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How I Spent My Summer Vacation

Searching for gravitational force violations and a theoretical fifth force

The Torsion Pendulum - Basic Properties

The torsion pendulum has the useful property of being able to detect small differences in forces exerted on a body. To understand the mechanism behind this, consider a bar with two different bodies attached to either end hanging from a thin fiber. If parallel, equal forces are exerted on both test bodies, we expect to see no resulting torque on the device, while unequal forces cause the pendulum to spin.[1] Provided we know the torsion constant κ of the fiber, we can calculate the net difference in force depending on the amplitude of spin of the pendulum.

The Eöt-wash group at University of Washington – Seattle uses the sensitivity of torsion pendulums to search for small short-range forces that depend on the properties of the materials they are acting on. Two particular forces that could be observed in this way are forces that couple to intrinsic electron spin and new gravitational forces that violate the Inverse Square Law or Weak Equivalence Principle at a small length scale.

Violating the Inverse Square Law

Theory predicts that there could be another boson aside from the graviton that mediates gravitational force. Called the Yukawa interaction, its existence requires that an additional term be added to the currently accepted gravitational force equation, resulting in the equation below:

$$F = \frac{Gm_1m_2}{r^2}\left[\,1 + \alpha\,(1 + r/\lambda)\,e^{-r/\lambda}\,\right]$$

This theory takes our currently accepted force law ($F = \text{const.}/r^2$) to be an approximation of a more precise description of gravitational force.[2]

Violating the Weak Equivalence Principle

The weak equivalence principle defines gravitational mass of an object to be identical to inertial mass. However, the equations of inertial and gravitational force share no parameters, making it surprising that these two masses can be one and the same. Given two materials with the same weight (gravitational force) but different composition, it is possible that they have different inertial masses, leading one material to experience a different net acceleration than the other.

Spin-Coupled Forces

In examining the possible results of CP-violating scenarios, physicists have theorized the existence of particles that could serve as the mediators carrying a fifth force that has yet to be measured. The nature of this force has been described by two spin-dependent potential equations:

$$V \,\alpha\, \boldsymbol{\sigma_e} \bullet \mathbf{r}$$

$$V \,\alpha\, \boldsymbol{\sigma_1} \bullet \boldsymbol{\sigma_2}$$

In the first equation, the force is a result of electron spin $\boldsymbol{\sigma_e}$ coupling with a point mass. In the second equation, it is a result of the spin of one source $\boldsymbol{\sigma_1}$ coupling with the spin of another $\boldsymbol{\sigma_2}$. If

[1] "The Eöt-Wash Group: How does a Torsion Balance work so well?" http://www.npl.washington.edu/eotwash/intro/how.html

[2] E.G. Adelberger, B.R. Heckel, and A.E. Nelson. "Tests of the Gravitational Inverse-Square Law." http://www.npl.washington.edu/eotwash/publications/pdf/review.pdf. p. 4-5.

the force on an object does indeed depend on the object's intrinsic spin, it may be possible to use a pendulum with large spin dipole moments on the torsion balance to detect this force. [3]

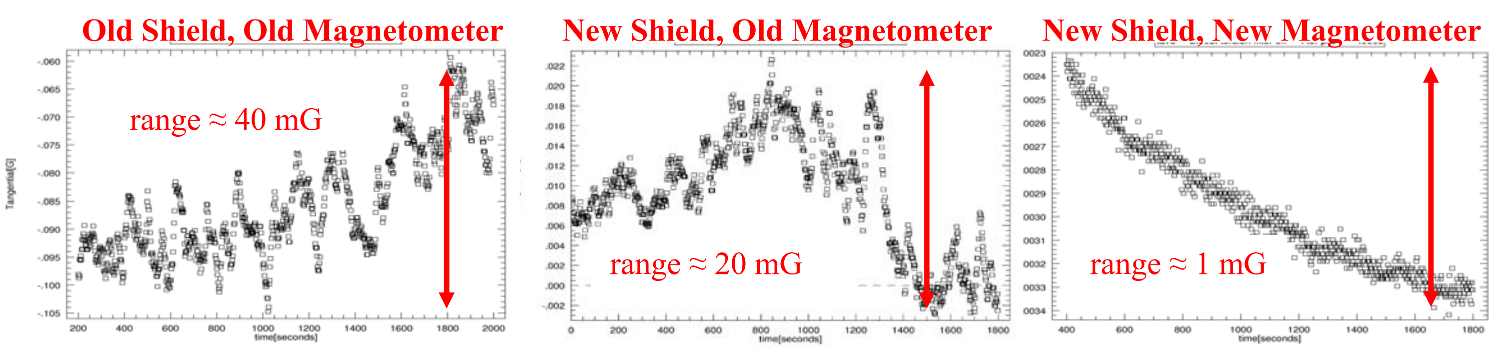
The Spin Pendulum

Several different designs for spin dipole pendulums have already been proposed and tested at the University of Washington. The design currently being implemented by graduate student William Terrano involves a 20-sided puck composed of 2 different permanent magnets cut into trapezoids of uniform size and attached in an alternating pattern. The metals are $SmCo_5$ and AlNiCo. The large number of polarized electron spins in AlNiCo give it its magnetization while the orbital angular momentum of electrons in $SmCo_5$ are responsible for about half of its magnetization. The difference in electron spin of the two materials gives the pendulum 10 spin dipole moments. If we now place this pendulum above another ring of the same design, an observed torque on the spin dipole aligning it with the spin moment of the other pendulum and minimizing potential would indicate the presence of a new spin-coupled force.

Construction of the Spin Pendulum – Eliminating External Magnetic Field

It is necessary for the Spin Pendulum to have minimal external magnetic field as electromagnetic force would give the pendulum an unwanted torque. The physical and magnetic uniformity of the trapezoidal pieces was strictly regulated in order to create a puck with circular magnetization and no magnetic leakage field. After assembly, any detected leakage fields were eliminated by passing that segment of the pendulum through a strong magnetic field created by permanent magnets.

The ability to magnetize the spin pendulum was hindered by poor magnetic shielding and a Hall probe magnetometer with significant background noise. In my first weeks in Seattle, I worked to construct a more effective magnetic shielding box and purchased a magnetoresistance magnetometer more suitable for detecting small magnetic fields. The integrity of the original magnetic shield had been compromised by numerous large holes that were in the shield and the welding that had been done to attach the ends together. My new magnetic shield design involved large sheets of mu-metal that were folded into a cube and securely attached together with screws. Below are three noise runs, the first of which was performed with the old shield and old magnetometer, the second of which uses the new shield and old magnetometer, and the third of which uses the new shield and new magnetometer.



The red arrows indicate the range of magnetic drift. The new shield was able to reduce magnetic error by a factor of two and the new magnetometer reduced the error by an additional factor of twenty.

[3] Cramer, Claire E. "A Torsion Balance Search for Spin-Coupled Forces." Diss. U of Washington, 2007. p. 6-7.

Improving Experimental Error: The Gravity Gradient Pendulum

The Eöt-Wash pendulums are unique in their ability to probe for small-magnitude forces. However, their sensitivity also makes them prone to movement as a result of known forces, particularly the changing gravitational field of objects such as people, tides, etc. To measure the significance of these environmental factors, I constructed the q2 pendulum, a pendulum with gravitational quadrupole moments (q_{21} and q_{22}) ten thousand times larger than the moments of typical Eöt-wash pendulums, making my pendulum particularly susceptible to effects of gravitational gradients. The base of my pendulum is aluminum while the extruding parts are made of denser copper. Placing most of the mass far from the origin on the horizontal and diagonal axes maximized q_{21} and q_{22}. Below is my final pendulum design with associated values of q_{21} and q_{22} (for calculations, see Appendix).

$q_{21} = 110.02$ gcm^2
$q_{22} = 62.79$ gcm^2

By calculating the gravitational quadrupole field of several environmental factors, I was able to roughly determine which factors would affect pendulum movement on a detectable scale (see Appendix). The pendulum movement caused by these gravitational fields can be subtracted out of the amplitude measurements of other Eöt-Wash pendulums, making the data easier to probe for a gravity-defying or spin-dependent force.

Gravitational Effects: Daily Traffic

Gravitational field estimates seemed to indicate that car and human traffic would have an observable effect on pendulum movement. I looked at daily fluctuations in pendulum movement using a computer program that tracks daily changes in the northward and westward directions. I then used the same program to compute weekday and weekend amplitude and took the difference between the two. The analysis provided strong evidence for daily fluctuations:

Direction 1: $\theta_1 = -100 +/- 13$ nrad
Direction 2: $\theta_2 = -29 +/- 13$ nrad

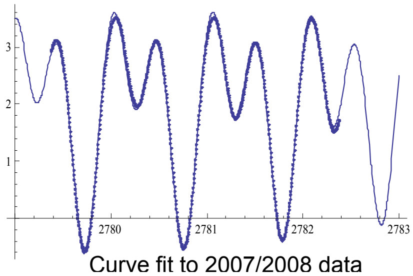
These once daily oscillations can be attributed to an increase in lab traffic during the workday and decrease as people leave for the evening. There was also some evidence for a change in amplitudes between weekdays and weekends, though more data collected over the next few weeks will make these results more conclusive:

$\Delta\theta$ ($\theta_{weekday} - \theta_{weekend}$):
Direction 1: $\Delta\theta_1 = -23 +/- 25$ nrad
Direction 2: $\Delta\theta_2 = -30.4 +/- 25$ nrad

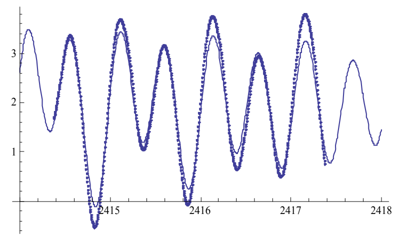
Gravitational Effects: Puget Sound Tides

Gravitational field estimates also suggested that the gravity gradient caused by the raising and lowering of the Puget Sound Tides should affect pendulum oscillation. I developed a function that predicts the height of the tides based on previous research on Puget Sound tidal frequency and data describing local tides collected from NOAA.[4] Below left is the data I used to find the tidal function. Below right is the function compared to data taken from 2006.

[4] NOAA "Tsunami Capable Tide Stations - Seattle." http://www.co-ops.nos.noaa.gov/tsunami/



Curve fit to 2007/2008 data



Compared to 2006 data

I used a computer program to measure the correlation between the function's predictions and my pendulum movement. Elliot Bay is the part of the Puget Sound closest to the university, so I expected a signal in both the northward and eastward directions:

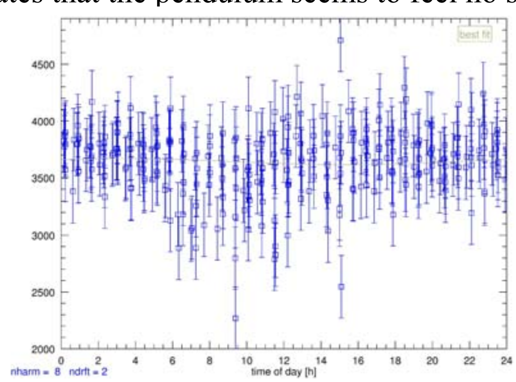
Direction 1: θ_1 = -22.1 +/- 11.9 nrad

Direction 2: θ_2 = 7.25 +/- 11.9 nrad

These observations suggest some correlation between the tides and pendulum amplitude and are on the same order of magnitude as predicted by the equation for amplitude (see appendix). However, more data needs to be collected and error bars need to decrease before correlation can be assumed with confidence.

Gravitational Effects: Sun and Moon

My gravitational quadrupole field calculations indicated that the sun and moon would not have a detectable effect on pendulum amplitude (see Appendix). While more data still needs to be collected before lunar effect, or lack thereof, can be experimentally determined, we can attempt to detect solar influence by tracking pendulum amplitude over the course of a day and comparing it to the sun's movement. The below graph maps the correlation between pendulum and solar movement. If the two were correlated, we would expect to see a sinusoidal curve. However, the flatline indicates that the pendulum seems to feel no significant solar gravitational gradient.



Summary

Based on my gravitational quadrupole field calculations and data collected by my large-q_{2m} pendulum, I am able to provide strong evidence that gravity gradients created by nearby traffic have an influence on pendulum amplitude. There is also evidence for the effect of the Puget Sound tides on pendulum amplitude. Results should become more conclusive as more data is collected and will permit us to subtract this error from the movement of other Eöt-wash pendulums, probing deeper for fundamental force violations and detection of a new spin-coupled force.

Appendix:
Gravitational Quadrupole Moment and Implications for Pendulum Amplitude

The expected amplitude of the pendulum can be found using the equation below:

$$\Theta_g = T_g / \kappa = -4\pi i G/\kappa \sum \frac{1}{2l+1} \sum m q_{lm} Q_{lm} \times e^{-im\varphi}$$ [5]

where κ is the torsion constant of the fiber, q_{lm} is the $l$mth order multipole moment, Q_{lm} is the $l$mth order multipole field. Gravitational dipoles cannot exist and the pendulum was designed so that the value of q_{20} is zero. Therefore, in considering the amplitude of the pendulum we look only at the gravitational effects of the quadrupole field on the q2 pendulum: $(l,m) = \{(2,1), (2,2)\}$.

The expressions for q_{lm} and Q_{lm} are:

$$q_{lm} = \int \rho_p(r)\, r^l\, Y^*_{lm}(r)\, d^3r$$
$$Q_{lm} = \int \rho_p(r')\, r'^{-(l+1)}\, Y_{lm}(r')\, d^3r'$$ [6]

And if we take the source of the gravity gradient as a single point source and add the appropriate the spherical harmonic expressions into the equation, the quadrupole moments and fields are:

$$q_{21} = -\sqrt{(15/8\pi)} \int \rho_p(r)\, r^2 \sin\theta \cos\theta\, e^{-i\varphi}\, d^3r$$
$$q_{22} = \sqrt{(15/32\pi)} \int \rho_p(r)\, r^2 \sin^2\theta\, e^{-2i\varphi}\, d^3r$$
$$Q_{21} = -\sqrt{(15/8\pi)}\, m/r'^3 \sin\theta \cos\theta$$
$$Q_{22} = \sqrt{(15/32\pi)}\, m/r'^3 \sin^2\theta$$

The torsion balance is capable of detecting $\Theta_g \geq 5$ nrad/day, implying that for the q2 pendulum with $q_{21} = 110.02$ gcm^2 and $q_{22} = 62.79$ gcm^2, only $Q_{2m} \geq 2 \times 10^{-5}$ g/cm^3 will cause a detectable change in pendulum amplitude. We use the above equations to calculate the gravitational quadrupole field of several massive objects.

[5] Su, Yue. "A New Test of the Weak Equivalence Principle." Diss. U of Washington, 1992. p. 20.
[6] Su, Yue. p. 19.

Chart of Gravitational Quadrupole Field of Various Objects

Object	Mass (g)	Theta (degrees)	Distance (cm)	Q_{21} (g/cm^3)	Q_{22} (g/cm^3)
Sun - closest approach	1.99×10^{33}	90	1.50×10^{13}	0	2.3×10^{-7}
Sun – 12pm on 8/6/08	1.99×10^{34}	20	1.52×10^{13}	-1.4×10^{-7}	2.6×10^{-8}
Moon - closest approach	7.35×10^{25}	90	3.56×10^{10}	0	6.3×10^{-7}
Moon – 12pm on 8/6/08	7.35×10^{26}	83	3.95×10^{10}	-1.1×10^{-7}	4.6×10^{-7}
Alex next to pendulum	5.44×10^{4}	90	200	0	2.6×10^{-3}
Office above pendulum	7.48×10^{4}	18.2	320.8	-5.2×10^{-4}	8.5×10^{-5}
Office diagonally above pendulum	2.18×10^{5}	50.2	476.1	-7.7×10^{-4}	4.6×10^{-4}
Parking Lot Cars	7.80×10^{6}	7	3000	-2.7×10^{-5}	1.7×10^{-6}
UPS Delivery Truck	4.1×10^{6}	2	2700	-5.6×10^{-6}	9.8×10^{-8}

Puget Sound Tides	Mass (high tide, grams)	Mass (low tide, grams)	Distance (cm)	Q_{22} (high tide)	Q_{22} (low tide)	ΔQ_{22}
Mass centered in middle of Elliot Bay	9.31x1014	8.80×10^{14}	7.83×10^{5}	7.49×10^{-4}	7.08×10^{-4}	4.10×10^{-5}
Mass centered at farthest point of Elliot Bay	9.31x1014	8.80×10^{14}	1.00×10^{6}	3.60×10^{-4}	3.40×10^{-4}	2.00×10^{-5}

The sun and moon seem to have inadequate gravitational moments to affect the pendulum. However, it seems that the tides and human traffic will have a measurable effect on pendulum amplitude. Based on the calculated ΔQ_{22} of the Puget Sound tides, we can tidal forces to affect pendulum amplitude Θ_g on the order of 10 nrad:

$$\Theta_g = -8\pi G/\kappa \times 2/5 \times q_{22} \times Q_{22}$$
$$= -16\pi G/5\kappa \times 62.79\ gcm^2 \times 4.10\times10^{-5}\ g/cm^3$$
$$\approx 10\ \text{nrad}$$