Nuclear forces and their impact on structure, reactions and astrophysics

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Lectures for Week 3

- **M.** Many-body problem and basis considerations (as); Many-body perturbation theory (rjf)
- **T.** Neutron matter and astrophysics (as); Operators (rjf)
- **W.** Nuclear matter (rjf); Student presentations
- **Th.** Impact on (exotic) nuclei (as); Student presentations
- **F.** Impact on fundamental symmetries (as); From forces to density functionals (rjf)

Outline

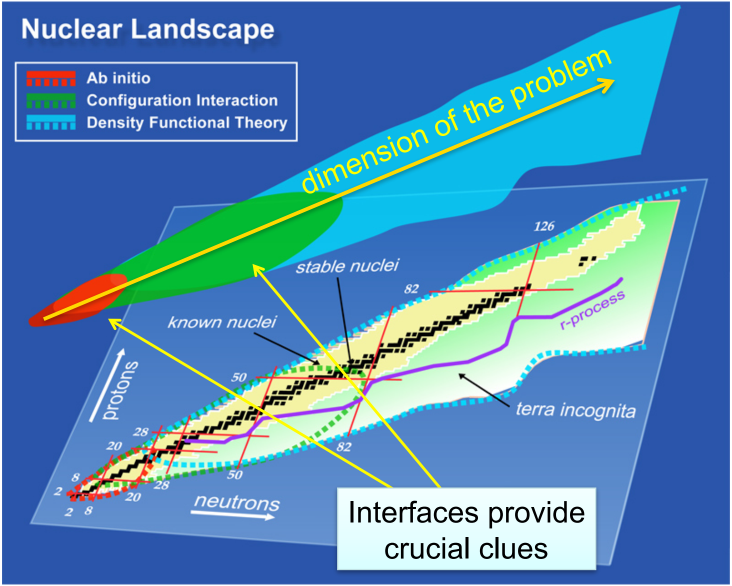
Many-body methods: Selected results

Dilute, natural, Fermi system

Bethe-Brueckner-Goldstone Power Counting

Teaser for MBPT applied in finite nuclei

Overlapping theory methods cover all nuclei

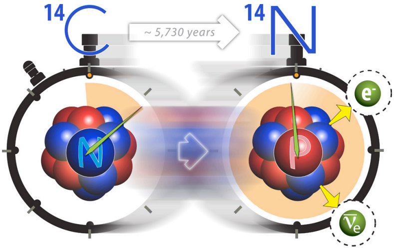


Interfaces provide crucial clues

U.S. DEPARTMENT OF ENERGY | Office of Science

"Why does Carbon-14 live so long?"

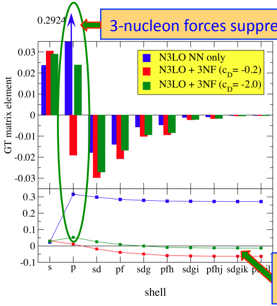
Carbon-14 dating relies on ~5,730 year half-life, but other light nuclei undergo similar beta decay with half-lives less than a day!



UNEDF SciDAC Collaboration
Universal Nuclear Energy Density Functional

- Members of UNEDF collaboration made microscopic nuclear structure calculations to solve the puzzle
- Used systematic chiral Hamiltonian from low-energy effective field theory of QCD
- Key feature: consistent *3-nucleon* interactions

3-nucleon forces suppress critical component compared to 2-nucleon forces only



net decay rate is very small

- Solutions of ^{14}C and ^{14}N through Hamiltonian diagonalization
- 100-fold reduction in Gamow-Teller transition matrix element

Calculations enabled by high-performance computing through INCITE program

- Dimension of matrix solved for 8 lowest states: ~ 1×10^9
- Solution took ~ 6 hours on 215,000 cores on Cray XT5 Jaguar at ORNL

INCITE Advancing America's Science and Industrial Competitiveness

Science ref.: Physical Review Letters **106**, 202502 (2011)
Computational ref.: Procedia Computer Science **1**, 97 (2010)

Coupled cluster method [from D. Dean, G. Hagen, T. Papenbrock]

Asymmetry dependence and spectroscopic factors

- Spectroscopic factors are not observables
- They are extracted from a cross section based on a specific structure and reaction model
- Structure and reaction models needs to be consistent!

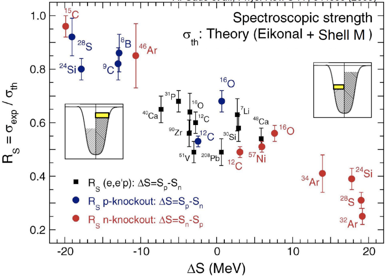
Theoretical cross section:

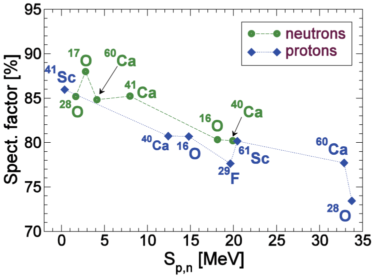
Reaction theory

$$\sigma(j^\pi) = \left(\frac{A}{A-1}\right)^N C^2 S(j^\pi)\sigma_{sp}(j, S_N + E_x[j^\pi])$$

Structure theory

A. Gade et al., Phys. Rev. C 77, 044306 (2008)



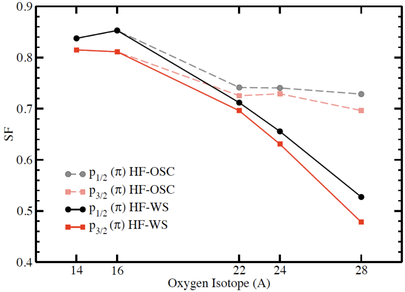


C. Barbieri, W.H.Dickhoff, Int. Jour. Mod. Phys. A24, 2060 (2009).

Self-consistent green's function method show rather weak asymmetry dependence for the spectroscopic factor.

Coupled cluster method [from D. Dean, G. Hagen, T. Papenbrock]

Quenching of spectroscopic factors for proton removal in neutron rich oxygen isotopes

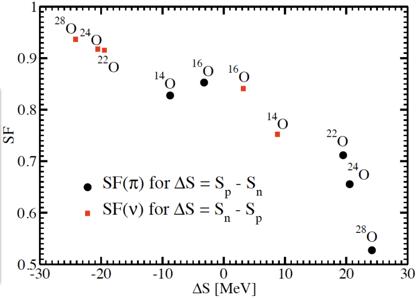


Spectroscopic factor is a useful tool to study correlations towards the dripline.

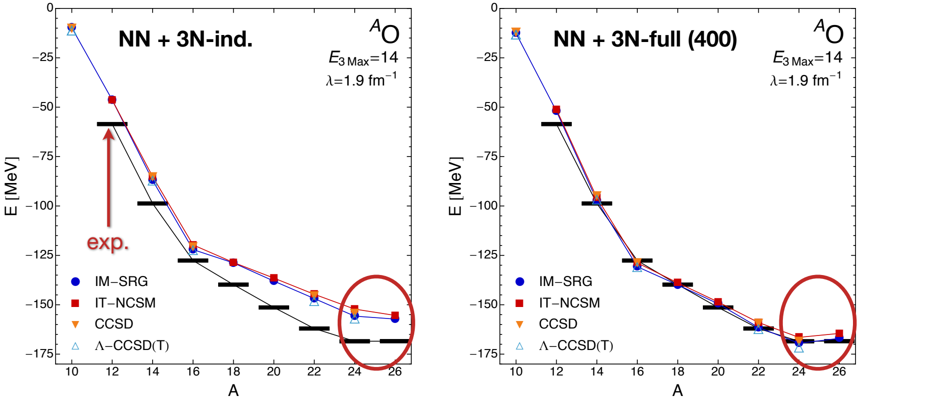
SF for proton removal in neutron rich ^{24}O show strong “quenching” pointing to large deviations from a mean-field like picture.
G. Hagen et al Phys. Rev. Lett. 107, 032501 (2011).

Strong asymmetry dependence on the SF for proton and neutron removal in neutron rich oxygen isotopes.

SF~1 for neutron removal while protons are strongly correlated SF ~0.6-0.7 in $^{22,24,28}O$



Oxygen chain with multi-reference IM-SRG [H. Hergert]

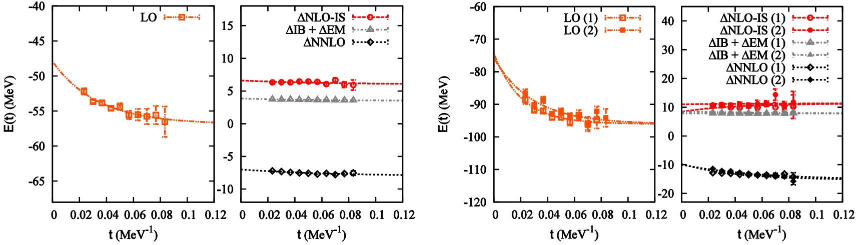


- ref. state: number-projected Hartree-Fock-Bogoliubov vacuum
- results (mostly) insensitive to choice of generator for same H^{od}
- **consistency between different many-body methods**

Ground states of ^{8}Be and ^{12}C [E. Epelbaum]

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations for ^{8}Be and ^{12}C, L=11.8 fm



Ground state energies (L=11.8 fm) of ^{4}He, ^{8}Be, ^{12}C & ^{16}O

	^{4}He	^{8}Be	^{12}C	^{16}O
LO $[Q^0]$, in MeV	$-28.0(3)$	$-57(2)$	$-96(2)$	$-144(4)$
NLO $[Q^2]$, in MeV	$-24.9(5)$	$-47(2)$	$-77(3)$	$-116(6)$
NNLO $[Q^3]$, in MeV	$-28.3(6)$	$-55(2)$	$-92(3)$	$-135(6)$
Experiment, in MeV	-28.30	-56.5	-92.2	-127.6

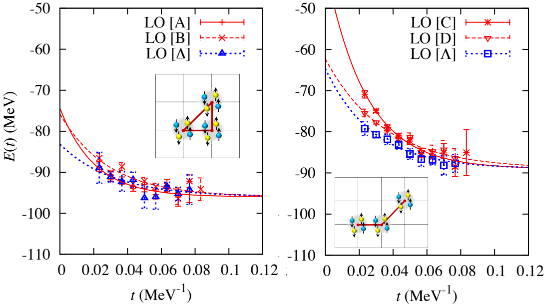
Hoyle State [E. Epelbaum]

EE, Krebs, Lähde, Lee, Meißner, PRL 106 (2011) 192501; PRL 109 (2012) 252501

Lattice results for low-lying even-parity states of ^{12}C

	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	$-96(2)$	$-94(2)$	$-89(2)$	$-88(2)$
NLO	$-77(3)$	$-74(3)$	$-72(3)$	$-70(3)$
NNLO	$-92(3)$	$-89(3)$	$-85(3)$	$-83(3)$
Exp	-92.16	-87.72	-84.51	$-82(1)$

Probing (α-cluster) structure of the 0_1^+, 0_2^+ states



RMS radii and quadrupole moments

	LO	Experiment
$r(0_1^+)$ [fm]	2.2(2)	2.47(2) [26]
$r(2_1^+)$ [fm]	2.2(2)	–
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6(2)	6(3) [27]
$r(0_2^+)$ [fm]	2.4(2)	–
$r(2_2^+)$ [fm]	2.4(2)	–
$Q(2_2^+)$ [$e\,\mathrm{fm}^2$]	$-7(2)$	–

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Many-body methods: Selected results

Dilute, natural, Fermi system

Bethe-Brueckner-Goldstone Power Counting

Teaser for MBPT applied in finite nuclei

EFT for "Natural" Short-Range Interaction

- A simple, general interaction is a sum of delta functions and derivatives of delta functions. In momentum space,

$$\langle \mathbf{k}|V_{\rm eft}|\mathbf{k}'\rangle = C_0 + \frac{1}{2}C_2(\mathbf{k}^2+\mathbf{k}'^2) + C_2'\mathbf{k}\cdot\mathbf{k}' + \cdots$$

- Or, $\mathcal{L}_{\rm eft}$ has most general local (contact) interactions:

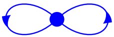
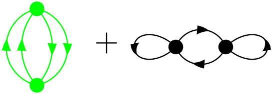
$$\mathcal{L}_{\rm eft} = \psi^\dagger\Big[i\frac{\partial}{\partial t} + \frac{\overrightarrow{\nabla}^2}{2M}\Big]\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}\big[(\psi\psi)^\dagger(\psi\overleftrightarrow{\nabla}^2\psi) + \text{ h.c.}\big]$$
$$+ \frac{C_2'}{8}(\psi\overleftrightarrow{\nabla}\psi)^\dagger\cdot(\psi\overleftrightarrow{\nabla}\psi) - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$$

- Dimensional analysis $\Longrightarrow C_{2i} \sim \dfrac{4\pi}{M}R^{2i+1}\,, \quad D_{2i} \sim \dfrac{4\pi}{M}R^{2i+4}$

Effective Field Theory Ingredients

Ref: Hammer, rjf [nucl-th/0702040]

1. Use the most general $\mathcal{L}$ with low-energy dof's consistent with global and local symmetries of underlying theory
   - $\mathcal{L}_{\rm eft} = \psi^\dagger \big[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M}\big]\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$
2. Declaration of regularization and renormalization scheme
   - natural $a_0 \Longrightarrow$ dimensional regularization and min. subtraction
3. Well-defined power counting $\Longrightarrow$ small expansion parameters
   - use the separation of scales $\Longrightarrow \frac{k_{\rm F}}{\Lambda}$ with $\Lambda \sim 1/R \Longrightarrow k_{\rm F}a_0$, etc.

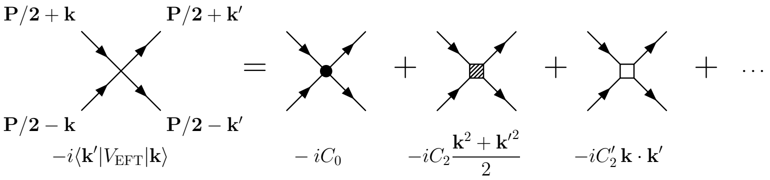
$\mathcal{O}(k_{\rm F}^6)$:  $\mathcal{O}(k_{\rm F}^7)$: 

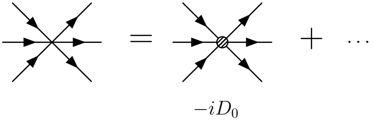
$$\mathcal{E} = \rho\frac{k_{\rm F}^2}{2M}\left[\frac{3}{5} + \frac{2}{3\pi}(k_{\rm F}a_0) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_{\rm F}a_0)^2 + \cdots\right]$$

- cleanly recovers perturbative free-space ERE and in-medium energy density (including logs), plus error estimates

Feynman Rules for EFT Vertices

$$\mathcal{L}_{\rm eft} = \psi^\dagger \Big[i\frac{\partial}{\partial t} + \frac{\overrightarrow{\nabla}^2}{2M} \Big] \psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}\Big[(\psi\psi)^\dagger(\psi \overleftrightarrow{\nabla}^2 \psi) + \text{ h.c.}\Big]$$
$$+ \frac{C_2'}{8}(\psi \overleftrightarrow{\nabla} \psi)^\dagger \cdot (\psi \overleftrightarrow{\nabla} \psi) - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$$





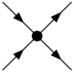
Renormalization

- Reproduce $f_0(k)$ in perturbation theory (Born series):

$$f_0(k) \propto a_0 - i a_0^2 k - (a_0^3 - a_0^2 r_0/2) k^2 + \mathcal{O}(k^3 a_0^4)$$

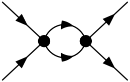
- Consider the leading potential $V^{(0)}_{\rm EFT}(\mathbf{x}) = C_0 \delta(\mathbf{x})$ or

$$\langle \mathbf{k}| V^{(0)}_{\rm eft} |\mathbf{k}'\rangle \Longrightarrow$$



$$\Longrightarrow C_0$$

- Choosing $C_0 \propto a_0$ gets the first term. Now $\langle \mathbf{k}| V G_0 V |\mathbf{k}'\rangle$:



$$\Longrightarrow C_0 M \int \frac{d^3q}{(2\pi)^3} \frac{1}{k^2 - q^2 + i\epsilon} C_0 \longrightarrow \infty!$$

$\Longrightarrow$ Linear divergence!

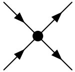
Renormalization

- Reproduce $f_0(k)$ in perturbation theory (Born series):

$$f_0(k) \propto a_0 - i a_0^2 k - (a_0^3 - a_0^2 r_0/2) k^2 + \mathcal{O}(k^3 a_0^4)$$

- Consider the leading potential $V_{\mathrm{EFT}}^{(0)}(\mathbf{x}) = C_0 \delta(\mathbf{x})$ or

$$\langle \mathbf{k} | V_{\mathrm{eft}}^{(0)} | \mathbf{k}' \rangle \Longrightarrow$$



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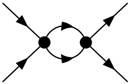


$$\Longrightarrow \int^{\Lambda_c} \frac{d^3 q}{(2\pi)^3} \frac{1}{k^2 - q^2 + i\epsilon} \longrightarrow \frac{\Lambda_c}{2\pi^2} - \frac{ik}{4\pi} + \mathcal{O}(\frac{k^2}{\Lambda_c})$$

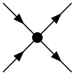
$\Longrightarrow$ If cutoff at Λ_c, then can absorb into C_0, but all powers of k^2

Renormalization

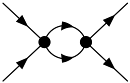
- Reproduce $f_0(k)$ in perturbation theory (Born series):

$$f_0(k) \propto a_0 - i a_0^2 k - (a_0^3 - a_0^2 r_0/2) k^2 + \mathcal{O}(k^3 a_0^4)$$

- Consider the leading potential $V_{\rm EFT}^{(0)}(\mathbf{x}) = C_0 \delta(\mathbf{x})$ or

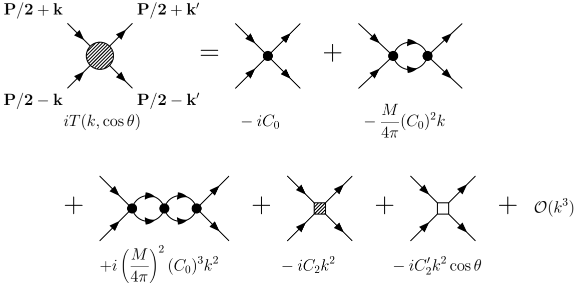
$$\langle \mathbf{k} | V_{\rm eft}^{(0)} | \mathbf{k}' \rangle \Longrightarrow$$  $$\Longrightarrow C_0$$

- Choosing $C_0 \propto a_0$ gets the first term. Now $\langle \mathbf{k} | V G_0 V | \mathbf{k}' \rangle$:

 $$\Longrightarrow \int \frac{d^D q}{(2\pi)^3} \frac{1}{k^2 - q^2 + i\epsilon} \xrightarrow{D \to 3} -\frac{ik}{4\pi}$$

Dimensional regularization with minimal subtraction
$\Longrightarrow$ only one power of k!

- Dim. reg. + minimal subtraction $\Longrightarrow$ simple power counting:



- Matching in free space:

$$C_0 = \frac{4\pi}{M} a_0 = \frac{4\pi}{M} R \,, \quad C_2 = \frac{4\pi}{M} \frac{a_0^2 r_0}{2} = \frac{4\pi}{M} \frac{R^3}{3} \,, \quad \cdots$$

- Recovers effective range expansion order-by-order with perturbative diagrammatic expansion
  - one power of k per diagram
  - estimate truncation error from dimensional analysis

Noninteracting Fermi Sea at $T = 0$

- Put system in a large box ($V = L^3$) with periodic bc's
  - spin-isospin degeneracy ν (e.g., for nuclei, $\nu = 4$)
  - fill momentum states up to Fermi momentum $k_{\rm F}$

$$N = \nu \sum_{\mathbf{k}}^{k_{\rm F}} 1 \; , \qquad E = \nu \sum_{\mathbf{k}}^{k_{\rm F}} \frac{\hbar^2 k^2}{2M}$$

- Use: $\int F(k)\, dk \approx \sum_i F(k_i)\Delta k_i = \sum_i F(k_i)\frac{2\pi}{L}\Delta n_i = \frac{2\pi}{L}\sum_i F(k_i)$
- In 1-D:

$$N = \nu \frac{L}{2\pi}\int_{-k_{\rm F}}^{+k_{\rm F}} dk = \frac{\nu k_{\rm F}}{\pi} L \implies \rho = \frac{N}{L} = \frac{\nu k_{\rm F}}{\pi}; \quad \frac{E}{L} = \frac{1}{3}\frac{\hbar^2 k_{\rm F}^2}{2M}\rho$$

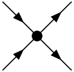
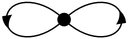
- In 3-D:

$$N = \nu \frac{V}{(2\pi)^3}\int^{k_{\rm F}} d^3k = \frac{\nu k_{\rm F}^3}{6\pi^2} V \implies \rho = \frac{N}{V} = \frac{\nu k_{\rm F}^3}{6\pi^2}; \quad \frac{E}{V} = \frac{3}{5}\frac{\hbar^2 k_{\rm F}^2}{2M}\rho$$

- Volume/particle $V/N = 1/\rho \sim 1/k_{\rm F}^3$, so spacing $\sim 1/k_{\rm F}$

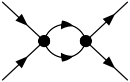
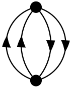
Energy Density From Summing Over Fermi Sea

- Leading order $V^{(0)}_{\rm EFT}(\mathbf{x}) = C_0\delta(\mathbf{x}) \Longrightarrow V^{(0)}_{\rm EFT}(\mathbf{k},\mathbf{k}') = C_0$

 $\Longrightarrow$ 

$$\mathcal{E}_{\rm LO} = \frac{C_0}{2}\nu(\nu-1)\left(\sum_{\mathbf{k}}^{k_{\rm F}} 1\right)^2 \propto a_0 k_{\rm F}^6$$

- At the next order, we get a linear divergence again:

 $\Longrightarrow$ 

$$\mathcal{E}_{\rm NLO} \propto \int_{k_{\rm F}}^{\infty} \frac{d^3q}{(2\pi)^3}\,\frac{C_0^2}{k^2-q^2}$$

- Same renormalization fixes it! Particles $\longrightarrow$ holes

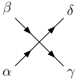
$$\int_{k_{\rm F}}^{\infty}\frac{1}{k^2-q^2} = \int_0^{\infty}\frac{1}{k^2-q^2} - \int_0^{k_{\rm F}}\frac{1}{k^2-q^2} \xrightarrow{D\to 3} -\int_0^{k_{\rm F}}\frac{1}{k^2-q^2} \propto a_0^2 k_{\rm F}^7$$

Feynman Rules for Energy Density at $T = 0$

- $T = 0$ Energy density $\mathcal{E}$ is sum of *Hugenholtz* diagrams
  - same vertices as free space (same renormalization!)
- Feynman rules:

1 Each line is assigned conserved $\widetilde{k} \equiv (k_0, \mathbf{k})$ and $[\omega_{\mathbf{k}} \equiv k^2/2M]$

$$iG_0(\widetilde{k})_{\alpha\beta} = i\delta_{\alpha\beta}\left(\frac{\theta(k - k_{\mathrm{F}})}{k_0 - \omega_{\mathbf{k}} + i\epsilon} + \frac{\theta(k_{\mathrm{F}} - k)}{k_0 - \omega_{\mathbf{k}} - i\epsilon}\right)$$

2 

$\longrightarrow (\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma})$ (if spin-independent)

3 After spin summations, $\delta_{\alpha\alpha} \to -\nu$ in every closed fermion loop.

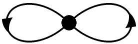
4 Integrate $\int d^4k/(2\pi)^4$ with $e^{ik_0 0^+}$ for tadpoles

5 Symmetry factor $i/(S\prod_{l=2}^{l_{\max}}(l!)^k)$ counts vertex permutations and equivalent l–tuples of lines

Power Counting

- Power counting rules
  1. for every propagator (line): M/k_{F}^2
  2. for every loop integration: k_{F}^5/M
  3. for every n–body vertex with $2i$ derivatives: $k_{\mathrm{F}}^{2i}/M\Lambda^{2i+3n-5}$
- Diagram with V_{2i}^n n–body vertices scales as $(k_{\mathrm{F}})^\beta$ *only*:

$$\beta = 5 + \sum_{n=2}^{\infty}\sum_{i=0}^{\infty}(3n+2i-5)V_{2i}^n \,.$$

- e.g., $\mathcal{O}\left(k_{\mathrm{F}}^6\right)$:  $\Longrightarrow V_0^2 = 1$

$$\Longrightarrow \beta = 5 + (3\cdot 2 + 2\cdot 0 - 5)\cdot 1 = 6 \Longrightarrow \mathcal{O}(k_{\mathrm{F}}^6)$$

Power Counting

- Power counting rules
  1. for every propagator (line): M/k_{F}^2
  2. for every loop integration: k_{F}^5/M
  3. for every n–body vertex with $2i$ derivatives: $k_{\mathrm{F}}^{2i}/M\Lambda^{2i+3n-5}$
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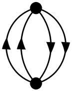
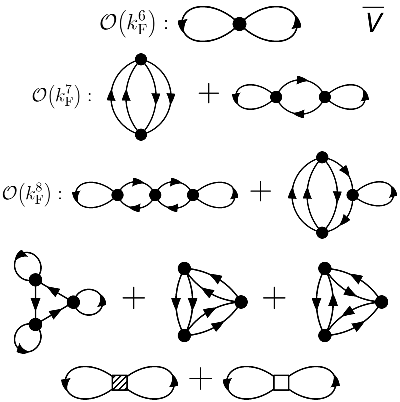
$$\beta = 5 + \sum_{n=2}^{\infty}\sum_{i=0}^{\infty}(3n+2i-5)V_{2i}^n \,.$$

- e.g.,  $\Longrightarrow V_0^2 = 2$

$$\Longrightarrow \beta = 5 + (3\cdot 2 + 2\cdot 0 - 5)\cdot 2 = 7 \Longrightarrow \mathcal{O}(k_{\mathrm{F}}^7)$$

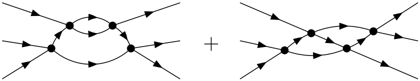
$T = 0$ Energy Density from Hugenholtz Diagrams



$$
\begin{aligned}
\frac{E}{V} = \rho \frac{k_{\rm F}^2}{2M} \Big[& \frac{3}{5} + (\nu - 1)\frac{2}{3\pi}(k_{\rm F} a_0) \\
& + (\nu - 1)\frac{4}{35\pi^2}(11 - 2\ln 2)(k_{\rm F} a_0)^2 \\
& + (\nu - 1)\big(0.076 + 0.057(\nu - 3)\big)(k_{\rm F} a_0)^3 \\
& + (\nu - 1)\frac{1}{10\pi}(k_{\rm F} r_0)(k_{\rm F} a_0)^2 \\
& + (\nu + 1)\frac{1}{5\pi}(k_{\rm F} a_p)^3 + \cdots \Big]
\end{aligned}
$$

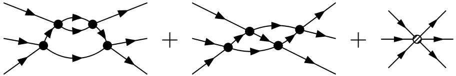
Looks Like a Power Series in $k_{\rm F}$! Is it?

- New logarithmic divergences in 3–3 scattering

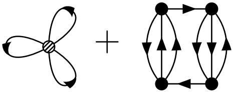


$\propto (C_0)^4 \ln(k/\Lambda_c)$

- Changes in Λ_c must be absorbed by 3-body coupling $D_0(\Lambda_c)$
  $\Longrightarrow D_0(\Lambda_c) \propto (C_0)^4 \ln(a_0\Lambda_c) + \text{const.}$ [Braaten & Nieto]

$\frac{d}{d\Lambda_c}\Big[$  $\Big] = 0 \Longrightarrow$ fixes coefficient!

- What does this imply for the energy density?

$\mathcal{O}\big(k_{\rm F}^9 \ln(k_{\rm F})\big)$:  $+ \; \ldots \propto (\nu-2)(\nu-1)\,(k_{\rm F}a_0)^4 \ln(k_{\rm F}a_0)$

Summary: Dilute Fermi System with Natural a_0

- The many-body energy density is perturbative in $k_{\mathrm{F}} a_0$
  - efficiently reproduced by the EFT approach
- Power counting $\Longrightarrow$ error estimate from omitted diagrams
- Three-body forces are inevitable in a low-energy effective theory
  - and not unique $\Longrightarrow$ they depend on the two-body potential
- The case of a natural scattering length is under control for a uniform system
  - What about a finite # of fermions in a trap? (DFT!)
  - What if the scattering length is not natural?

Outline

Many-body methods: Selected results

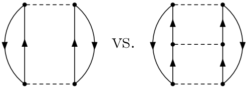
Dilute, natural, Fermi system

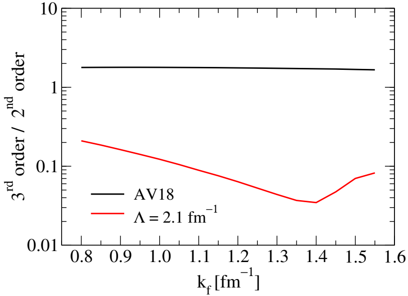
Bethe-Brueckner-Goldstone Power Counting

Teaser for MBPT applied in finite nuclei

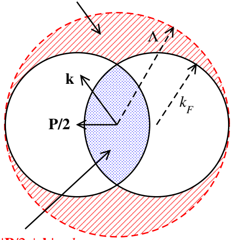
Bethe-Brueckner-Goldstone Power Counting

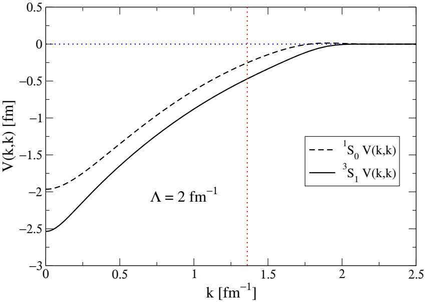
Strong short-range repulsion
$\Longrightarrow$ Sum V ladders $\Longrightarrow$ G





$V_{\text{low }k}$ momentum dependence + phase space $\Longrightarrow$ perturbative





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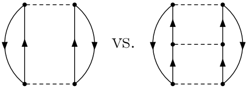

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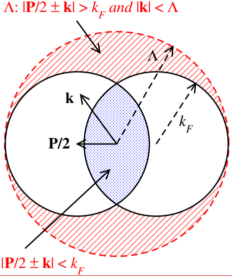
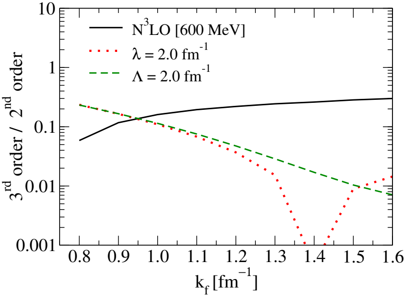
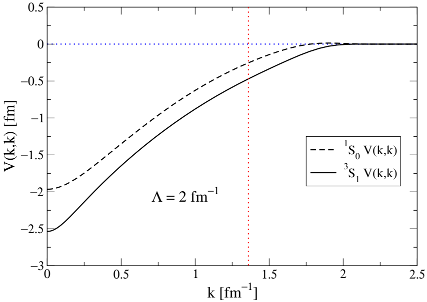
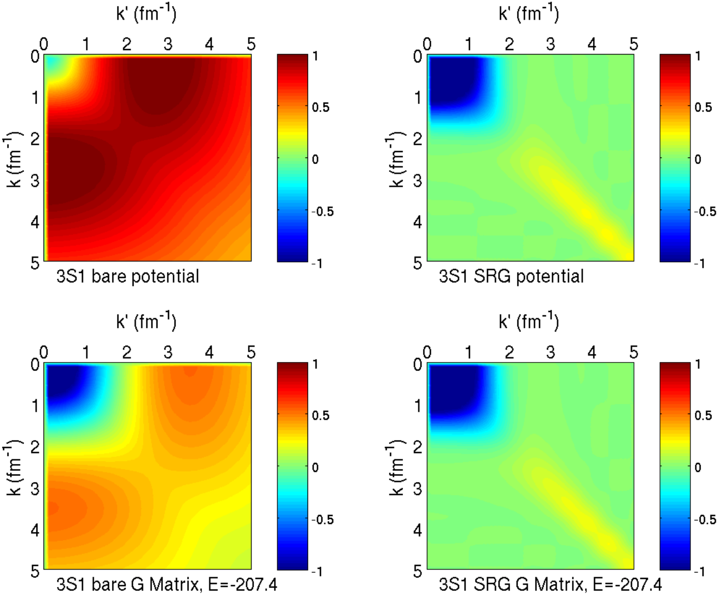






Compare Potential and G Matrix: AV18



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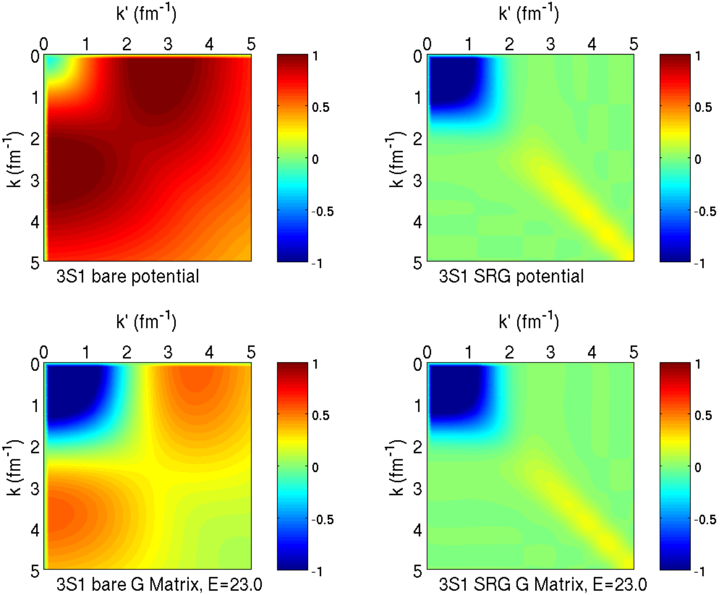
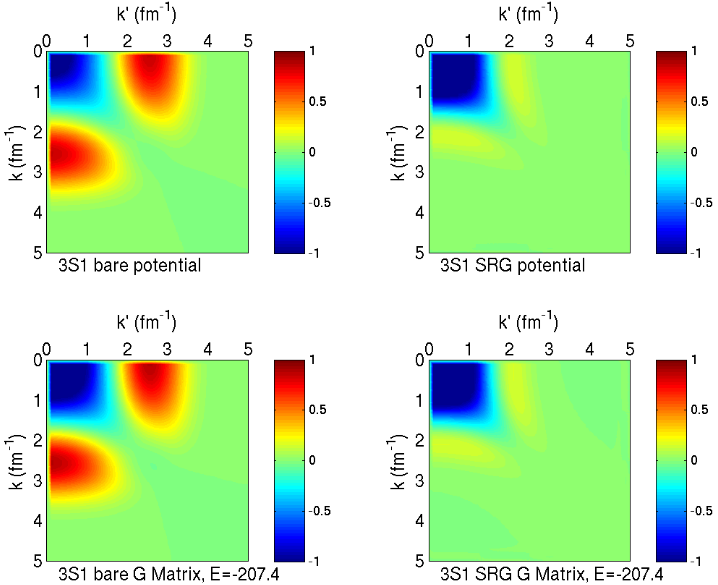


Compare Potential and G Matrix: N^{3}LO (500 MeV)



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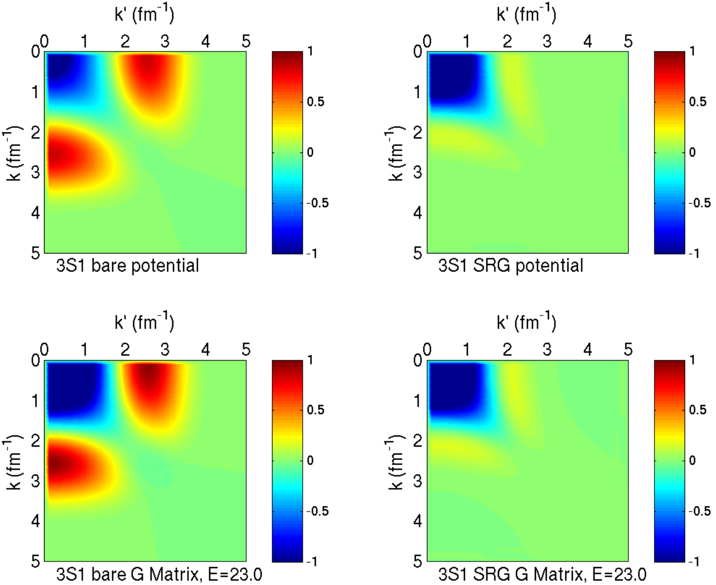
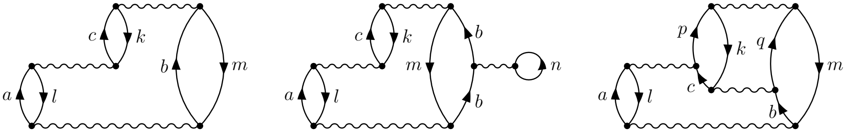


Hole-Line Expansion Revisited (Bethe, Day, ...)

- Consider ratio of fourth-order diagrams to third-order:

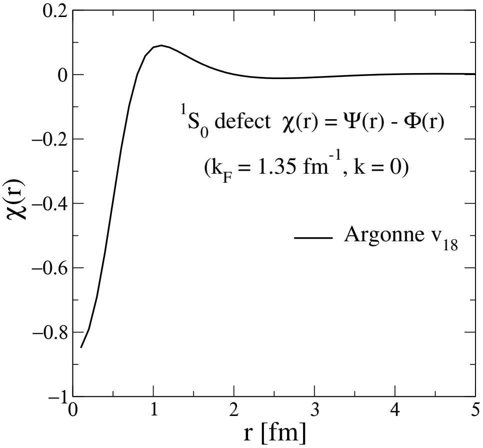


- "Conventional" G matrix still couples low-k and high-k
  - no new hole line $\Longrightarrow$ ratio $\approx -\chi(\mathbf{r}=0) \approx -1 \Longrightarrow$ sum all orders
  - add a hole line $\Longrightarrow$ ratio $\approx \sum_{n \leq k_{\rm F}} \langle bn|(1/e)G|bn\rangle \approx \kappa \approx 0.15$
- Low-momentum potentials decouple low-k and high-k
  - add a hole line $\Longrightarrow$ still suppressed
  - no new hole line $\Longrightarrow$ also suppressed (limited phase space)
  - freedom to choose single-particle $U \Longrightarrow$ use for Kohn-Sham

$\Longrightarrow$ Density functional theory (DFT) should work!

Two-Body Correlations at Nuclear Matter Density

- Defect wf $\chi(r)$ for particular kinematics ($k = 0$, $P_{\rm cm} = 0$)
- AV18: “Wound integral” provides expansion parameter



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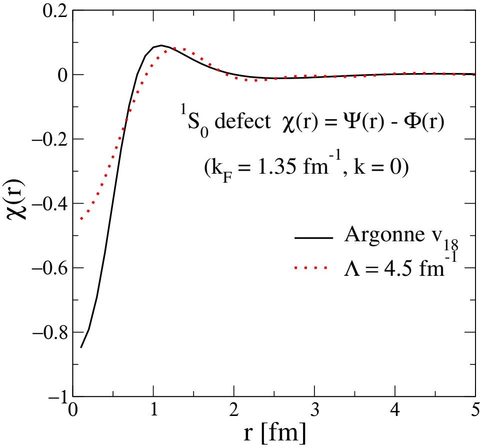

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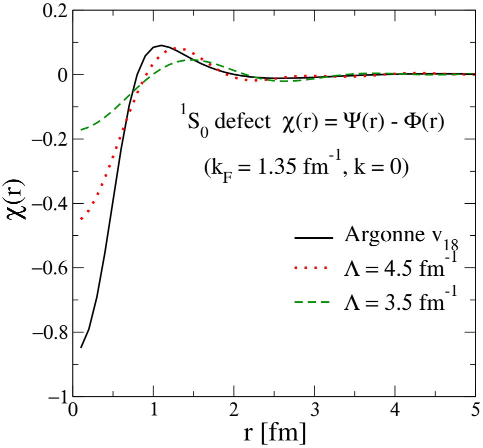

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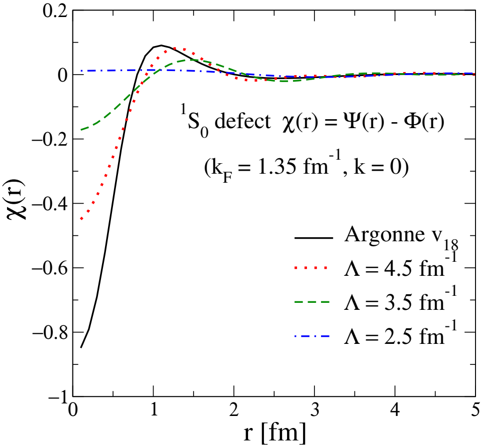
- Tensor ($^3S_1$) $\Longrightarrow$ larger defect



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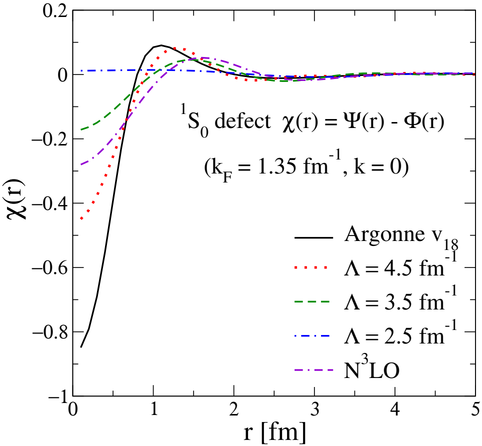

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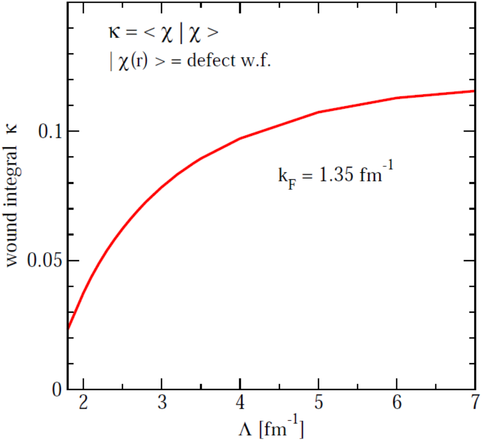
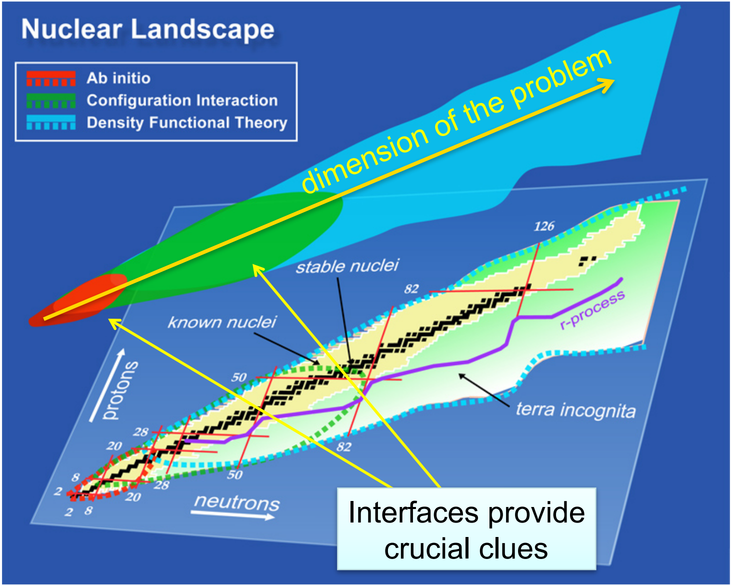
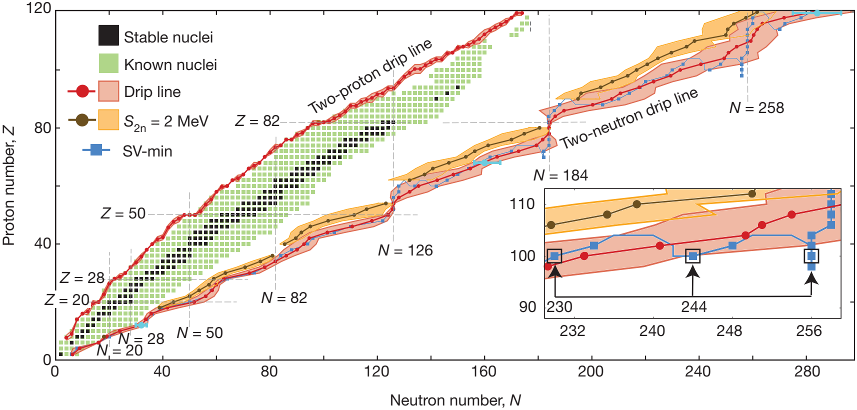


Overlapping theory methods cover all nuclei



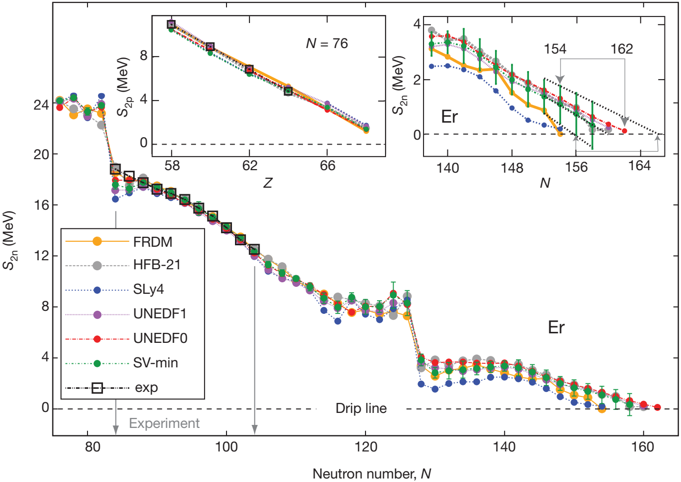
"The limits of the nuclear landscape"

J. Erler et al., Nature **486**, 509 (2012)



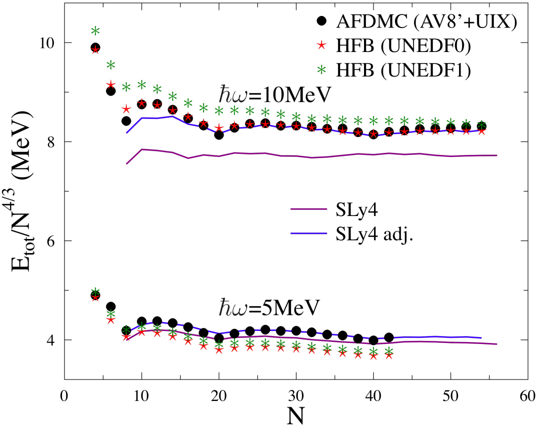
- Proton and neutron driplines predicted by Skyrme EDFs
  - Total: 6900 ± 500 nuclei with $Z \leq 120$ (≈ 3000 known)
  - Estimate systematic errors by comparing models

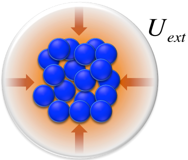
"The limits of the nuclear landscape"



- Two-neutron separation energies of even-even erbium isotopes
  - Compare different functionals, with uncertainties of fits
  - Dependence on neutron excess poorly determined (cf. driplines)

Impact of forces: Use *ab initio* pseudo-data

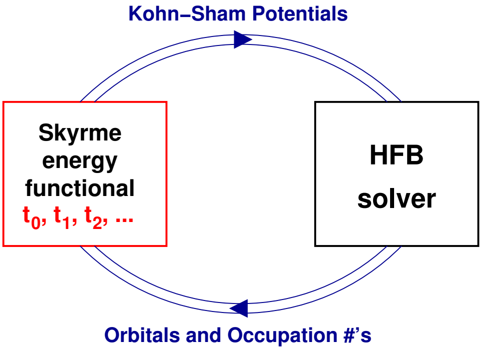




- Put neutrons in a harmonic oscillator trap with $\hbar\omega$ (cf. cold atoms!)
- Calculate exact result with AFDMC [S. Gandolfi, J. Carlson, and S.C. Pieper, Phys. Rev. Lett. 106, 012501 (2011)] (or with other methods)
- UNEDF0 and UNEDF1 functionals improve over Skyrme SLy4!

Self-consistent Skyrme EDF and beyond

$$\mathcal{E}_{Skyrme} = \frac{\tau}{2M} + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^{2+\alpha} + \frac{1}{16}(3t_1 + 5t_2)\rho\tau + \frac{1}{64}(9t_1 - 5t_2)|\nabla\rho|^2 + \cdots$$



- Kohn-Sham DFT $\Longrightarrow$ iterate to self-consistency
- Looks like dilute, natural functional!
- Low-energy QCD: NDA power counting?
- Use DME to put in pion exchange from χEFT

Schematic equations to solve self-consistently:

$$V_{\text{KS}}(\mathbf{r}) = \frac{\delta E_{\text{int}}[\rho]}{\delta\rho(\mathbf{r})} \quad \Longleftrightarrow \quad [-\frac{\nabla^2}{2m} + V_{\text{KS}}(\mathbf{x})]\psi_\alpha = \varepsilon_\alpha\psi_\alpha \quad \Longrightarrow \quad \rho(\mathbf{x}) = \sum_\alpha n_\alpha|\psi_\alpha(\mathbf{x})|^2$$

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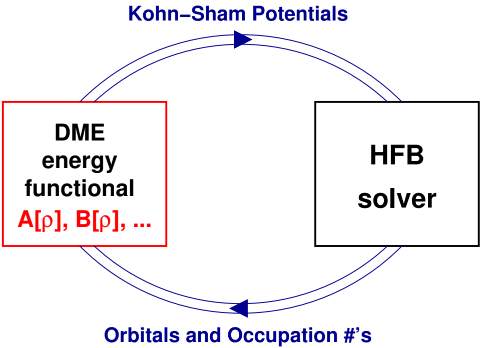

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Many-body methods: Selected results

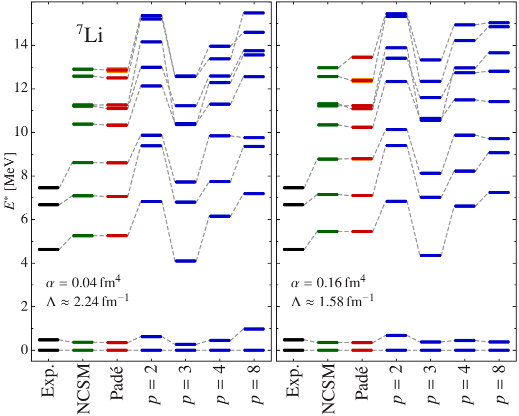
Dilute, natural, Fermi system

Bethe-Brueckner-Goldstone Power Counting

Teaser for MBPT applied in finite nuclei

High-order Rayleigh-Schrödinger MBPT in finite nuclei

- R. Roth et al.
- Excitation energies in ^{7}Li
- Degenerate R-S MBPT
- SRG with two resolutions from N^{3}LO 2NF
- Fixed HO model space



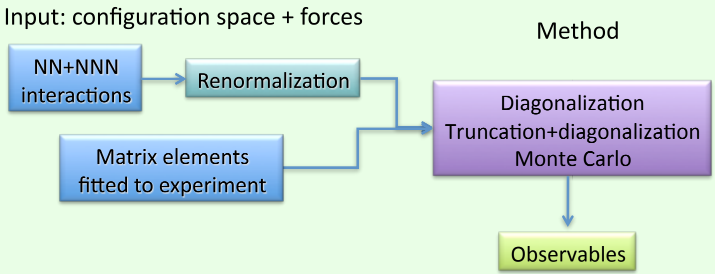
Order $p = 2$, 3, 4, and 8 compared to experiment, exact NCSM calculations, and the Padé resummed result

$\Longrightarrow$ note the good agreement of the last two!

The shell model revisited

Configuration interaction techniques

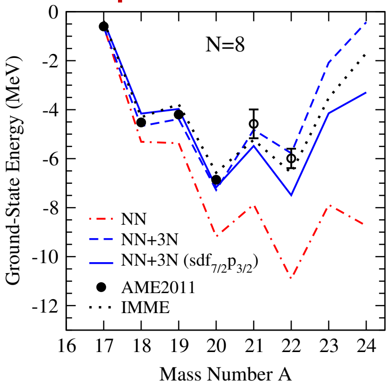
- light and heavy nuclei
- detailed spectroscopy
- quantum correlations (lab-system description)

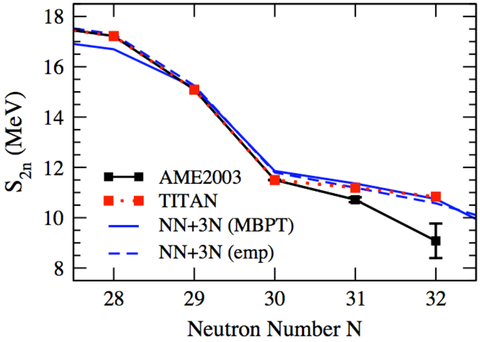


- Direct comparison with experiment
- Pseudo-data to inform reaction theory and DFT

Confronting theory and experiment to both driplines

- Precision mass measurements test impact of chiral 3NF
- Proton rich [Holt et al., arXiv:1207.1509]
- Neutron rich [Gallant et al., arXiv:1204.1987]
- Many new tests possible!





- Shell model description using chiral potential evolved to $V_{\mathrm{low}\,k}$ plus 3NF fit to $A = 3, 4$
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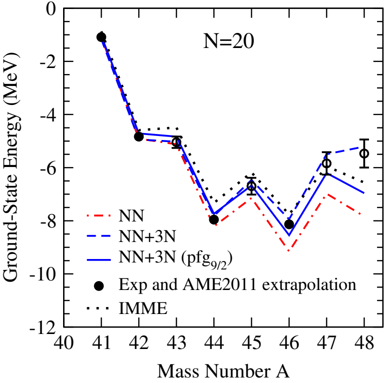

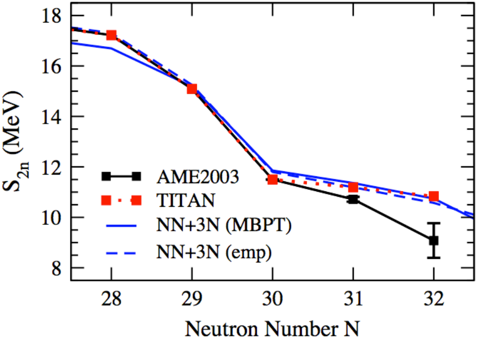

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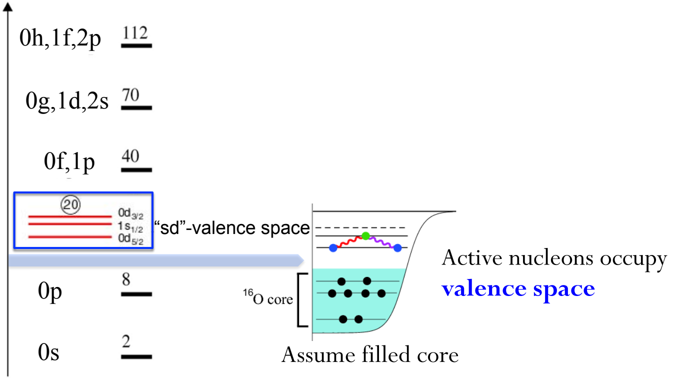
Non-empirical shell model [from J. Holt]

Solving the Nuclear Many-Body Problem

Nuclei understood as many-body system starting from closed shell, add nucleons

Interaction and energies of valence space orbitals from original $V_{\text{low }k}$

This alone does not reproduce experimental data



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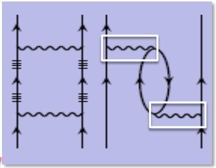
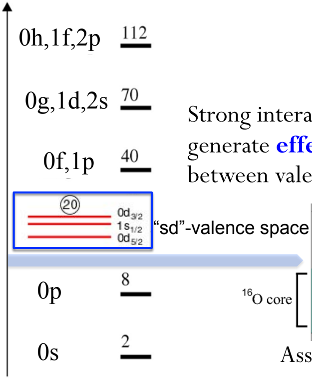
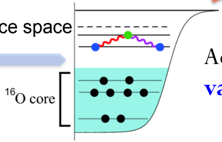
Hjorth-Jensen, Kuo, Osnes (1995)





Strong interactions with core generate **effective interaction** between valence nucleons



Active nucleons occupy **valence space**

Assume filled core

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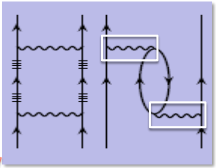
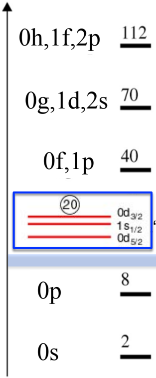
Effective two-body matrix elements
Single-particle energies (SPEs)

Hjorth-Jensen, Kuo, Osnes (1995)





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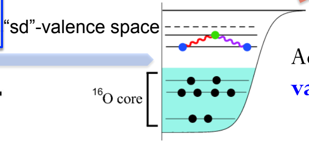

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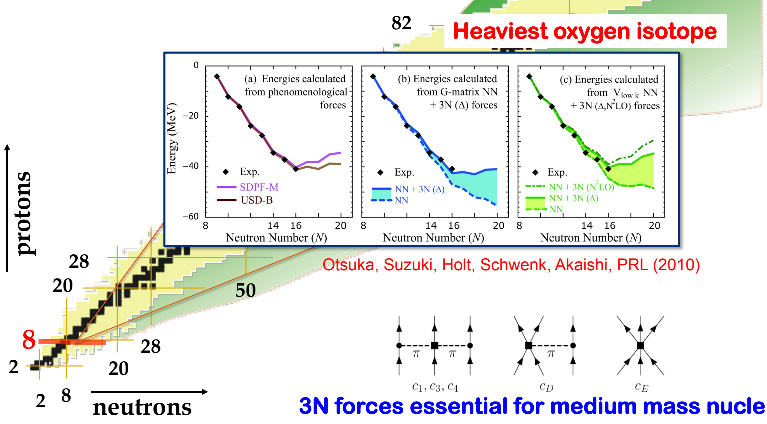
Chiral 3NFs meet the shell model [from J. Holt]

Drip Lines and Magic Numbers: The Evolving Nuclear Landscape

Important in light nuclei, nuclear matter…

What are the limits of nuclear existence?

How do magic numbers form and evolve?



Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010)

3N forces essential for medium mass nuclei

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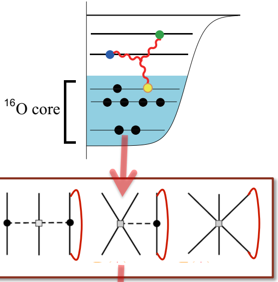
3N Forces for Valence-Shell Theories

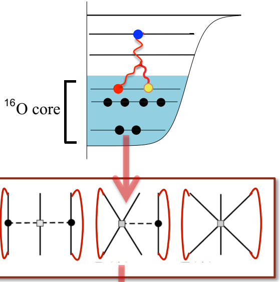
Normal-ordered 3N: contribution to valence neutron interactions

Effective two-body



$$\langle ab | V_{3N,\mathrm{eff}} | a'b' \rangle = \sum_{\alpha = core} \langle \alpha ab | V_{3N} | \alpha a'b' \rangle$$

Effective one-body



$$\langle a | V_{3N,\mathrm{eff}} | a' \rangle = \frac{1}{2} \sum_{\alpha\beta = core} \langle \alpha\beta a | V_{3N} | \alpha\beta a' \rangle$$

Combine with microscopic NN: eliminate empirical adjustments

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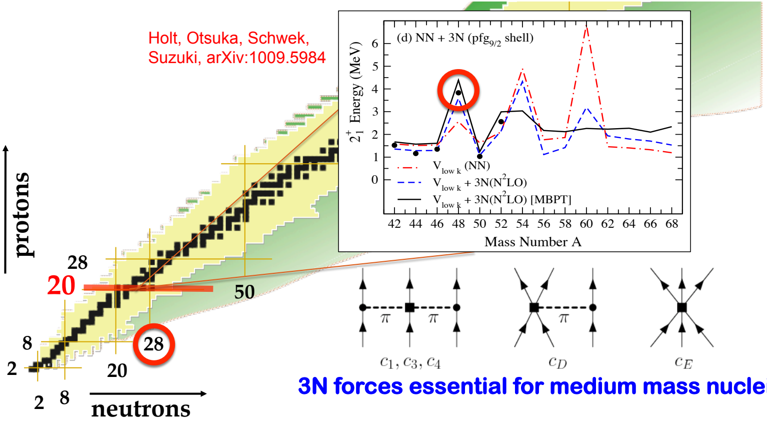
How do magic numbers form and evolve?

N=28 magic number in calcium

Holt, Otsuka, Schwek, Suzuki, arXiv:1009.5984



3N forces essential for medium mass nuclei