Nuclear forces and their impact on structure, reactions and astrophysics

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July, 2013

Lectures for Week 2

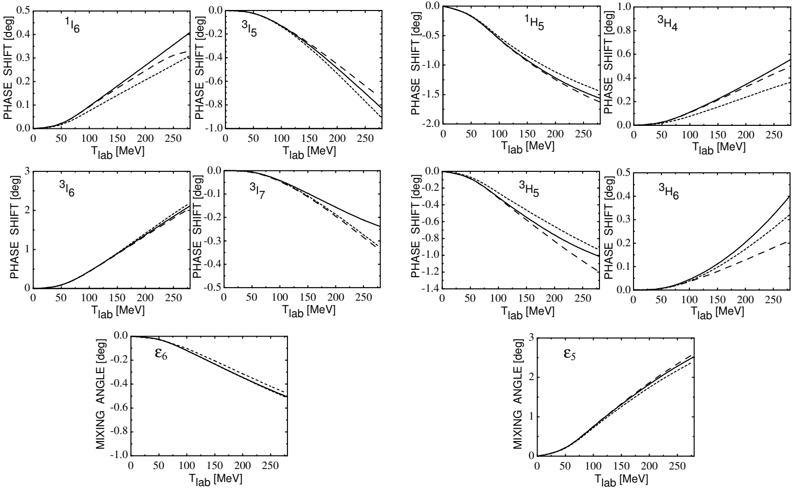
- **M.** Chiral EFT 1 (as); χ-symmetry in NN scattering, QCD 2 (rjf)
- **T.** Chiral EFT 2 (rjf); Three-nucleon forces 1 (as)
- **W.** Renormalization group 1 (rjf); Forces from LQCD, hyperon-nucleon (as)
- **Th.** Renormalization group 2 (rjf); Nuclear forces and electroweak interactions (as)
- **F.** Many-body overview (rjf); Three-nucleon forces 2 (as)

Outline

- High partial waves and chiral symmetry
- Lattice QCD
- NN from LQCD (slides from recent talk by S. Aoki)

Figures from Kaiser, Brockmann, Weise, Nucl. Phys. A 625, 758 (1997)

Key: dashed = OPE; full = OPE iterated + irred. 2π; others = NN PSA



[What is a mixing angle?] What do you conclude about pion exchange?

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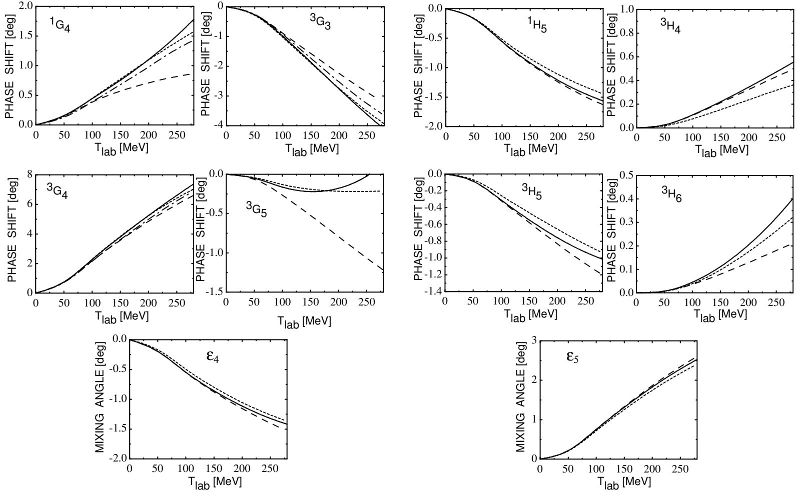

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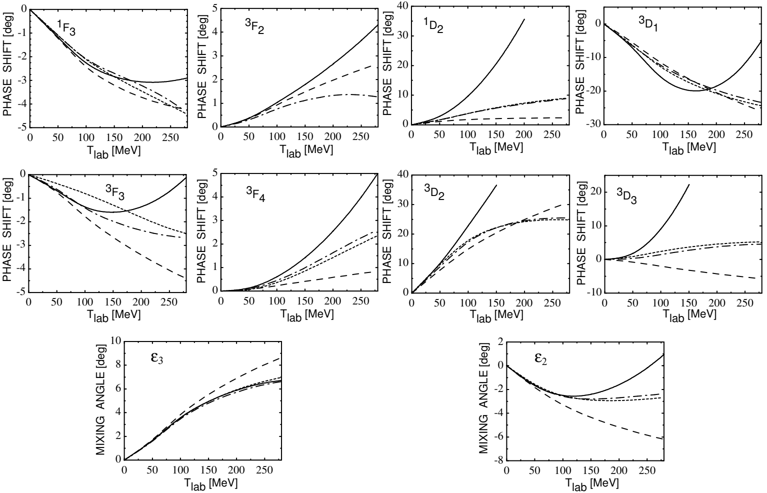
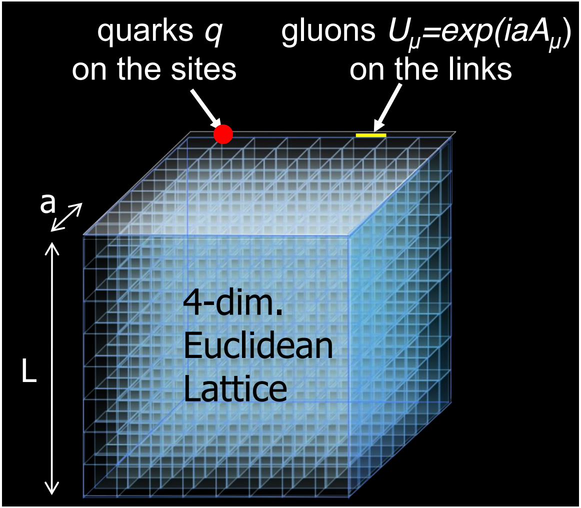


Why does OPE work better at low energy and high *L*?

Outline

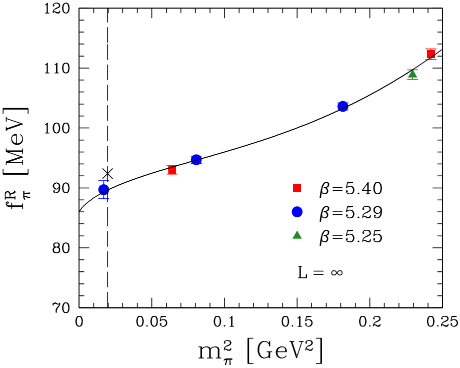
- High partial waves and chiral symmetry
- **Lattice QCD**
- NN from LQCD (slides from recent talk by S. Aoki)

Visualization of the lattice [from T. Hatsuda]



QCDSF LQCD calculations of f_π and g_A [arXiv:1302.2233]

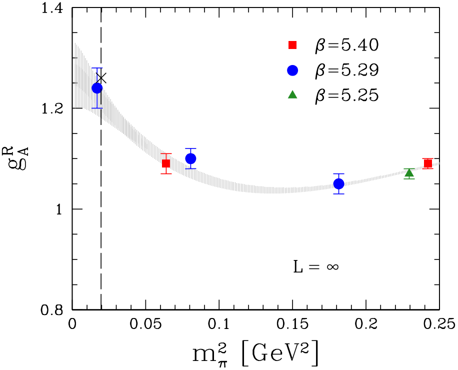
- "Two flavors of nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions and Wilson plaquette action"
- 3 lattice spacings: $a = 0.076$, 0.071, and 0.060 fm
- Renormalized $f_\pi^R = 89.7 \pm 1.5 \pm 1.8$ MeV at $m_\pi = 130$ MeV
- Solid line is fit to chiral ansatz $\Longrightarrow$ also determines $\bar{l}_4$ from ChPT



Very encouraging but are all the systematic errors under control?

QCDSF LQCD calculations of f_π and g_A [arXiv:1302.2233]

- g_A is known experimentally from neutron β decay
- Renormalized $g_A^R = 1.24 \pm 0.04$ at $m_\pi = 130\,\text{MeV}$
- Shaded band is *fit* to ChEFT $g_A^R(m_\pi) \Longrightarrow$ note rapid rise



Very encouraging but are all the systematic errors under control?

Outline

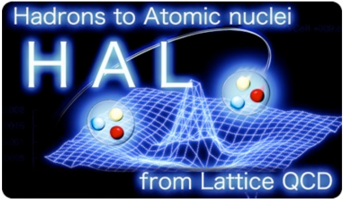
High partial waves and chiral symmetry

Lattice QCD

NN from LQCD (slides from recent talk by S. Aoki)

Extensions of the HAL QCD approach to inelastic and multi-particle scatterings in lattice QCD

Sinya Aoki
University of Tsukuba

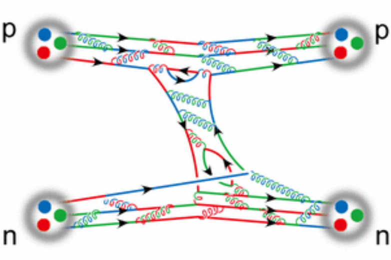


HAL QCD Collaboration

INT Workshop INT-15-53W
“Nuclear Reactions from Lattice QCD”
Institute for Nuclear Theory, University of Washington , Seattle, USA,
March 11-12, 2013

1. Introduction

HAL QCD approach to Nuclear Force



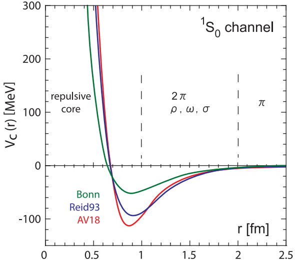
Potentials in QCD ?

What are “potentials” (quantum mechanical objects) in quantum field theories such as QCD ?

“Potentials” themselves can NOT be directly measured. cf. running coupling in QCD

scheme dependent, ambiguities in inelastic region

experimental data of scattering phase shifts → potentials, but not unique



“Potentials” are still useful tools to extract observables such as scattering phase shift.

↓

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

HAL QCD strategy

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Step 1

define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

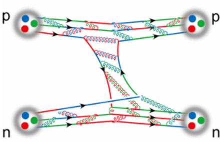
Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

$$W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

energy

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator



Key Property 1

Lin et al., 2001; CP-PACS, 2004/2005

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

$r = |\mathbf{r}| \to \infty$

$\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD !

How can we extract it ?

cf. Luescher's finite volume method

Step 2

define non-local but energy-independent “potential” as

$\mu = m_N/2$

reduced mass

$$\left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underline{U(\mathbf{x},\mathbf{y})}\varphi_{\mathbf{k}}(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

non-local potential

Key Property 2

A non-local but energy-independent potential can be constructed as

inner product

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\mathrm{th}}} \left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x})\eta^{-1}_{\mathbf{k},\mathbf{k}'}\varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

$\varphi_{\mathbf{k}}$ is linearly independent.

For $^\forall W_{\mathbf{p}} < W_{\mathrm{th}} = 2m_N + m_\pi$ (threshold energy)

$$\int d^3y\, U(\mathbf{x},\mathbf{y})\phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(x)\eta^{-1}_{\mathbf{k},\mathbf{k}'}\eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right]\varphi_{\mathbf{p}}(x)$$

Note 1: Potential satisfying this is not unique.

Note2: Non-relativistic approximation is NOT used. We just take the equal-time frame.

Step 3

expand the non-local potential in terms of derivative as $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta^3(\mathbf{x}-\mathbf{y})$

$$V(\mathbf{x}, \nabla) = \underset{\text{LO}}{V_0(r)} + \underset{\text{LO}}{V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})} + \underset{\text{LO}}{V_T(r)S_{12}} + \underset{\text{NLO}}{V_{\text{LS}}(r)\mathbf{L}\cdot\mathbf{S}} + \underset{\text{NNLO}}{O(\nabla^2)}$$

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$

$V_A(\mathbf{x})$ local and energy independent coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

Step 4

extract the local potential. At LO, for example, we simply have

$$V_{\mathrm{LO}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

Step 5

solve the Schroedinger Eq. in the infinite volume with this potential.

⇨ phase shifts and binding energy below inelastic threshold

(We can calculate the phase shift at all angular momentum.)

$\delta_L(k)$ exact by construction

$\delta_L(p \neq k)$ approximated one by the derivative expansion

expansion parameter

$$\frac{W_p - W_k}{W_{\mathrm{th}} - 2m_N} \simeq \frac{\Delta E_p}{m_\pi}$$

We can check a size of errors at LO of the expansion.
We can improve results by extracting higher order terms in the expansion.

2. Results from lattice QCD

Ishii *et al.* (HALQCD), PLB712(2012) 437.

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\}\underline{\overline{\mathcal{J}}(t_0)}|0\rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t)N(\mathbf{x},t)\} \sum_{n,s_1,s_2} \underline{|2N, W_n, s_1, s_2\rangle\langle 2N, W_n, s_1, s_2|}\overline{\mathcal{J}}(t_0)|0\rangle \ + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2}\varphi^{W_n}(\mathbf{r})e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2|\overline{\mathcal{J}}(0)|0\rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0\underline{\varphi^{W_0}(\mathbf{r})}e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii *et al.* (HALQCD), PLB712(2012) 437

normalized 4-pt Correlation function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

⇩

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + \underline{U} - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

energy-independent

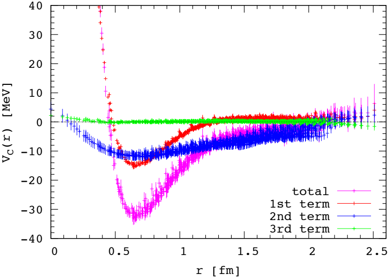
Leading Order

potential

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st **2nd** **3rd** **total**

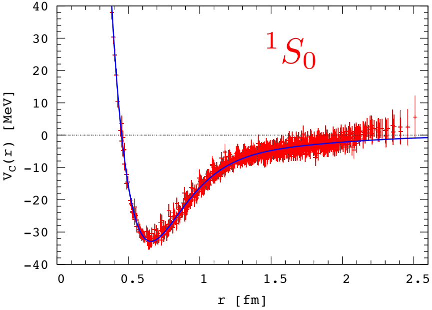
3rd term(relativistic correction) is negligible.



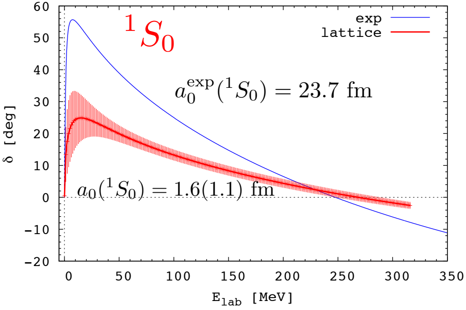
Ground state saturation is no more required. (advantage over finite volume method.)

2+1 flavor QCD, spin-singlet potential $m_\pi \simeq 700$ MeV a=0.09fm, L=2.9fm

NN potential



phase shift



Qualitative features of NN potential are reproduced.

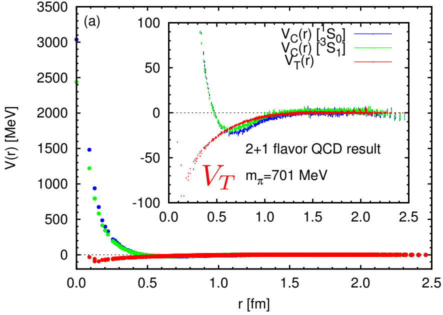
(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

It has a reasonable shape.
The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

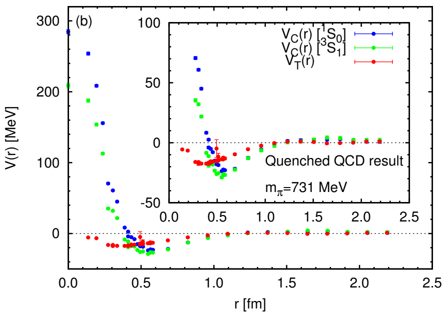
Potentials

full QCD



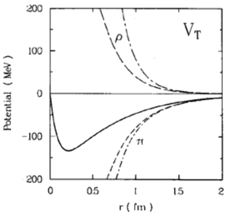
$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm

quenched QCD



$a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

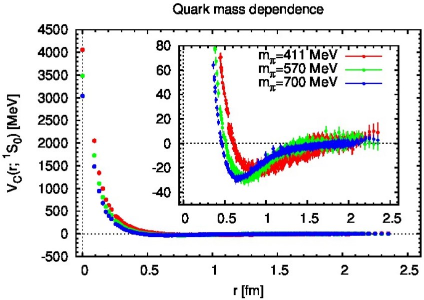
- no repulsive core in the tensor potential.
- the tensor potential is enhanced in full QCD

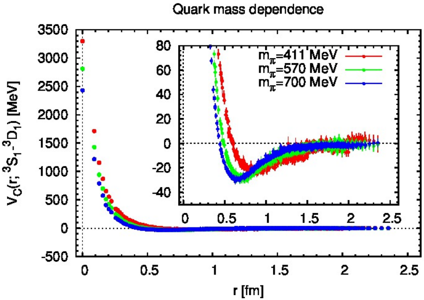


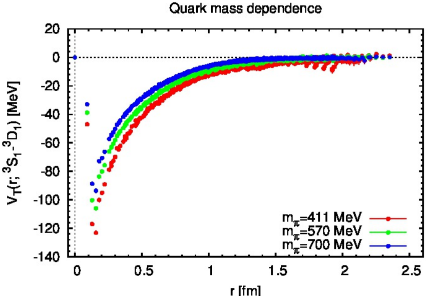
from
R.Machleidt,
Adv.Nucl.Phys.**19**

Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Quark mass dependence (full QCD)







- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?
- both repulsive core and attractive pocket are also grow as the pion mass decreases.

Potentials for the negative parity sector

$$V_{NN}^{(I)}(\vec{r},\vec{\nabla}) = \underbrace{V_0^{(I)}(r) + V_\sigma^{(I)}(r)\cdot(\vec{\sigma}_1\cdot\vec{\sigma}_2)} + \underset{\textbf{LO}}{V_T^{(I)}(r)\cdot S_{12}} + \underset{\textbf{NLO}}{V_{LS}^{(I)}(r)\cdot\vec{L}\cdot\vec{S}} + O(\nabla^2)$$

LO

$$V_C(r) \equiv V_0(r) + V_\sigma(r)\cdot(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$
$$= \begin{cases} V_0(r) - 3V_\sigma(r) & \text{for S=0} \\ V_0(r) + V_\sigma(r) & \text{for S=1} \end{cases}$$

S=0,P=+ (I=1)	S=1,P=+ (I=0)	S=0,P=– (I=0)	S=1,P=– (I=1)
$V_C(r)$	$V_C(r), V_T(r), V_{LS}(r)$	$V_C(r)$	$V_C(r), V_T(r), V_{LS}(r)$

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2{-}^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO

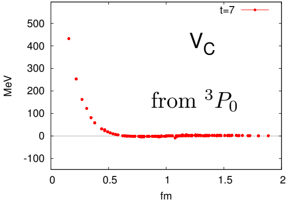
Preliminary Results (full QCD)

Murano et al. (HAL QCD), lat2012

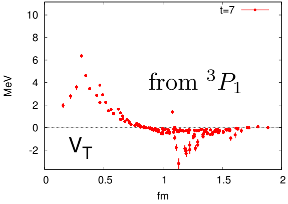
2-flavor QCD, a=0.16 fm

$m_\pi \simeq 1.1\ \text{GeV}$

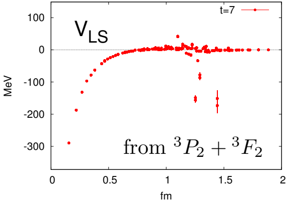
LO



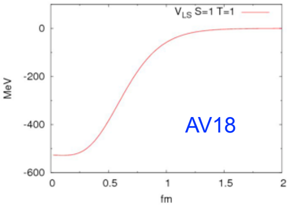
LO



NLO



Very weak !



HAL QCD Spin-orbit force from Lattice QCD

K. Murano et al, arXiv:1305.2293