	3.36	3.96	3.97
	3.50		
	3.71		
3.85	3.84		
	3.98		
4.09	4.08		
4.15	4.15		
	4.22		
	4.27		
4.34			
	4.41		
4.44	4.45		

Table 1: **Comparison of predicted and observed modes in the test cavity.**

be worth while investigating in a more thorough manner. Optimization of end plate shape is imperitive before any such design can be incorporated into the actual ADMX setup, and there are many parameters that have yet to be tested or optimized. Another extension of these results would be to investigate the effect of dielectrics in the cavity. Poisson/Superfish has capabilities for handling this problem as well, so at least the modeling aspect of this extension should not be difficult.

While the method of ridging end plates holds great appeal for ADMX, it does not necessarily replace the need for tuning rods in the cavity. In particular, while ridging the end plates provides an effective means of lowering mode frequencies by a fixed amount, adjusting this frequency shift in the course of an experiment by, say, changing the depth of the ridges poses a substantial engineering challenge (namely, how to maintain good electrical contact between the ridges and the rest of the cavity, to allow flow of surface currents). Eventually, the most effective tuning mechanism for ADMX is likely to be a combination of tuning rods *and* ridged end plates, and thus the final analysis must take into account both features.