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A Cryogenic Torsion Balance

Equivalence Principle

As the famous (and probably apocryphal) story goes, sometime around 1600 Galileo performed an experiment in which he simultaneously dropped balls of different masses off of the Leaning Tower of Pisa, demonstrating that the balls took the same amount of time to reach the ground.

This was one of the earliest experiments relating to the Equivalence Principle (EP), which states that a uniform gravitational field is (locally) indistinguishable from a uniformly accelerated reference frame. With a little thought, one can see that the EP implies that in a uniform gravitational field, the trajectory of a particle in free fall depends only on the initial position and velocity of the particle, and not on the particle's mass or composition. This implication is known as the "Universality of Free Fall," abbreviated UFF. It is the UFF that Galileo's Leaning Tower experiment was testing. [1]

The UFF can be understood in the Newtonian description of gravity as requiring that the inertial mass of an object be exactly proportional to the gravitational mass of the object. Newton's Second Law,

$$F = m_i a$$

relates the inertial mass m_i of an object to the force on an object and the acceleration the object experiences due to that force. Newton's Law of Gravitation,

$$F_g = G\frac{m_s m_g}{r^2} = m_g g$$

gives the gravitational force on a body with gravitational mass m_g separated from source mass m_s by a distance r. We consider the situation in which m_s and r are very nearly constant (as happens near the surface of the Earth) and summarize all constants in the equation by calling them "g." Thus, we can see that when gravity is the only force acting on an object, the acceleration due to gravity will be

$$a = \frac{m_g}{m_i} g$$

If we assume that m_i and m_g are exactly proportional, we might as well define this ratio to be 1, i.e. we say that inertial mass is exactly equal to gravitational mass. Thus, tests of UFF are often considered to be tests of whether $m_i = m_g$ for all matter.

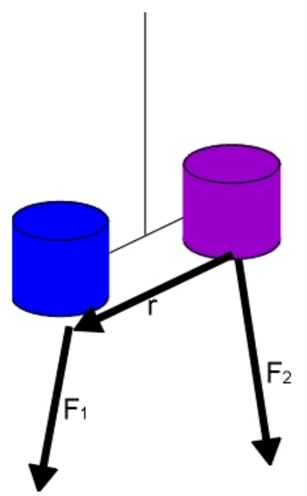
Einstein was struck by the EP and used it as a starting point when attempting to develop a relativistic theory of gravity. His resulting theory, General Relativity, indeed assumes that UFF holds. General Relativity is the most successful theory of gravity to date; any test of UFF is also a test of General Relativity.

Perhaps the most exciting reason to continue to test UFF is that modern attempts to unify quantum theory and General Relativity predict that UFF, and hence EP, *is* violated, albeit at a small scale. Thus, extremely precise measurements of UFF are one of the best ways to test new quantum theories of gravity. (A new weak interaction that couples to something other than mass, such as baryon number, would also show up in a UFF experiment as a "violation" of EP.)

There are several ways to test UFF. As noted above, one method is simply to drop objects in the Earth's gravitational field and watch for any differential acceleration. Another method, utilized by Newton among others, is to suspend different masses from swinging pendulums of equal arm lengths and determine the periods of the pendulums. To date, however, the most sensitive tests of UFF have been performed using torsion pendulums, first used by Eötvös in the early twentieth century to confirm UFF to within a few parts in a billion. The latest UFF experiment from the Eöt-Wash group using a rotating torsion pendulum confirms the equivalence principle (with a range extending from infinity down to approximately one meter) to within a few parts in ten trillion. [2]

Torsion Pendulums

Torsion pendulums are used in tests of the equivalence principle because they are extraordinarily sensitive to slight angles in the *net* forces on test bodies. This sensitivity comes from their almost uninhibited ability to rotate; the restoring torque from ultra-thin torsion fibers used in these experiments is extremely small.



We consider the cartoon diagram (see left) to be a simplified torsion pendulum, where two bodies of different composition hang opposite one another. We presume that there is some "source mass" which gravitationally acts on the pendulum, producing forces on the test bodies that contribute to the net forces F_1 and F_2 on the bodies. The magnitude of the torque T along the fiber is given by

$$T = \frac{(F_1 \times F_2) \cdot r}{F_1 + F_2}$$

This means the pendulum will rotate if the net forces on the test bodies aren't parallel, i.e. if there is a difference in the horizontal accelerations of the bodies. Here, horizontal is with respect to the vertical line defined by the torsion fiber. In tests of EP, this means that the gravitational field acting on the pendulum must have a horizontal component relative to the pendulum; this is achieved in various ways, depending on the source mass being used. (E.g., with the Earth as a source mass, the inertial force caused by the Earth's rotation makes a horizontal gravity component appear.) In a uniform gravitational field (and in the absence of stray forces that would couple differently to the two different test bodies, such as electromagnetic forces), there will be no torque, even if the two test bodies on either side of the pendulum have different masses or are made of different materials. [6]

Modern torsion balance EP experiments feature a source mass that rotates with respect to the pendulum with some period; this reduces uncertainty when data is collected over many periods. In this case, one would monitor the deflection angle of the torsion pendulum as the source mass moves around the pendulum to see if there were any systematic deflections with the same period (and phase) as the period of source mass rotation.

Challenge: Gravity Gradients

One of the leading issues with torsion balance experiments in practice is the presence of gravity gradients near the Earth's surface. In a non-uniform gravitational field, the geometry of a torsion pendulum can lead to unwanted torques that would occur with the same frequency as a true EP-violating signal (even if EP is not violated). Furthermore, there may be torques that have a period different from the one of interest for EP-violation, but which are strong enough to introduce unwanted background effects into the data. In order to reduce these unwanted signals, one must either make the test pendulum insensitive to these gradients or one must cancel the gradients with compensators (or both).

Expansion of the gravitational potential energy between a source mass and a test mass in spherical harmonics gives a very useful expression relating the geometry of the test pendulum to the torques one could expect due to the gravity gradients produced by a particular source mass. We let the test mass (the pendulum) have mass density $\rho_p(\overrightarrow{r})$ and the source mass have mass density $\rho_S(\overrightarrow{r}')$. Then the potential energy of the test pendulum from the field of the source mass is given by

$$W = -G \int d^3r \rho_p(\overrightarrow{r}) \int d^3r' \rho_S(\overrightarrow{r}') \frac{1}{|\overrightarrow{r} - \overrightarrow{r}'|} = -4\pi G \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{m=-l}^{+l} q_{lm} Q_{lm}$$

where $q_{lm} = \int \rho_p(\overrightarrow{r}) r^l Y^*_{lm}(\hat{r}) d^3r$ and $Q_{lm} = \int \rho_S(\overrightarrow{r}') r'^{-(l+1)} Y_{lm}(\hat{r}') d^3r'$. The important point is that one has expressed the potential energy as a sum of terms, each of which is a product of something that depends only on the test mass ($q_{\rm lm}$, called the "*lm*th order multipole moment) and something that depends only on the source mass ($Q_{\rm lm}$, called the "*lm*th order multipole field"). Furthermore, for a source mass that is relatively far away from the pendulum, the terms in the double sum fall off in magnitude rapidly; thus, only needs to worry about the first few low-order multiple moments and fields. One can then differentiate the potential energy with respect to the rotational angle ϕ of the pendulum to find the expected torque, and thus the expected angular deflection, due to the effects of the multipole moments coupling to nearby gravitational multipole fields [3],[5].

In order to minimize these unwanted torques in EP experiments, gravity compensators can be built around the pendulum in order to cancel unwanted gradients. In addition, the torsion pendulums themselves are designed to have very small low-order multipole moments.

(One can calculate using the above formulas that in a free-hanging pendulum, the q_{11} moment will always vanish, since the center of mass of the pendulum will always hang along the line of the torsion fiber. This confirms that in a uniform gravitational field with no EP violation, there would be no torque on a pendulum, even if it were asymmetrical.)

Cryogenic Experiment

One of the main limitations on the sensitivity of modern torsion balance experiments is the difficulty in resolving extremely tiny angles of deflection of the pendulum. Thermal noise in the torsion fiber is the main limiting factor here. The torque noise in a torsion fiber is give by

$$S_{\tau}^{\frac{1}{2}}(f) = \sqrt{\frac{4k_B T\kappa}{2\pi Q f}}$$

where T is the temperature of the torsion fiber, κ is the torsion constant, Q is the quality factor of the wire and f is the frequency of the signal under consideration. Cryogenic temperatures not only reduce T, but also increase the quality factor Q of some torsion fiber materials by several orders of magnitude. The combination of these two factors could reduce torque noise by a factor of 40, significantly reducing the statistical uncertainty in torsion balance EP experiments [1]. In Frank Fleischer's particular setup, the torsion fiber will be somewhat shorter than those typically used in Eöt-Wash experiments, which increases the value of the torsion constant κ in the above torque noise expression. Even when taking this into account, however, Frank's apparatus has the potential to reduce torque noise by at least an order of magnitude.

For Frank's experiment, the source mass will be the Sun and/or the galactic center. Thus, the relative source/test mass rotation is provided by the rotation of the Earth. The advantage of this is that the rotation is very smooth; furthermore, the experiment will be sensitive to EP violations with long ranges, which are of particular theoretical interest. In addition, the setup for the experiment is simpler than those that use close-range source masses, as gravity compensators and rotation tables are not required. The disadvantage of this is that there are many systematic effects that also have a daily period, such as human traffic, tidal changes and temperature changes.

Figure 1 below illustrates the basic design of the cryogenic apparatus:

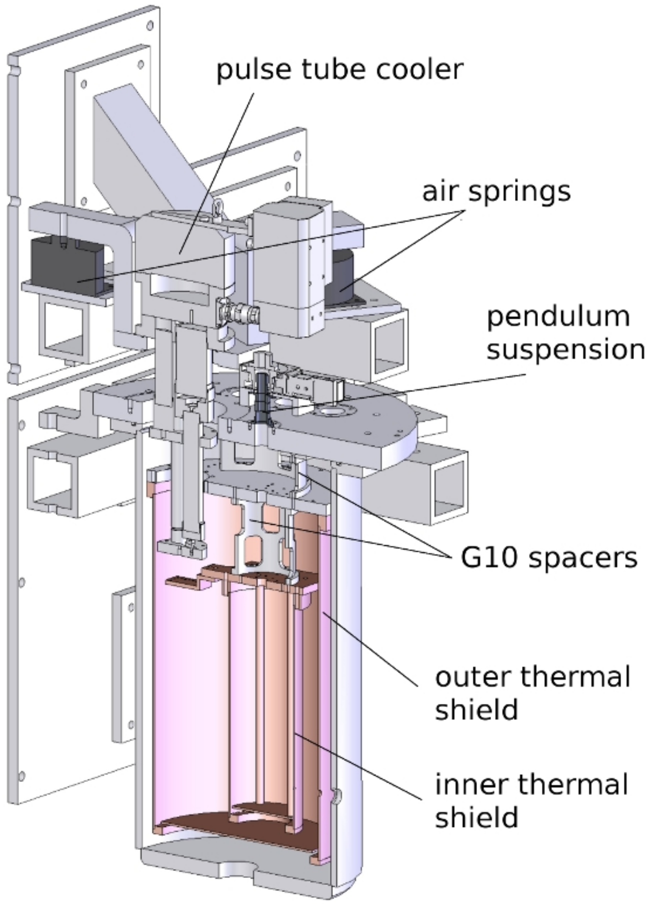


Figure 1: A cutaway view illustrating the heart of the apparatus. The autocollimator, not shown, will be situated on the side of the vacuum chamber opposite the wall mounting. The outer thermal shield will be cooled down to approximately 90 Kelvin and the inner thermal shield will be cooled to about 6 Kelvin. (In a recent test of the apparatus, the outer shield got down to 80 K and the inner shield down to 6 K.)

The basic idea behind the design is to cool the torsion fiber down to a few degrees Kelvin. In the above picture, the pendulum fiber will attach to the pendulum suspension, and the pendulum itself will hang down inside the inner thermal shield, at the level of the autocollimator hole (near bottom of vacuum chamber, opposite wall). For the cooling mechanism, a pulse tube cooler ("cold head") was chosen over liquid cryogens for both convenience and cost. The main challenge in using this apparatus is that the pulse tube cooler creates (by torsion balance standards) strong vibrations during operation. Several features of the design are in place to minimize this unwanted noise (as well as noise produced by other sources, such as vibrations from seismic activity).

The hollow vacuum chamber is rigidly attached to the adjacent wall, which sits against a hillside. The separate support for the pulse tube cooler is also attached to this wall, but it is *not* rigidly connected to the vacuum chamber. Furthermore, the cooler does not rest directly on its supports, but rather on air springs, which helps decouple its vibrations. The heat links that connect the pulse cooler to the outer and inner thermal shields (not shown in the above diagram) are made of flexible copper wires, which further decouples the cold head vibrations.

A pendulum in this apparatus will be suspended from a torsion fiber that connects to a magnetic damper disk (see Figure 2 below).

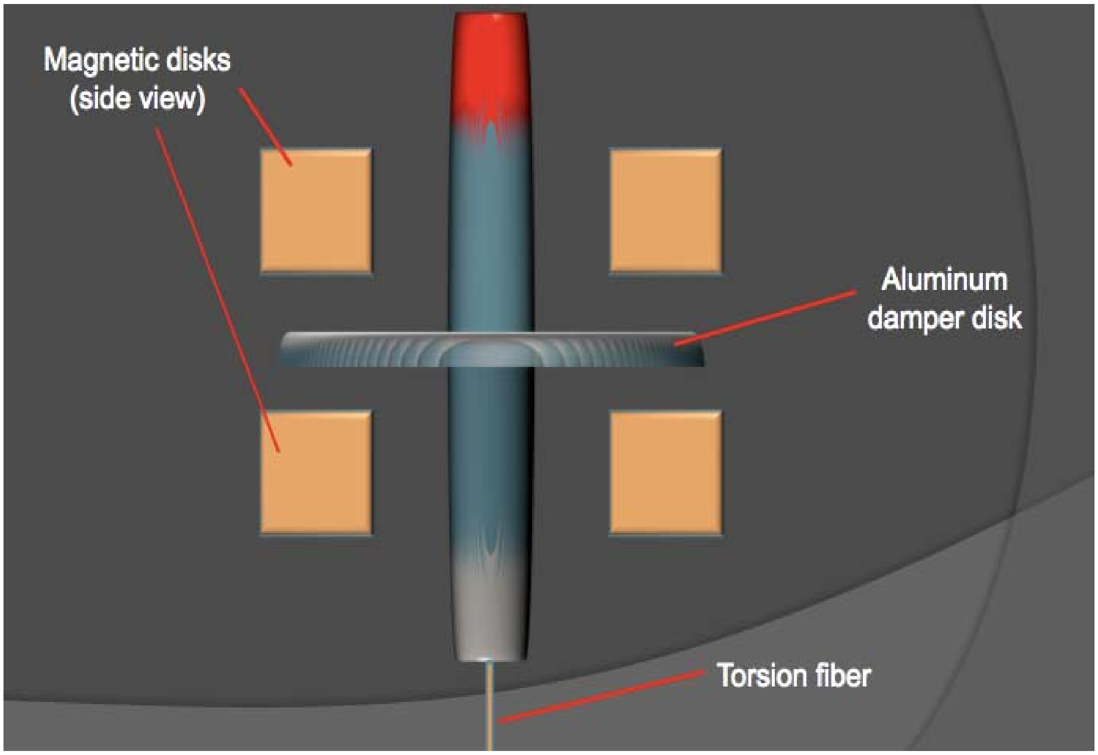


Figure 2: Side view of damper disk. Permanent ring magnets (labeled “magnetic disks” above) sit above and below the damper disk. The aluminum damper disk is rotationally symmetric, so fiber rotation is unaffected. Any other type of fiber movement will move the damper disk relative to the magnetic field caused by the magnetic disks, creating eddy currents that dampen out the motion.

The damper disk sits between two powerful magnets in such a way that rotational pendulum motion is unimpeded; a simple rotation will not change the position of the circular damper disk relative to the magnetic field. However, “swinging” motion and “wobble” motion, both of which involve a back-forth motion in the torsion fiber (and both of which are undesirable), *will* cause the damper disk to move relative to the surrounding magnetic field, inducing eddy currents that dampen out the motion. In addition, any up-down “bounce” motion in the pendulum will partially couple with larger motions in a copper bellows attached above the damper disk (not shown in the above picture); these larger movements will then be dampened out by the damper disk.

I spent my first few weeks at UW working on various parts in the CENPA machine shop for this apparatus. Several of the parts I made were associated with this damper; for example, I made an iron yoke that encases the damper and magnets.

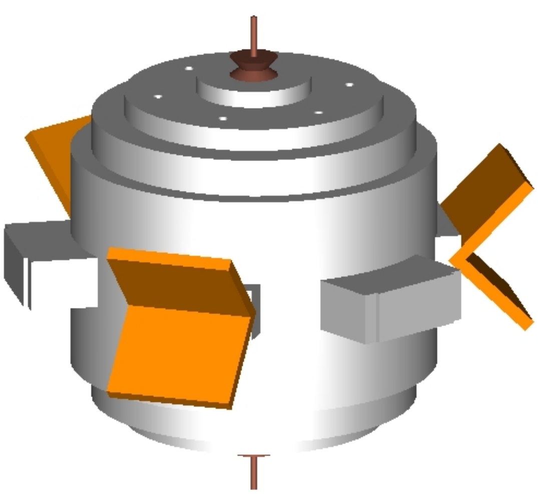
During my last couple of days at UW, I helped put together the parts for the damper disk piece in preparation for a test of the damper disk.

Design of Test Pendulum

The purpose of a test pendulum is to check for undesired systematic effects (especially those that could give a false "UFF violating signal") and to see whether the noise in the apparatus is down to an acceptably low level. (That is, this particular pendulum is testing the cryogenic apparatus; it is *not* used for EP data collection.) In this apparatus the cold head has the potential to be a major source of noise; the test pendulum will thus also check to see whether this source of vibration has been sufficiently damped.

In order to minimize any effect that might be due to actual UFF violation, a test pendulum should not have a composition dipole. In addition, a test pendulum should be made as insensitive as possible to low-order gravitational couplings (see above section on gravity gradients).

I designed a test pendulum for Frank Fleischer's apparatus with these points in mind. Using the moment-calculating program MULTIN along with the visualization program AutoCAD, I created a design featuring 120-degree rotational symmetry about the z-axis and reflection symmetry across the x-y plane (see picture below). In making this design, I had to take into account the feasibility of actually constructing the pendulum in a machine shop. Although these practical considerations put some limits on how small the low-order moments could be, with some trial and error I was able to come up with a design that is satisfactory. The chart (below-right) shows a list of the important low-order moments for this pendulum.



Moment of Inertia	141	g*cm^2
q_{20}	$-3.8*10^{-5}$	g*cm^2
q_{30}	$-1.5*10^{-4}$	g*cm^2
q_{33}	$-7.1*10^{-4}$	g*cm^2
q_{40}	$3.7*10^{-2}$	g*cm^2
q_{43}	$-4.4*10^{-4}$	g*cm^2

In the above-left picture, the grayish-silver colored material is Aluminum 7075. On the top and bottom of the pendulum are copper attachment screws, the top one of which connects to the torsion fiber. (The screw on the bottom is included solely for symmetry reasons.) Three gold-coated 90° mirrors are evenly spaced around mid-plane of the pendulum. Also along the mid-plane, one opposite each gold mirror, are three aluminum blocks that help to cancel some of the low-order moments. (One of these blocks is hidden from view in the above image.) The central aluminum cylinder, which has a diameter of 1.43″, is hollow, with a wall thickness of 0.18″; this was done in order to give the pendulum a larger moment of inertia without increasing its mass significantly past 70 grams, the mass of a typical Eot-Wash EP test pendulum. (The actual mass of this test pendulum is slightly less than 72 grams.) The pendulum has twelve air holes that extend from the inner hollow section to the outside. Six of these air holes are placed evenly around the top of the pendulum (these are visible in the above picture), with the other six in the corresponding positions on the bottom. The air holes are necessary because the pendulum will be in vacuum. In addition, the air holes were useful in the design process, since small tweaks in the q_{lm} moments were made possible by small adjustments in the air hole positions.

Mirror Shaft

While we waited for the test pendulum to be built by the CENPA instrument shop, Frank decided to get a somewhat cruder measure of the amount of noise in the apparatus by inserting a shaft with a mirror attached into the position normally occupied by a pendulum and torsion fiber. This would give us a rough idea of whether or not the vibration caused by the pulse cooler was being sufficiently damped; if we were to find that the pulse tube cooler caused vibrations above the thermal noise level at *room* temperature, then certainly we could not hope to operate at the thermal noise level at 6 Kelvin without modifications to the setup.

In addition to getting a rough measure of the noise, the mirror shaft allows one to calibrate the autocollimator. By rotating the mirror through a known angle, one can turn the voltage readouts from the autocollimator photodetector into an accurate measure of the displacement angle ϕ.

Future of Project

Once the test pendulum is built, test runs will be conducted to see whether the noise in the system has been reduced down to the thermal level expected at 6 Kelvin. It is suspected that several adjustments to the system will have to be made before this happens, as the exquisite sensitivity of torsion balances makes them quite prone to environmental vibrations. Though the damping of the pulse tube cooler will likely be the main challenge to overcome, one must also consider possible noise from other sources, such as seismic activity. In addition, EP tests using the Sun or galactic center as the source mass will by necessity have signals of interest with the period of a day or a sidereal day, respectively. This makes these experiments particularly prone to environmental factors with a daily period, such as temperature fluctuation, the movement of the tides and human traffic at and around the CENPA lab.

In addition to noise reduction, it is likely that several different torsion fiber materials will be tried before one is chosen for use in an EP experiment. Once a fiber is picked, a new pendulum will be designed and constructed for use in the actual EP experiment. (This pendulum, unlike the test pendulum, will have a composition dipole.) It is hoped that this cryogenic setup will improve the current bounds on EP-violation (for interaction ranges between 1 AU and infinity) by an order of magnitude.

Finally, this project will lay the groundwork for future cryogenic torsion balance experiments looking at things other than EP violation. Although modification to the apparatus would be required, it is possible that this setup could be used for investigation of new spin-dependent forces or even for short-range tests of the inverse square law. In any case, this project will give other experimenters some practical knowledge about working with a cryogenic torsion balance.

References

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