Nuclear forces and their impact on structure, reactions and astrophysics

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Ohio State University

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Lectures for Week 3

- **M.** Many-body problem and basis considerations (as);
  Many-body perturbation theory (rjf)
- **T.** Neutron matter and astrophysics (as); MBPT + Operators (rjf)
- **W.** Operators + Nuclear matter (rjf); Student presentations
- **Th.** Impact on (exotic) nuclei (as); Student presentations
- **F.** Impact on fundamental symmetries (as); From forces to density functionals (rjf)

Outline

Teaser: Skyrme vs. pionless, perturbative functional

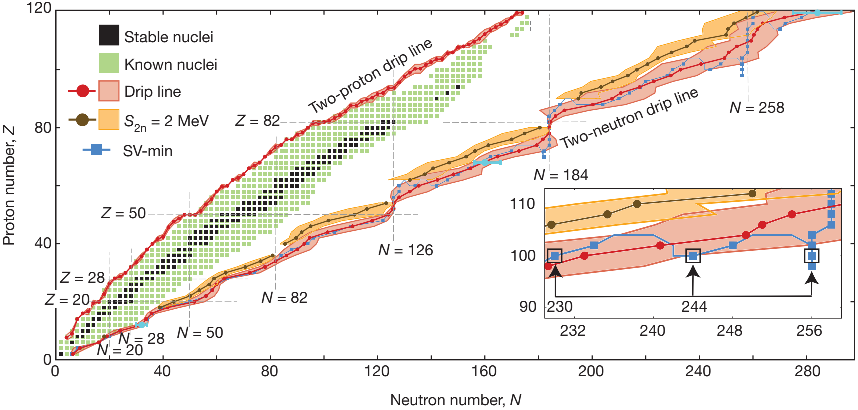
Bethe-Brueckner-Goldstone Power Counting

Preview for MBPT applied in finite nuclei

Operators and resolution

"The limits of the nuclear landscape"

J. Erler et al., Nature **486**, 509 (2012)



- Proton and neutron driplines predicted by Skyrme EDFs
  - Total: 6900 ± 500 nuclei with $Z \leq 120$ (≈ 3000 known)
  - Estimate systematic errors by comparing models

Teaser: Comparing Skyrme and natural, pionless Functionals

- Textbook Skyrme EDF (for $N = Z$) $[\rho = \langle\psi^\dagger\psi\rangle, \tau = \langle\nabla\psi^\dagger\cdot\nabla\psi\rangle]$

$$E[\rho,\tau,\mathbf{J}] = \int d^3x \left\{ \frac{\tau}{2M} + \frac{3}{8}t_0\rho^2 + \frac{1}{16}(3t_1 + 5t_2)\rho\tau + \frac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2 - \frac{3}{4}W_0\rho\nabla\cdot\mathbf{J} + \frac{1}{16}t_3\rho^{2+\alpha} + \cdots \right\}$$

- Natural, pionless $\rho\tau\mathbf{J}$ energy density functional for $\nu = 4$

$$E[\rho,\tau,\mathbf{J}] = \int d^3x \left\{ \frac{\tau}{2M} + \frac{3}{8}C_0\rho^2 + \frac{1}{16}(3C_2 + 5C_2')\rho\tau + \frac{1}{64}(9C_2 - 5C_2')(\nabla\rho)^2 - \frac{3}{4}C_2''\rho\nabla\cdot\mathbf{J} + \frac{c_1}{2M}C_0^2\rho^{7/3} + \frac{c_2}{2M}C_0^3\rho^{8/3} + \frac{1}{16}D_0\rho^3 + \cdots \right\}$$

- Same functional as dilute Fermi gas with $t_i \leftrightarrow C_i$?
  - Is Skyrme missing non-analytic, NNN, long-range (pion), (and so on) terms? (But NDA works: C_i's are natural!)
  - Isn't this a "perturbative" expansion?

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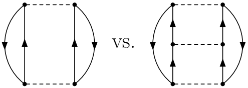
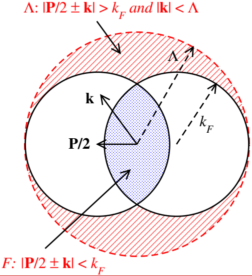
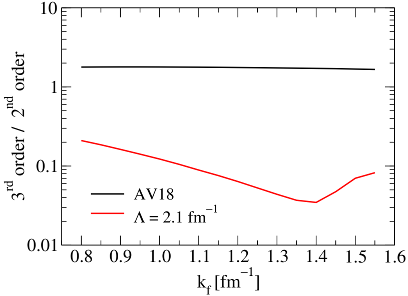
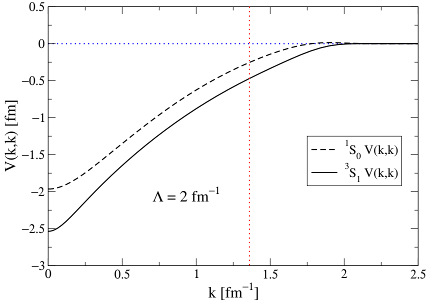
Strong short-range repulsion
$\Longrightarrow$ Sum V ladders $\Longrightarrow$ G



$V_{\text{low }k}$ momentum dependence + phase space
$\Longrightarrow$ perturbative







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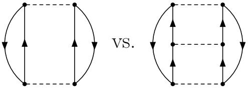

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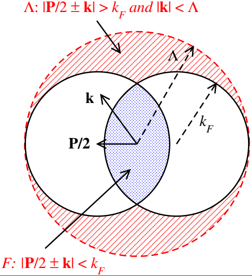
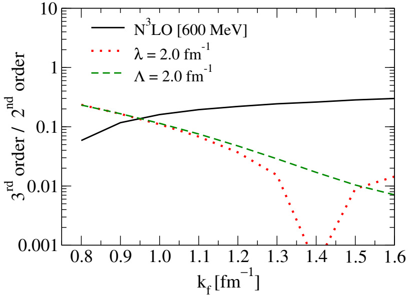
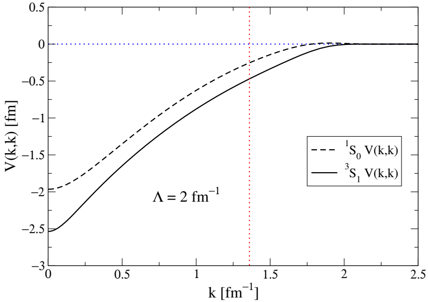
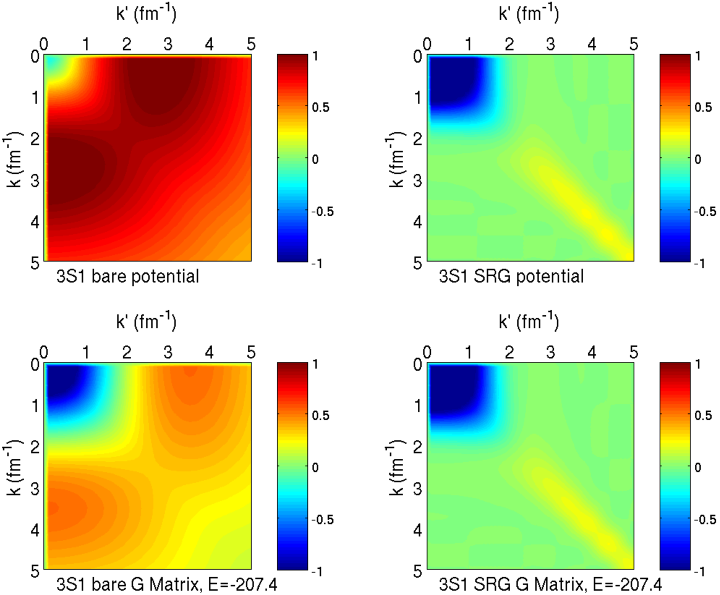






Compare Potential and G Matrix: AV18



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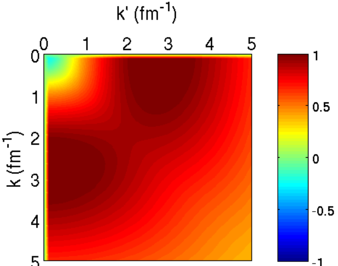
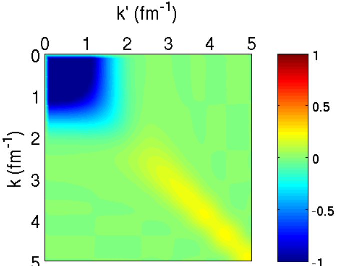
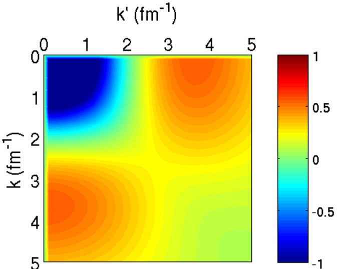


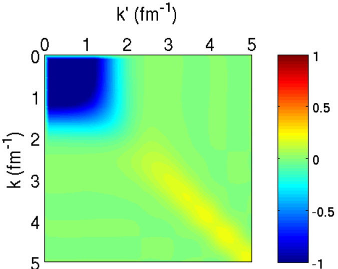
3S1 bare potential



3S1 SRG potential

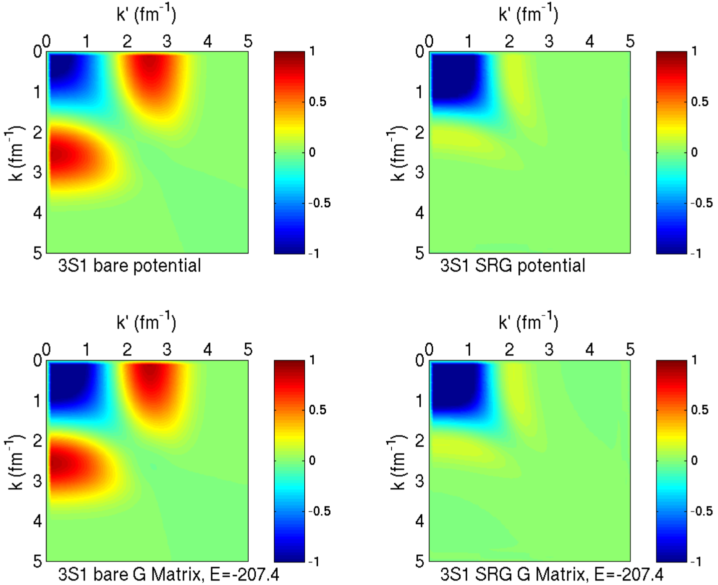


3S1 bare G Matrix, E=23.0



3S1 SRG G Matrix, E=23.0

Compare Potential and G Matrix: N^3LO (500 MeV)



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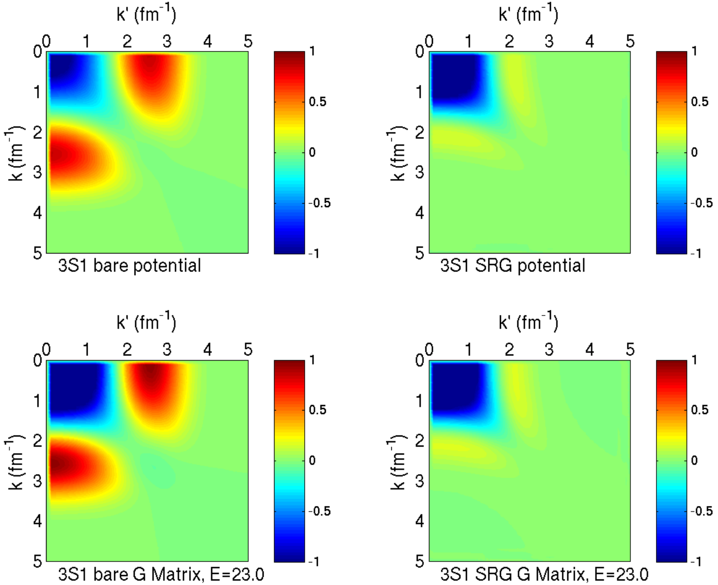
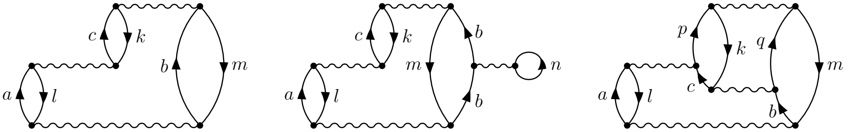


Hole-Line Expansion Revisited (Bethe, Day, . . .)

- Consider ratio of fourth-order diagrams to third-order:

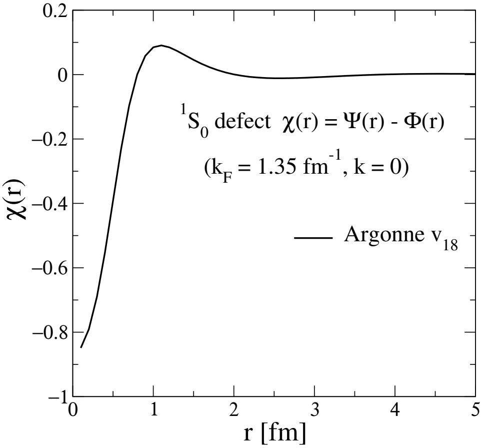


- "Conventional" G matrix still couples low-k and high-k
  - no new hole line $\Longrightarrow$ ratio $\approx -\chi(\mathbf{r}=0) \approx -1 \Longrightarrow$ sum all orders
  - add a hole line $\Longrightarrow$ ratio $\approx \sum_{n\leq k_{\rm F}} \langle bn|(1/e)G|bn\rangle \approx \kappa \approx 0.15$
- Low-momentum potentials decouple low-k and high-k
  - add a hole line $\Longrightarrow$ still suppressed
  - no new hole line $\Longrightarrow$ also suppressed (limited phase space)
  - freedom to choose single-particle $U \Longrightarrow$ use for Kohn-Sham

$\Longrightarrow$ Density functional theory (DFT) should work!

Two-Body Correlations at Nuclear Matter Density

- Defect wf $\chi(r)$ for particular kinematics ($k = 0$, $P_{\rm cm} = 0$)
- AV18: "Wound integral" provides expansion parameter



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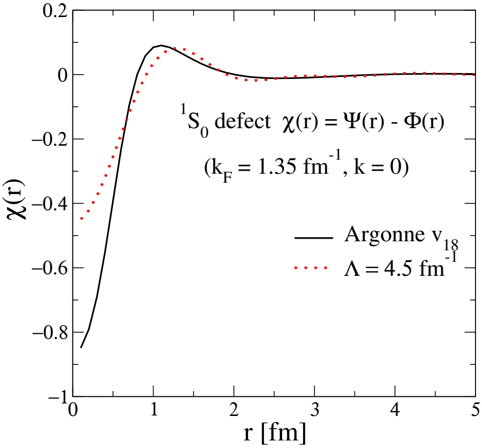

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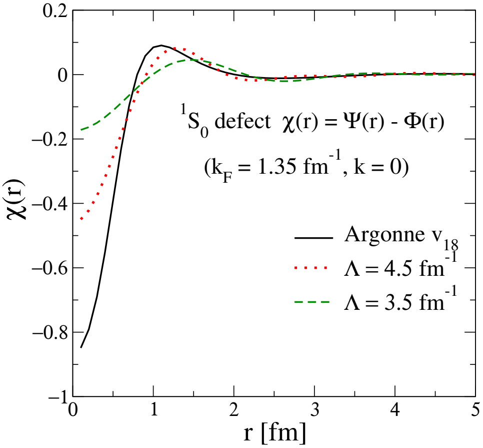

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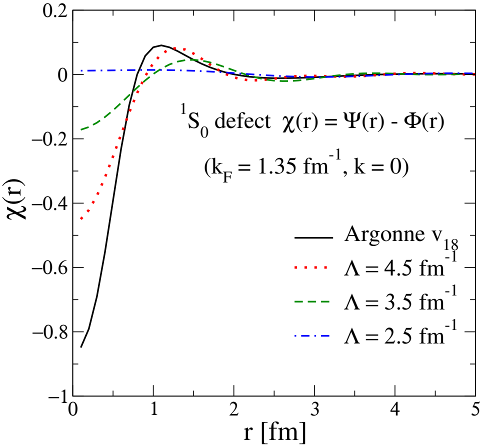
- Tensor ($^3S_1$) $\Longrightarrow$ larger defect



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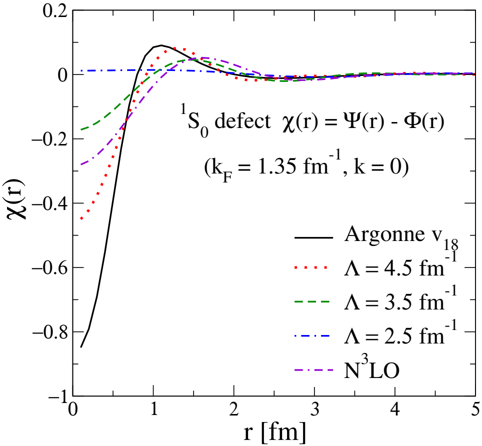

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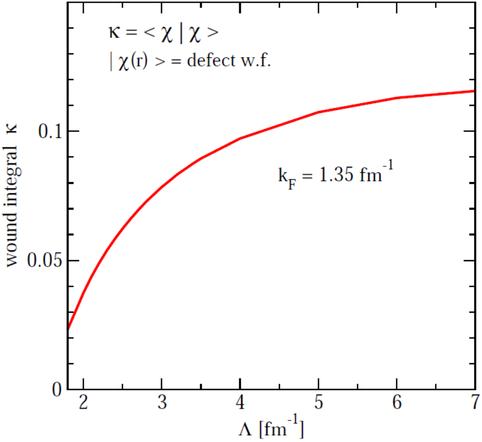

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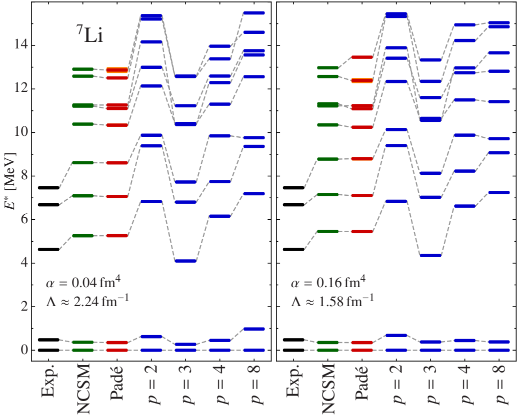
Operators and resolution

High-order Rayleigh-Schrödinger MBPT in finite nuclei

- R. Roth et al.
- Excitation energies in ^{7}Li
- Degenerate R-S MBPT
- SRG with two resolutions from N^{3}LO 2NF
- Fixed HO model space



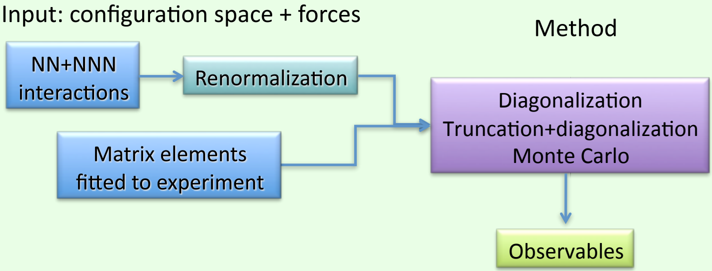
Order $p = 2$, 3, 4, and 8 compared to experiment, exact NCSM calculations, and the Padé resummed result

$\Longrightarrow$ note the good agreement of the last two!

The shell model revisited

Configuration interaction techniques

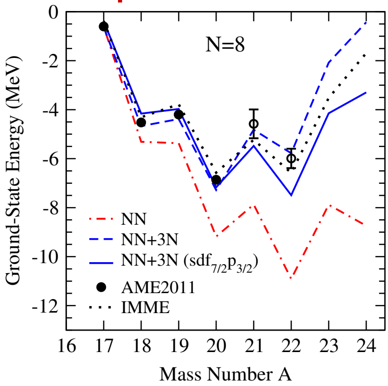
- light and heavy nuclei
- detailed spectroscopy
- quantum correlations (lab-system description)

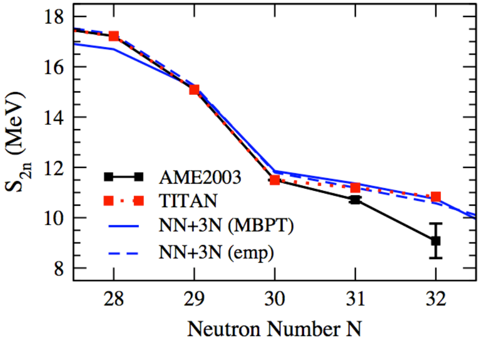


- Direct comparison with experiment
- Pseudo-data to inform reaction theory and DFT

Confronting theory and experiment to both driplines

- Precision mass measurements test impact of chiral 3NF
- Proton rich [Holt et al., arXiv:1207.1509]
- Neutron rich [Gallant et al., arXiv:1204.1987]
- Many new tests possible!





- Shell model description using chiral potential evolved to $V_{\mathrm{low}\,k}$ plus 3NF fit to $A = 3, 4$
- Excitations outside valence space included in 3rd order MBPT

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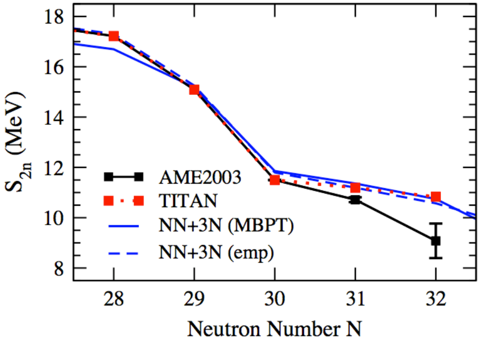

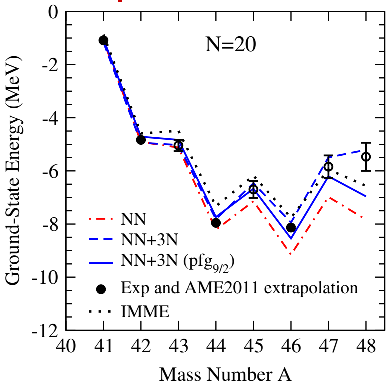

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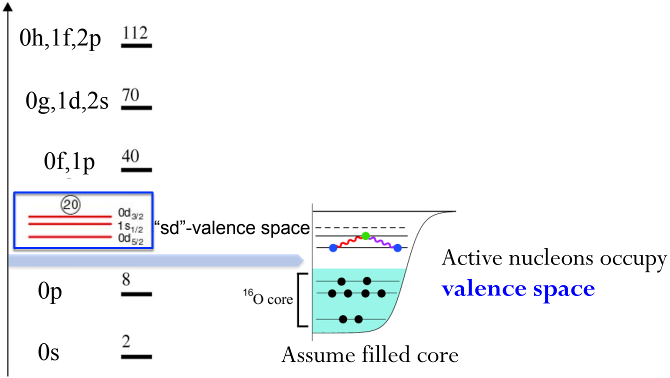
Non-empirical shell model [from J. Holt]

Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Interaction and energies of valence space orbitals from original $V_{\text{low }k}$

This alone does not reproduce experimental data



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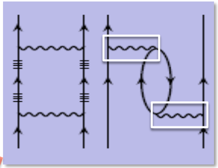
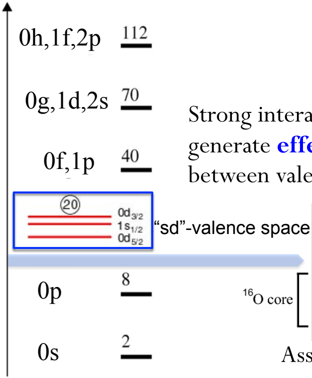
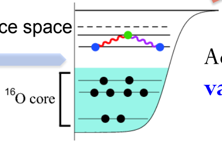
This alone does not reproduce experimental data – allow explicit breaking of core

Hjorth-Jensen, Kuo, Osnes (1995)





Strong interactions with core generate **effective interaction** between valence nucleons



Assume filled core

Active nucleons occupy **valence space**

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Interaction and energies of valence space orbitals from original $V_{\text{low }k}$

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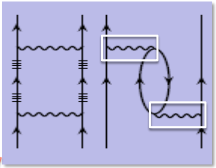
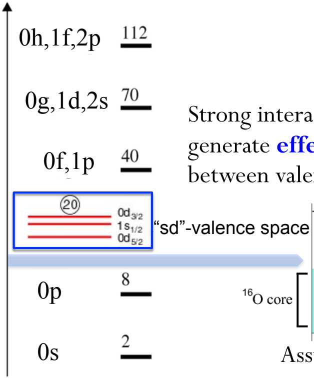
Effective two-body matrix elements

Single-particle energies (SPEs)

Hjorth-Jensen, Kuo, Osnes (1995)





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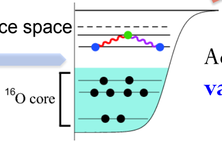

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Assume filled core

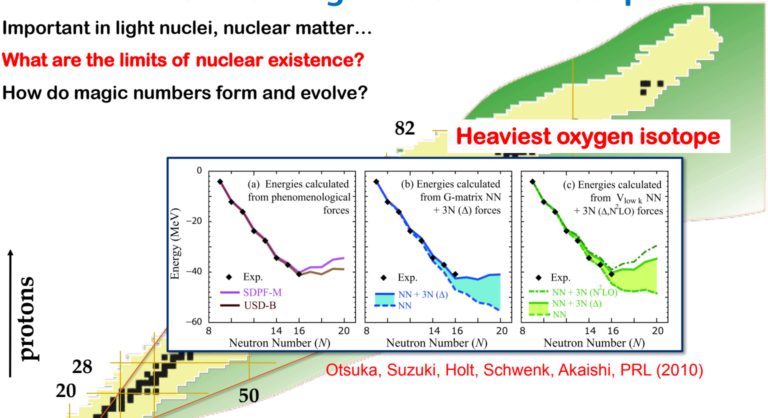
Chiral 3NFs meet the shell model [from J. Holt]

Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

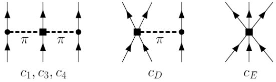
Important in light nuclei, nuclear matter…

What are the limits of nuclear existence?

How do magic numbers form and evolve?



Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)



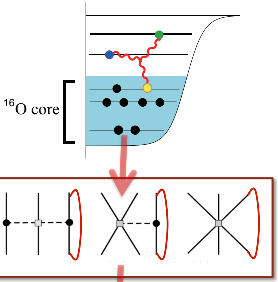
3N forces essential for medium mass nuclei

Chiral 3NFs meet the shell model [from J. Holt]

3N Forces for Valence-Shell Theories

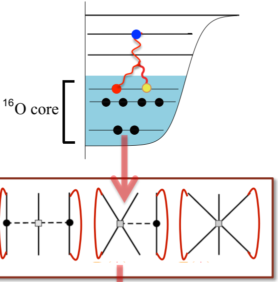
Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



$$\langle ab | V_{3N,\mathrm{eff}} | a'b' \rangle = \sum_{\alpha=core} \langle \alpha ab | V_{3N} | \alpha a'b' \rangle$$

Effective one-body



$$\langle a | V_{3N,\mathrm{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta=core} \langle \alpha\beta a | V_{3N} | \alpha\beta a' \rangle$$

Combine with microscopic NN: eliminate empirical adjustments

Chiral 3NFs meet the shell model [from J. Holt]

Drip Lines and Magic Numbers: 3N Forces in Medium-Mass Nuclei

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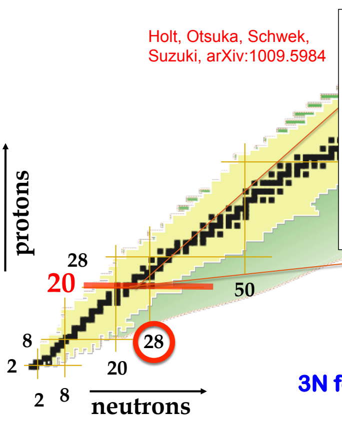
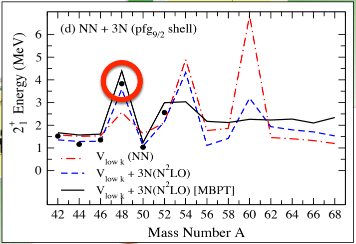
What are the limits of nuclear existence?

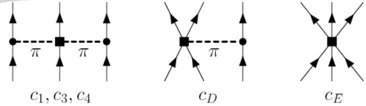
How do magic numbers form and evolve?

N=28 magic number in calcium

Holt, Otsuka, Schwek, Suzuki, arXiv:1009.5984







3N forces essential for medium mass nuclei

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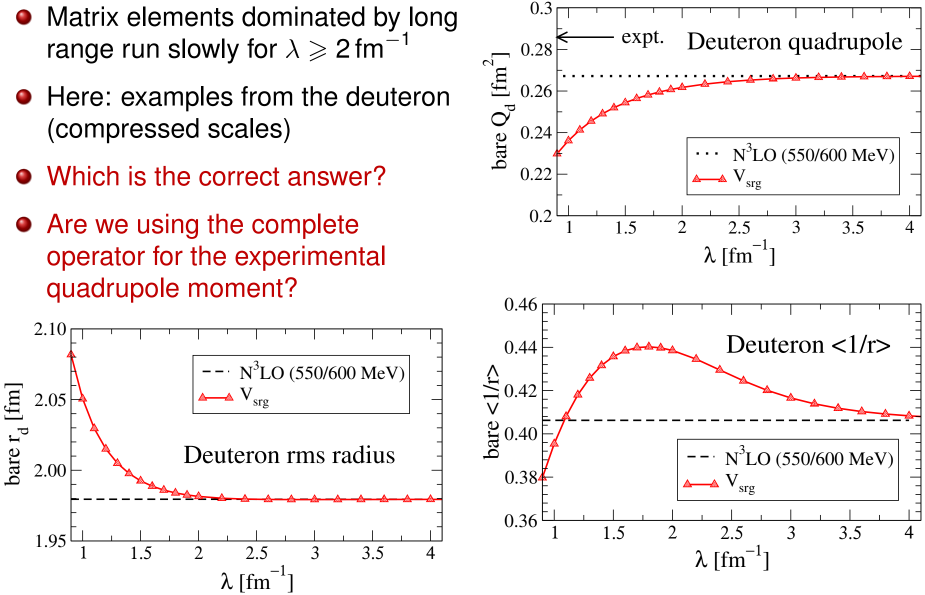
Operators and resolution

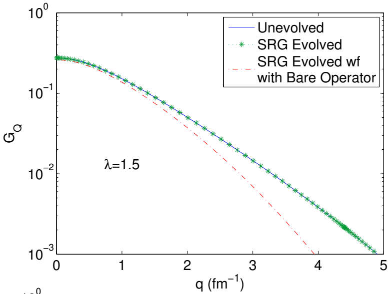
Unevolved long-distance operators change slowly with λ

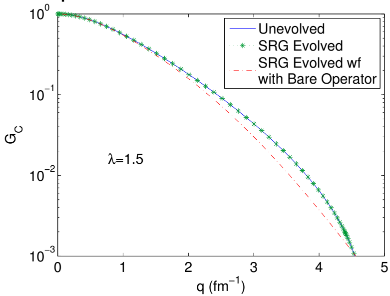
- Matrix elements dominated by long range run slowly for $\lambda \geqslant 2\,\mathrm{fm}^{-1}$
- Here: examples from the deuteron (compressed scales)
- Which is the correct answer?
- Are we using the complete operator for the experimental quadrupole moment?

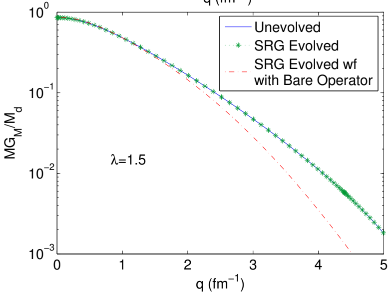


Deuteron electromagnetic form factors

- G_C, G_Q, G_M in deuteron with chiral EFT at leading order (Valderrama et al.)
- NNLO 550/600 MeV potential
- Unchanged at low q with unevolved operators
- Independent of λ with evolved operators







‘Non-observables’ vs. Scheme-dependent observables

- Some quantities are *in principle* not observable
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- Directly measurable quantities are “clean” observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!

‘Non-observables’ vs. Scheme-dependent observables

- Some quantities are *in principle* not observable
  - T.D. Lee: “The root of all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; these will be called ‘non-observables’.”
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- Directly measurable quantities are “clean” observables
  - E.g., cross sections and energies
  - Note: Association with a Hermitian operator is not enough!
- Scale- and scheme-dependent derived quantities
  - Critical questions to address for each quantity:
    - What is the ambiguity or convention dependence?
    - Can one convert between different prescriptions?
    - Is there a consistent extraction from experiment such that they can be compared with other processes and theory?
  - Physical quantities can be *in-practice* clean observables if scheme dependence is negligible (e.g., $(e, 2e)$ from atoms)
  - How do we deal with dependence on the Hamiltonian?

Partial list of 'non-observables' references

- *Equivalent Hamiltonians in scattering theory*, H. Ekstein, (1960)
- *Measurability of the deuteron D state probability*, J.L. Friar, (1979)
- *Problems in determining nuclear bound state wave functions*, R.D. Amado, (1979)
- *Nucleon nucleon bremsstrahlung: An example of the impossibility of measuring off-shell amplitudes*, H.W. Fearing, (1998)
- *Are occupation numbers observable?*, rjf and H.-W. Hammer, (2002)
- *Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors*, A.M. Mukhamedzhanov and A.S. Kadyrov, (2010)
- *Non-observability of spectroscopic factors*, B.K. Jennings, (2011)
- *How should one formulate, extract, and interpret 'non-observables' for nuclei?*, rjf and A. Schwenk, (2010) [in J. Phys. G focus issue on Open Problems in Nuclear Structure Theory, edited by J. Dobaczewski]

Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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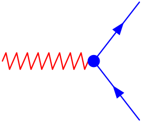
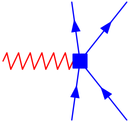
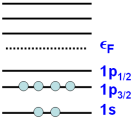
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states.
  So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- [Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only. E.g., see rjf, Hammer, PLB **531**, 203 (2002).]
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

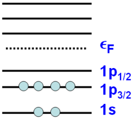
All pieces mix with unitary transformation

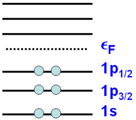
- A one-body current becomes many-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger \; = $  $+\, \alpha$  $+ \cdots$

- New wf correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+ \cdots \Longrightarrow Z$ 

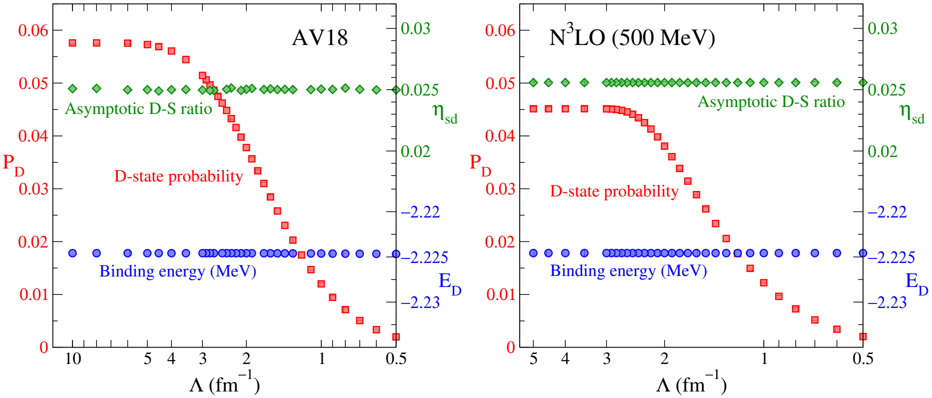
$+\, \alpha$ 

$+ \cdots$

  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger |\Psi_n^{A-1}\rangle$
  - Thus *spectroscopic factors* are scale dependent

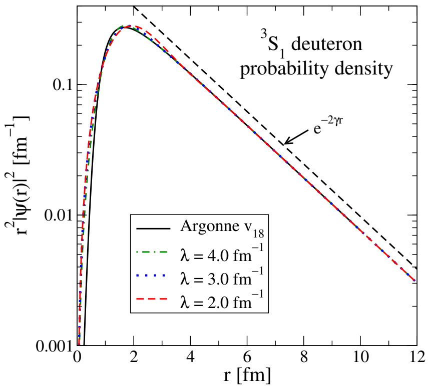
- Final state interactions (FSI) are also modified by $\widehat{U}$
- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, …

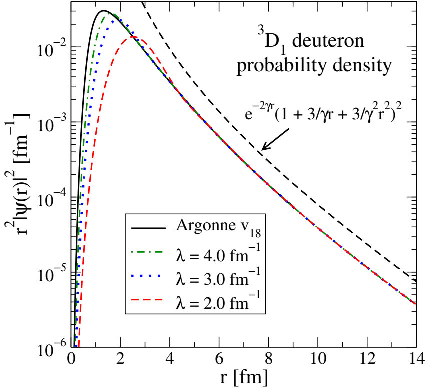
Deuteron scale-(in)dependent observables



- $V_{\text{low }k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)
- Plan: Make analogous calculations for $A > 2$ quantities (like SFs)

Why are ANC's different? Coordinate space



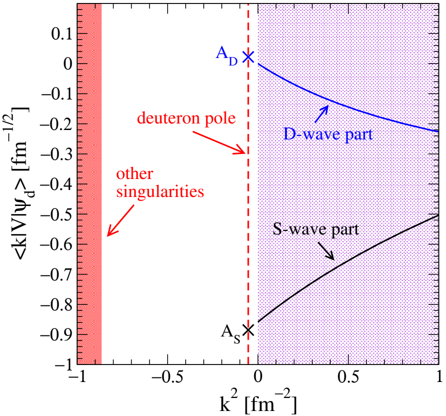


- ANC's, like phase shifts, are asymptotic properties
  $\Longrightarrow$ short-range unitary transformations do not alter them
  [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle\mathbf{k}|\psi_n\rangle+\langle\mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle\mathbf{k}|\psi_n\rangle$$
$$\Longrightarrow \langle\mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle\mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$
2. $$\langle\mathbf{r}|\psi_n\rangle = \int\frac{d^3k}{(2\pi)^3}e^{i\mathbf{k}\cdot\mathbf{r}}\langle\mathbf{k}|\psi_n\rangle \xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$
3. integral dominated by pole from 1.
4. extrapolate $\langle\mathbf{k}|V|\psi_n\rangle$ to $k^2=-\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Next vertex singularity at $-(\gamma+m_\pi)^2 \Longrightarrow$ same for FSI

How should one choose a scale/scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - Low-k potential: improve CI or MBPT convergence, or to make microscopic connection to shell model or . . .
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology?
- Cleanest extraction from experiment
  - Can one “optimize” validity of impulse approximation?
  - Ideally extract at one scale, evolve to others using RG
- Plan: use range of scales to test calculations and physics
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguities (e.g., in spectroscopic factors)
  - Match Hamiltonians and operators (EFT) and then use RG

Operator flow in practice [e.g., see arXiv:1008.1569]

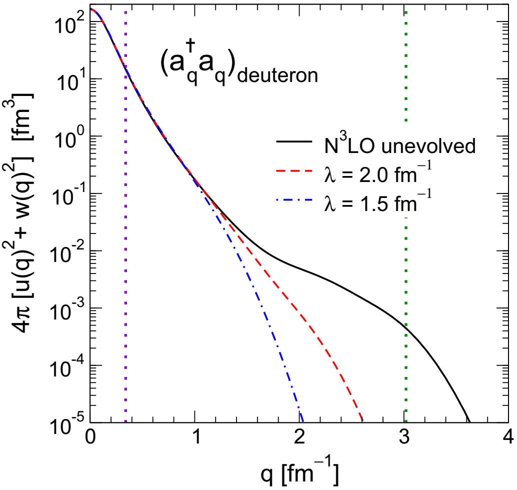
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

so O_s evolves via

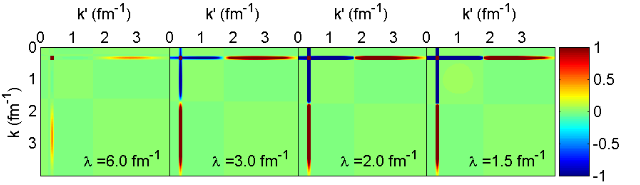
$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$ or solve dU_s/ds flow
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\mathrm{fm}^{-1}$ in deuteron

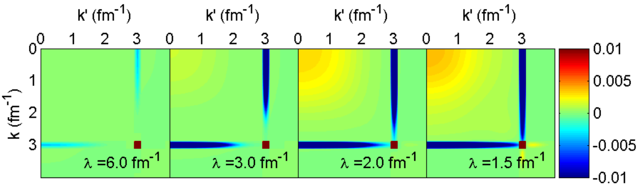


High and low momentum operators in deuteron

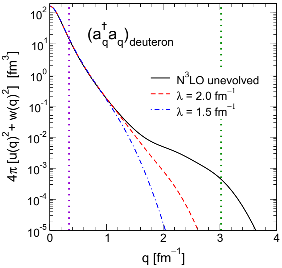
- **Integrand** of $\langle U a_q^\dagger a_q U^\dagger \rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



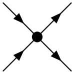
- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



- Momentum distribution

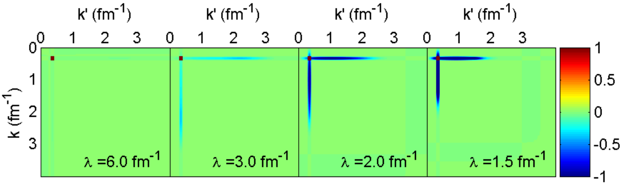


- One-body operator does not evolve (for “standard” SRG)
- Induced two-body operator ≈ regularized delta function:



High and low momentum operators in deuteron

- **Integrand** of $\langle\psi_d|\,(U a^\dagger_q a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\text{fm}^{-1}$



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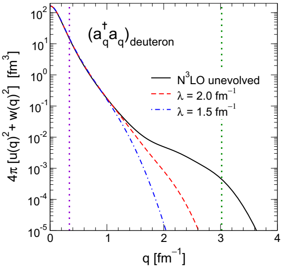

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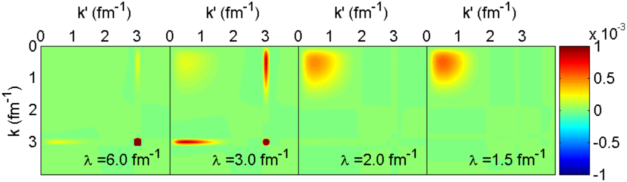
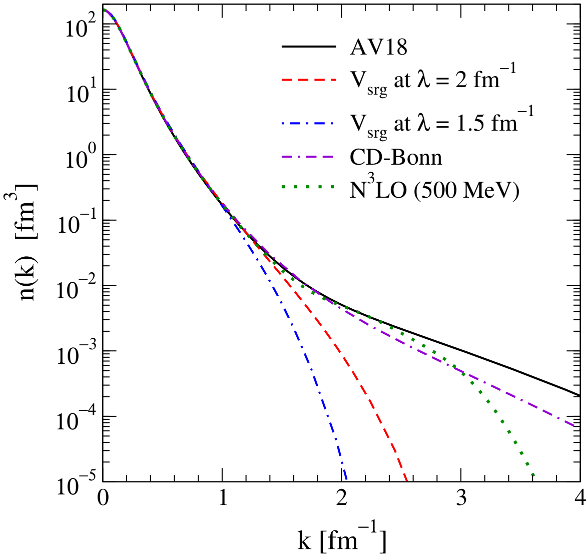


- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2\rangle$, $\langle Q_d\rangle$, $\langle 1/r\rangle$ $\langle \frac{1}{r}\rangle$, $\langle G_C\rangle$, ...

Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



- E.g., extract from electron scattering?
- Scale- and scheme-dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- No region where $1/a_s \ll k \ll 1/R$ (cf. large k limit for unitary gas)

What parts of wf's can be extracted from experiment?

- Measurable: asymptotic (IR) properties like phase shifts, ANC's
- Not observables, but well-defined theoretically given a Hamiltonian: interior quantities like spectroscopic factors
  - These depend on the scale and the scheme
  - Extraction from experiment requires robust factorization of structure and reaction; only the combination is scale/scheme independent (e.g., cross sections) [What if weakly dependent?]
- What about the high-momentum tails of momentum distributions?
  - Consider cold atoms in the unitary regime
  - Compare to nuclear case
- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)
- So might expect Hamiltonian- and resolution-dependent but *A*-independent high-momentum tails of wave functions
  - Universal extrapolation for different *A*, but λ_{SRG} dependent