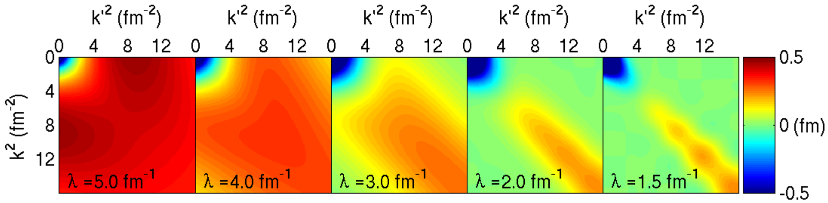
Atomic Nuclei at Low Resolution

Dick Furnstahl

Department of Physics
Ohio State University

July, 2013



Outline

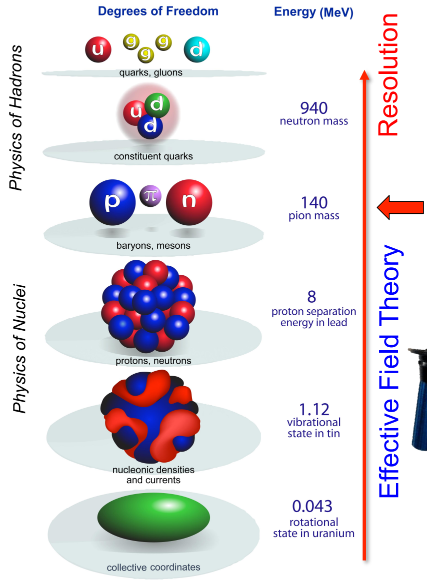
Outline

Overview

Lowering the resolution: Similarity RG in practice

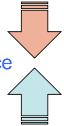
SRG Basics

Connecting degrees of freedom with EFT and RG



Hot and dense quark-gluon matter

Hadron structure



Hadron-Nuclear interface

Nuclear structure
Nuclear reactions

Nuclear astrophysics
New standard model

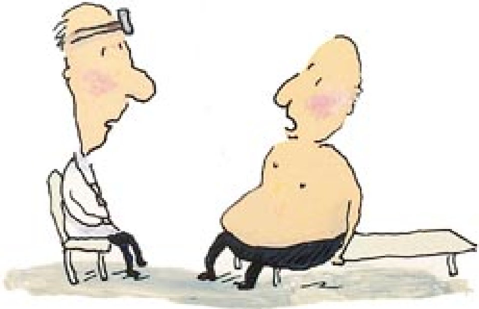
Applications of nuclear science

Low resolution makes physics easier + efficient

- Weinberg's Third Law of Progress in Theoretical Physics:
  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

Low resolution makes physics easier + efficient

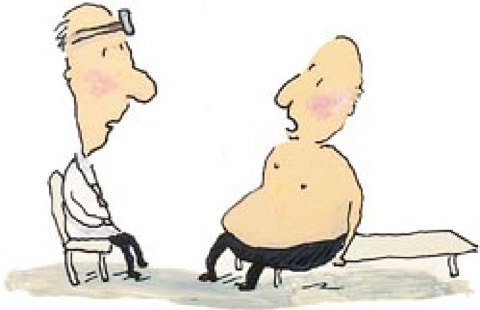
- Weinberg's Third Law of Progress in Theoretical Physics: *"You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"*
- There's an old joke about a doctor and patient ...



Patient: Doctor, doctor, it hurts when I do this!

Low resolution makes physics easier + efficient

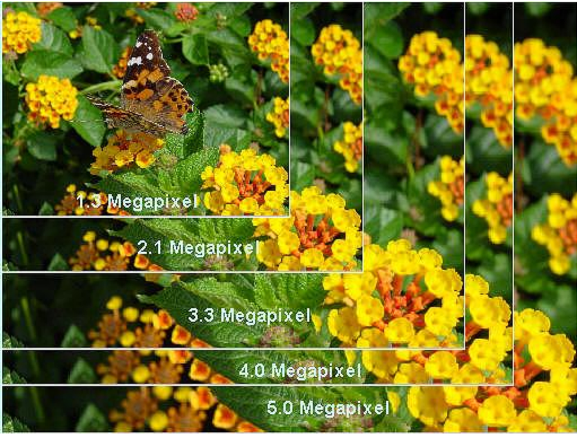
- Weinberg's Third Law of Progress in Theoretical Physics:
  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"
- There's an old joke about a doctor and patient ...



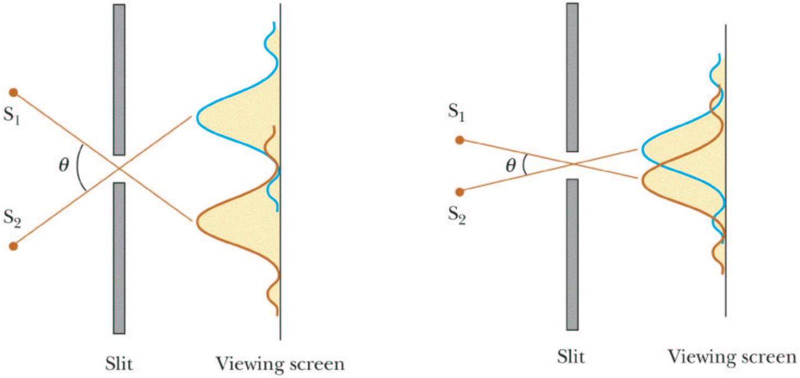
Patient: Doctor, doctor, it hurts when I do this!
Doctor: Then don't do that.

Digital resolution: Higher resolution is better (?)

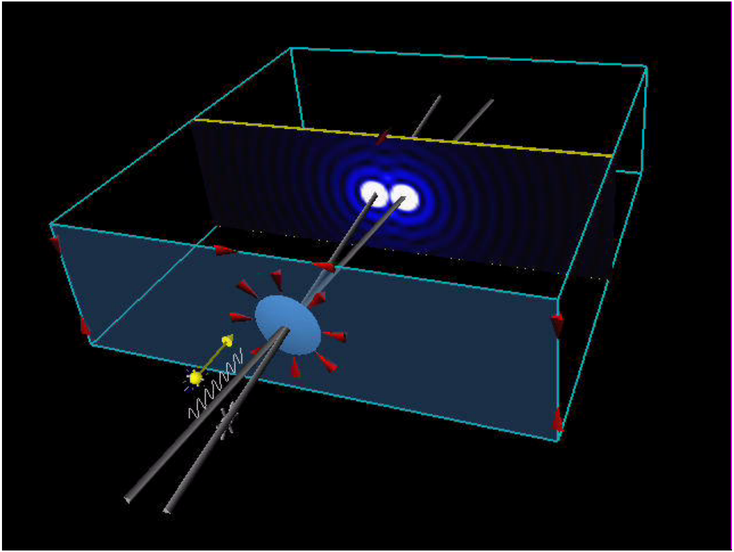
- Computer screens, printers, digital cameras, TV's . . .
- Higher resolution $\Longrightarrow$ more pixels
- Pixel size $\ll$ characteristic scale $\Longrightarrow$ greater detail



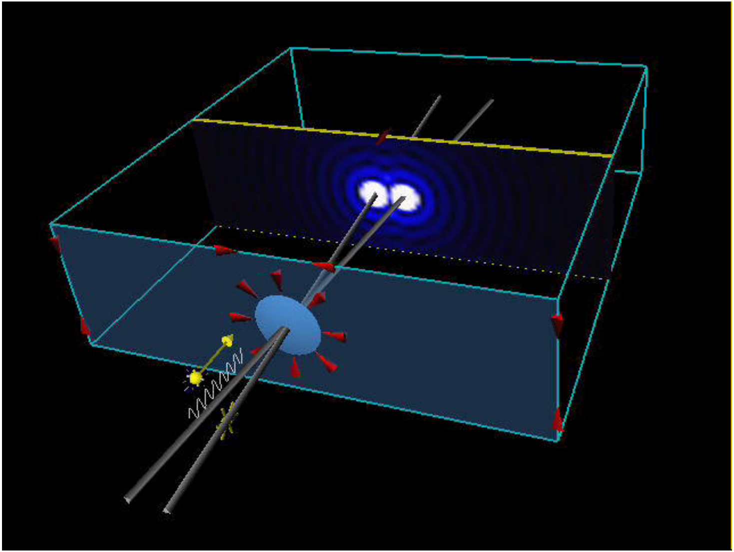
Diffraction and resolution



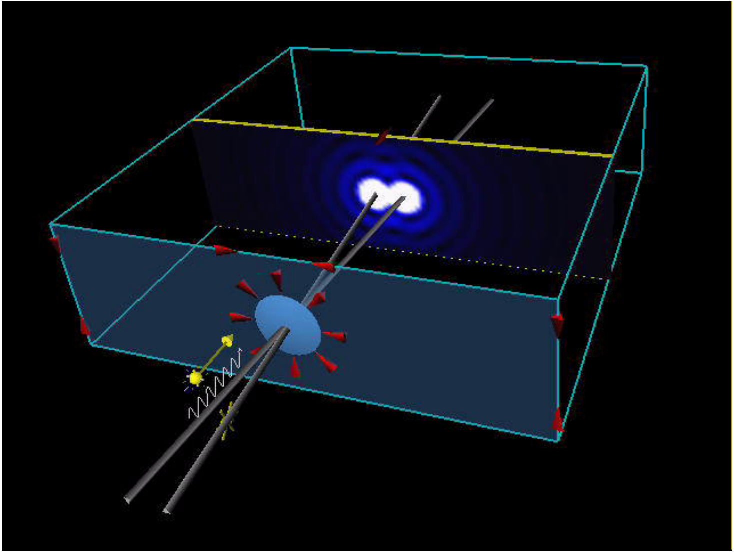
Wavelength and resolution



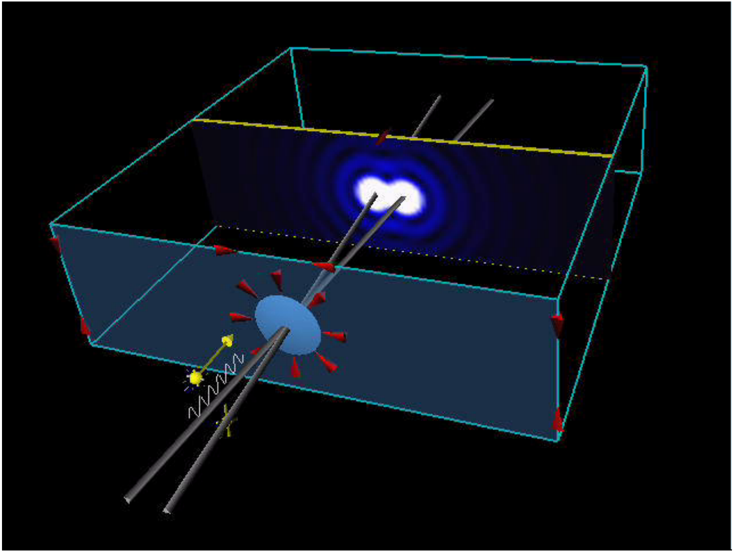
Wavelength and resolution



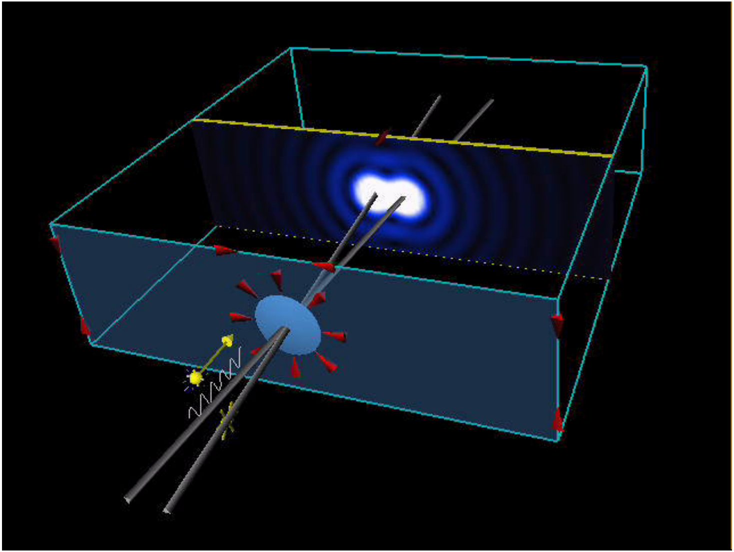
Wavelength and resolution



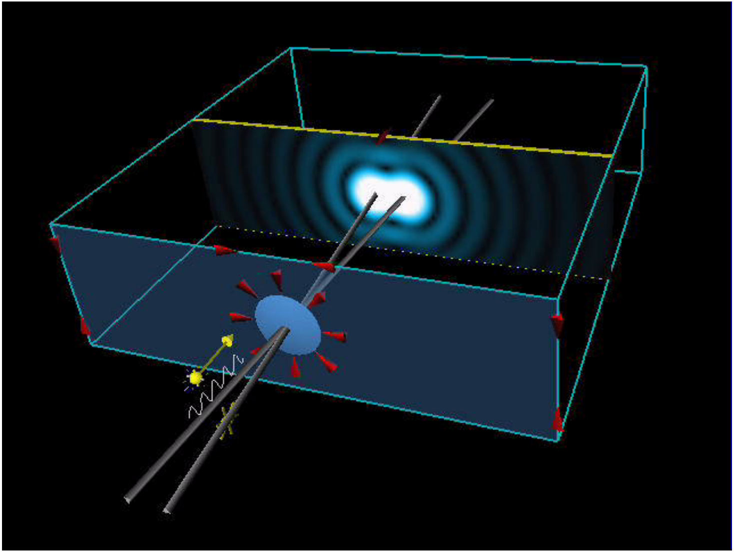
Wavelength and resolution



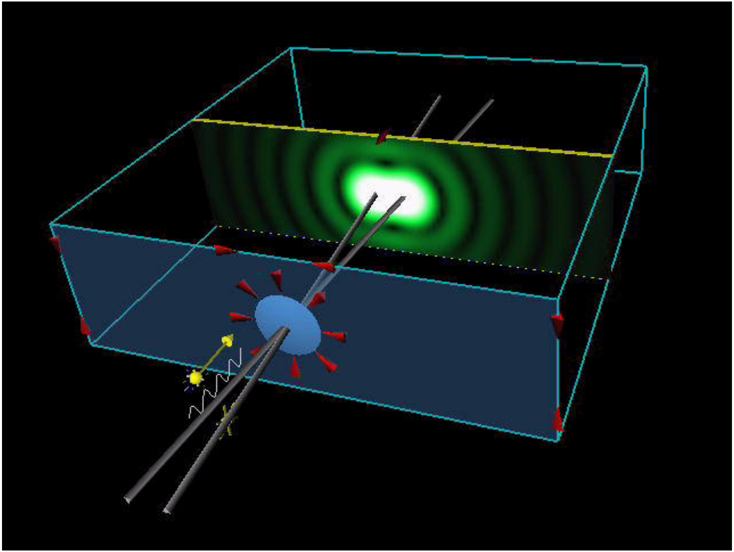
Wavelength and resolution



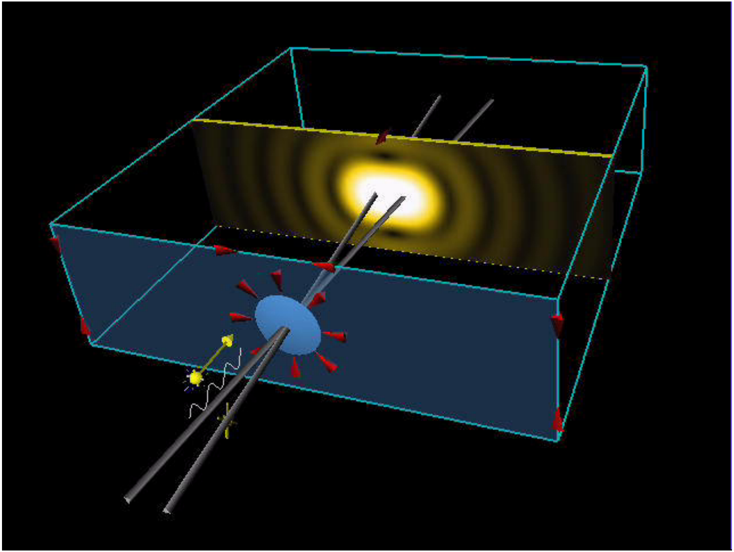
Wavelength and resolution



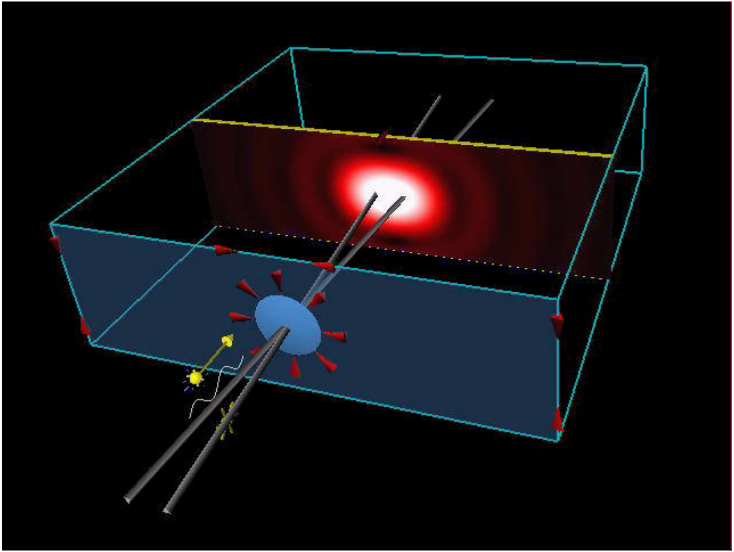
Wavelength and resolution



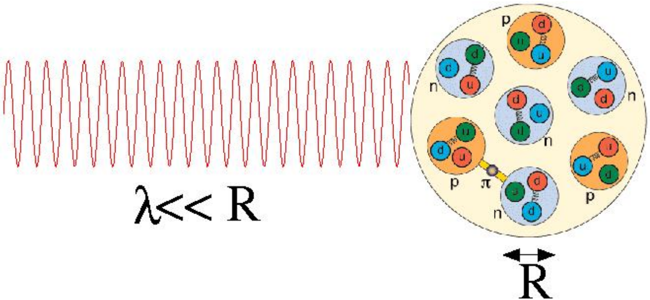
Wavelength and resolution



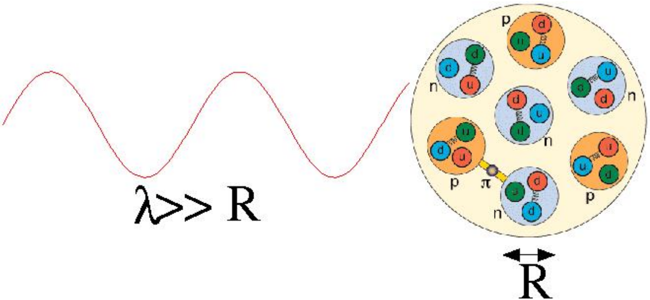
Wavelength and resolution



Principle of *any* effective low-energy description

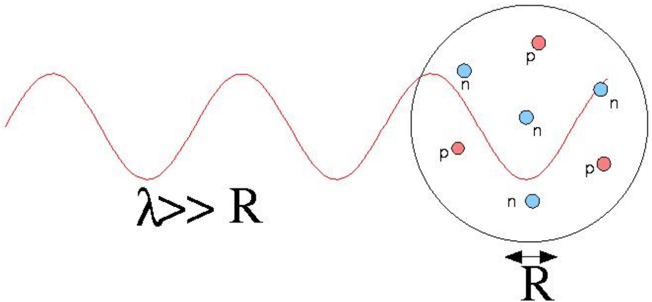


Principle of *any* effective low-energy description



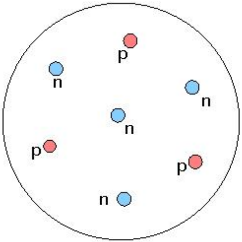
- If system is probed at low energies, fine details not resolved

Principle of *any* effective low-energy description



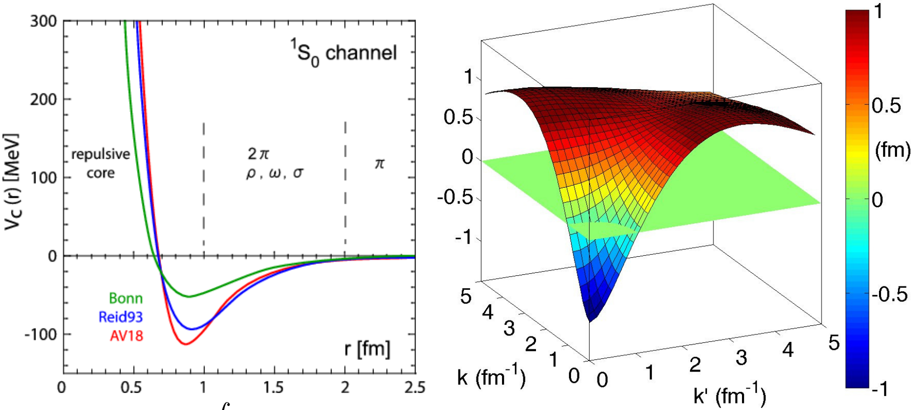
- If system is probed at low energies, fine details not resolved
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables
  - Could be a model or systematic (e.g., effective field theory)
- *Physics interpretation often changes with resolution!*

Principle of *any* effective low-energy description



- If system is probed at low energies, fine details not resolved
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables
  - Could be a model or systematic (e.g., effective field theory)
- *Physics interpretation often changes with resolution!*
- Low density ⇔ low interaction energy ⇔ low resolution (?)

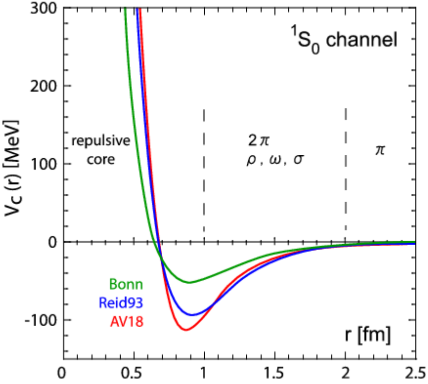
Why is textbook nuclear physics so hard?

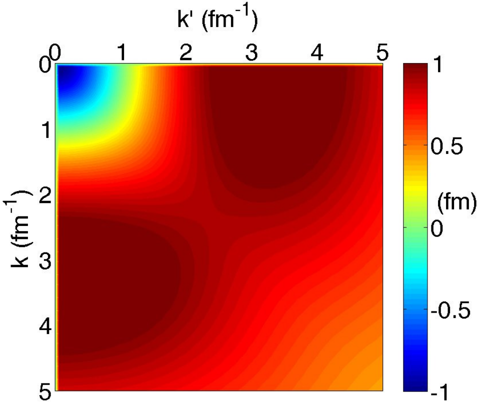


$$V_{L=0}(k,k') = \int d^3r\, j_0(kr)\, V(r)\, j_0(k'r) = \langle k|V_{L=0}|k'\rangle \implies V_{kk'} \text{ matrix}$$

- Momentum units ($\hbar = c = 1$): typical relative momentum in large nucleus $\approx 1\,\text{fm}^{-1} \approx 200\,\text{MeV}$ but ...
- Repulsive core $\implies$ large high-k ($\gtrsim 2\,\text{fm}^{-1}$) components

Why is textbook nuclear physics so hard?

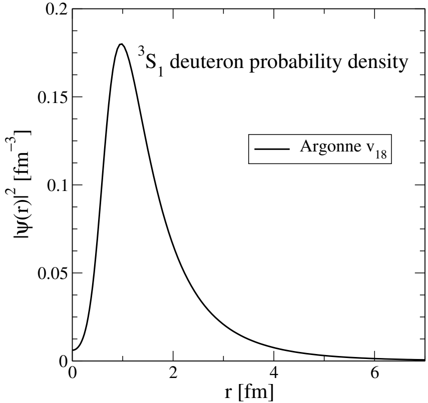


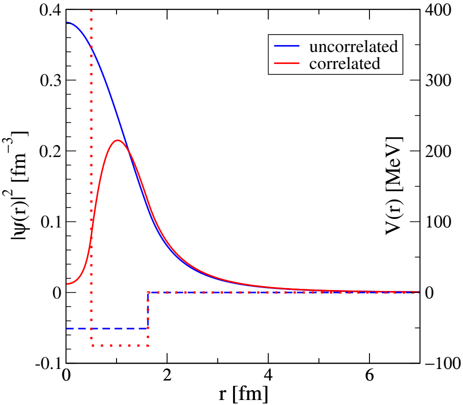


$$V_{L=0}(k,k') = \int d^3r\, j_0(kr)\, V(r)\, j_0(k'r) = \langle k|V_{L=0}|k'\rangle \implies V_{kk'} \text{ matrix}$$

- Momentum units ($\hbar = c = 1$): typical relative momentum in large nucleus $\approx 1\,\text{fm}^{-1} \approx 200\,\text{MeV}$ but ...
- Repulsive core $\Longrightarrow$ large high-k ($\gtrsim 2\,\text{fm}^{-1}$) components

Consequences of a repulsive core

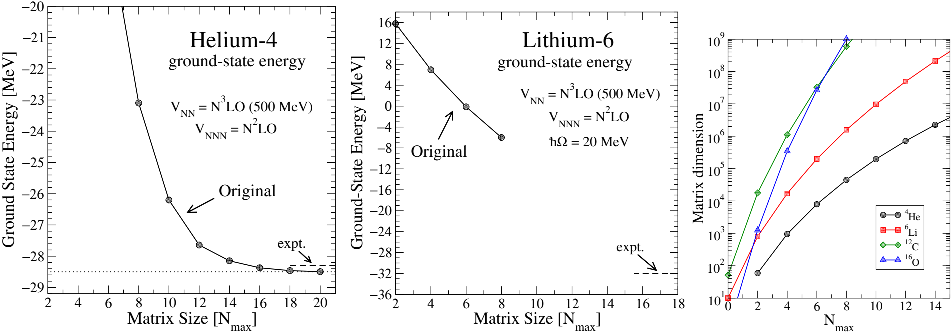




- Probability at short separations suppressed $\Longrightarrow$ "correlations"
- Short-distance structure $\Leftrightarrow$ high-momentum components
- Greatly complicates expansion of many-body wave functions

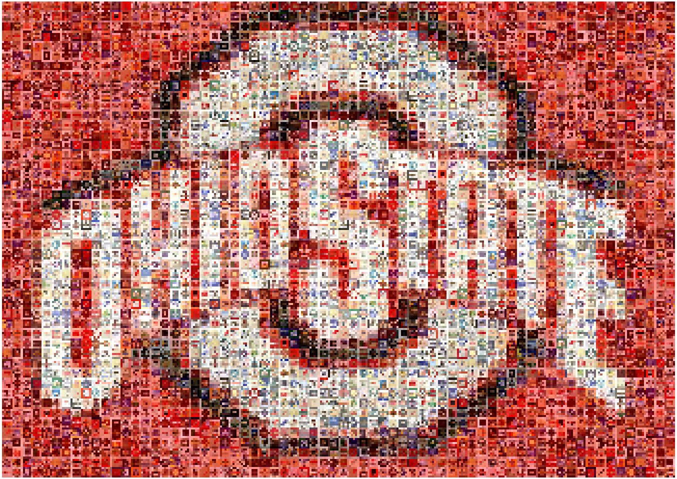
Many short wavelengths $\Longrightarrow$ Large matrices

- Harmonic oscillator basis with N_{max} shells for excitations
- Graphs show convergence for *soft* chiral EFT potential (although not at optimal $\hbar\Omega$ for ^{6}Li)



- Factorial growth of basis with $A \Longrightarrow$ limits calculations
- Too much resolution from potential $\Longrightarrow$ mismatch of scales

What if your theory has too much resolution?



What if your theory has too much resolution?

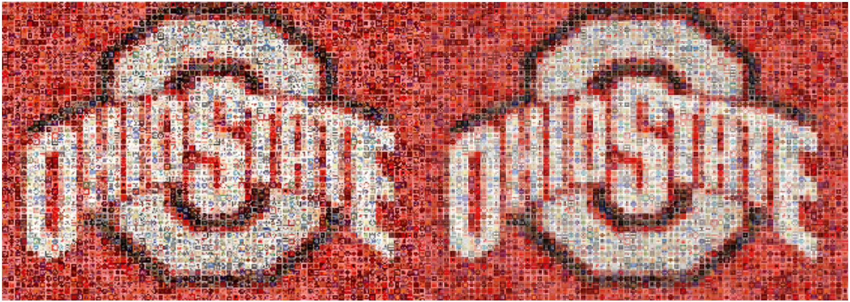
What if your theory has too much resolution?

What if your theory has too much resolution?

Claim: Nuclear physics with textbook $V(\mathbf{r})$ is like using beer coasters!

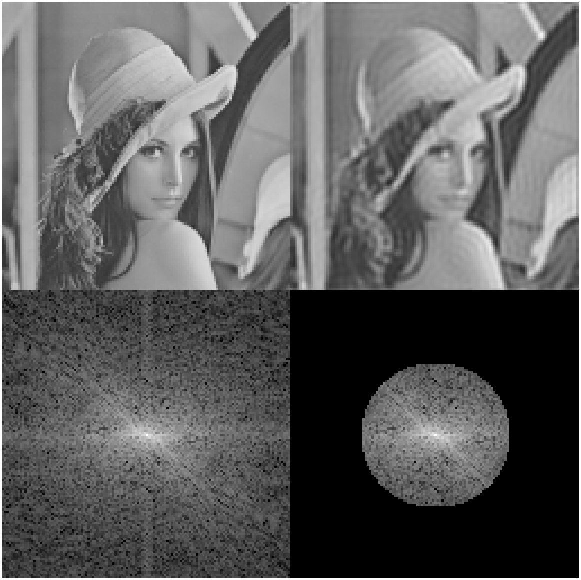
Less painful to use a low-resolution version!

High resolution Low resolution



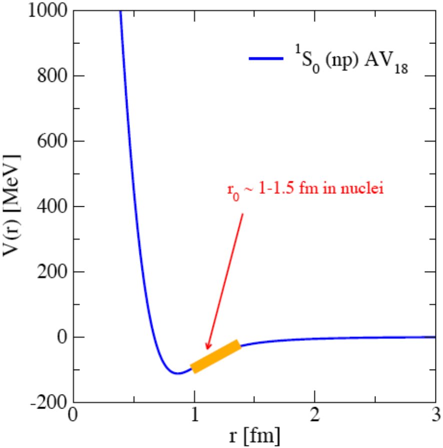
- Can greatly reduce storage without distorting message
- Resolution was lowered here by “block spinning”
- Alternative: apply a low-pass filter

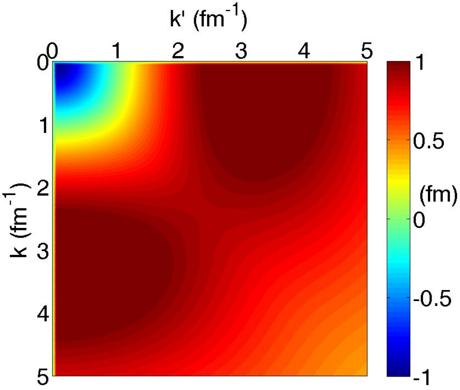
Low-pass filter on an image



- Much less information needed
- Long-wavelength info is preserved

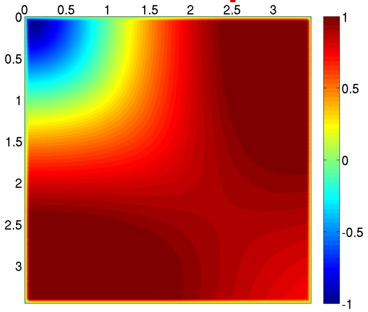
Try a low-pass filter on nuclear $V(r)$

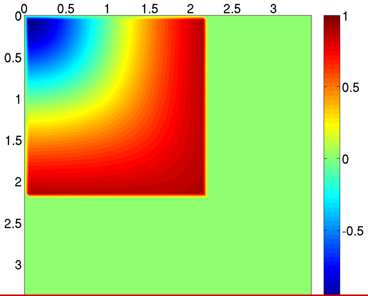


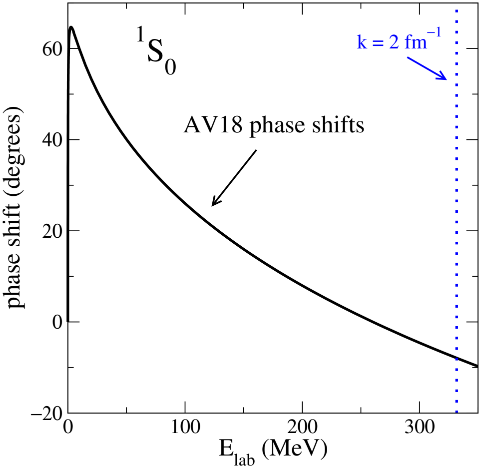


$\Longrightarrow$ Set to zero high momentum ($k \geqslant 2\,\mathrm{fm}^{-1}$) matrix elements and see the effect on low-energy observables

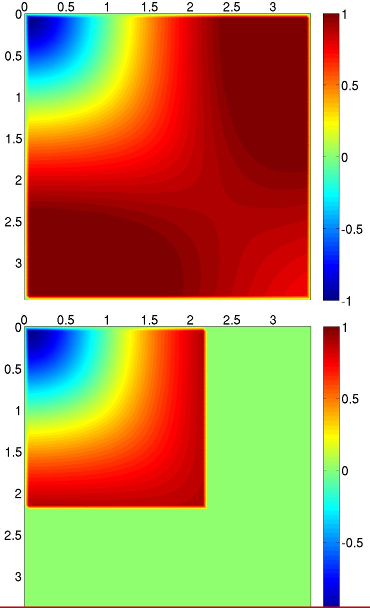
Effect of low-pass filter on observables

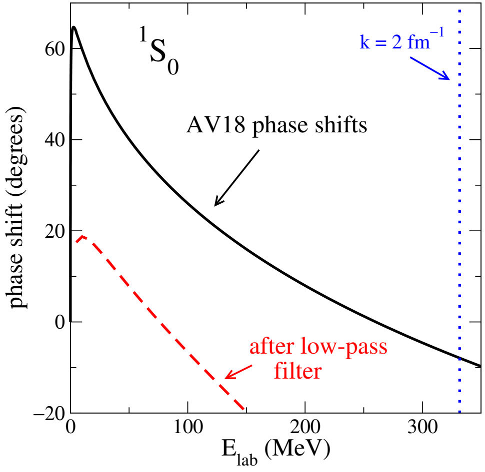






Effect of low-pass filter on observables





Why did our low-pass filter fail?

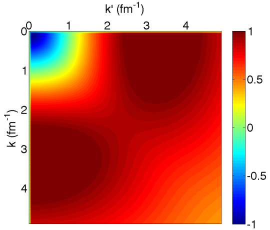
- Basic problem: low k and high k are coupled (mismatched dof's!)
- E.g., perturbation theory for (tangent of) phase shift:

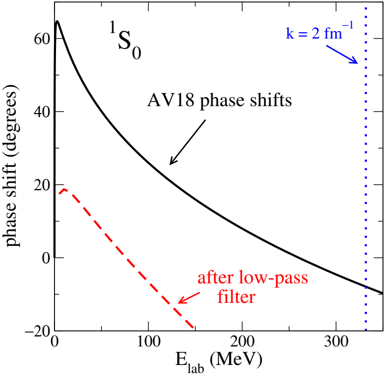
$$\langle k|V|k\rangle + \sum_{k'} \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2 - k'^2)/m} + \cdots$$

- Solution: Unitary transformation of the H matrix $\Longrightarrow$ decouple!

$$\begin{aligned} E_n &= \langle\Psi_n|H|\Psi_n\rangle \qquad U^\dagger U = 1 \\ &= (\langle\Psi_n|U^\dagger)UHU^\dagger(U|\Psi_n\rangle) \\ &= \langle\widetilde{\Psi}_n|\widetilde{H}|\widetilde{\Psi}_n\rangle \end{aligned}$$

- Here: Decouple using RG





S. Weinberg on the Renormalization Group

- From “Why the Renormalization Group is a good thing”
  “The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you’re talking about are the relevant degrees of freedom for the problem at hand.”

S. Weinberg on the Renormalization Group

- From "Why the Renormalization Group is a good thing"
  "The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- Improving perturbation theory in high-energy physics
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Universality in critical phenomena
  - Filter out short-distance degrees of freedom

S. Weinberg on the Renormalization Group

- From “Why the Renormalization Group is a good thing”
  “The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you’re talking about are the relevant degrees of freedom for the problem at hand.”
- Improving perturbation theory in high-energy physics
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Universality in critical phenomena
  - Filter out short-distance degrees of freedom
- Simplifying calculations of nuclear structure/reactions
  - Make nuclear physics look more like quantum chemistry!
  - Like other RG applications, gains can seem like magic
- RG violates the First Law of Progress in Theoretical Physics
  Conservation of Information: “You will get nowhere by churning equations” $\Longrightarrow$ *but with RG you do!*

Outline

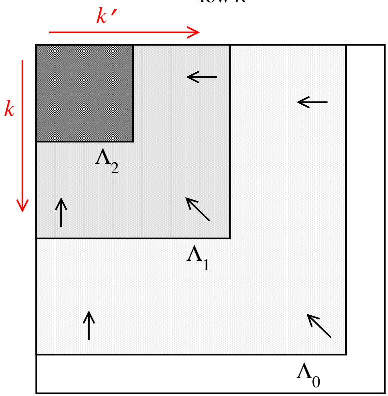
Overview

Lowering the resolution: Similarity RG in practice

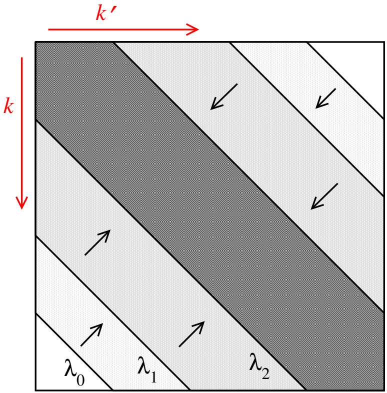
SRG Basics

Two ways to decouple with RG equations

"$V_{\mathrm{low}\,k}$"



Similarity RG



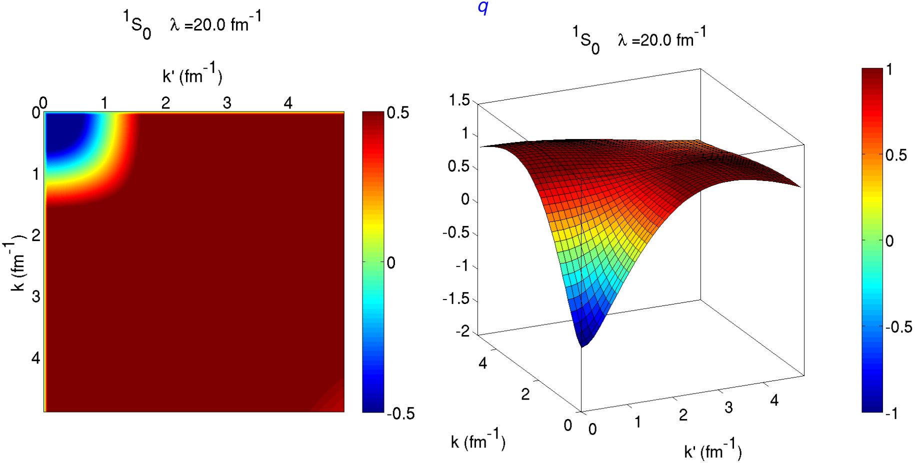
- Lower a cutoff Λ_i in k, k', e.g., demand $dT(k,k';k^2)/d\Lambda = 0$
- Drive the Hamiltonian toward diagonal with "flow equation" [Wegner; Glazek/Wilson (1990's)]

$\Longrightarrow$ Both tend toward universal low-momentum interactions!

Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

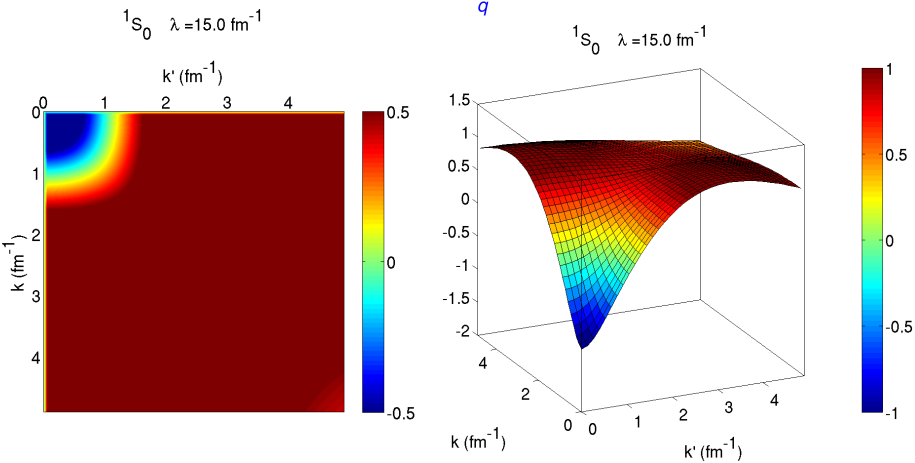
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

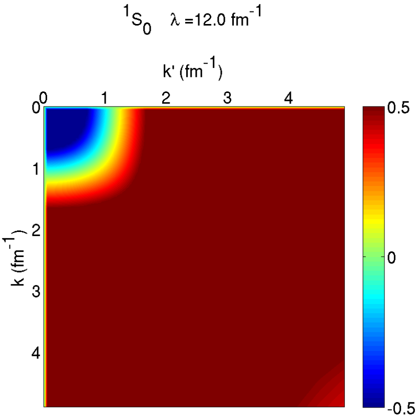
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

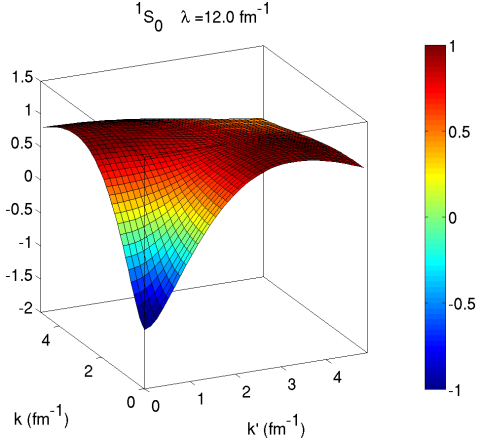


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

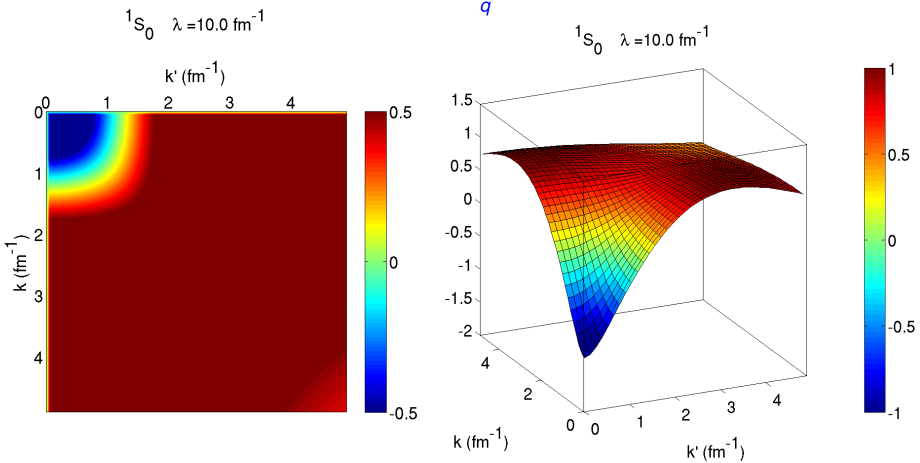




Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

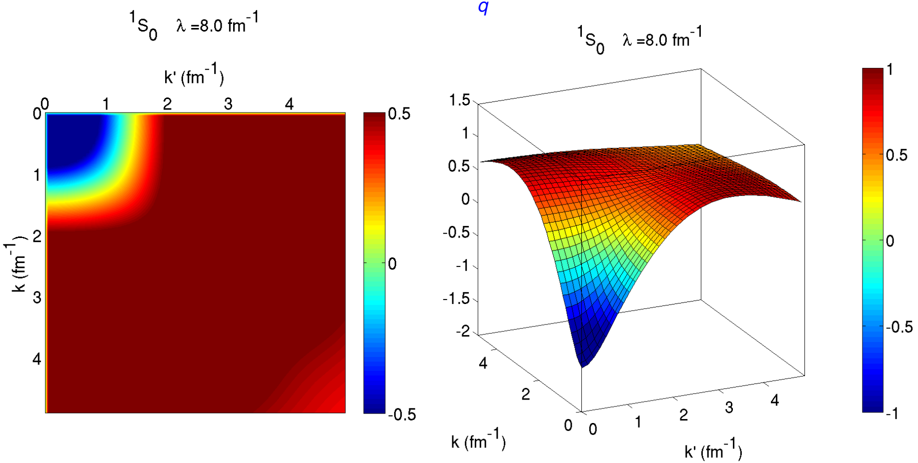
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

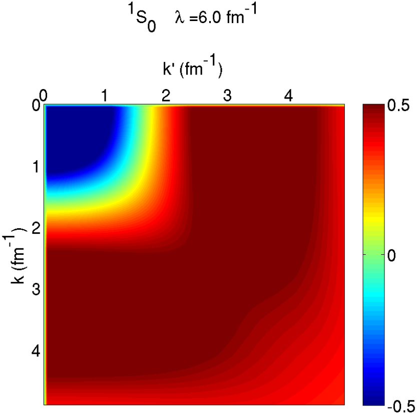
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

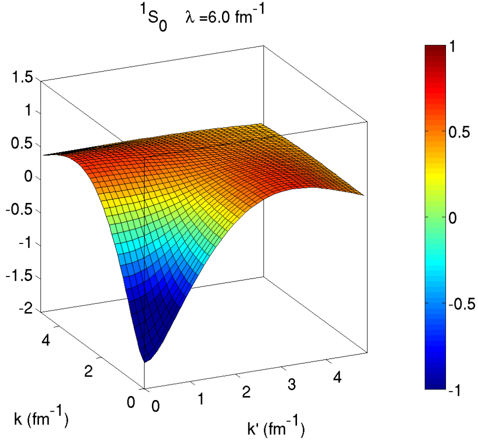


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

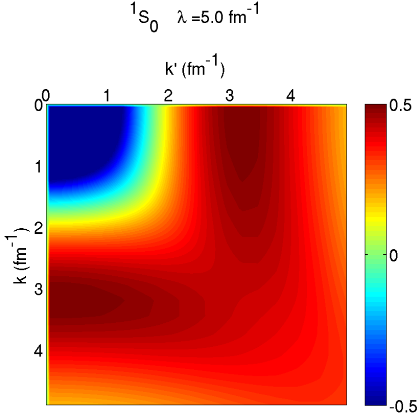


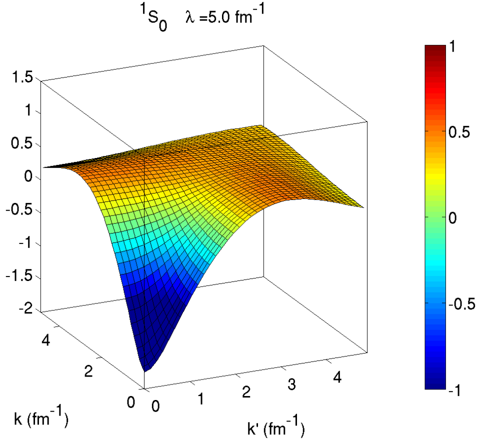


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

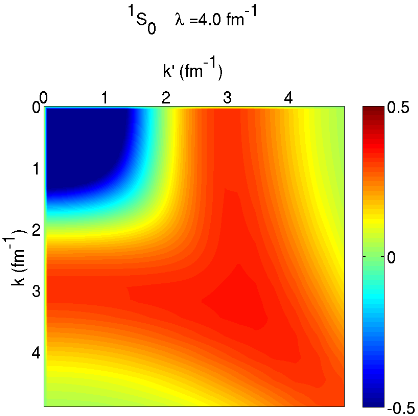


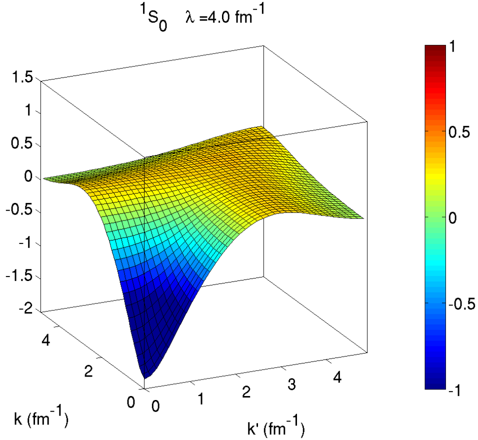


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

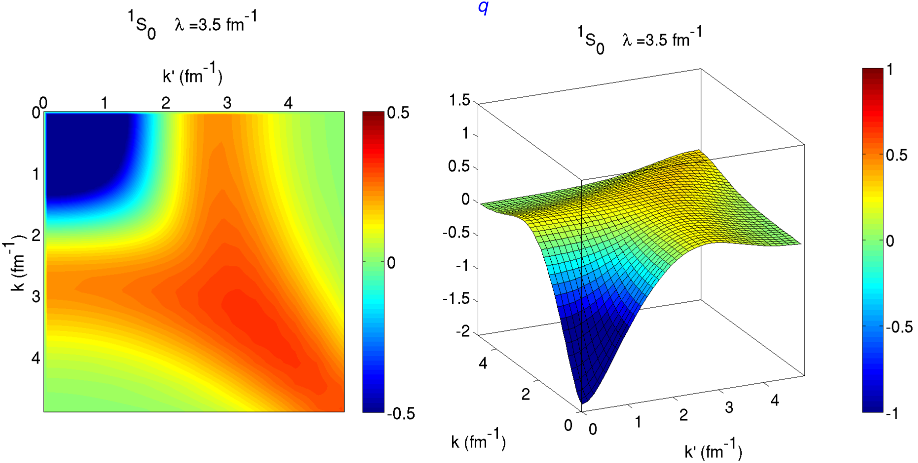




Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

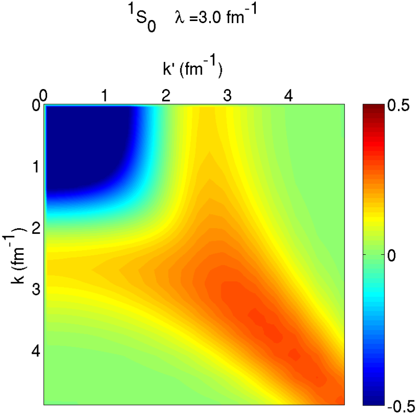
$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

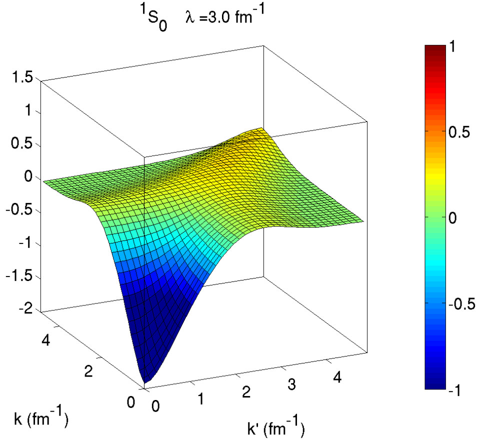


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

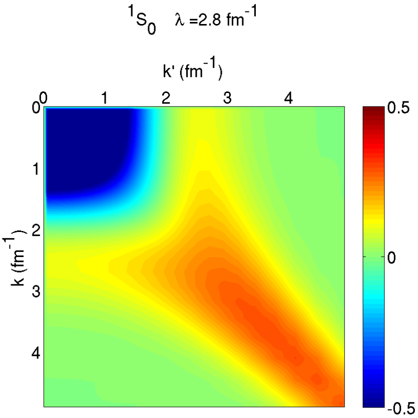


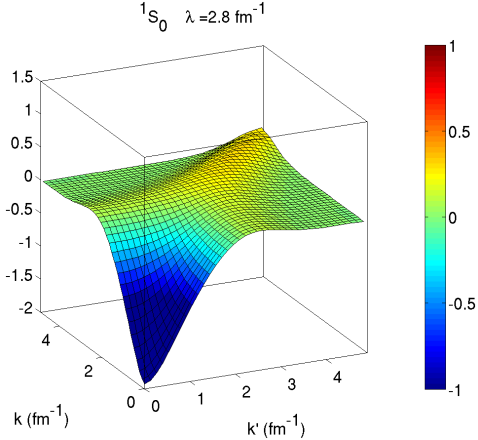


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

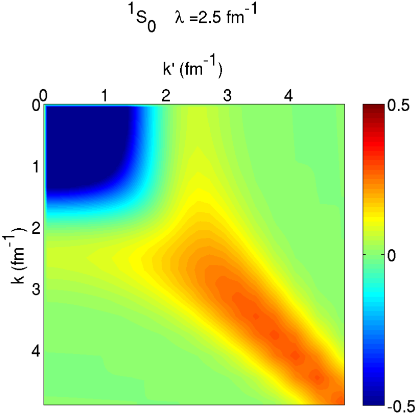


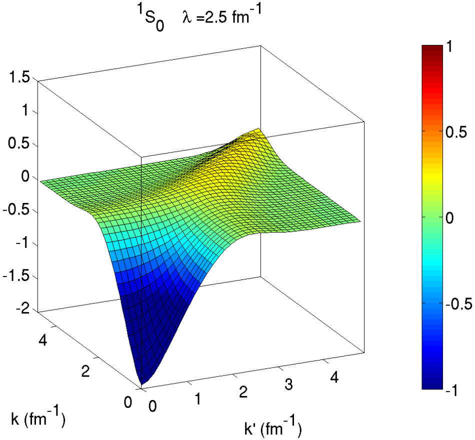


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

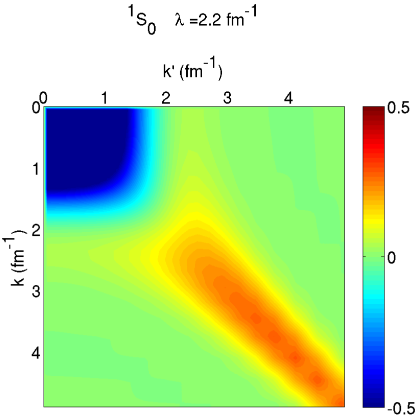


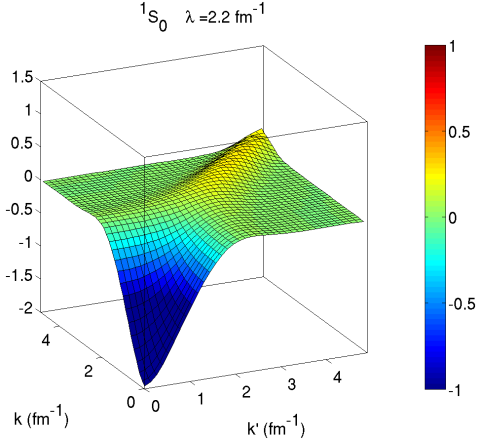


Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

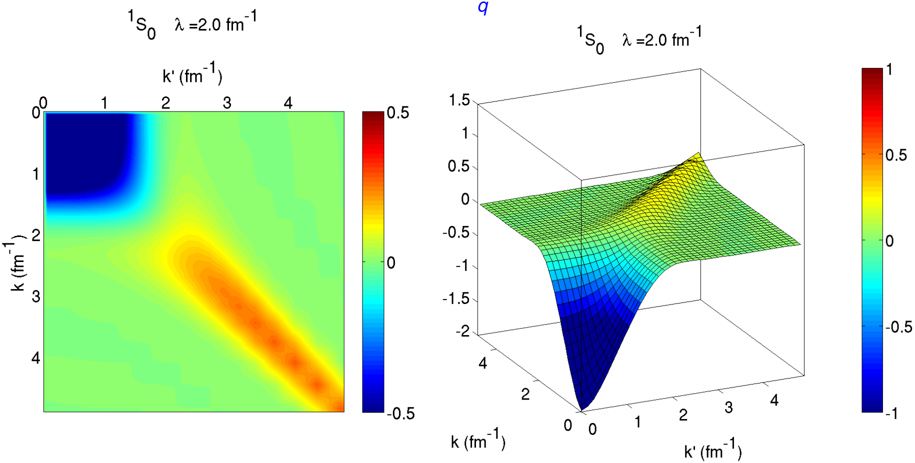




Flow equations in action: NN only

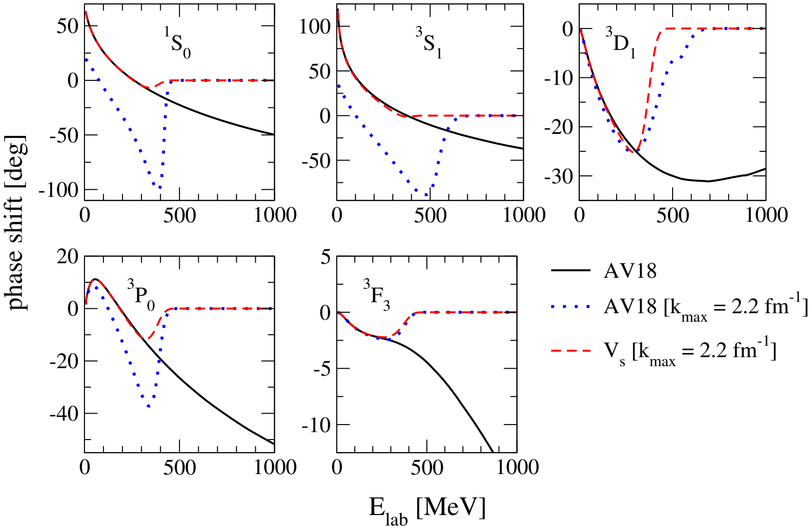
- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

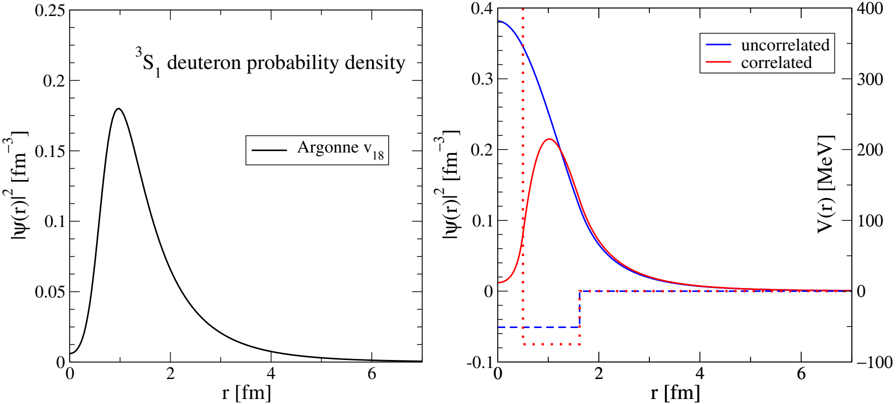


Low-pass filters work! [Jurgenson et al. (2008)]

- Phase shifts with $V_s(k, k') = 0$ for $k, k' > k_{\rm max}$

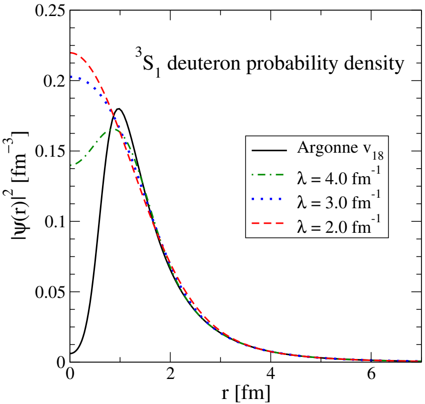


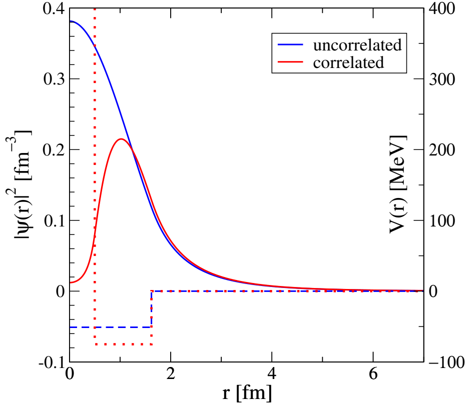
Consequences of a repulsive core revisited



- Probability at short separations suppressed $\Longrightarrow$ "correlations"
- Short-distance structure $\Leftrightarrow$ high-momentum components
- Greatly complicates expansion of many-body wave functions

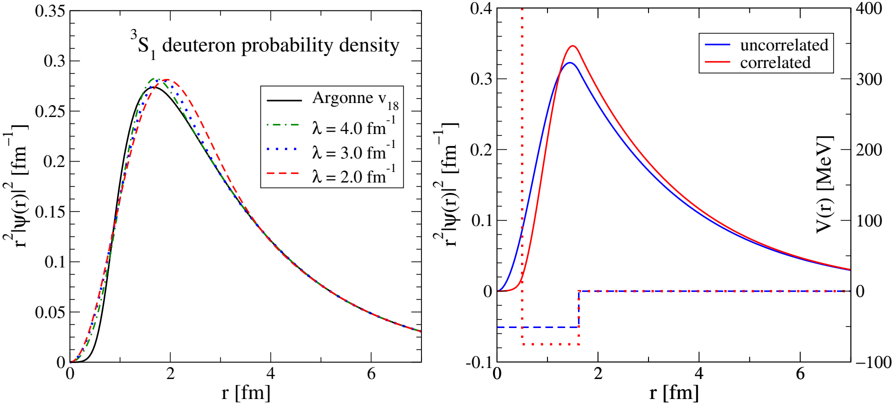
Consequences of a repulsive core revisited





- Transformed potential $\Longrightarrow$ no short-range correlations in wf!
- Potential is now non-local: $V(\mathbf{r})\psi(\mathbf{r}) \longrightarrow \int d^3\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')$
  - A problem for Green's Function Monte Carlo approach
  - Not a problem for many-body methods using HO matrix elements

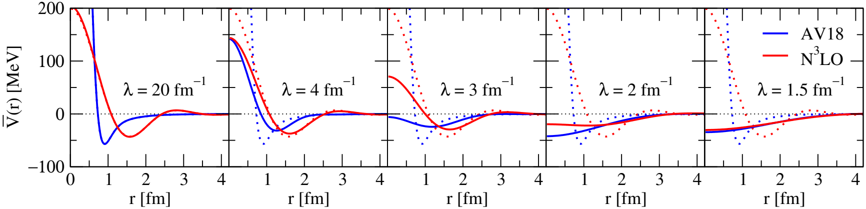
Consequences of a repulsive core revisited



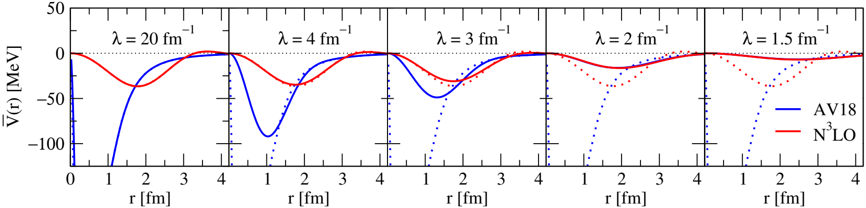
- Transformed potential $\Longrightarrow$ no short-range correlations in wf!
- Potential is now non-local: $V(\mathbf{r})\psi(\mathbf{r}) \longrightarrow \int d^3\mathbf{r}'\, V(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')$
  - A problem for Green's Function Monte Carlo approach
  - Not a problem for many-body methods using HO matrix elements

Visualizing the softening of NN interactions

- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r,r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]

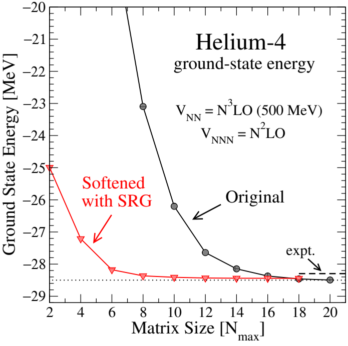


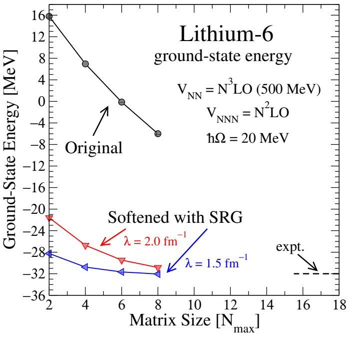
- Tensor part (S-D mixing) [graphs from K. Wendt]

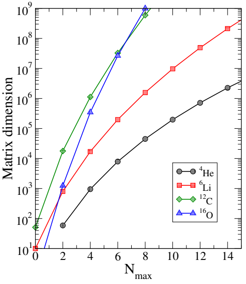


Many short wavelengths $\Longrightarrow$ Large matrices

- Harmonic oscillator basis with $N_{\rm max}$ shells for excitations
- Graphs show convergence for *soft* chiral EFT potential and evolved SRG potentials (including NNN)







- Better convergence, but rapid growth of basis still a problem (solution: importance sampling of matrix elements [R. Roth])

Outline

Basics: SRG flow equations [arXiv:0912.3688]

- Transform an initial hamiltonian, $H = T + V$:

$$H_s = U_s H U_s^\dagger \equiv T + V_s \ ,$$

where s is the *flow parameter*. Differentiating wrt s:

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \text{with} \qquad \eta_s \equiv \frac{dU_s}{ds} U_s^\dagger = -\eta_s^\dagger \ .$$

- η_s is specified by the commutator with Hermitian G_s:

$$\eta_s = [G_s, H_s] \ ,$$

which yields the unitary flow equation (T held fixed),

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s] \ .$$

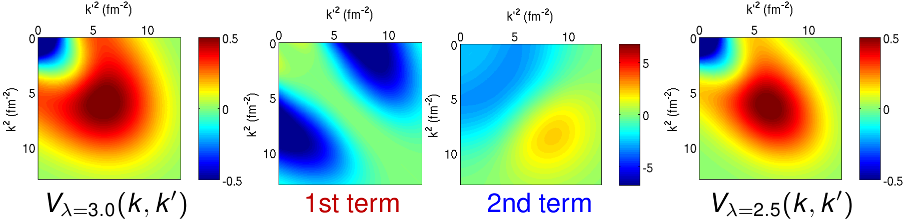
- G_s determines flow $\Longrightarrow$ many choices (T, H_D, H_{BD}, ...)

Flow in momentum basis with $\eta(s) = [T, H_s]$

- For $A = 2$, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \qquad \text{with} \quad T_{\rm rel}|k\rangle = \epsilon_k |k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



- First term drives ${}^1S_0$ V_λ toward diagonal:

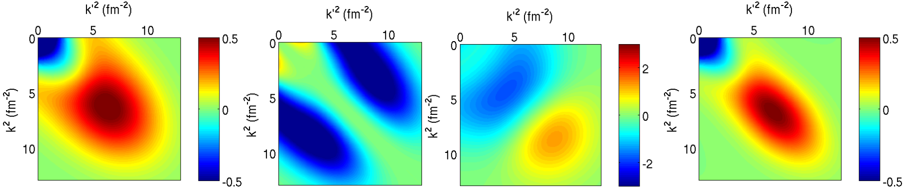
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \ldots$$

Flow in momentum basis with $\eta(s) = [T, H_s]$

- For $A = 2$, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \quad \text{with} \quad T_{\rm rel}|k\rangle = |\epsilon_k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$



$V_{\lambda=2.5}(k,k')$ 1st term 2nd term $V_{\lambda=2.0}(k,k')$

- First term drives $^1S_0$ V_λ toward diagonal:

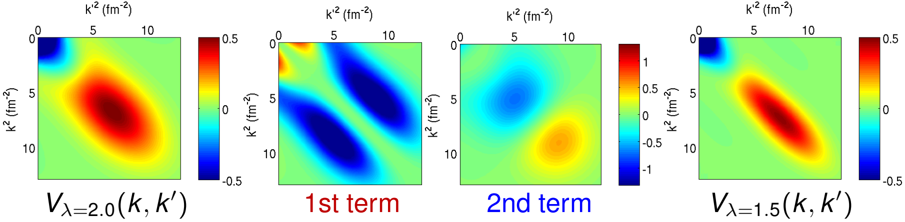
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \ldots$$

Flow in momentum basis with $\eta(s) = [T, H_s]$

- For $A = 2$, project on rel. momentum states $|k\rangle$, but generic

$$\frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \quad \text{with} \quad T_{\rm rel}|k\rangle = |\epsilon_k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$

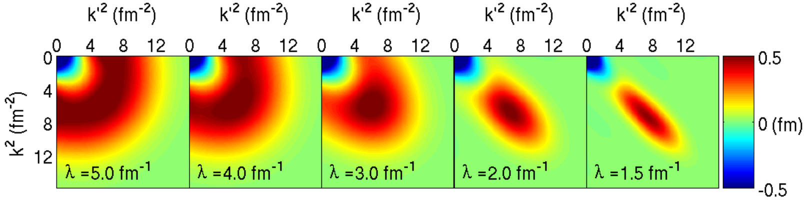


- First term drives $^1S_0$ V_λ toward diagonal:

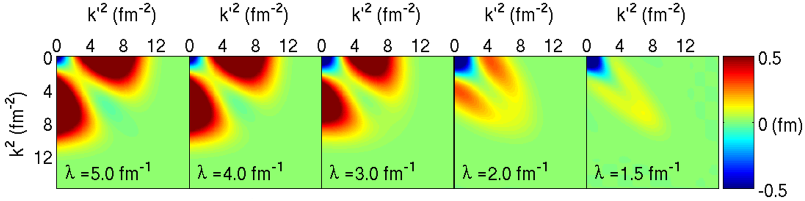
$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \ldots$$

Flow of different N^{3}LO chiral EFT potentials

- $^1S_0$ from N^{3}LO (500 MeV) of Entem/Machleidt



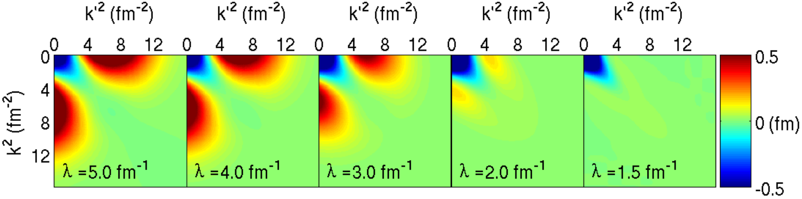
- $^1S_0$ from N^{3}LO (550/600 MeV) of Epelbaum et al.



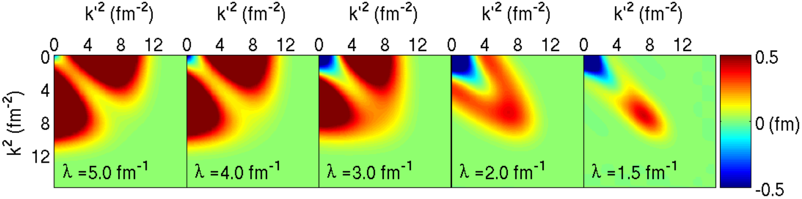
- Significant decoupling even for “soft” EFT interaction

Flow of different N^{3}LO chiral EFT potentials

- $^3S_1$ from N^{3}LO (500 MeV) of Entem/Machleidt

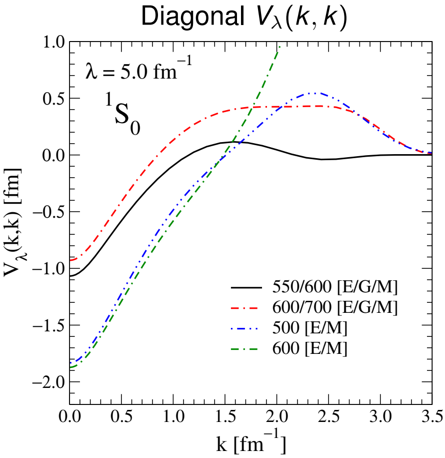


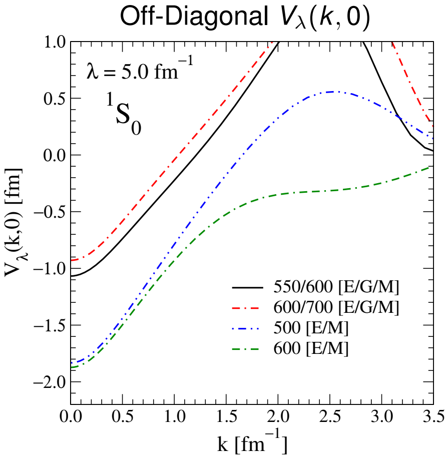
- $^3S_1$ from N^{3}LO (550/600 MeV) of Epelbaum et al.



- Significant decoupling even for “soft” EFT interaction

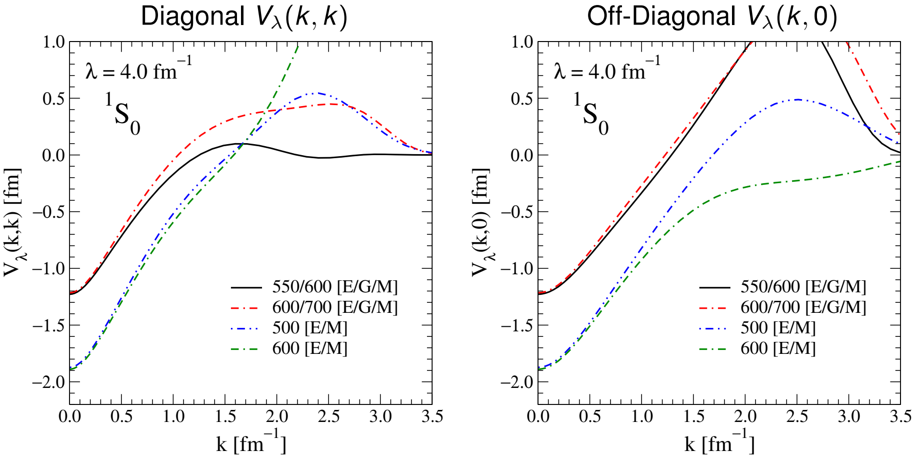
Run to lower λ via SRG $\Longrightarrow$ ≈Universal V_{NN}





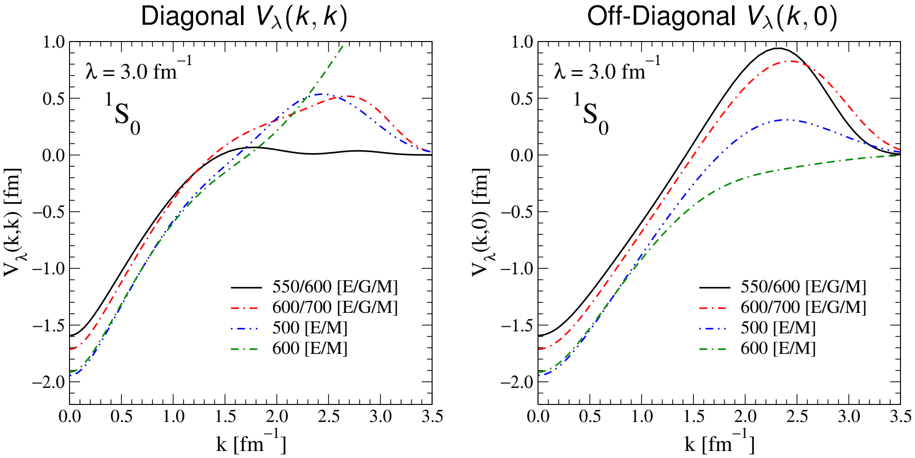
- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

Run to lower λ via SRG $\Longrightarrow$ ≈Universal V_{NN}



- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

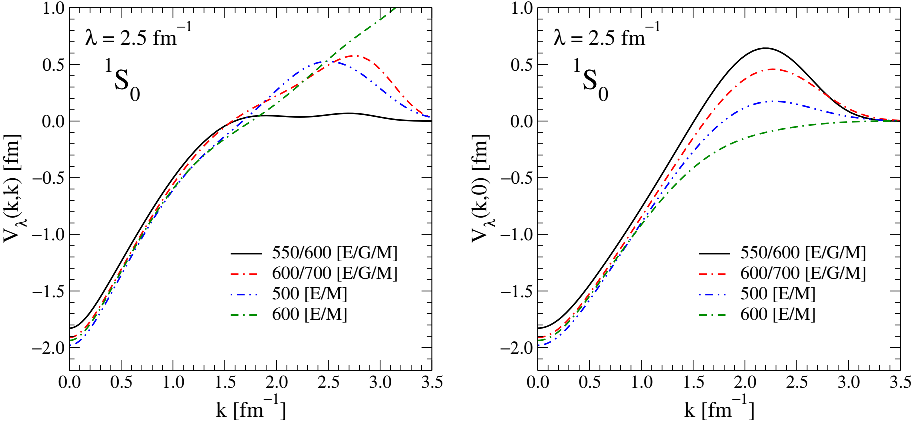
Run to lower λ via SRG $\Longrightarrow$ ≈Universal V_{NN}



- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

Run to lower λ via SRG $\Longrightarrow$ ≈Universal V_{NN}

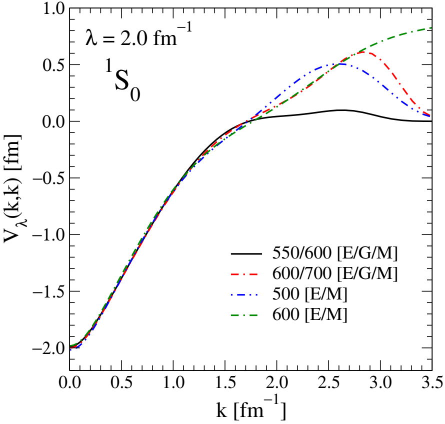
Diagonal $V_\lambda(k,k)$ Off-Diagonal $V_\lambda(k,0)$



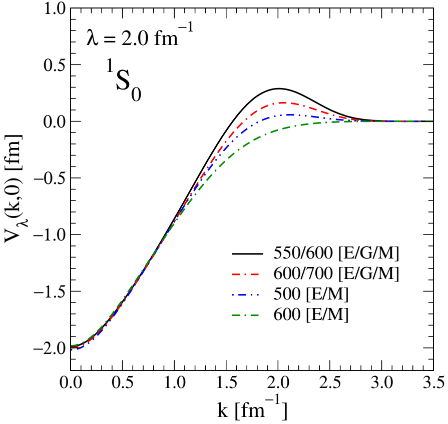
- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

Run to lower λ via SRG $\Longrightarrow$ $\approx$Universal V_{NN}

Diagonal $V_\lambda(k,k)$

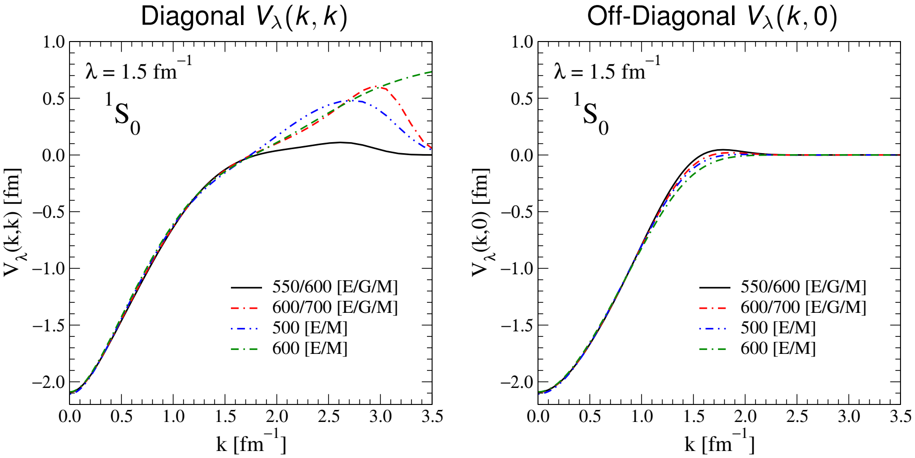


Off-Diagonal $V_\lambda(k,0)$



- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

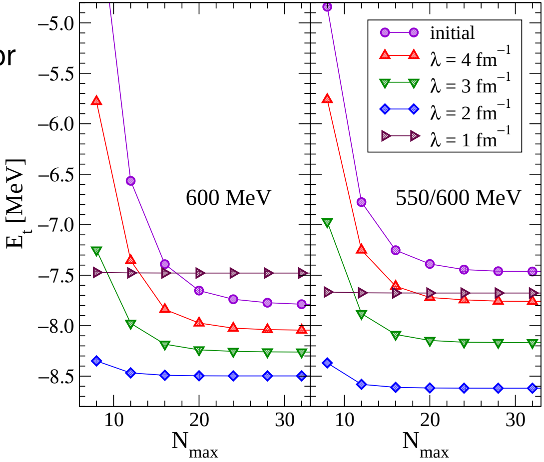
Run to lower λ via SRG $\Longrightarrow \approx$Universal V_{NN}



- Apparently different NN potentials flow to common V_{NN}
- Do NNN interactions evolve to universal form? [Hebeler: yes!]

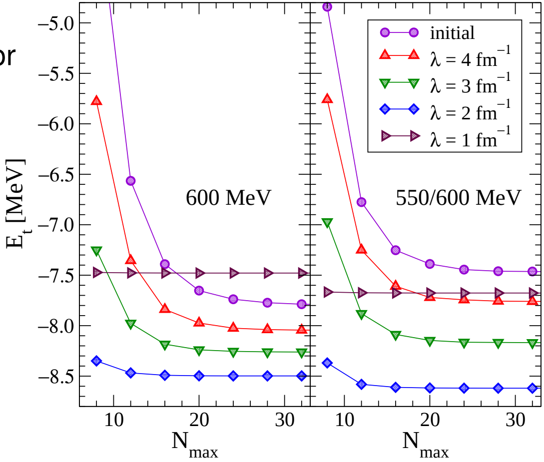
Variational calculations in three-nucleon systems

- Triton ground-state energy vs. size of harmonic oscillator basis ($N_{\max}\hbar\omega$ excitations)
- Rapid convergence as λ decreases
- Note softening already at $\lambda = 3\,\text{fm}^{-1}$ with N^3LO EFT $\Lambda = 600\,\text{MeV} = 3\,\text{fm}^{-1}$
- Different binding energies!



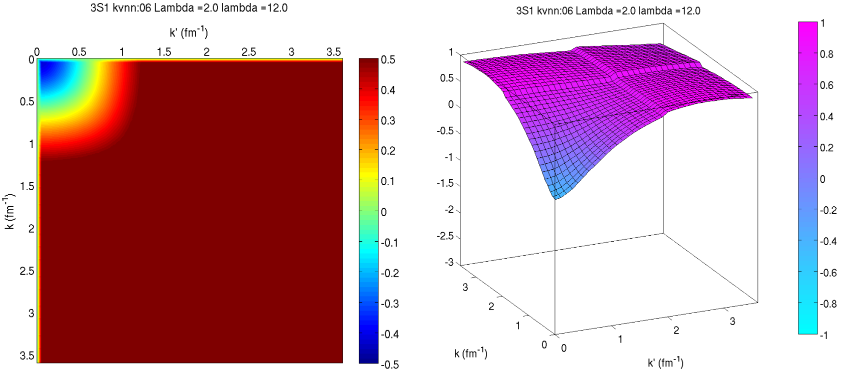
Variational calculations in three-nucleon systems

- Triton ground-state energy vs. size of harmonic oscillator basis ($N_{\rm max}\hbar\omega$ excitations)
- Rapid convergence as λ decreases
- Note softening already at $\lambda = 3\,\text{fm}^{-1}$ with N^3LO EFT $\Lambda = 600\,\text{MeV} = 3\,\text{fm}^{-1}$
- Different binding energies!
- Nuclear matter doesn't saturate at low λ



Block diagonalization via SRG $[G_s = H_{BD}]$

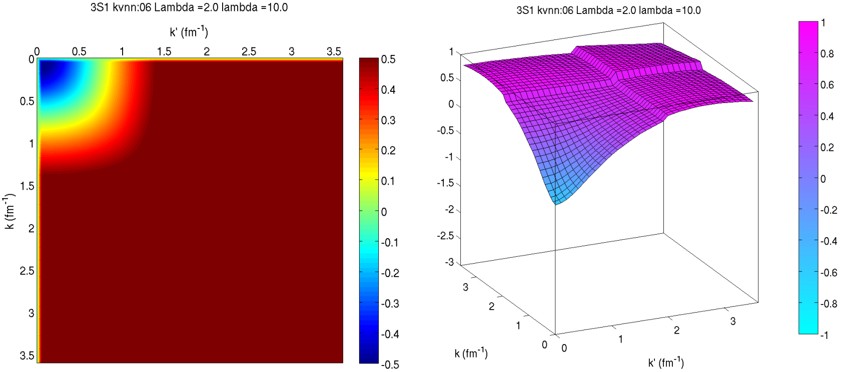
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

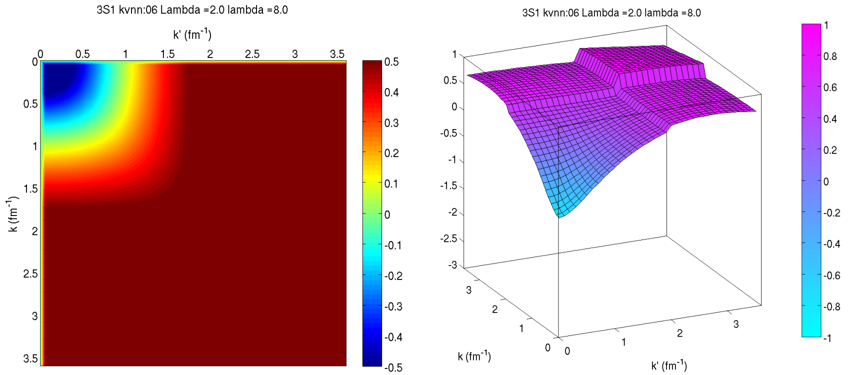
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$



- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

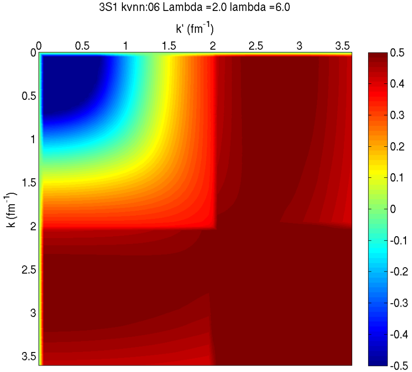
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

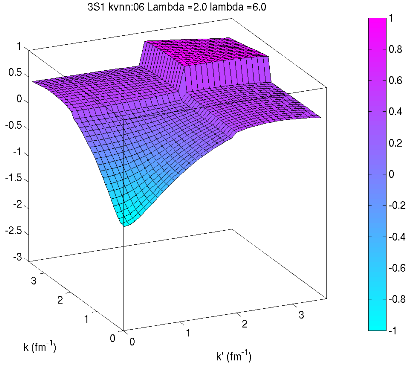


- What are the best generators for nuclear applications?

Block diagonalization via SRG $[G_s = H_{BD}]$

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

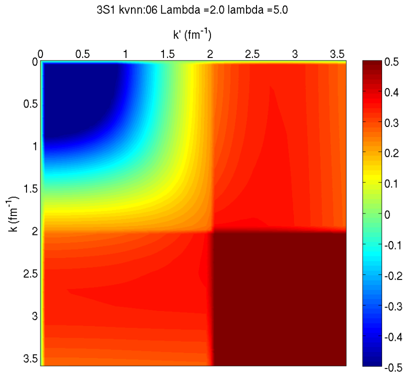


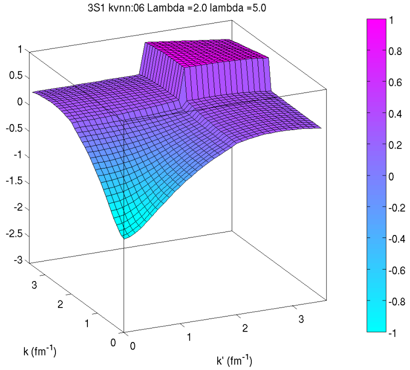


- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

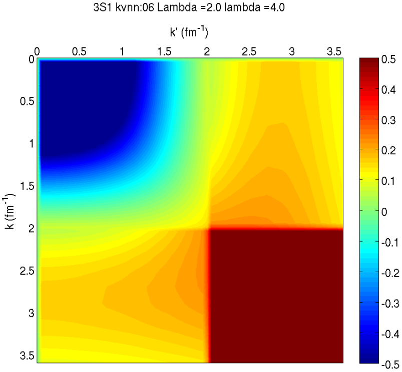


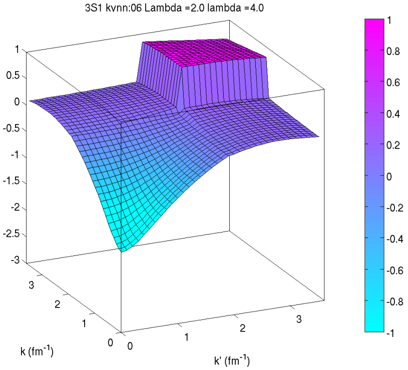


- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

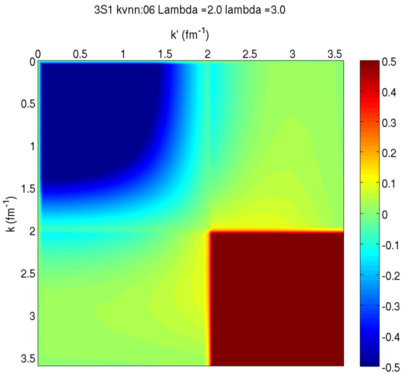


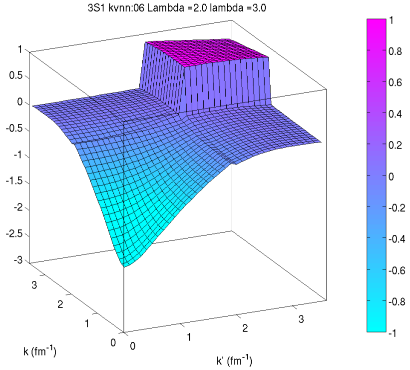


- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

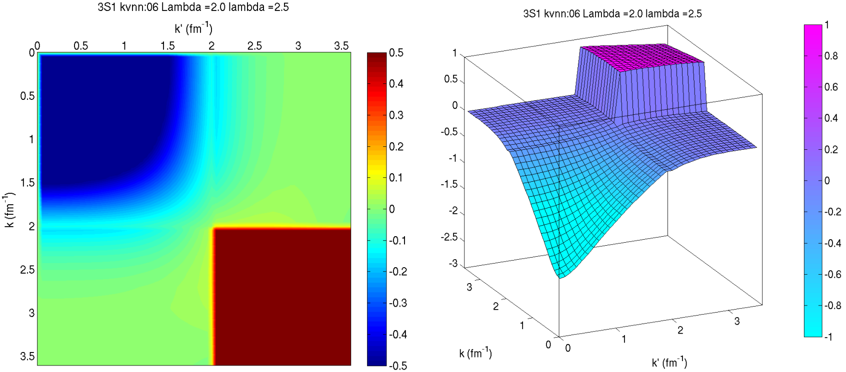




- What are the best generators for nuclear applications?

Block diagonalization via SRG $[G_s = H_{BD}]$

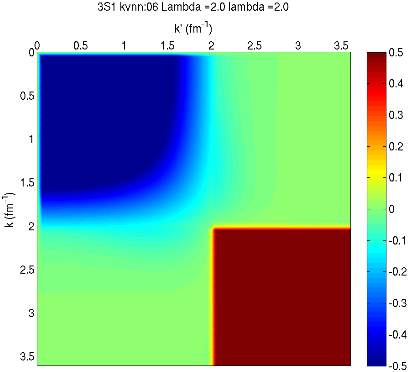
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

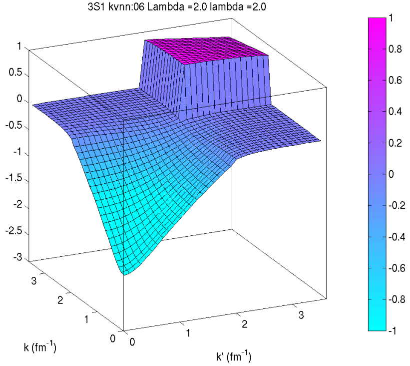


- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

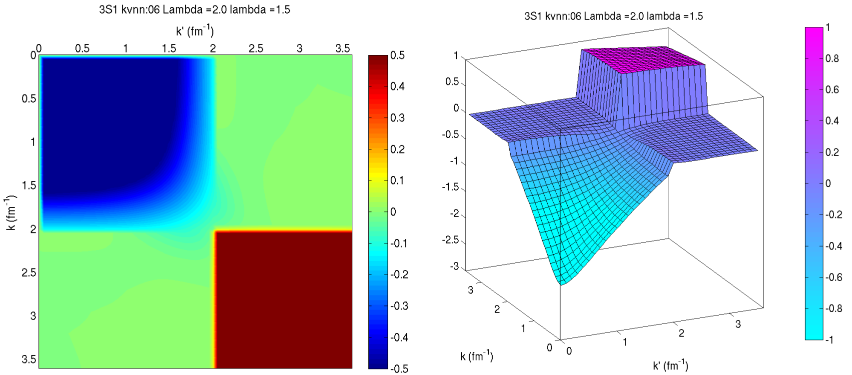




- What are the best generators for nuclear applications?

Block diagonalization via SRG [$G_s = H_{BD}$]

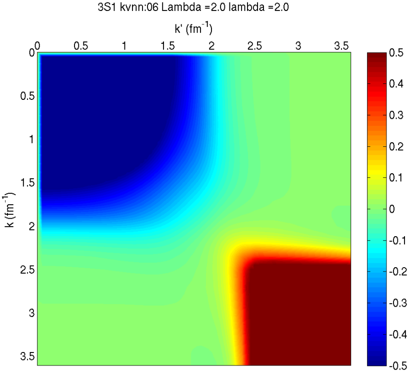
- Can we get a $\Lambda = 2\,\text{fm}^{-1}$ $V_{\text{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$

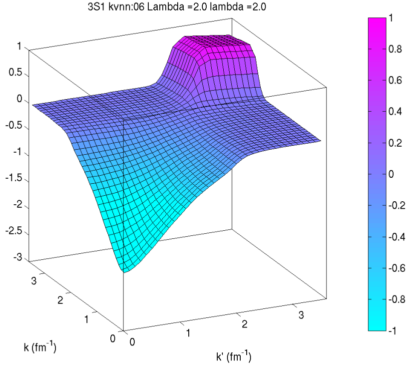


- What are the best generators for nuclear applications?

Block diagonalization via SRG $[G_s = H_{BD}]$

- Can we get a $\Lambda = 2\,\mathrm{fm}^{-1}$ $V_{\mathrm{low}\,k}$-like potential with SRG?
- Yes! Use $\frac{dH_s}{ds} = [[G_s, H_s], H_s]$ with $G_s = \begin{pmatrix} PH_sP & 0 \\ 0 & QH_sQ \end{pmatrix}$





- What are the best generators for nuclear applications?