Nuclear forces and their impact on structure, reactions and astrophysics

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Ohio State University

July, 2013

Lectures for Week 2

- **M.** Many-body problem and basis considerations (as); Many-body perturbation theory (rjf)
- **T.** Neutron matter and astrophysics (as); Operators 1 (rjf)
- **W.** Operators 2, nuclear matter (rjf); Student presentations
- **Th.** Impact on (exotic) nuclei (as); Student presentations
- **F.** Impact on fundamental symmetries (as); From forces to density functionals (rjf)

Outline

Overview of nuclei leading to nuclear matter

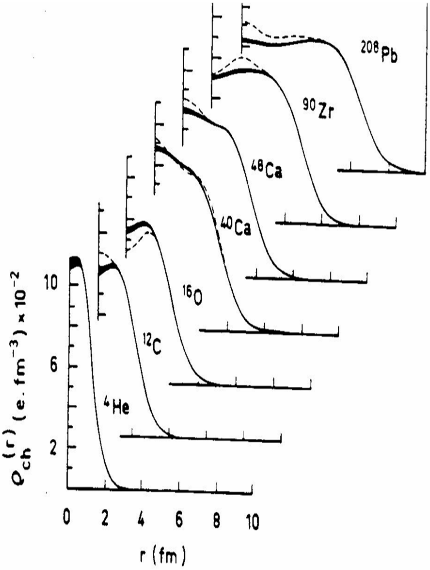
Nuclear matter calculations

Operators and resolution

What do (ordinary) nuclei look like?

- Charge densities of magic nuclei (mostly) shown
- Proton density has to be "unfolded" from $\rho_{\text{charge}}(r)$, which comes from elastic electron scattering
- Roughly constant interior density with $R \approx (1.1\text{–}1.2\,\text{fm}) \cdot A^{1/3}$
- Roughly constant surface thickness

$\Longrightarrow$ Like a liquid drop!



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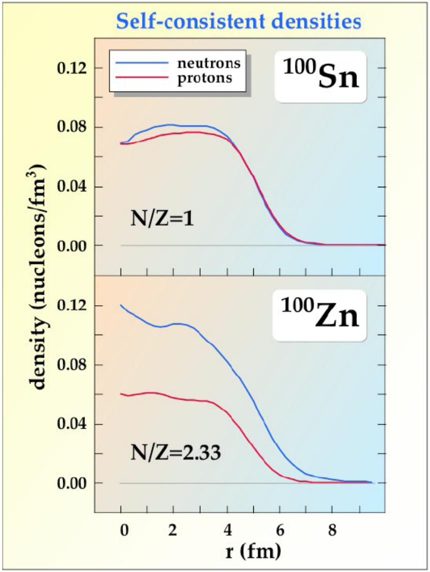
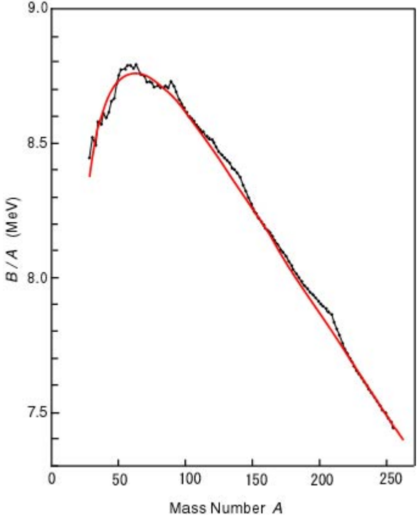


Semi-empirical mass formula $(A = N + Z)$

$$E_B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\rm sym} \frac{(N-Z)^2}{A} + \Delta$$

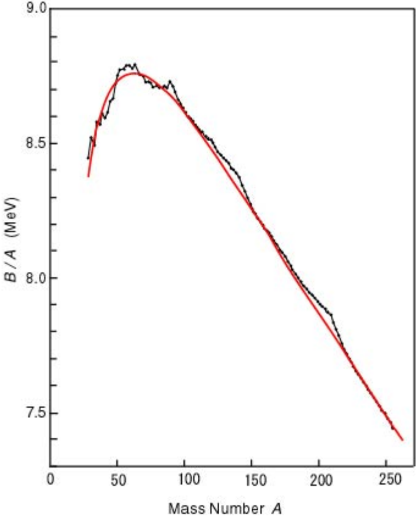
- Many predictions!
- Rough numbers: $a_v \approx 16\,\text{MeV}$, $a_s \approx 18\,\text{MeV}$, $a_C \approx 0.7\,\text{MeV}$, $a_{\rm sym} \approx 28\,\text{MeV}$
- Pairing $\Delta \approx \pm 12/\sqrt{A}\,\text{MeV}$ (even-even/odd-odd) or 0 [or $43/A^{3/4}$ MeV or ...]
- Surface symmetry energy: $a_{\rm surf\ sym}(N-Z)^2/A^{4/3}$
- Much more sophisticated mass formulas include shell effects, etc.



Semi-empirical mass formula per nucleon

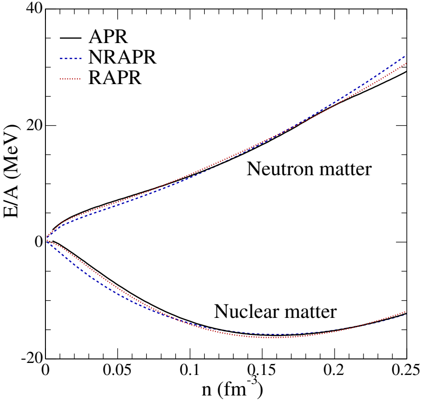
$$\frac{E_B(N,Z)}{A} = a_v - a_s A^{-1/3} - a_C \frac{Z^2}{A^{4/3}} - a_{\rm sym} \frac{(N-Z)^2}{A^2}$$

- Divide terms by $A = N + Z$
- Rough numbers:
  $a_v \approx 16\,\text{MeV}$, $a_s \approx 18\,\text{MeV}$,
  $a_C \approx 0.7\,\text{MeV}$, $a_{\rm sym} \approx 28\,\text{MeV}$
- Surface symmetry energy:
  $a_{\rm surf\ sym}(N-Z)^2/A^{7/3}$
- Now take $A \to \infty$ with Coulomb $\to$ 0 and fixed N/A, Z/A
- Surface terms negligible



Nuclear and neutron matter energy vs. density

[Akmal et al. calculations shown]



- Uniform with Coulomb turned off
- Density n (or often ρ)
- Fermi momentum $n = (\nu/6\pi^2)k_{\mathrm{F}}^3$
- Neutron matter ($Z = 0$) has positive pressure
- Symmetric nuclear matter ($N = Z = A/2$) saturates
- *Empirical* saturation at about $E/A \approx -16\,\text{MeV}$ and $n \approx 0.17 \pm 0.03\,\text{fm}^{-3}$

Outline

Overview of nuclei leading to nuclear matter

Nuclear matter calculations

Operators and resolution

Recent activity on nuclear matter (from INSPIRE) . . .

- A. Carbone, A. Rios and A. Polls, “Symmetric nuclear matter with chiral three-nucleon forces in the self-consistent Green’s functions approach,” arXiv:1307.1889 [nucl-th].
- T. Katayama and K. Saito, “Properties of dense, asymmetric nuclear matter in Dirac-Brueckner-Hartree-Fock approach,” arXiv:1307.2067 [nucl-th].
- T. Inoue *et al.* [HAL QCD Collaboration], “Equation of State for Nucleonic Matter and its Quark Mass Dependence from the Nuclear Force in Lattice QCD,” arXiv:1307.0299 [hep-lat].
- G. Baardsen, A. Ekstrm, G. Hagen and M. Hjorth-Jensen, “Coupled Cluster studies of infinite nuclear matter,” arXiv:1306.5681 [nucl-th].
- G. Colucci, A. Sedrakian and D. H. Rischke, “Impact of relativistic chiral one-pion exchange on nuclear matter properties,” arXiv:1303.1270 [nucl-th].
- J. A. Oller, “Chiral effective field theory for nuclear matter,” PoS QNP **2012**, 134 (2012).
- N. Kaiser, “Chiral four-body interactions in nuclear matter,” Eur. Phys. J. A **48**, 135 (2012) [arXiv:1209.4556 [nucl-th]].
- M. Baldo and H. R. Moshfegh, “Correlations in nuclear matter,” Phys. Rev. C **86**, 024306 (2012) [arXiv:1209.2270 [nucl-th]].

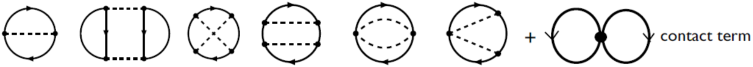
Chiral Dynamics of Nuclear Matter

Munich Group (Kaiser, Fritsch, Holt, Weise, . . .)

- Basic idea: ChPT loop expansion becomes EOS expansion:

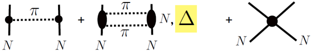
$$E(k_F) = \sum_{n=2}^{\infty} k_F^n \, f_n(k_F/m_\pi, \Delta/m_\pi) \qquad [\Delta = M_\Delta - M_N \approx 300\,\text{MeV}]$$

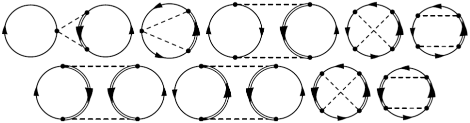
- 1st pass: N's and π's $\Longrightarrow$ count k_F's by medium insertions



  - Saturation from *Pauli-blocking* of iterated 1π-exchange
  - Problems with single-particle and isospin properties and . . .

- 2nd pass: include $\pi N\Delta$ dynamics:

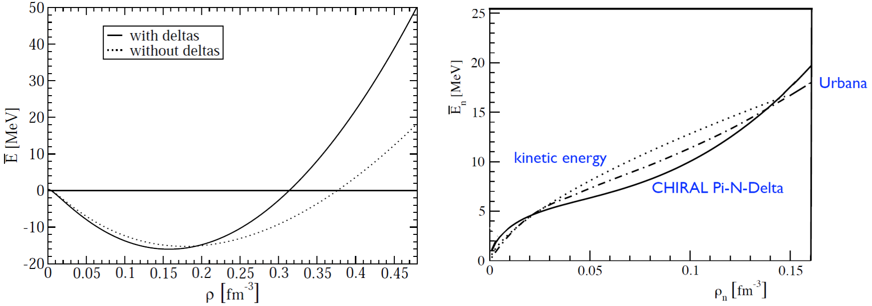




Chiral Dynamics of Nuclear Matter (cont.)

Munich Group (Kaiser, Fritsch, Holt, Weise, . . .)

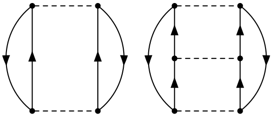
- 3-Loop: Fit nuclear matter saturation, predict neutron matter



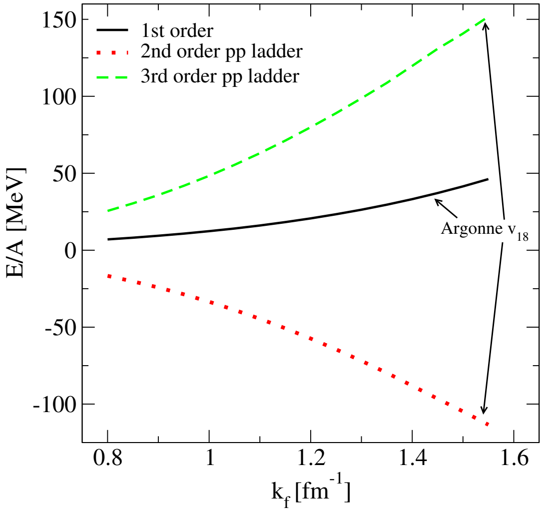
- Substantial improvement in s.p. properties, spin-stability, . . .
- Issues for perturbative chiral expansion of nuclear matter:
  - higher orders, convergence? power counting?
  - relation of LEC's to free space EFT?

Nuclear matter with NN ladders only [nucl-th/0504043]

- Brueckner ladders order-by-order



- Repulsive core $\Longrightarrow$ series diverges
- Usual solution: resum into G-matrix then do hole-line expansion



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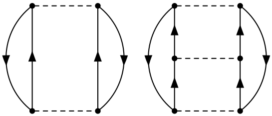

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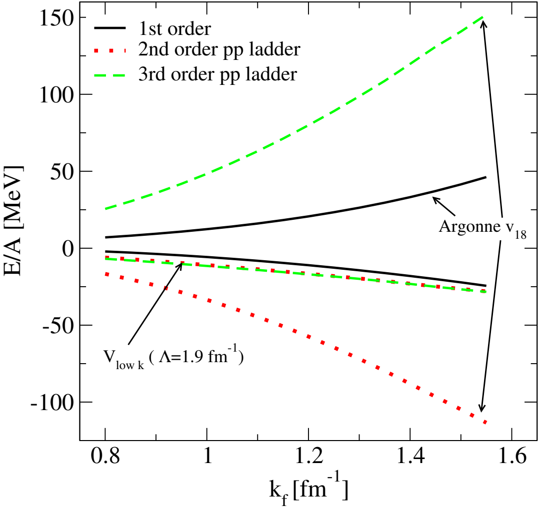
- $V_{\text{low } k}$ converges, so can choose U for DFT



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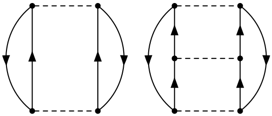

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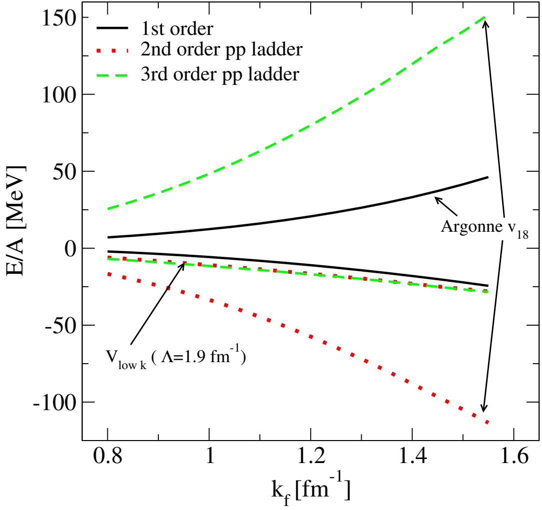
- No saturation in sight!



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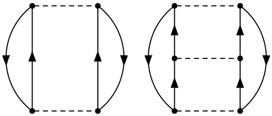
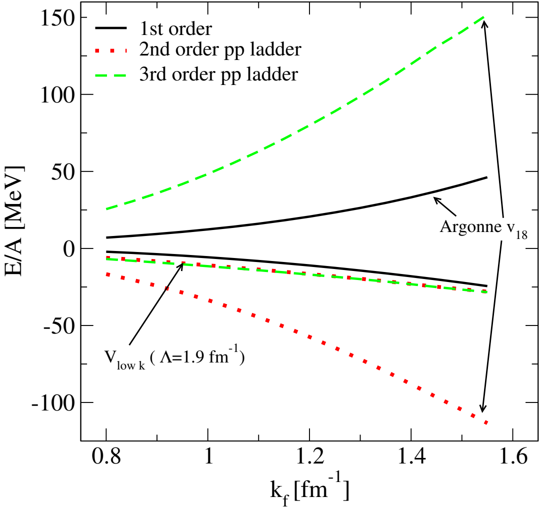
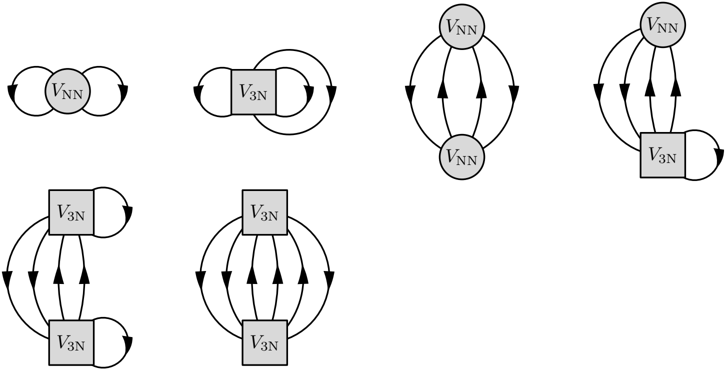


- Repulsive core $\Longrightarrow$ series diverges
- Usual solution: resum into G-matrix then do hole-line expansion
- $V_{\text{low }k}$ converges, so can choose U for DFT
- No saturation in sight!
- But now add 3-body!



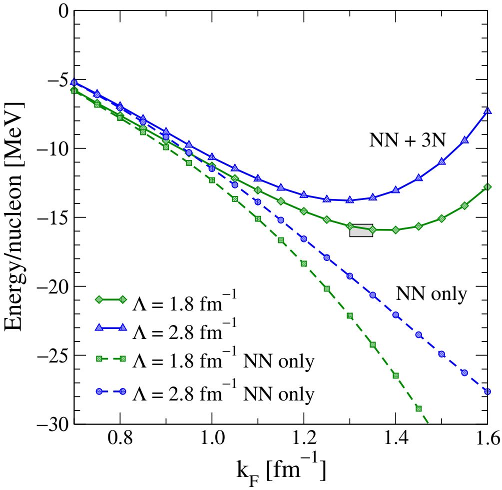
Diagrams for MBPT to second order



Diagrams contributing to the energy per particle up to second order in MBPT, taking two- and three-body interactions into account.

Energy per particle in SNM vs. Fermi momentum

- Compare NN-only results to NN+3NF
- Two representative NN cutoffs
- Fixed 3N cutoff
- 3N constants fit to few-body nuclei $\Longrightarrow$ prediction!
- Hebeler et al. (2011)



There's nothing new under the sun ...

- Is the idea that repulsive three-nucleon forces could be the dominant nm saturation mechanism a new one?
- Consider this quote:
  "... if we accept the potentials ... as a semiphenomenological working basis for our calculations, we find that the many-body forces, and in particular the three-body repulsion, provide a satisfactory qualitative understanding of nuclear saturation."
- Where does it come from?

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- Consider this quote:
  "... if we accept the potentials ... as a semiphenomenological working basis for our calculations, we find that the many-body forces, and in particular the three-body repulsion, provide a satisfactory qualitative understanding of nuclear saturation."
- Where does it come from? Drell and Huang, 1953!

PHYSICAL REVIEW VOLUME 91, NUMBER 6 SEPTEMBER 15, 1953

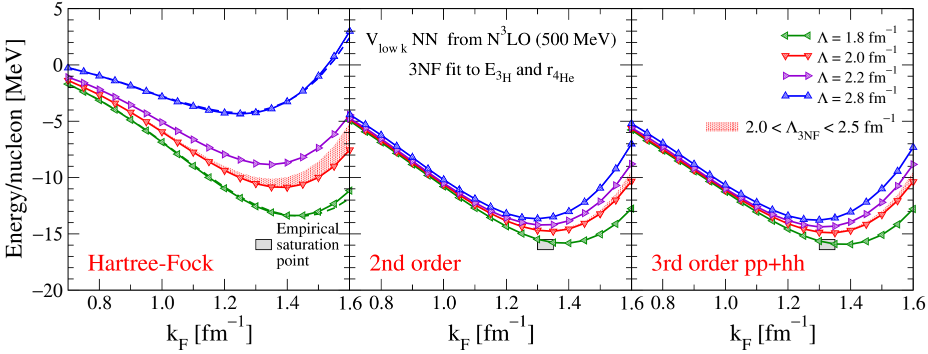
Many-Body Forces and Nuclear Saturation*†

S. D. DRELL AND KERSON HUANG‡
Department of Physics and Laboratory of Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received June 10, 1953)

- Disclaimer: Pion forces, but not chiral symmetry! . . .

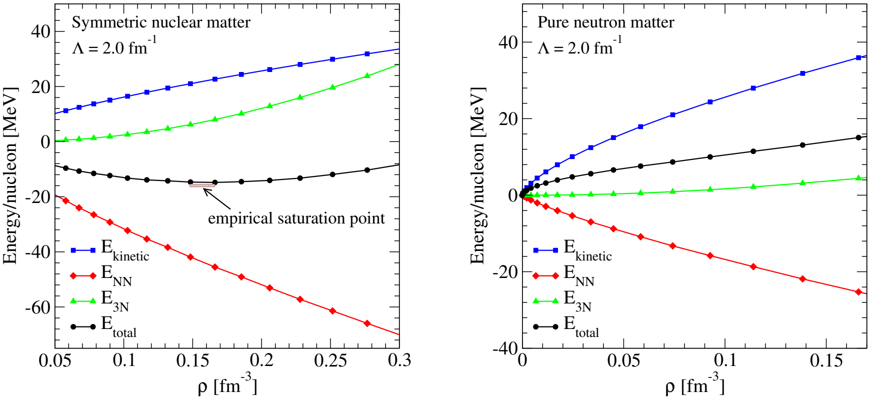
Low resolution calculations of nuclear matter

- Evolve NN by RG to low momentum, fit NNN to $A = 3, 4$
- Predict nuclear matter in MBPT [Hebeler et al. (2011)]



- Cutoff dependence at 2nd order significantly reduced
- 3rd order contributions are small
- Remaining cutoff dependence: many-body corrections, 4NF?

Hierarchy of many-body contributions to SNM and PNM



- E_{NN} denotes the energy contributions from NN interactions
- E_{3N} all contributions which include at least one 3N interaction
- Discussion questions in the exercises!

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Overview of nuclei leading to nuclear matter

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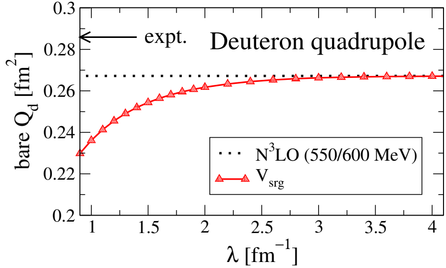
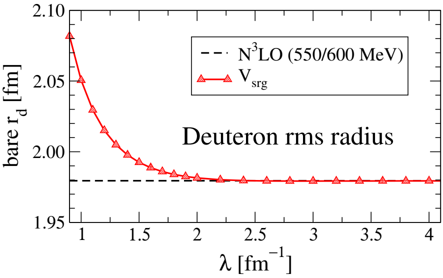
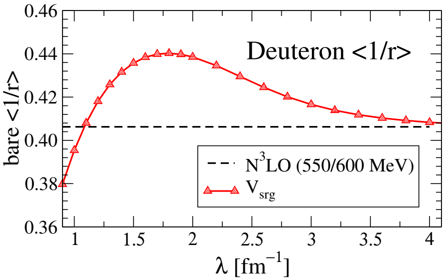
Unevolved long-distance operators change slowly with λ

- Matrix elements dominated by long range run slowly for $\lambda \geqslant 2\,\mathrm{fm}^{-1}$
- Here: examples from the deuteron (compressed scales)
- Which is the correct answer?
- Are we using the complete operator for the experimental quadrupole moment?

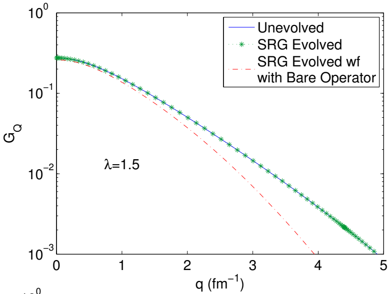


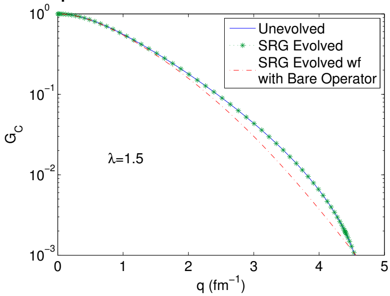


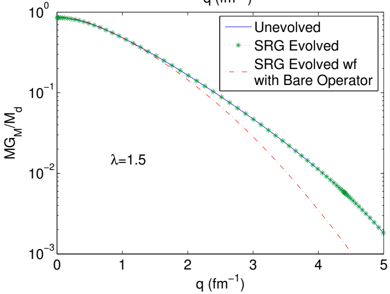


Deuteron electromagnetic form factors

- G_C, G_Q, G_M in deuteron with chiral EFT at leading order (Valderrama et al.)
- NNLO 550/600 MeV potential
- Unchanged at low q with unevolved operators
- Independent of λ with evolved operators







Operator flow in practice [e.g., see arXiv:1008.1569]

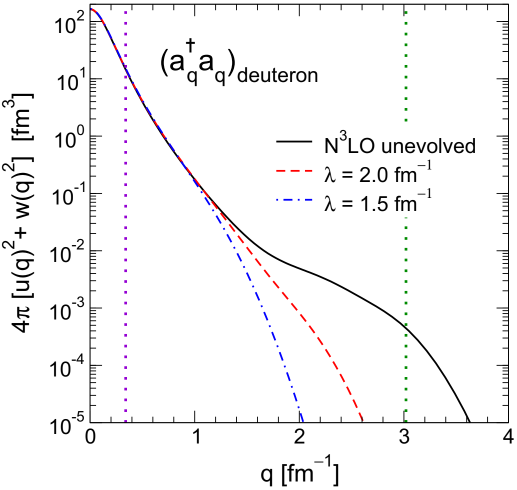
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

so O_s evolves via

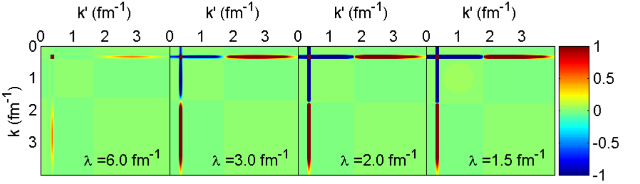
$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$ or solve dU_s/ds flow
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\text{fm}^{-1}$ in deuteron

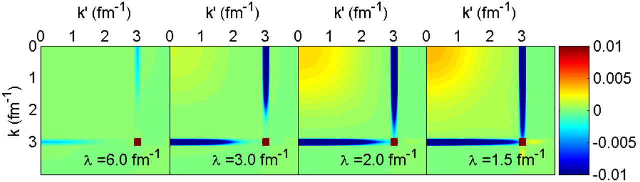


High and low momentum operators in deuteron

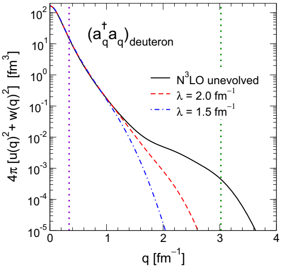
- **Integrand** of $(Ua_q^\dagger a_q U^\dagger)$ for $q = 0.34\,\text{fm}^{-1}$



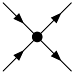
- **Integrand** for $q = 3.02\,\text{fm}^{-1}$



- Momentum distribution



- One-body operator does not evolve (for “standard” SRG)
- Induced two-body operator ≈ regularized delta function:



High and low momentum operators in deuteron

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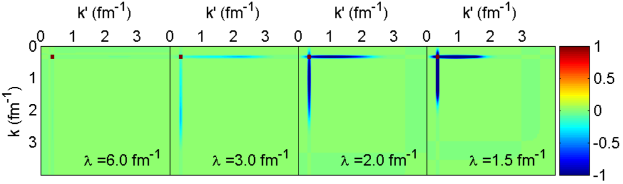

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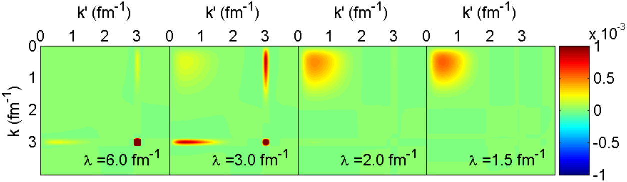

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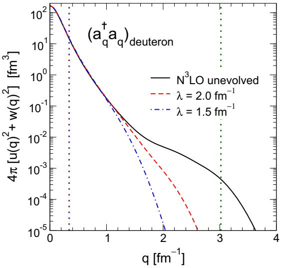
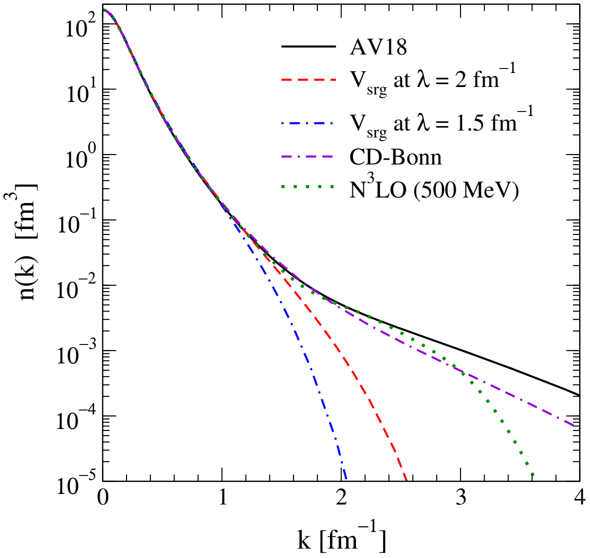


- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$ $\left\langle \frac{1}{r}\right\rangle$, $\langle G_C\rangle$, ...

Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



- E.g., extract from electron scattering?
- Scale- and scheme-dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- No region where $1/a_s \ll k \ll 1/R$ (cf. large k limit for unitary gas)

‘Non-observables’ vs. Scheme-dependent observables

- Some quantities are *in principle* not observable
  - T.D. Lee: “The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”
  - E.g., you can’t measure absolute position or time or a gauge

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- Directly measurable quantities are “clean” observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!

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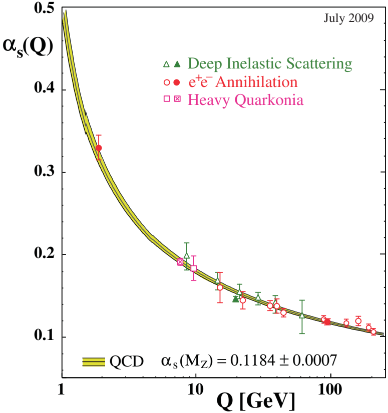
- Directly measurable quantities are "clean" observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!
- Scale- and scheme-dependent derived quantities
  - Critical questions to address for each quantity:
    - What is the ambiguity or convention dependence?
    - Can one convert between different prescriptions?
    - Is there a consistent extraction from experiment such that they can be compared with other processes and theory?
  - Physical quantities can be *in-practice* clean observables if scheme dependence is negligible (e.g., $(e, 2e)$ from atoms)
  - How do we deal with dependence on the Hamiltonian?

Partial list of ‘non-observables’ references

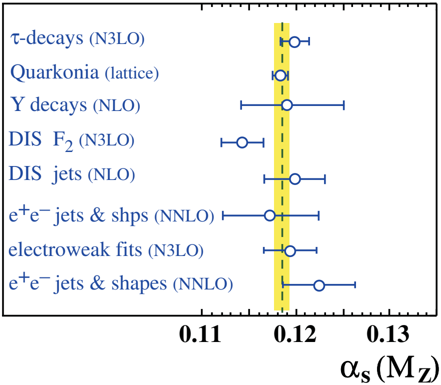
- *Equivalent Hamiltonians in scattering theory*, H. Ekstein, (1960)
- *Measurability of the deuteron D state probability*, J.L. Friar, (1979)
- *Problems in determining nuclear bound state wave functions*, R.D. Amado, (1979)
- *Nucleon nucleon bremsstrahlung: An example of the impossibility of measuring off-shell amplitudes*, H.W. Fearing, (1998)
- *Are occupation numbers observable?*, rjf and H.-W. Hammer, (2002)
- *Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors*, A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)
- *Non-observability of spectroscopic factors*, B.K. Jennings, (2011)
- *How should one formulate, extract, and interpret ‘non-observables’ for nuclei?*, rjf and A. Schwenk, (2010) [in J. Phys. G focus issue on Open Problems in Nuclear Structure Theory, edited by J. Dobaczewski]

Measuring the QCD Hamiltonian: Running $\alpha_s(Q^2)$



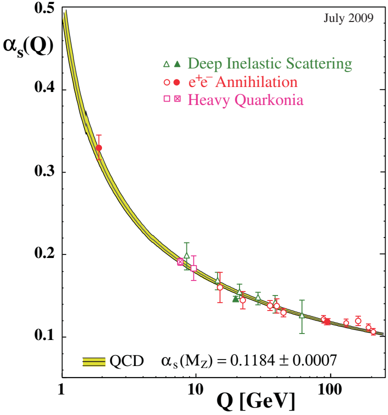
- The QCD coupling is *scale* dependent ("running"): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\mathrm{QCD}})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\mathrm{MS}}$)

- Extractions from experiment can be compared (here at M_Z):



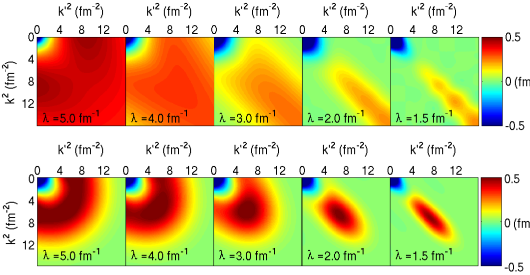
- cf. QED, where $\alpha_{em}(Q^2)$ is effectively constant for soft Q^2:
  $\alpha_{em}(Q^2 = 0) \approx 1/137$
  ∴ fixed H for quantum chemistry

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



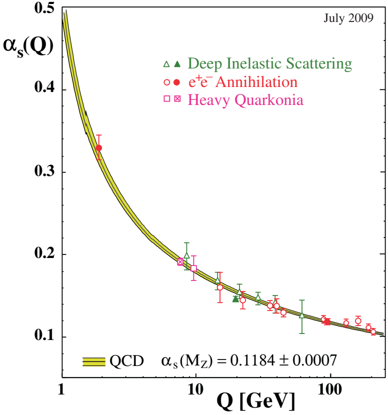
- The QCD coupling is *scale* dependent (cf. low-E QED): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\rm MS}$)

- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\text{low }k}$) running of initial potential with λ (decoupling or separation scale)



- Scheme dependence: AV18 vs. N^{3}LO (plus associated 3NFs)
- But all are (NN) phase equivalent!
- Shift contributions between interaction and sums over intermediate states

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



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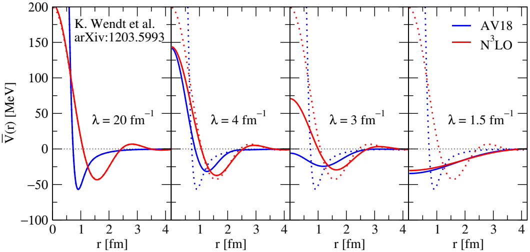
- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{{\rm low}\,k}$) running of initial potential with λ (decoupling or separation scale)
- Project non-local NN potential to visualize: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$



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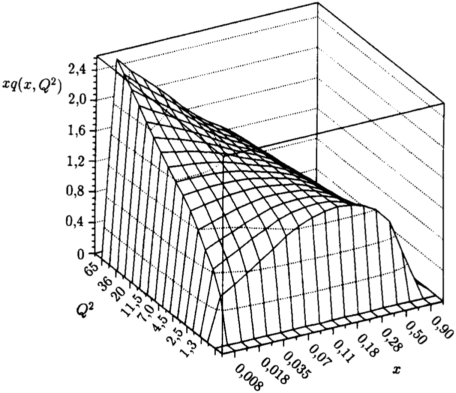
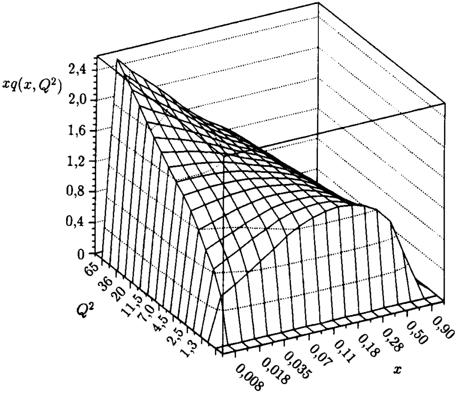
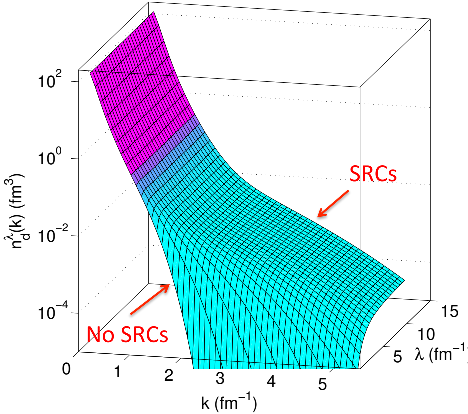
Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x\, q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

Parton vs. nuclear momentum distributions





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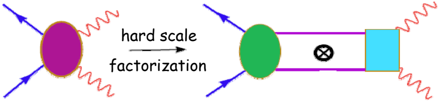
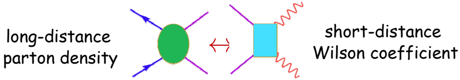
- Deuteron momentum distribution is scale *and* scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Factorization: high-E QCD vs. low-E nuclear

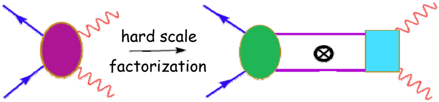


$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Q^2 running of $f_a(x, Q^2)$ comes from choosing μ_f to optimize extraction from experiment

Factorization: high-E QCD vs. low-E nuclear



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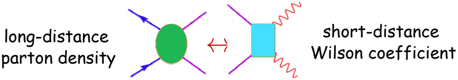

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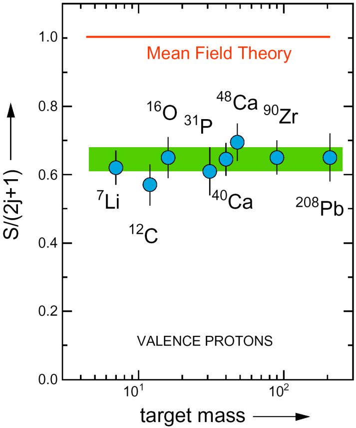
- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section — Structure model: spectroscopic factor — Reaction model: single-particle cross section

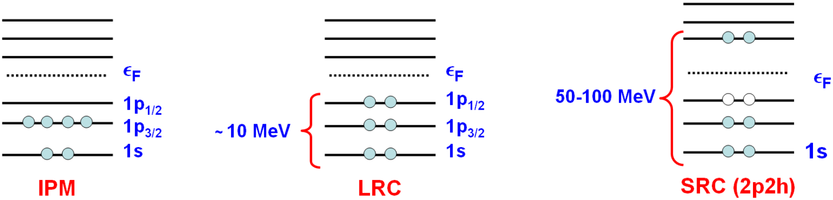
$$\sigma^{if} = \sum_{|J_f - J_i| \le j \le J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What does it mean to be *consistent* between structure and reaction models? Treat separately? (No!)
- How does scale/scheme dependence come in?
- What are the trade-offs? (Does simpler structure always mean much more complicated reaction?)

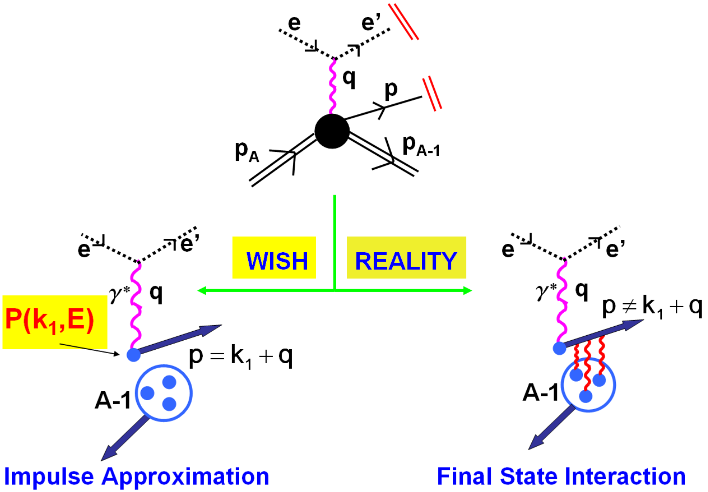
Scale/scheme dependence: spectroscopic factors



- Spectroscopic factors for valence protons have been extracted from $(e, e'p)$ experimental cross sections (e.g., NIKHEF 1990's at left)
- Used as canonical evidence for "correlations", particularly short-range correlations (SRC's)
- But if SFs are scale/scheme dependent, how do we explain the cross section?

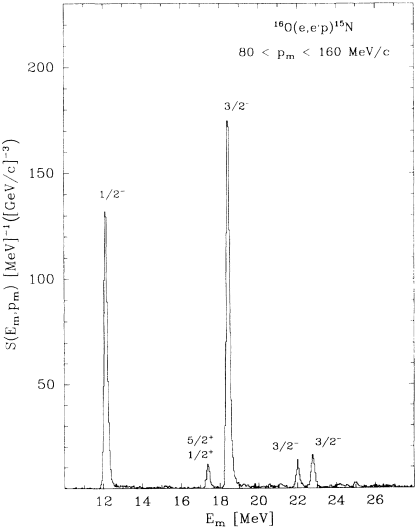


Standard story for $(e, e'p)$ [from C. Ciofi degli Atti]

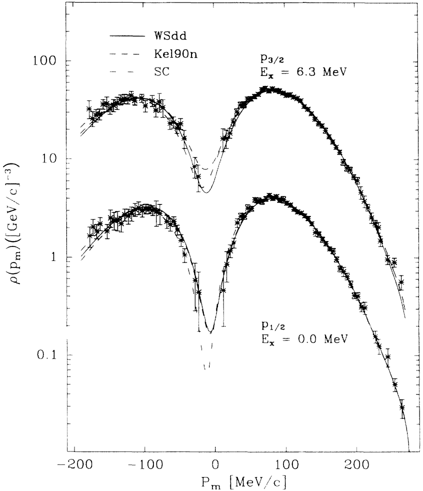


- In IA: "missing" momentum $p_m = k_1$ and energy $E_m = E$
- Choose E_m to select a discrete final state for range of p_m
- FSI treated as managable *add-on* theoretical correction to IA

(Assumed) factorization of $(e, e'p)$ cross section



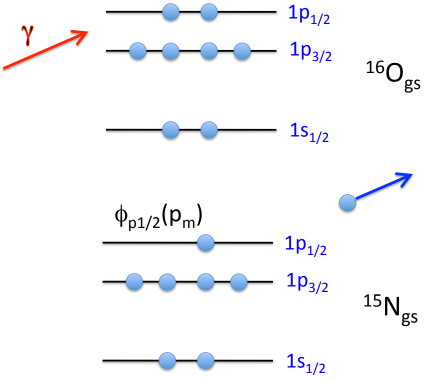
Missing energy spectrum for $^{16}O(e, e'p)^{15}N$ [Leuschner (1994)]

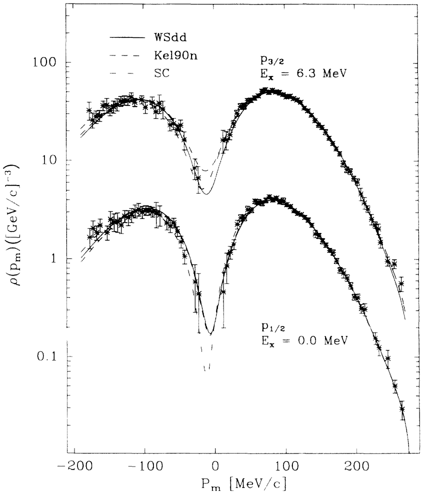


$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \;\propto |\phi_\alpha(\mathbf{p}_m)|^2$$

$\Longrightarrow p_{1/2}$ spectroscopic factor ≈ 0.63

(Assumed) factorization of $(e, e'p)$ cross section





- Knock out $p_{1/2}$ proton from ^{16}O to ^{15}N ground state in IPM
- Adjust s.p. well depth and radius to identify $\phi_\alpha(\mathbf{p}_m)$
- Final state interactions (FSI) added using optical potential(s)

$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \;\; \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

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Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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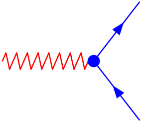
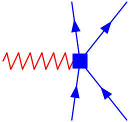
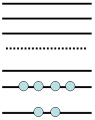
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states.
  So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- [Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only. E.g., see rjf, Hammer, PLB **531**, 203 (2002).]
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

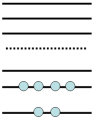
All pieces mix with unitary transformation

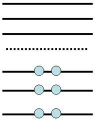
- A one-body current becomes many-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger =$  $+\ \alpha$  $+ \cdots$

- New wf correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+ \cdots \Longrightarrow Z$ 

$+\ \alpha'$ 

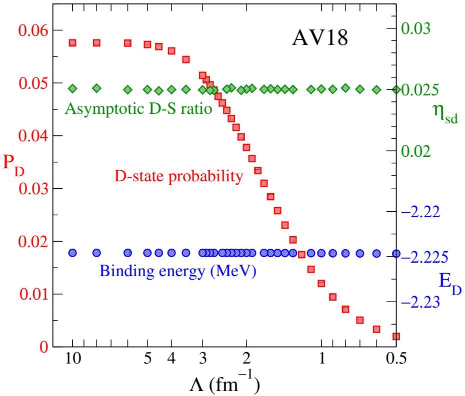
$+ \cdots$

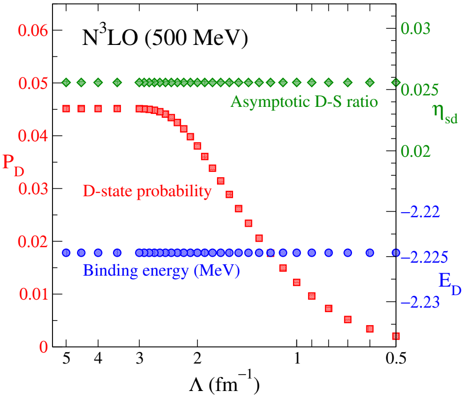
  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - Thus *spectroscopic factors* are scale dependent

- Final state interactions (FSI) are also modified by $\widehat{U}$

- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, ...

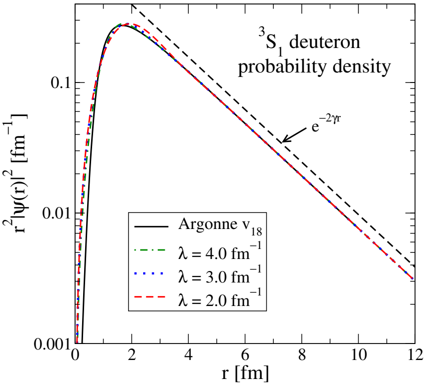
Deuteron scale-(in)dependent observables

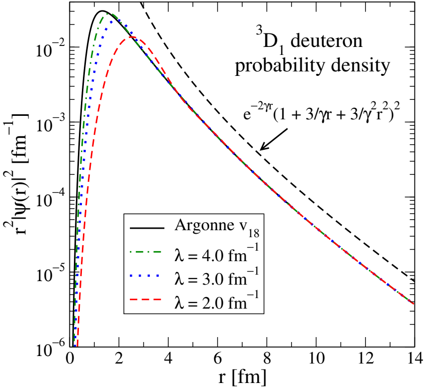




- $V_{\text{low }k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)
- Plan: Make analogous calculations for $A > 2$ quantities (like SFs)

Why are ANC's different? Coordinate space



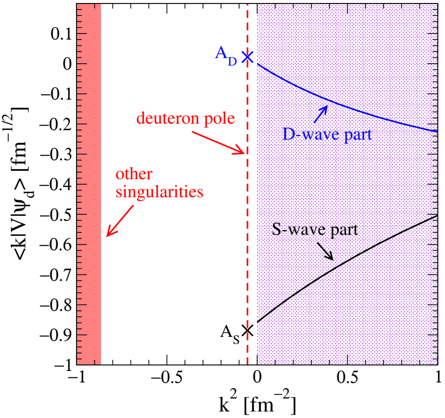


- ANC's, like phase shifts, are asymptotic properties
  $\Longrightarrow$ short-range unitary transformations do not alter them
  [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle + \langle \mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle$$
$$\Longrightarrow \langle \mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle \mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$
2. $$\langle \mathbf{r}|\psi_n\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}\langle \mathbf{k}|\psi_n\rangle \xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$
3. integral dominated by pole from 1.
4. extrapolate $\langle \mathbf{k}|V|\psi_n\rangle$ to $k^2 = -\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Next vertex singularity at $-(\gamma + m_\pi)^2 \Longrightarrow$ same for FSI

Final comments and questions

- Summary (and follow-up) points
  - While scale and scheme-dependent observables can be (to good approximation) unambiguous for *some* systems, they are often (generally?) not for nuclei!
  - Scale/scheme includes *consistent* Hamiltonian and operators. How dangerous is it to treat experimental analysis in pieces?
  - Unitary transformations reveal *natural* scheme dependence
  - Parton distribution functions as a paradigm
    $\Longrightarrow$ Can we have controlled factorization at low energies?
- Questions for which RG/EFT perspective + tools can help
  - How should one choose a scale/scheme?
  - Can we (should we) use a reference Hamiltonian?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What is the role of short-range/long-range correlations?
  - How do we match Hamiltonians and operators?
  - When is the assumption of one-body operators viable?
  - . . . and many more!

How should one choose a scale/scheme?

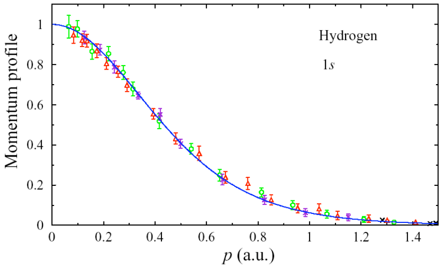
- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - Low-*k* potential: improve CI or MBPT convergence, or to make microscopic connection to shell model or . . .
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology?
- Cleanest extraction from experiment
  - Can one “optimize” validity of impulse approximation?
  - Ideally extract at one scale, evolve to others using RG
- Plan: use range of scales to test calculations and physics
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguities (e.g., in spectroscopic factors)
  - Match Hamiltonians and operators (EFT) and then use RG

What parts of wf's can be extracted from experiment?

- Measurable: asymptotic (IR) properties like phase shifts, ANC's
- Not observables, but well-defined theoretically given a Hamiltonian: interior quantities like spectroscopic factors
  - These depend on the scale and the scheme
  - Extraction from experiment requires robust factorization of structure and reaction; only the combination is scale/scheme independent (e.g., cross sections) [What if weakly dependent?]
- What about the high-momentum tails of momentum distributions?
  - Consider cold atoms in the unitary regime
  - Compare to nuclear case
- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)
- So might expect Hamiltonian- and resolution-dependent but *A*-independent high-momentum tails of wave functions
  - Universal extrapolation for different *A*, but λ_{SRG} dependent

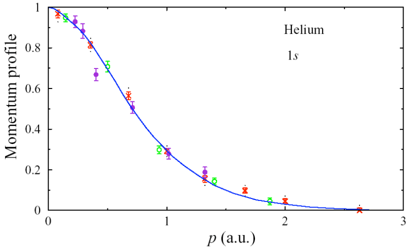
When are wave functions measurable? [W. Dickhoff]

Atoms studied with the (e,2e) reaction



$$\varphi_{1s}(p) = 2^{3/2}\pi \frac{1}{(1+p^2)^2}$$

Hydrogen 1s wave function
"seen" experimentally
Phys. Lett. 86A, 139 (1981)



And so on for other atoms ...

Helium
in Phys. Rev. A8, 2494 (1973)

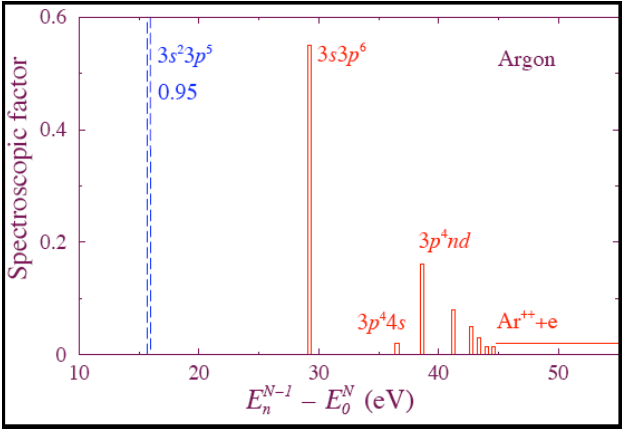
- But compare approximations for $(e, 2e)$ on atoms to those for $(e, e'p)$ on nuclei! (Impulse approx., FSI, vertex, . . .)

Spectroscopic factors in atoms

For a bound final N-1 state the spectroscopic factor is given by $$S = \int d\vec{p} \left| \left\langle \Psi_n^{N-1} \left| a_{\vec{p}} \right| \Psi_0^N \right\rangle \right|^2$$

For H and He the 1s electron spectroscopic factor is 1

For Ne the valence 2p electron has S=0.92 with two additional fragments, each carrying 0.04, at higher energy.



Argon
3p and 3s
strength

Closed-shell atoms
n(α) = 0 or 1

One-body scattering, small scheme dependence ⟹ robust SF

When can you measure a potential?

- Think about quantum mechanical convolution for energy

$$E = \int d\mathbf{x}\, \Psi^*(\mathbf{x})(T+V)\Psi(\mathbf{x})$$

  - (Schematic: e.g., here $\mathbf{x} = \{\mathbf{x}_1, \mathbf{x}_2\}$)
  - When can we isolate $H = T + V$ from $|\Psi(\mathbf{x})|^2$?
- Need very heavy particles or long-distances so that wave functions can be approximated as delta functions
- Examples
  - classical limit (e.g., gravitational potential)
  - heavy quark potential on a lattice
  - Coulomb potential in atoms/molecules
- In nuclear case, can change both $\Psi(\mathbf{x})$ and $V(\mathbf{x})$ at short distance and leave E unchanged $\Longrightarrow$ not measurable
- In field theory formulation, freedom to shift between interaction vertex and propagator for exchanged particle

Impulse approximation

- The discussion always starts with: "If we assume . . . "
  - Usually that the impulse approximation is good (one-body current and one active nucleon), and increasingly good with larger momentum transfer
  - Final state interactions neglected (and then assumed to be accounted for in a model-independent way)
- This brings to mind some quotes:
  - "If my grandmother had wheels, she'd be a bicycle."
  - "Hope is not a plan!" (or a reliable guide to experiment)
- How well the impulse approximation works depends on the system and probe (process dependent)
  - Works well: electron scattering from atoms, neutron scattering from liquid helium (??? maybe not in detail)
  - Large corrections: nuclear reactions!
- Should we choose a scheme in which the impulse approximation is best satisfied?