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Characterization of the magnetic field of a cosine coil distribution

Andrew McClung

University of Washington Physics REU

Department of Physics, Carleton College

I. Introduction

The birth of electric dipole moment (EDM) experiments can be traced to a letter by Purcell and Ramsey published in *Physical Review* in 1950.[1] In their letter they maintain that, though the existence of a permanent electric dipole moment would violate parity (then thought to be an inviolable, fundamental symmetry), "there is no compelling reason for excluding this possibility." Observation of parity violation first occurred six years later, when C. S. Wu found that the electrons from the beta decay of cobalt-60 were preferentially emitted in the direction of nuclear spin.[2]

Today instances of both P- and the more egregious CP-violation are accounted for in the Standard Model, but not in quantities sufficient to explain certain fundamental phenomena. For instance, the preponderance of matter over antimatter in our universe requires CP-violation beyond the predictions of the Standard Model.[3] Unlike those of the Standard Model, EDM magnitude predictions from Supersymmetry, a potential extension for the Standard Model, are well within experimental reach.[3]

The experiment at the University of Washington searches for a permanent EDM in a diamagnetic isotope of mercury. The current upper bound for an EDM in mercury-199 is 3.1×10^{-29} e cm, C. L. 95%. Longer spin coherence lifetimes reduce the uncertainty in precession frequency, resulting in a more precise EDM measurement.[3] As spin coherence lifetimes depend on the uniformity of the magnetic field, any effort to create a more uniform field increases the sensitivity of the EDM measurement. This summer I sought to accurately map the magnetic field near the center of the coils used for the University of Washington EDM experiment and identify the largest field gradient, which will, through future auxiliary correction coils, improve the overall uniformity of the magnetic field.

II. Experiment at the University of Washington

The experiment at the University of Washington compares the Larmor precession frequencies of two spin polarized vapor cells to obtain evidence for permanent EDMs. A particle with magnetic moment μ in a magnetic field $\mathbf{B}$ experiences a torque

$$\tau_B = \mu \times \mathbf{B}. \tag{1}$$

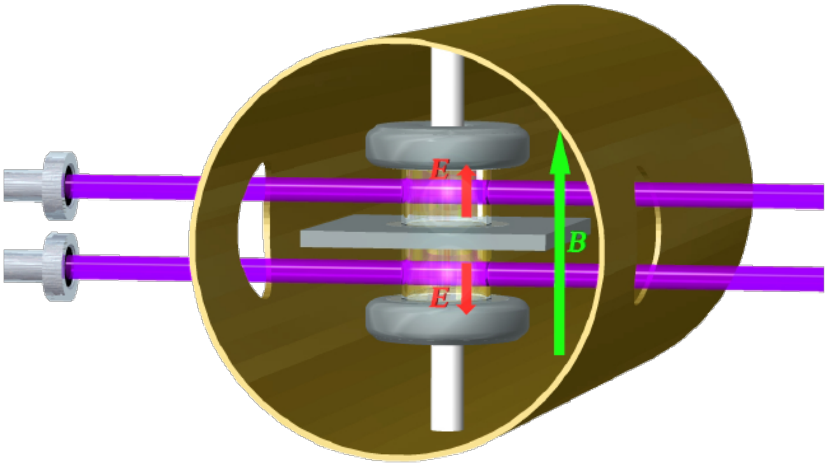

Figure 1: The top mercury cell is subject to parallel **E**- and **B**-fields, while for the bottom cell the **E**- and **B**-fields are antiparallel.(*Image: William Clark Griffith*)[3]

Similarly, a particle with an electric dipole moment d in an electric field $\mathbf{E}$ experiences a torque

$$\tau_E = d \times \mathbf{E}. \tag{2}$$

A permanent EDM would necessarily lie along the axis of spin.[3] Thus, if a permanent EDM exists, mercury atoms subject to the same magnetic field but opposite electric fields would experience different torques and would hence precess at different frequencies. This is at the core of the University of Washington experiment: two mercury gas cells are subjected to the same magnetic but opposite electric fields, as seen in Figure 1.

After spin-polarizing the cells, precession frequency measurements are made by passing a linearly polarized probe light through the cell. The spin-polarized cell rotates the plane of polarization through an angle proportional to the dot product of the direction of probe light propagation, $\hat{k}$, and the direction of the atomic polarization vector, $\mathbf{P}_a$. The probe light exiting the mercury cells is passed through a linear polarizer, after which its intensity is recorded. Because of the torque due to the magnetic field, $\mathbf{P}_a$ rotates at the precession frequency, which is recorded in the intensity data.

Over time, the spin polarization decays, leading to an attenuating signal, as seen in Figure 2.[3] The uncertainty in a single precession is inversely proportional to the spin coherence lifetime,[4] hence the sensitivity of EDM measurements will increase with longer polarization lifetimes. One of the major factors in the length of the spin coherence lifetime is the homogeneity of the magnetic field—any variation across the cell will lead to slightly different precession frequencies, giving rise in the long run to decoherence.

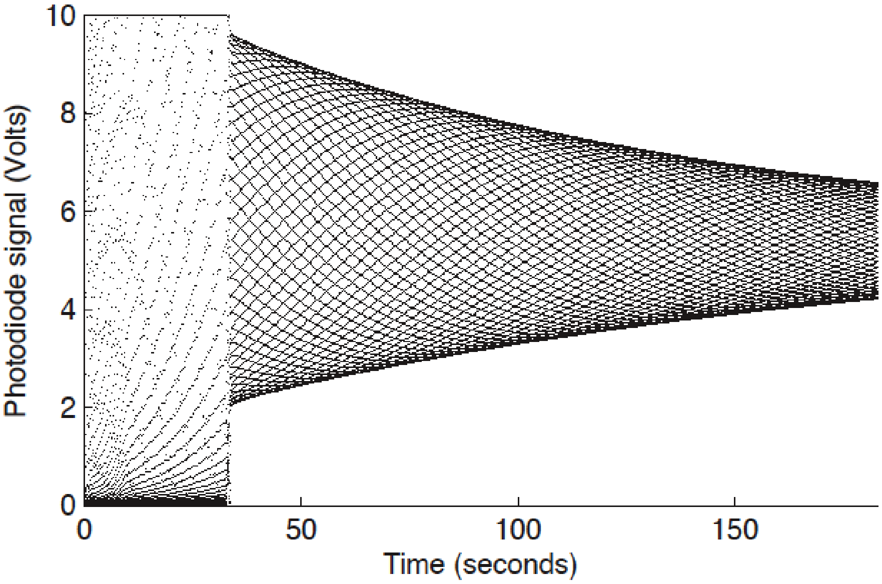

Figure 2: After an initial pump sequence, the spin polarization decays exponentially, resulting in an attenuating signal.(*Image: Matthew Swallows*)[4]

III. Mapping the magnetic field

My assignment this summer was to characterize the magnetic fields of the coils used for the University of Washington's EDM experiment. To achieve the homogeneous field necessary for the experiment, a cylinder wrapped with magnet wire in a cosine distribution, as depicted in Figure 3, is used. In addition to the main vertical field, there are windings that create axial and transverse fields. All three fields are independently variable using a current driver. To minimize external effects, the coil is placed in three layers of magnetic shielding, which must be degaussed prior to data collection to eradicate any accrued field inhomogeneities.

A. Data collection

The magnetic field was measured with a one dimensional flux gate magnetometer (depicted in Figure 4) attached by an aluminum rod to three orthogonal translators which allowed for accurate positioning within the coil. The flux gate could be oriented in three orthogonal directions corresponding to a right-handed coordinate system with the positive y-direction pointing along the axis of the cylinder.[†] The output from the flux gate was sent to a computer data acquisition program, which took the mean of the output from the flux gate over ten second intervals with a sampling rate of 50 hertz.

Field measurements were taken on a cubic lattice of 27 points spaced .5 cm apart, with the origin of the cube at the center of the coil. Because of Maxwell's equations for magnetism, only two orthogonal orientations of the flux gate were necessary to characterize a field. The requisite pair of orthogonal data sets

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[†] This is, confusingly, not the coordinate system depicted in Figure 3, which I was unaware of when I came up with my convention.

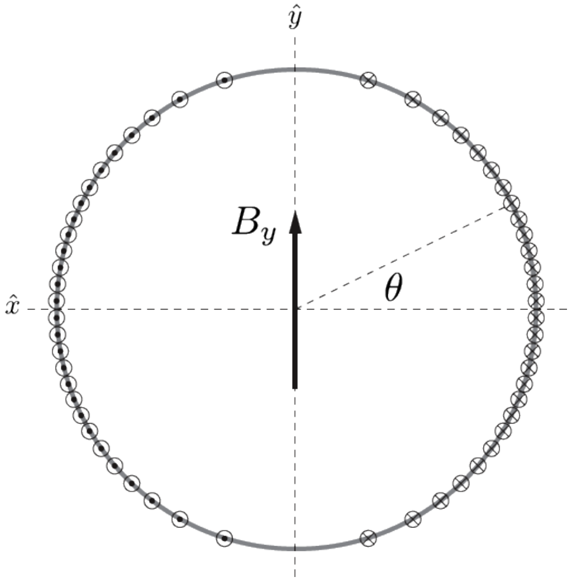

Figure 3: Wires distributed in a $\cos\theta$ distribution approximate a uniform field within the cylinder. (*Image: Matthew Swallows*)[4]

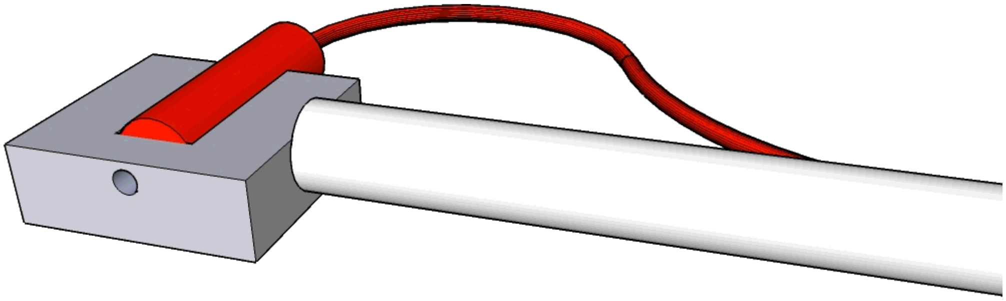

Figure 4: The mounting plate for the flux gate magnetometer was designed to accommodate measurements in the x, y, and z directions.

were taken on vertical, axial, and transverse fields.

B. Numerical methods

The second-order, three-dimensional Taylor series expansion requires the knowledge of 27 derivatives at a certain point $\mathbf{a}$. To expand about the point $\mathbf{a} = (0, 0, 0)$, these derivatives needed to be approximated. For functions which vary slowly with respect to the lattice spacing, a relatively good approximation is that the derivative at the midpoint $\mathbf{m}$ between two lattice points $\mathbf{l}$ and $\mathbf{n}$ is given by

$$\frac{B(\mathbf{n}) - B(\mathbf{l})}{\mathbf{n} - \mathbf{l}}. \tag{3}$$

First-order partial derivatives can be approximated by choosing $\mathbf{l}$ and $\mathbf{n}$ to differ only in one direction. In general, nth-order partial derivatives can be iterated from the lattice of $(n-1)$th-order partial derivatives.

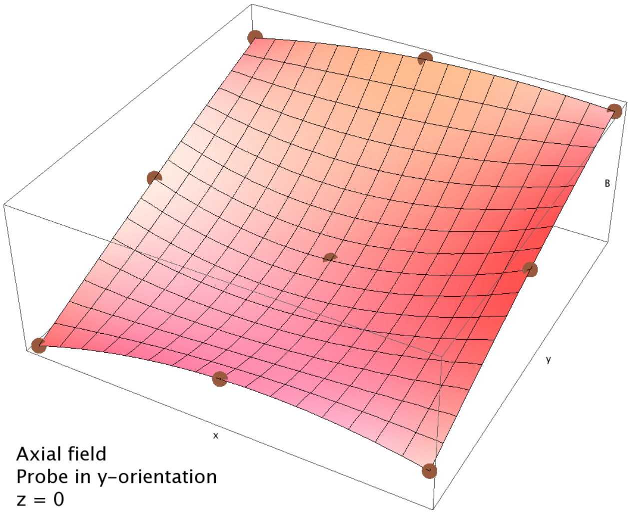

Figure 5: The cross-section of the calculated axial B_y field, taken at $z = 0$

Using Maxwell's equations for magnetism and the Taylor series expansion for two orthogonal probe orientations, a complete set of partial derivatives for B_x, B_y, and B_z could be determined. It can be shown that a complete set of partial derivatives determines the field up to a constant.

IV. Results

Using the methods described above, maps of B_x, B_y, and B_z were determined (each up to a constant factor) for the vertical, axial, and transverse fields. A comparison of measured B_y points and the calculated B_y field is shown in Figure 5.

V. Conclusion and future work

The field interpolation algorithms devised were successful in mapping a local segment of the magnetic field of a cosine coil distribution. In the long run, these data and algorithms may provide the necessary field gradient information to implement additional corrective fields. Before that point, it would be desirable to create take data with larger lattices in order to gain information about the behavior of the field over larger distance scales. Maps must also be made for the coil currently in use in the experiment, which, due to geometrical restrictions, would require a creative data acquisition scheme to be statistically relevant.

Acknowledgments

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References

[1] E. M. Purcell and N. F. Ramsey, Physical Review **78**, 807 (1950).

[2] D. Griffiths, *Introduction to Elementary Particles*, Wiley-VCH, 2nd edition, 2008.

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[4] M. Swallows, *A search for the permanent electric dipole moment of mercury-199*, PhD thesis, University of Washington, 2007.