Effects of Tiling Sky on Weak Lensing Correlation

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ABSTRACT

In order to investigate the distribution and properties of dark energy and dark matter in the universe, two-point correlation function for image ellipticity and galaxy count over opening angle has to be analyzed. Aside from the distortion caused by the atmospheric turbulence, instrument-induced artifacts will influence the correlation function as well. Therefore, it is important to quantify these artificial effects to study the limitations of the surveys by the telescope to ensure that the artifacts of tiling the sky are significantly subtle and negligible compared to weak-lensing signals. For the simplicity of correlation computation, we simulate the positions of numerous galaxies with the position of the field's center thereof. The effects by various factors, such as limiting magnitude and seeing, will all be tested with the field's centers dithered and not dithered respectively. Eventually, it is discovered that the galaxy count correlation will indeed fluctuate by around 1% of its overall average, due to different numbers of galaxies caused by different limiting magnitudes in field's centers. Similarly, with some approximations, the difference in seeing in each field will also affect the ellipticity correlation by roughly 3%. In addition, to dither the field's centers will help reduce the fluctuation in fine structure in both cases. In order to extract the artifacts derived from tiling the sky, all these effects on weak-lensing and galaxy-count correlation have to be realized and taken into account.

1. Introduction

The way to tile the sky depends on the locations and emphasis of the telescopes and also varies with the filters of a telescope.

Here we need to figure out the way to normalize correlation in this following simpler case and apply it to a real case afterward. Regardless of the location and filters, a telescope can theoretically observe half of a sky, a perfect hemisphere. Hence, we first generate a certain number of 9.6-square-degree field's centers with a regular pattern to tessellate the entire sky and in every field's center there sits a delta-function dot of light as a simulation for all the galaxies in it. We then compute the two-point correlation function over the ensemble of delta functions as a function of opening angle. Subsequently, all the field's centers will be dithered by random directions and distances, and the correlation function will be again computed. By running averages over a certain interval throughout the dithered correlation function, we try to flatten out the dithered correlation function's tiny spikes and obtain the main structure as normalization. Nonetheless, the normalized correlations turn out to be greatly influenced by the artifacts of normalization. Similarly, the correlation functions for field's centers from r-band cronos 92 will also be computed to see these artifacts. Conclusively, we use numbers of pairs at each angle as normalization instead.

On the other hand, the tessellation of the sky on a telescope is not necessarily a regular pattern on a perfect hemisphere but instead it depends on the location and the filters. To simulate a real telescope with the factors of location and filters, we analyze the Cronos 92 simulation for Large Synoptic Survey Telescope (LSST; e.g., Tyson & Angel 2001; Tyson 2002). Ultimately, to learn how much various factors will affect the weak-lensing and galaxy-count correlation, we will weight each field's center respectively with different parameters, such as seeing and limiting magnitudes.

For the simulation of the effects by seeing, we choose r-band because the beams in r-band have shorter wavelengths, which will be less affected by atmosphere and optics. Although the infrared is even less affected, the background microwaves turn out to dominate the signals. Nevertheless, to be consistent with the papers where equations and constants are obtained, we use i-band instead to simulate the effects by limiting magnitudes.

2.1 Simulation and Procedures for Perfect Hemisphere Tessellation with Delta Functions

On the condition that there is no gap between images, we totally generate 3,266 field's centers over a hemisphere with a regular pattern. For each correlation function, we choose every two points on the sky, calculate the opening angle subtended at the telescope by these two points, and store the value at the opening angle thereof, which is defined in Eqn. (1), where D(α) and α respectively stands for delta function and opening angel. Accordingly, the two-point correlation as a function of opening angle, shown in Fig. 1, will be obtained.

$$\mathrm{C_{pp}} = < D(0) \times \mathrm{D}(\alpha) > \quad (1)$$

Then all 3,266 field's centers are dithered by different random directions and distances, where the probability is uniformly distributed in each field. After the same process of computation for correlation function, the graph for dithered field's centers will be obtained as shown in Fig. 2.

In order to obtain the main structure as temporary normalization, we have to totally smooth out the fine structure by running averages over every ± 0.9-degree interval throughout the plot. Therefore, the graph for normalization is derived as shown in Fig. 3.

With this main structure, we can further normalize the correlation function for dithered and non-dithered field's centers. To normalize the correlation, we divide the Fig. 1 and Fig. 2 by the values of Fig. 3 at each corresponding angle, and thus Fig. 5 & 6 are obtained respectively. Based on Fig. 5 & 6, we can see an obvious fluctuation of artifacts totally caused by this normalization. In addition, according to the difference between Fig. 3 & 4, a huge error will also occur as well, if the correlation derived from a perfectly smooth distribution on a perfect hemisphere is used as normalization.

In order to extract the artifacts in normalization and see the effects of various factors directly, we therefore need use the numbers of pairs at each opening angle, which are Fig. 2 & 3 themselves, to be the normalization instead. Therefore, by definition, we will currently get value 1 at any angles, but it finally allows us to observe the exact differences when we plug in the seeing and limiting magnitude.

2.2 Figures Analysis for Perfect Hemisphere Tessellation with Delta Functions

In Fig. 1 and 5, it can easily be realized that many significantly huge spikes exist twice as high as the main structure nearby. Since the field's centers are generated with a regular pattern on each layer, it will cause such large numbers of combinations at certain angles. In addition, due to the fact that the field's centers are generated on only 37 distinct layers on the hemisphere with certain declinations, it brings even more periodic patterns to the huge spikes.

As the entire field's centers are dithered randomly in RA and Dec directions, the even distribution and certain layers are rooted out manually. To dither the field's centers will obviously help minimize the regular pattern of two-point correlation function, especially at small angles, the angles of interest. Nonetheless, the number of field's centers is still a finite number so the small spikes in fine structure are inevitable.

For the dithered field's centers, the ratio of the fluctuation to the magnitude of the correlation function is roughly 10^{-1} *to* 10^{-2}. In this simple case of simulation, we may try to predict the results of various factors to be 1% to 10% before plugging in various factors, and this prediction will later on be tested by the subsequent analysis normalized by numbers of pairs at each opening angle.

3.1 Simulation and Procedures for LSST Cronos 92 with Delta Functions

There are 3,277 distinct field's centers in r-band Cronos 92 covering the sky as shown in Fig. 7.

As for this Cronos 92, the sky coverage is no longer a perfect hemisphere so it is possible to form a correlation structure different from the one for field's centers on a perfect hemisphere.

Through the same computation discussed in §2.1, a non-dithered correlation as a function of opening angle is obtained as shown in Fig. 8.

In order to extract the artifacts of regular pattern in tessellation of the sky, we also try to dither every field's center by different random directions and distances and the correlation turns out to be the histogram shown in Fig. 9.

Likewise, we still apply the moving-average method to smooth out its fine structure. The interval over which the average is being calculated is also ±0.9 degree. Hence, we will obtain the normalization as shown in Fig. 10.

With this main structure, we can normalize the correlation function for dithered and non-dithered field's centers. To normalize the correlation, we divide the Fig. 8 & 9 by 10 at each corresponding angle to get Fig. 11 & 12 respectively. Based on Fig. 11 & 12, similar to hemisphere tessellation, we will be more or less able to observe the effects on correlation function by the way we tile the sky under the limitation of the artifacts by normalization. However, since the influences by various factors on the correlation need to be studied exactly, the normalization by Fig. 11 & 12 themselves is required when the delta functions are substituted by the other parameters.

3.2 Figures Analysis for LSST Cronos 92 with Delta Functions

In Fig. 8 and 11, because the field's centers in r-band for Cronos 92 have to be located according to various factors, the pattern used to tile the sky is much less regular. As a result, albeit non-dithered field's centers still result in many spikes, there is no more clearly periodic pattern and the highest spike, around 12,000, is a lot lower than the highest one in Fig. 1, which is close to 20,000. (The areas under these two curves are close enough for their spikes to be compared.) Thus, the regularity and extremely huge spikes issues are resolved in Cronos 92 simulation.

Likewise, to dither the field's centers significantly reduces the oscillation of the correlation function caused by artifacts. The ratio of the fluctuation to the magnitude of the correlation function is also roughly $10^{-1}\ to\ 10^{-2}$. This result agrees with the hemisphere simulation, which will later on be tested by the subsequent analysis normalized by numbers of pairs at each opening angle.

4.1 Simulation and Procedures for LSST Cronos 92 with Limiting Magnitude

Here the field's centers are no longer weighted by delta functions, but by numbers of galaxies in the corresponding field's centers, which can be derived from the limiting magnitudes extracted from i-band in Cronos 92 database.

For the simplicity, first we convert the apparent magnitudes, m, to the numbers of galaxies, N, by a straight line fit approximation (Peebles 1993) defined in Eqn. (2), and now the correlation function is defined in Eqn. (3). With the same process of correlation computation and the normalization by number of pairs at each angle, the Fig. 13 is generated.

$$\frac{dN}{dm} = 10^{-5.70\pm0.10+0.6m} \quad (2)$$

$$\mathrm{C_{pp}} = < N(0) \times \mathrm{N}(\alpha) > \quad (3)$$

Due to the fact that dimmer galaxies can be observed while more visits of observation are taken, the coadded depths (Ivezi´c et al. 2008) have to be calculated from Eqn. (4) and will replace the limiting magnitudes in order to graph the Fig. 14.

$$\mathrm{m_{coadded}} = 1.25 \sum 10^{\frac{m5}{1.25}} \quad (4)$$

Further we apply the Malmquist bias, or so-called Eddington bias (Teerikorpi 2004), to convert the limiting magnitudes to numbers of galaxies more accurately, and the Malmquist bias in this particular case is defined as

$$\log N(m) = \log N_{obs}(m) - \frac{1}{2}\frac{\sigma^2\alpha^2}{\log e} \quad (5)$$

$$\sigma^2 = \sigma_{rand}^2 + \sigma_{sys}^2 \quad (6)$$

$$\sigma_{rand}^2 = (0.04-\gamma)x + \gamma x^2 \quad (7)$$

where $x = 10^{04(m-m_5)}$. Here m_5 is the 5σ depth and, given counts to $i = 0.4$ as N_0, $\gamma = 0.039$, $\alpha = 0.4$, and $\sigma_{sys} = 0$.

Finally a more reliable Fig. 15 is obtained, which shows the effects of different limiting magnitudes on galaxy counts correlation analysis. Similarly, the dithered correlation is also computed to get Fig. 16.

4.2 Figures Analysis for LSST Cronos 92 with Limiting Magnitude

The first two figures, Fig. 13 & 14, are simply showing that, out of two necessary steps, coadded depth and Malmquist bias, the latter step converts the limiting magnitudes closer to the real numbers of galaxies than the former.

Here we have come to our conclusions that different limiting magnitudes appearing from the tessellation of the sky will cause the galaxy count correlation a fluctuation of 1 % for non-dithered, and 0.1% for dithered field's centers, which is smaller than our prediction from the correlation weighted with delta functions.

Therefore, caused by tiling the sky, these effects of the fluctuation in the fine structure and the lift of the curve in small angles in main structure have to be taken into account while the galaxy count correlation analysis.

5.1 Simulation and Procedures for LSST Cronos 92 with Seeing

Here we start weighting the field's centers with ellipticity, ϵ, defined in Eqn. (8) where a and b are major and minor axes respectively. According to the Eqn. (9), the size of galaxies will further be smeared by seeing extracted from Cronos 92.

We assume the initial ellipticity in each field is 10^{-3} and average angular size of the galaxies is 2 arcsec. Hence, when we plug in the seeing in r-band, mostly around 0.7 arcsec, the ellipticity will be smeared to be smaller values, different from field to field.

$$\epsilon = 1 - \frac{b}{a} \qquad (8)$$

$$\left.\begin{aligned} a_{smeared} &= \sqrt{a_{initial}^2 + Seeing^2} \\ b_{smeared} &= \sqrt{b_{initial}^2 + Seeing^2} \end{aligned}\right\} \qquad (9)$$

Since we can only smear the ellipticity by one value of seeing, we have computed the correlation with two separate sets of data, first and last seeing in each field in the database, for comparison. Eventually, we have generated Fig 17 & 18 for non-dithered, and 19 & 20 for dithered, all of which are still normalized by being divided by numbers of pairs at each angle.

5.2 Figure Analysis for LSST Cronos 92 with Seeing

Here are some errors in this simulation, because in the real ellipticity correlation analysis, a much smaller region, instead of the entire field image, will be used as a unit field and ellipticity is a vector, instead of scalar defined in Eqn. (5).

Regardless of those uncertainties, then the fluctuation caused by difference in the seeing is approximately 3% for non-dithered and 1% for dithered, which agrees with the prediction from the correlation weighted with delta functions.

Unknown as the magnitudes of the uncertainties in this simulation are, but difference in seeing in each field will surly bring a certain amount of fluctuation, which might be a few percents, in the ellipticity correlation. Hence it is important to realize this effect and take it into account.

6. Conclusion

We have realized the huge errors and uncertainties that appear from the normalization by both moving-average method and perfectly-smooth-distribution method. As a result, the normalization for the correlation analysis has to be treated carefully.

Ultimately, it is discovered that the galaxy count correlation will indeed fluctuate by around 1% of its overall average, due to different numbers of galaxies caused by different limiting magnitudes in field's centers. To dither does help extract the artifacts of tiling the sky and thus minimize the fluctuation in fine structure to be 0.1 %.

Similarly, with some approximations, the difference in seeing in each field will also affect the ellipticity correlation by roughly 3%. In addition, to dither the field's centers will also help reduce the fluctuation in fine structure to be around 1 %.

In order to extract the artifacts derived from tiling the sky to ensure the weak-lensing signal is higher than the uncertainties caused by the tessellation of the sky, all these effects on weak-lensing and galaxy-count correlation have to be realized and taken into account.

REFERENCES

Ivezi´c, ˇ Z. et al. 2008b, astro-ph/0805.2366

Peebles P. J. E., 1993, Principles of Physical Cosmology, Princeton University Press, Princeton, p. 23-24

Teerikorpi P., 2004, A&A, 424