Nuclear forces and their impact on structure, reactions and astrophysics

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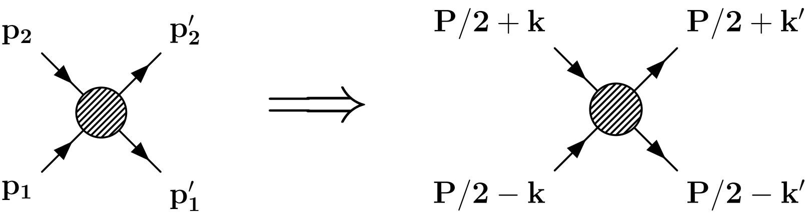
Lectures for Week 1

- **M.** QCD 1 (as); Scattering theory 1 (rjf)
- **T.** Nuclear forces 1 (rjf); Scattering theory 2 (as)
- **W.** Nuclear forces 2 (rjf); Renormalization and Universality (as)
- **Th.** Cold atoms and neutrons, QMC (ag);
  Tensor/spin-orbit forces, deuteron properties (rjf)
- **F.** QMC and chiral EFT interactions (ag);
  Three-body forces and halo nuclei (as)

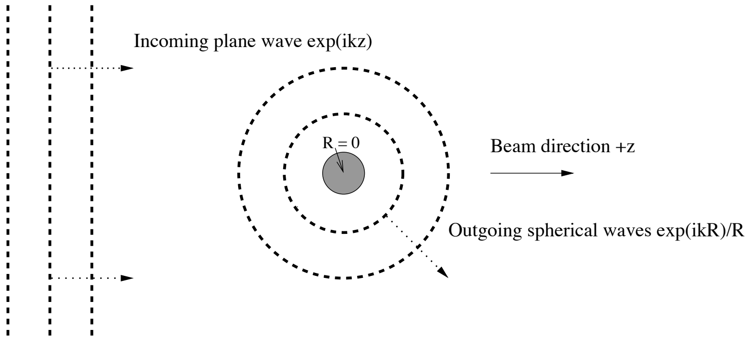
Outline

Scattering theory 1

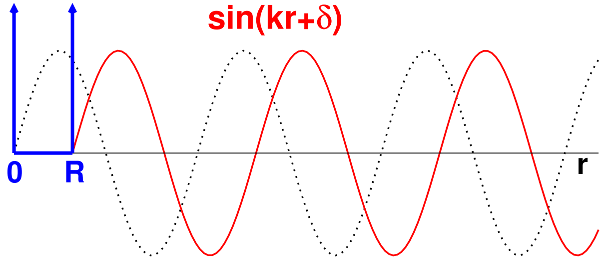
Kinematics for scattering in lab and relative coordinates

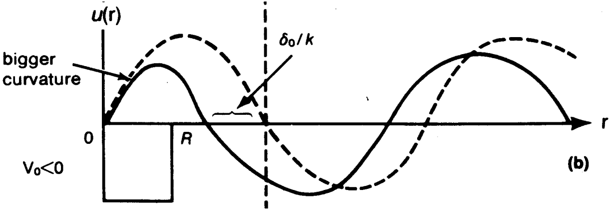


Scattering incident plane wave [From F. Nunes notes]



Repulsive and attractive phase shifts





Mathematica square well phase shifts: $n\pi$ ambiguity

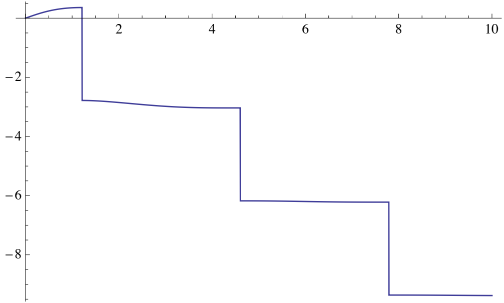
In[5]:=
```
deltaAnalytic[k_, V0_] :=
 ArcTan[Sqrt[Ek[k] / (Ek[k] + V0)] Tan[R Sqrt[2 μ (Ek[k] + V0)]]] - R Sqrt[2 μ Ek[k]]
```

Use Cell->Convert to->TraditionalForm to get it in a form easier to check we have entered it correctly:

$$\text{deltaAnalytic}(\text{k\_}, \text{V0\_}) := \tan^{-1}\left(\sqrt{\frac{\text{Ek}(k)}{\text{Ek}(k)+\text{V0}}}\tan\left(R\sqrt{2\,\mu\,(\text{Ek}(k)+\text{V0})}\right)\right) - R\sqrt{2\,\mu\,\text{Ek}(k)}$$

In[7]:=
```
Plot[deltaAnalytic[k, 0.5], {k, 0, 10}]
```

Out[7]=

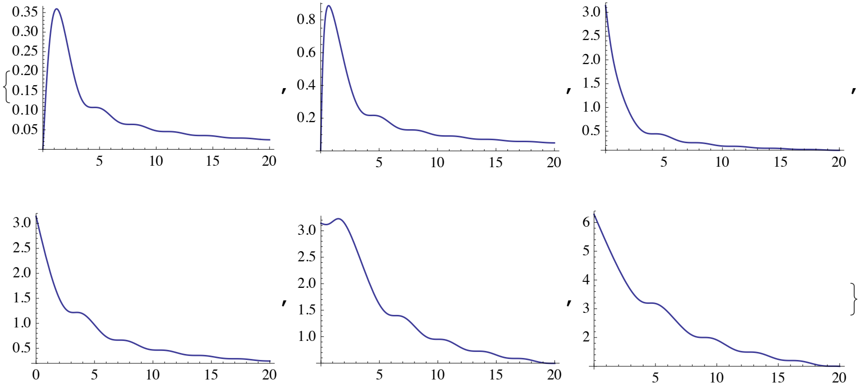


What is going on with the steps? Why are they there? Is the phase shift really discontinuous? How would you fix it?

Mathematica square well phase shifts: Levinson's theorem!

In[22]:=
```
Table[Plot[{deltaVPA[k, V0]}, {k, 0.001, 20}, PlotRange → Full],
 {V0, {0.5, 1.0, 2.0, 5.0, 10.0, 20.0}}]
(* Try for V0 = 0.5 to 20 *)
```

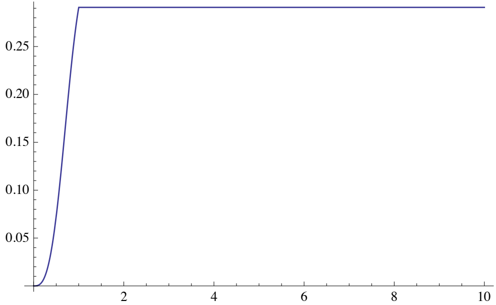
Out[22]=



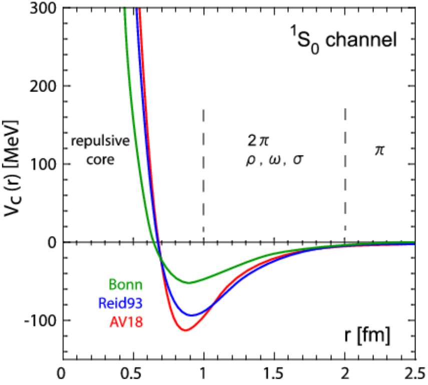
Variable Phase Approach equation solved in Mathematica

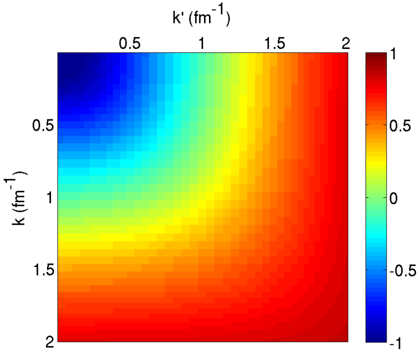
```
In[12]:= deltaVPA[k_, V0_] := (
   Rmax = 10; (* integrate out to Rmax; just need Rmax > R for square well *)
   ans = NDSolve[{deltarho'[r] == -(1 / k) 2 μ Vsw[r, V0] Sin[k r + deltarho[r]]^2,
      deltarho[0] == 0}, deltarho, {r, 0, Rmax}, AccuracyGoal → 6, PrecisionGoal → 6];
   (deltarho[r] /. ans)[[1]] /. r → Rmax (* evaluate at r=Rmax *)
  )

In[15]:= Plot[Evaluate[deltarho[r] /. ans], {r, 0, Rmax}]
```

Out[15]=



AV18


300
200
100
0
-100
Vc (r) [MeV]
0
0.5
1
1.5
2
2.5
r [fm]
$^1S_0$ channel
repulsive
core
2π
ρ, ω, σ
π
Bonn
Reid93
AV18


k' (fm^{-1})
0.5
1
1.5
2
k (fm^{-1})
1
0.5
0
-0.5
-1

Example: coordinate basis for *local* one-body potential

- Discretize $0 \le r \le R_{\max}$ with $r_i = i \times h$, where $h = R_{\max}/N$
- We can approximate the Schrödinger equation at point r_k as

$$-\frac{\hbar^2}{2M}\frac{u(r_k+h)-2u(r_k)+u(r_k-h)}{h^2} + V(r_k)u(r_k) = Eu(r_k) \; .$$

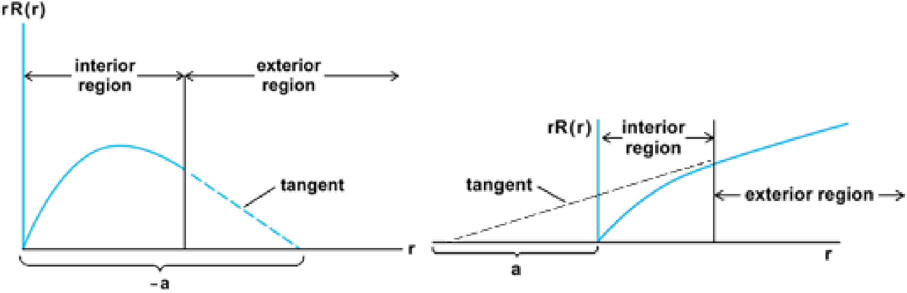
$$\text{or} \qquad -\frac{u_{k+1}-2u_k+u_{k-1}}{h^2} + V_k u_k = E u_k \; .$$

- In matrix form with $u_0 = 0$, $u_N \approx 0$, this is tri-diagonal ($\hbar = 2M = 1$):

$$\begin{pmatrix} \frac{2}{h^2}+V_1 & -\frac{1}{h^2} & 0 & \cdots & 0 \\ -\frac{1}{h^2} & \frac{2}{h^2}+V_2 & -\frac{1}{h^2} & & \vdots \\ 0 & -\frac{1}{h^2} & \ddots & & \vdots \\ \vdots & & & \ddots & -\frac{1}{h^2} \\ 0 & \cdots & \cdots & -\frac{1}{h^2} & \frac{2}{h^2}+V_{N-1} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ u_{N-1} \end{pmatrix} = E \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ u_{N-1} \end{pmatrix}$$

- If V is *non-local*, it has off-diagonal matrix elements in this basis

Identifying the S-wave scattering length a_0



From Filomena Nunes notes