# Evidence for a $U(1)_{B-L}$ Gauge Interaction in $\nu$ Oscillation Anomalies

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## Abstract

The recent short baseline neutrino experiments LSND and MiniBooNE exhibit an anomalous behavior unexplained by the standard theory of neutrino oscillations. The experiments indicate that neutrino oscillation probability depends on the both the energy scale and the chirality of the neutrinos concerned, and are consistent with the $U(1)_{B-L}$ gauge interaction model with 6 neutrinos and 3+2 mixing proposed by [1]. This paper will present a minimalistic 3+1 mixing described by the same model. A numerical analysis shows that the 3+1 mixing model provides an improved explanation for the LSND and MiniBooNE anomalies over the standard theory of neutrino oscillation. This model is moreover in good agreement with the long baseline data from MINOS.

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## I. INTRODUCTION

While the Standard Model (SM) does not include right-handed neutrinos, and by extension predicts massless neutrinos, recent observation of neutrino oscillation provides evidence that neutrinos have a nonzero mass. A standard seesaw mechanism of combined Dirac and Majorana mass terms has since been proposed to extend the SM to account for the evidence in favor of light but massive active neutrinos [2] [4] [10]. The three known neutrino flavors, $e$, $\mu$, and $\tau$, are each postulated to be a superposition of at least 3 known mass eigenstates, so for a neutrino of flavor $l$:

$$|\nu_l\rangle = \sum_{l,m} U_{l,m} |\nu_m\rangle \tag{1}$$

here the $U_{l,m}$ are the components of the leptonic mixing matrix $U$ [3]:

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \tag{2}$$

where $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$, $\theta_{12}, \theta_{13}, \theta_{23}$ are the standard mixing angles, and the CP-violating phase has been dropped. Experimental evidence indicates that neutrino masses are on the order of less than 1 eV with mass squared differences $\Delta m_{12}^2 = (8.0 \pm 0.3) \cdot 10^{-5}$ eV$^2$, $|\Delta m_{23}^2| = (2.5 \pm 0.2) \cdot 10^{-3}$ eV$^2$ [1] [2]. An ultrarelativistic limit on the Hamiltonian of a neutrino propagating in space is thus justified. If $\mathcal{M}$ is the mass matrix of the neutrino, the Hamiltonian in the mass basis is given by [9]:

$$H_m = \frac{\mathcal{M}^2}{2E} + V_W \tag{3}$$

where $V_W$ is due to the MSW effect and is nonzero only for a neutrino propagating in matter. Note that this is a relativistic expansion, and that we may drop terms proportional to the identity. The Hamiltonian can then be rewritten in the flavor basis using the leptonic mixing matrix $U$:

$$H_{fl} = U^\dagger H_m U \tag{4}$$

Which can then be used to calculate the oscillation probability of a neutrino of flavor $l$ into a flavor $l'$:

$$P_{l\to l'}(L,E) = |\langle \nu_{l'}(0)|\, \nu_l(L)\rangle|^2 \tag{5}$$

where $L$ is the baseline. For the case of only 2 neutrinos at an energy $E$, this formula reduces to [3]:

$$P_{l\to l'} = \sin^2 2\theta \sin^2\left(\frac{L\Delta M^2}{4E}\right) \tag{6}$$

The standard theory thus fails to distinguish between neutrinos and antineutrinos, and is moreover sensitive to changes in $\frac{L}{E}$, but not to individual changes in $E$ or $L$ in the absence of matter effects and CP violating phases for 3 neutrinos. We note that the CP violating phase has a near zero coefficient and can therefore be assumed to have a negligible affect on the oscillation probability. The recent experiments LSND and MiniBooNE are, neglecting the possibility of experimental misinterpretation, indicative of differences in neutrino oscillation probability depending on chirality and energy scale. If these experiments can be taken prima facie, a new interaction accounting for both factors should be consistent with the anomalous results of LSND and MiniBooNE. In this paper, I argue that the addition of a $U(1)_{B-L}$ gauge interaction to the SM reconciles these anomalies. This interaction gives rise to a new $B-L$ potential, which differentiates between left- and right-handed neutrinos and suppresses oscillations at high energies regardless of the baseline. This model will be shown to be in reasonable agreement with the initial short-baseline data of LSND and MiniBooNE, as well as with preliminary long baseline $\nu_\mu$ and $\bar{\nu}_\mu$ survival data from the MINOS experiment. Qualitative predictions for $\nu_e$ and $\bar{\nu}_e$ appearance at MINOS are made.

## II. LSND AND MINIBOONE

The LSND experiment produced a $\bar{\nu}_\mu$ beam of energies less than 100 MeV in a 30 m long collider, and recorded the number of $\bar{\nu}_e$ events at the detector. LSND saw an excess of $\bar{\nu}_e$ events at $3.8\sigma$ although it was sensitive to mass squared differences larger than 0.1 eV$^2$, with the best fit point at $\Delta M^2 = 1$ eV$^2$ [5]. As it is far larger than $\Delta m^2_{12}$ or $\Delta m^2_{23}$, it has been suggested that a heavier sterile neutrino might be contributing to the oscillations [2]. The MiniBooNE experiment, designed to test LSND, accelerated $\nu_\mu$ and detected $\nu_e$ events in a 541 m baseline, while keeping the $\frac{L}{E}$ ratio the same as that of LSND [6]. Since the standard probability formula exhibits a periodic $\frac{L}{E}$ dependence and fails to distinguish between neutrinos and anti-neutrinos, MiniBooNE was expected to exhibit the same excess of LSND in higher energies. The data from MiniBooNE, however, exhibits no evidence of oscillations above 475 MeV but saw an anomalous excess of $\nu_e$ events at $3\sigma$ in energies between 200-475

MeV [6].

These data suggest that the oscillation probability depends on baseline and energy individually rather than on their ratio, and that neutrinos and anti-neutrinos exhibit different oscillation patterns, which justifies adding an interaction that would depend separately on each of the required parameters and thus may be able to account for these anomalies. For this purpose, the model discussed in this paper utilizes a $U(1)_{B-L}$ gauge interaction mediated by a light gauge boson called the paraphoton. This gauge interaction will be shown to generate a potential which is sensitive to energy, baseline, and chirality.

## III. ADDING A $B - L$ INTERACTION

Extended SM theories that include neutrino mass generally introduce a seesaw mechanism similar to the quark model and include heavy sterile neutrinos [2] [4]. This model, introduced by [1], utilizes a miniseesaw mechanism with three light sterile neutrinos ($m \sim$ eV). The mass of the neutrinos is composed of both Dirac and Majorana mass terms. Note that this implies that neutrinos are Majorana particles and that $L$, and consequently $B - L$, are violated. In order to satisfy the phenomena seen at the range of short-baseline experiments, the paraphoton is required to have a mass on the order of tens of keV. For further discussion on the mass of the paraphoton, see [1]. Experimental constraints requiring high energy neutrinos to exhibit few oscillations imply that the Dirac mass matrix can be taken to be degenerate. In [1], the 6x6 mass matrix was taken to be

$$\mathcal{M} = \begin{pmatrix} 0 & mU \\ mU^\dagger & M \end{pmatrix} \tag{7}$$

where $m$ is the common Dirac mass, $U$ is the 3x3 leptonic mixing matrix, and $M$ is the diagonal 3x3 Majorana mass matrix. The potential matrix in the flavor basis is a block diagonal 6x6 matrix [1]:

$$\mathcal{V} = \begin{pmatrix} -V & 0 \\ 0 & V \end{pmatrix}$$

which remains unchanged under rotations of the active flavors. $V$ carries opposite signs for active and sterile neutrinos, and for active neutrinos and antineutrinos. This allows the $B - L$ potential to account for differences in oscillation probability of neutrinos and antineutrinos. More explicitly, consider the Hamiltonian in the flavor basis

$$H_{fl} = \frac{\mathcal{M}^2}{2E} + \mathcal{V} \tag{8}$$

Note that we set $\theta_{13} = 0.244, \theta_{12} = 0.591, \theta_{23} = 0.785$ [1] [4] [2]. If $V_{B-L} < 0$, as it is for antineutrinos, the Hamiltonian may become nearly degenerate at certain values of $E$, thus allowing for increased oscillations due to resonance, which allows the model to predict greater oscillations for antineutrinos than for neutrinos at specific energy ranges.

If $U_D$ is the matrix that diagonalizes $H_{fl}$ and $H_D$ is the diagonalized Hamiltonian in the flavor basis, i.e. $H_{fl} = U_D^\dagger H_D U_D$, then the standard oscillation formula may be used to calculate oscillation probabilities [1]:

$$P_{l\to l'} = \sum_{\alpha,\beta} |K_{ll',\alpha\beta}| \cos\left(\widetilde{\Delta}_{\alpha\beta} L - \arg\left(K_{ll',\alpha\beta}\right)\right) \tag{9}$$

$$K_{ll',\alpha\beta} = U_D^{l\alpha} U_D^{l'\alpha *} U_D^{l\beta *} U_D^{l'\beta}$$

$$\widetilde{\Delta}_{\alpha\beta} = \left|(H_D)_{\alpha\alpha} - (H_D)_{\beta\beta}\right|$$

While in principle the model allows for mixing between all 6 neutrinos, a numerical analysis reveals that a reasonable fit for the data of LSND, MiniBooNE, and MINOS may be obtained by allowing only for 3+1 mixing, assuming 2 sterile neutrinos with negligible mixing. The minimalistic 3+1 scenario is given preference for detailed study, although it should be noted that allowing the remaining two sterile neutrinos to mix can be reasonably expected to improve the fit of the data to the model. To specialize the general model to four neutrino mixing, we rewrite $H_{fl}$, omitting two of the sterile neutrinos.

$$\mathcal{M}'^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta^2 & 0 \\ 0 & 0 & 0 & M'^2 \end{pmatrix}, U_a = \begin{pmatrix} U & & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, U_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\theta) & \sin(\theta) \\ 0 & 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$U' = U_a \cdot U_s \tag{10}$$

We neglect the smaller mass squared difference responsible for solar neutrino oscillations in this analysis, because it does not produce significant effects at the energies and baselines we

[] [2] While $\theta_{13}$ has not been precisely determined, we use the experimentally preferred value 0.244 radians to simplify the numerical analysis.

consider.

$$\mathcal{V} = \begin{pmatrix} V_W & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{B-L} \end{pmatrix}$$

where $\theta$ is the sterile-active mixing angle. Note that the CP-violating phase and all terms proportional to the identity have been dropped. The 4x4 Hamiltonian in the flavor basis can then be calculated from equation (8), and we use equation (9) to find the probability of oscillation for a fixed point $(\delta, \theta, V_{B-L}, M)$ in parameter space.

**A Simplified Analytic Approach**

While the MSW term in the potential matrix $\mathcal{V}$ renders an analytic approach impractical, in the limit where $V_W \rightarrow 0$ - for a neutrino propagating in a vacuum, for instance - the oscillation or survival probabilities may be calculated analytically, as the nonzero entries of the Hamiltonian are then simply a 2x2 matrix. Note that numerically, $V_W$ is too small to produce significant effect for the experiments we analyze. We find the survival probability of a muon neutrino, and then proceed to the numerical analysis, which includes the MSW term.

The Hamiltonian in the flavor basis is, as before:

$$H = U_a U_s \frac{\mathcal{M}^2}{2E} U_s^T U_a^T + 2\lambda V_s$$

$$\lambda = \pm 1 \tag{11}$$

$$V_s = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{B-L} \end{pmatrix}$$

Since $U_a$ and $V_s$ commute:

$$H = U_a \left( U_s \frac{\mathcal{M}^2}{2E} U_s^T + 2\lambda V_s \right) U_a^T = U_a U_{\tilde{s}} H_D U_{\tilde{s}}^T U_{\tilde{s}}^T \tag{12}$$

where

$$U_{\tilde{s}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\widetilde{\theta}) & \sin(\widetilde{\theta}) \\ 0 & 0 & -\sin(\widetilde{\theta}) & \cos(\widetilde{\theta}) \end{pmatrix}$$

and

$$H_D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \widetilde{m}^2 & 0 \\ 0 & 0 & 0 & \widetilde{M}^2 \end{pmatrix} \tag{13}$$

where $\widetilde{\theta}$ is the diagonalizing angle and $\widetilde{m}^2, \widetilde{M}^2$ are the eigenvalues of $H$. As noted above, $H$ is effectively a 2x2 matrix

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m^2\cos(\theta)^2 + M^2\sin(\theta)^2 & (M^2 - m^2)\cos(\theta)\sin(\theta) \\ 0 & 0 & (M^2 - m^2)\cos(\theta)\sin(\theta) & M^2\cos^2(\theta) + m^2\sin^2(\theta) + 4\lambda V E \end{pmatrix} \tag{14}$$

while the inclusion of the MSW term would have allowed for nonzero entries in the first and second rows of $H$. The equivalence (12) allows us to solve for $\widetilde{\theta}$

$$\tan\left(2\widetilde{\theta}\right) = \frac{\sin(2\theta)}{\cos(2\theta) + \frac{4\lambda V E}{M^2 - m^2}} \tag{15}$$

and for $\widetilde{m}^2$ and $\widetilde{M}^2$:

$$\begin{aligned} \widetilde{m^2} &= \frac{M^2 + m^2}{2} + \frac{M^2 - m^2}{2}\left[\Lambda - \sqrt{1 + 2\Lambda\cos(2\theta) + \Lambda^2}\right] \\ \widetilde{M^2} &= \frac{M^2 + m^2}{2} + \frac{M^2 - m^2}{2}\left[\Lambda + \sqrt{1 + 2\Lambda\cos(2\theta) + \Lambda^2}\right] \\ \Lambda &= \frac{4\lambda V E}{M^2 - m^2} \end{aligned} \tag{16}$$

and the diagonalizing matrix $U_D = U_a U_{\tilde{s}}$ is finally:

$$U_D = \begin{pmatrix} c_{13} & 0 & c_{\widetilde{\theta}} s_{13} & s_{\widetilde{\theta}} s_{13} \\ -s_{13}s_{23} & c_{23} & c_{13}c_{\widetilde{\theta}}s_{23} & c_{13}s_{23}s_{\widetilde{\theta}} \\ -c_{23}s_{13} & -s_{23} & c_{13}c_{23}c_{\widetilde{\theta}} & c_{13}c_{23}s_{\widetilde{\theta}} \\ 0 & 0 & -s_{\widetilde{\theta}} & c_{\widetilde{\theta}} \end{pmatrix} \tag{17}$$

where the rows are flavor eigenstates and the columns are mass eigenstates. The survival probability for muon neutrinos is then

$$P_{\mu\to\mu} = 1-4\beta(1-\beta)\left[\cos^2\widetilde{\theta}\sin^2\left(\frac{\widetilde{m}^2 L}{4E}\right)+\sin^2\widetilde{\theta}\sin^2\left(\frac{\widetilde{m}^2 L}{4E}\right)\right]-\beta^2\sin^2 2\widetilde{\theta}\sin^2\left(\frac{\left(\widetilde{M}^2-\widetilde{m}^2\right)L}{4E}\right) \tag{18}$$

$$\beta^2 = c_{13}^2 s_{23}^2$$

and is demonstrably dependent on $E$, $L$, and the sign of $V_{B-L}$. This probability also exhibits a periodic dependence on the mixing angle with a period of $\pi$.

## IV. NUMERICAL ANALYSIS: LSND, MINIBOONE, AND MINOS

The numerical analysis incorporated $V_W$ in the Hamiltonian as given by equation (8). A constrained parameter space consisting of $(\delta, \theta, V_{B-L}, M')$ was then sampled for $2\cdot 10^6$ points. The Hamiltonian was numerically diagonalized for each point and the probability and $\chi^2$ for the data of each experiment (LSND neutrino and antineutrino data, MiniBooNE neutrino and anti-neutrino data, and MINOS neutrino and anti-neutrino data) was calculated, with 49 degrees of freedom. I note that due to the significant error margins on MiniBooNE and MINOS anti-neutrino data, the model was not significantly constrained by the data in these experiments. The range of $V_{B-L}$ and $M$ has been limited by the analysis of [1] to:

$$0 < V_{B-L} < 5 \text{ neV}$$

$$0.9 \text{ eV} < M' < 7 \text{ eV}$$

Note that the lower limit of $V$ did not prove to be constraining and was therefore dropped, while the range of $M'$ was expanded to account for the minimalistic use of only one sterile neutrino mixing, as opposed to the two used in [1]. I put a reasonable upper limit of 0.01 eV on $\delta^2$, while allowing $\theta$ to vary between 0 and $\pi$. The remaining parameter space is of volume (0.213 eV)$\cdot$(5 neV). Approximately 0.1% of the sampled points fit the LSND, MiniBooNE, and MINOS data, where the fitting points were required to satisfy $\chi^2/DOF \leq 2$.

The best-fit-point of the parameter space, $\delta = 0.034$ eV, $\theta = 3.06$ rad, $V_{B-L} = 4.6$ peV, $M = 1.99$ eV was found to have a combined $\chi^2_{tot}$ value of (91.1)/(49 DOF). In figures 1-10, we plot the $\chi^2$ distribution against different values of $\theta$ for the $\chi^2_{tot}$, as well as for each experiment individually to demonstrate that the data favors sterile-active mixing over the standard theory active-active mixing only. For every experiment, with the exception of the MiniBooNE antineutrinos where the data has high uncertainties, a nonzero value of $\theta$ was favored. The overall $\chi^2_{tot}$ is over 20 points lower for $\theta > 0.05$ than for $\theta = 0$, which is a sufficiently large difference to suggest non-negligible sterile-active mixing.

We use the best fit point to make a qualitative prediction for MINOS $\nu_e$ and $\bar{\nu}_e$ appearance in figure (number). We expect that a limited amount of $\nu_\mu \rightarrow \nu_e$ oscillations should be observable for energies below 0.5 GeV, while $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations are enhanced at higher energies and are more likely to be easily observed in the MINOS analysis range.

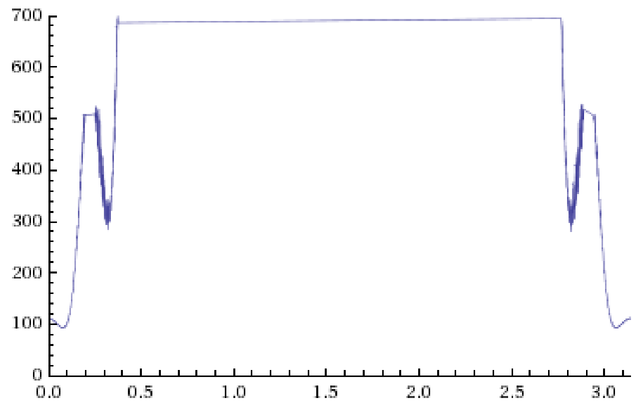


FIG. 1: $\chi^2_{tot}$ graphed against $\theta$. There is clear minimum at $\theta \sim 0.1$ and $\theta \sim \pi - 0.1$.

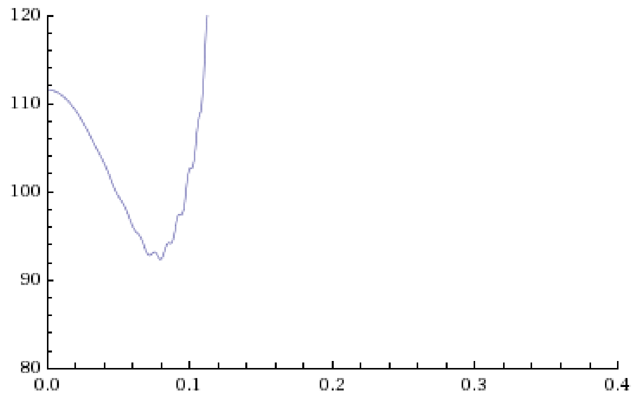


FIG. 2: The minimum values of $\chi^2_{tot}$ at small values of $\theta$. The minimum at small but nonzero values of $\theta$ shows that the 3+1 $B - L$ model's small active-sterile mixing is favored over the standard theory.

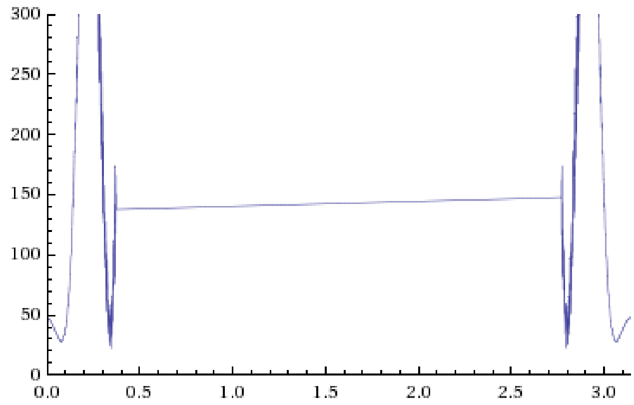


FIG. 3: The distribution of $\chi^2$ for the MINOS experiment muon neutrino survival data as a function of $\theta$. Minimum values are obtained at about $\theta = 0.1$, $\theta = \pi - 0.1$, $\theta = 0.3$ and $\theta = \pi - 0.3$

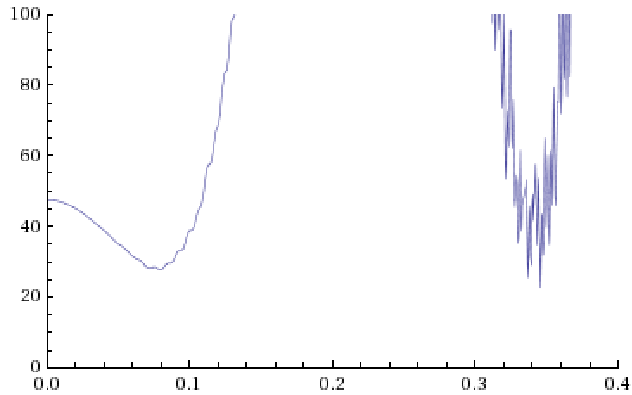


FIG. 4: Minima of $\chi^2$ for MINOS at small values of $\theta$. $\theta = 0$ is not a minimum.

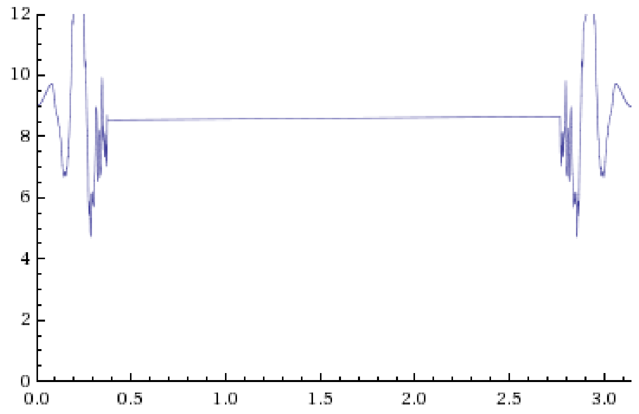


FIG. 5: The distribution of $\chi^2$ for the MINOS experiment muon antineutrinos survival data appears to favor the same $\theta$ values as MINOS neutrinos do.

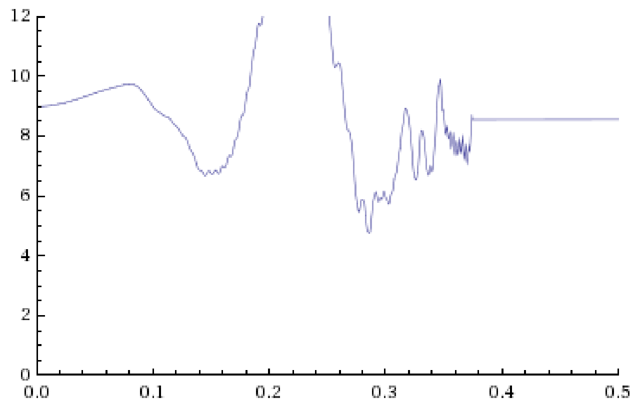


FIG. 6: Minima of $\chi^2$ for MINOS antineutrinos at small values of $\theta$. As for the MINOS neutrinos, $\theta = 0$ is not a minimum.

Since the graphs of $\chi^2$ are clearly symmetric about $\pi/2$, we limit the graphs below to that region for brevity.

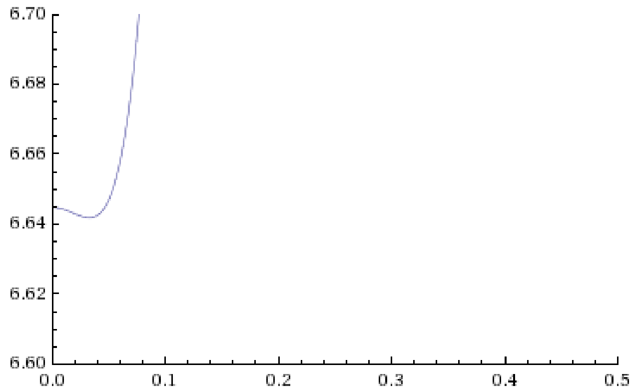


FIG. 7: The MiniBooNE experiment for muon neutrino survival $\chi^2$ distribution at small values of $\theta$. While the minimum is less pronounced in this distribution, it is still not centered at $\theta = 0$.

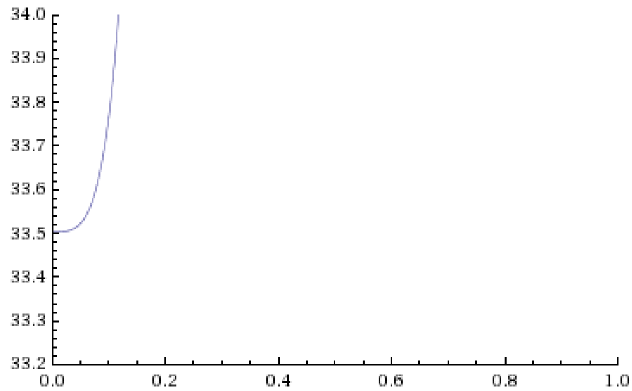


FIG. 8: The MiniBooNE experiment for muon antineutrino survival $\chi^2$ distribution at small values of $\theta$. The only experiment to favor a $\theta$ value of 0, it does not greatly undermine the hypothesis of small sterile-active mixing due to the preliminary nature of the data and its large error bars.

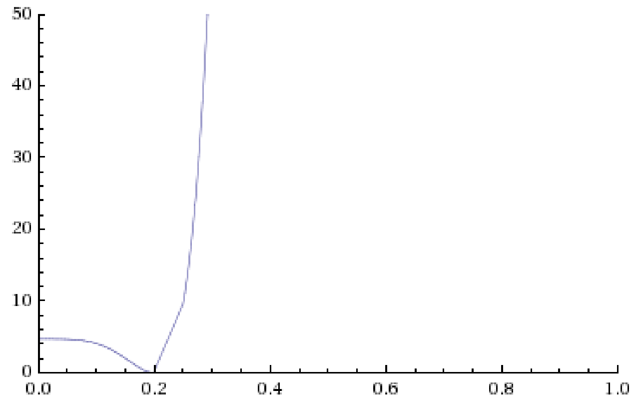


FIG. 9: The LSND experiment for electron neutrino appearance $\chi^2$ distribution at small values of $\theta$. LSND also favors a small nonzero mixing angle.

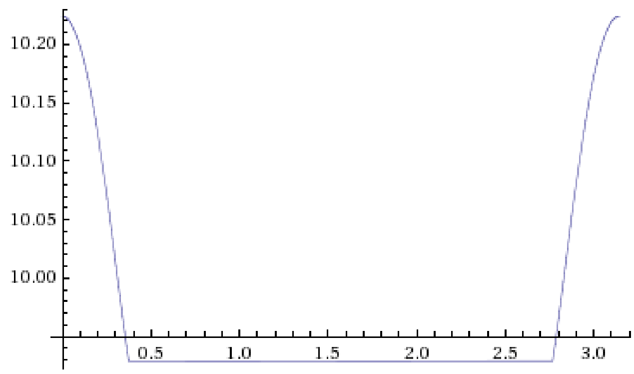


FIG. 10: The LSND experiment for electron antineutrino appearance $\chi^2$ distribution. LSND antineutrino data also appears to favor a small nonzero mixing angle [3].

Note that, as stated above, the oscillation probability is proportional to $\sin^2(2\theta)$, so $\theta = a$ and $\theta = \pi - a$ are both referred to as a small mixing.

We use the best fit point above to make a qualitative prediction for MINOS $\nu_e$ and $\bar{\nu}_e$ appearance in Figures 11 and 12.

[] [3] Note that LSND neutrino and antineutrino $\chi^2$ values were both based on one data point each.

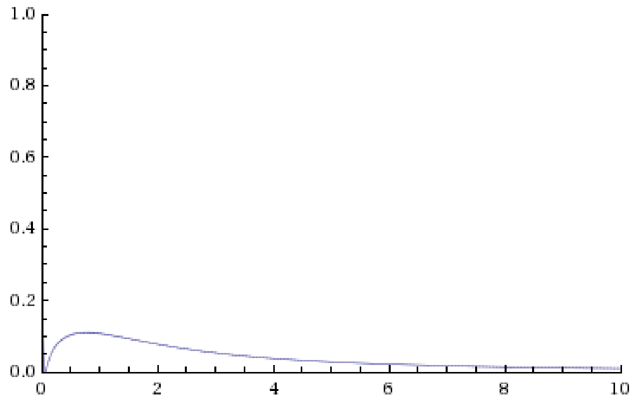


FIG. 11: Predictions for MINOS $\nu_e$ appearance probability, with energy in GeV

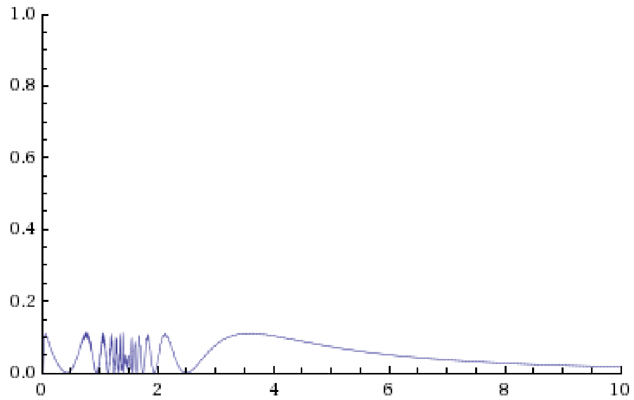


FIG. 12: Predictions for MINOS $\bar{\nu}_e$ appearance probability, with energy in GeV

We expect that a limited amount of $\nu_\mu \to \nu_e$ oscillations should be observable for energies below 0.5 GeV, while $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations are enhanced at higher energies and are more likely to be easily observed in the MINOS analysis range.

## V. SUMMARY

I have shown that a 3+1 minimalistic interpretation of a 3+3 $U(1)_{B-L}$ gauge interaction model provides a better explanation of the recent data of MiniBooNE, LSND, and MINOS than the standard theory does. The parameter space used has been constrained by the analysis of [1], which accounted for a number of other neutrino experiments. The predictions for future MINOS data expect low energy $nu_e$ appearance probability, which may not be observable at the MINOS analysis range, and higher energy $\bar{\nu}_e$ appearance probability, which should be observable at the MINOS analysis range.

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