Introduction to Effective Field Theories in QCD

bira

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Background by S. Hossenfelder

Outline

- Effective Field Theories
  - Introduction
  - What is Effective
  - Example: NRQED
  - Summary
- QCD at Low Energies
- Towards Nuclear Structure

References:

U. van Kolck, L.J. Abu-Raddad, and D.M. Cardamone,
Introduction to effective field theories in QCD,
in *New states of matter in hadronic interactions*
(Proceedings of the Pan American Advanced Studies Institute, 2002),
nucl-th/0205058

D.B. Kaplan,
Effective field theories,
Lectures at 7th Summer School in Nuclear Physics Symmetries,
Seattle, WA, 18-30 Jun 1995,
nucl-th/9506035

What Holds the Nucleus Together?

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind.

Hans A. Bethe 1953

“There are few problems in nuclear theoretical physics which have attracted more attention than that of trying to determine the fundamental interaction between two nucleons. It is also true that scarcely ever has the world of physics owed so little to so many …
… It is hard to believe that many of the authors are talking about the same problem or, in fact, that they know what the problem is.”

M. L. Goldberger

Midwestern Conference on Theoretical Physics, Purdue University, 1960

Nuclear Physics

The canons of tradition

I♪ Nuclei are essentially made out of non-relativistic nucleons (protons and neutrons), which interact via a potential

II♪ The potential is mostly two-nucleon, but there is evidence for smaller three-nucleon forces

III♪ Isospin is a good symmetry, except for a sizable breaking in two-nucleon scattering lengths and other, smaller effects

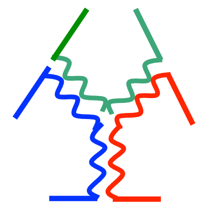
IV♪ External probes (*e.g.* photons) interact mainly with each nucleon, but there is evidence for smaller two-nucleon currents

but… WHY?

Quantum Chromodynamics

On the road to infrared slavery

1. Up, down quarks are relativistic, interacting via multi-gluon exchange
2. The interaction is a multi-quark process *e.g.*



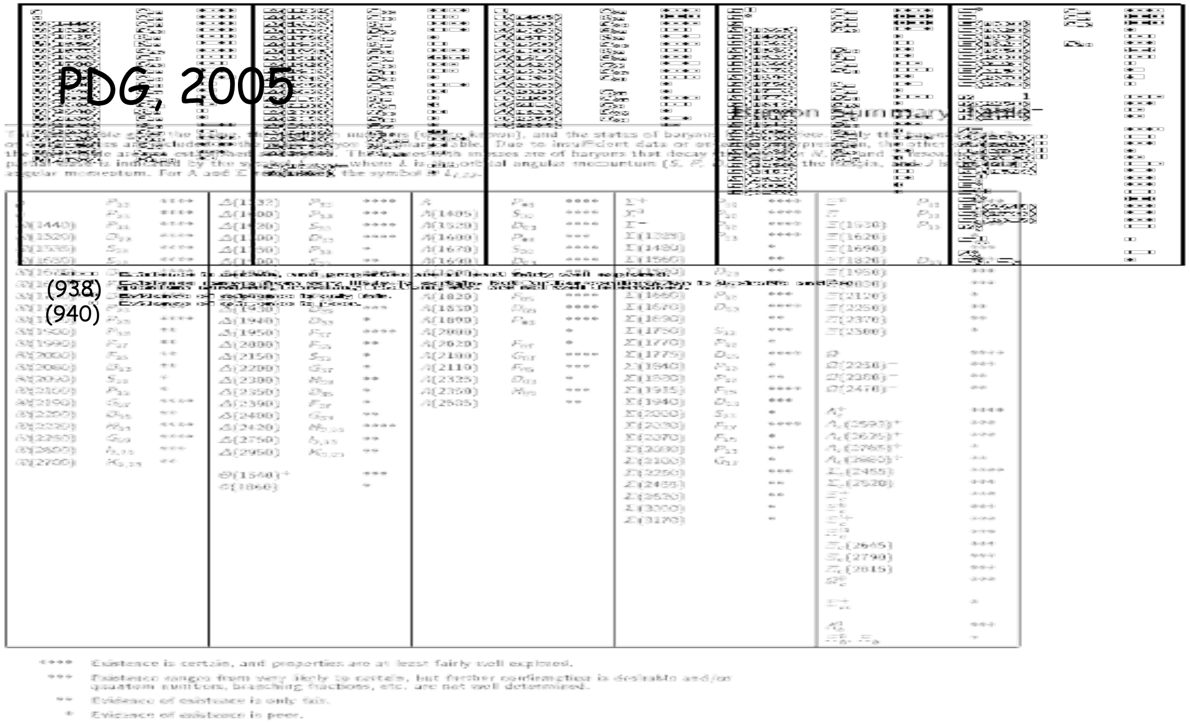
3. Isospin symmetry is not obvious: $\varepsilon = \frac{m_d - m_u}{m_d + m_u} \sim \frac{1}{3}$
4. External probes can interact with collection of quarks

difficulty

quarks and gluons **not** the most convenient

degrees of freedom at low energies

How does nuclear structure emerge from QCD?



Hadronic Scales

$$M_{QCD} \sim 1000 \text{ MeV}/c^2$$

Nucleus (g.s.)	J^P	I	$E\,(\mathrm{MeV})$	$\langle r^2 \rangle_{ch}^{1/2}\,(\mathrm{fm})$
$^2\mathrm{H}\,(d)$	1^+	0	-2.2246	$2.116(6)$
$^3\mathrm{H}\,(t)$	$\frac{1}{2}^+$	$\frac{1}{2}$	-8.482	$1.755(86)$
$^3\mathrm{He}$	$\frac{1}{2}^+$	$\frac{1}{2}$	-7.718	$1.959(34)$
$^4\mathrm{He}\,(\alpha)$	0^+	0	-28.296	$1.676(8)$
$^5\mathrm{He}$	$\frac{3}{2}^-$	$\frac{1}{2}$	$+0.9$	
$\cdots$			$\frac{E}{A}\,(\mathrm{MeV})$	$\frac{1}{k_F}\,(\mathrm{fm})$
Nuclear Matter	0^+	0	~ 16	0.73

Friar, '93

Nuclear Scales

$$Q \sim M_{nuc}c \sim 100\ \mathrm{MeV}/c$$

$$E \sim \frac{M_{nuc}^2}{M_{QCD}}c^2 \sim 10\ \mathrm{MeV}$$

$$r \sim \frac{\hbar}{M_{nuc}c} \sim 2\ \mathrm{fm}$$

Multi-scale problems

H atom

$$H = \left(\frac{p^2}{2m_e} - \frac{\alpha\hbar c}{r}\right)\left[1 + O\left(\frac{p^2}{m_e^2 c^2}; \frac{\hbar^2}{m_e^2 c^2 r^2}\right)\right] \qquad \alpha \equiv \frac{e^2}{4\pi\hbar c} \cong \frac{1}{137} \ll 1$$

$$\left.\begin{array}{l} r \sim R \\ p \sim \dfrac{\hbar}{R} \end{array}\right\} E(R) \sim \left(\frac{\hbar^2}{2m_e R^2} - \frac{e^2}{4\pi R}\right)$$

$$\frac{dE(R)}{dR} = 0 \Rightarrow R = \frac{\hbar}{\alpha m_e c}$$

Three scales

$$m_e c^2 = 0.5 \text{ MeV}$$

$$pc \sim \alpha m_e c^2 = 3.6 \text{ keV}$$

$$-E \sim \frac{p^2}{2m_e} \sim \frac{1}{2}\alpha^2 m_e c^2 = 13.6 \text{ eV}$$

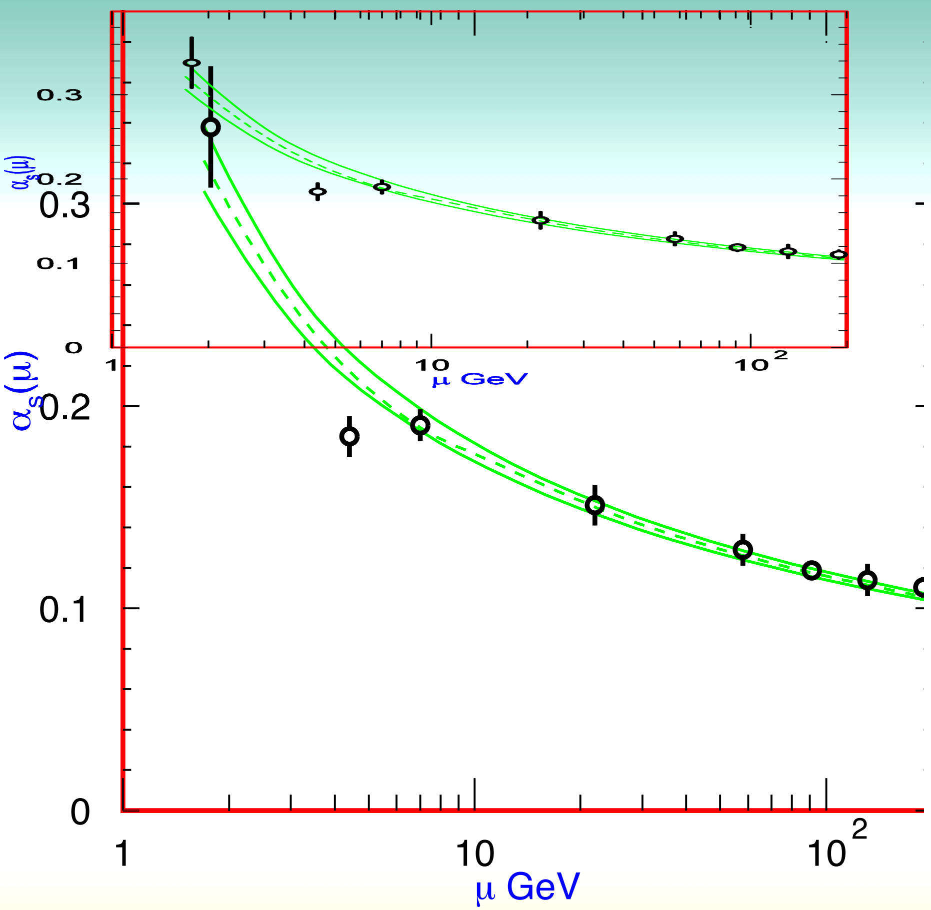
(each step down: α, α)

(from now on, units such that $\hbar = 1, c = 1$)

However…

no obvious small coupling
in nuclear forces.

QCD
"fine-structure"
constant



Needed: method that does <u>not</u>

rely on small couplings

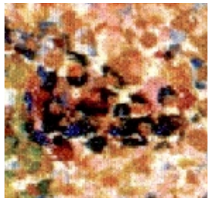
➡ EFFECTIVE FIELD THEORY

I do not believe that scientific progress is always best advanced by keeping an altogether open mind. It is often necessary to forget one's doubts and to follow the consequences of one's assumptions wherever they may lead ---the great thing is not to be free of theoretical prejudices, but to have the right theoretical prejudices. And always, the test of any theoretical preconception is in where it leads.

S. Weinberg, *The First Three Minutes*
1972

Ingredients

- Relevant degrees of freedom



Ingredients

- Relevant degrees of freedom

choose the coordinates that fit the problem

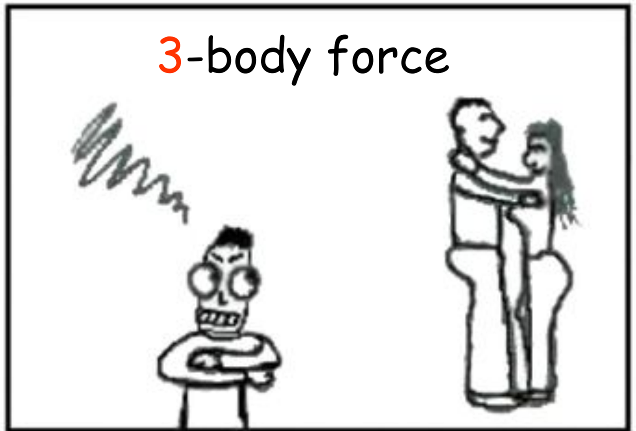
- All possible interactions

Example: Earth-moon-satellite system

$R_m \simeq 1.7$ Mm $\quad d \simeq 384$ Mm $\quad R_E \simeq 6.4$ Mm

2-body forces → 2+3-body forces

change in resolution



Wikipedia

Ingredients

- Relevant degrees of freedom

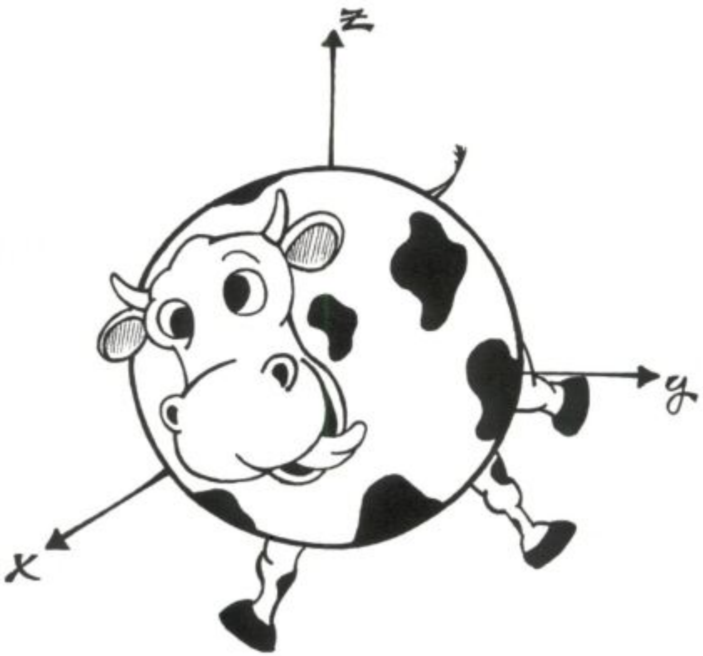
choose the coordinates that fit the problem

- All possible interactions

what is not forbidden is compulsory

- Symmetries

A farmer is having trouble with a cow whose milk has gone sour. He asks three scientists—a biologist, a chemist, and a physicist—to help him. The biologist figures the cow must be sick or have some kind of infection, but none of the antibiotics he gives the cow work. Then, the chemist supposes that there must be a chemical imbalance affecting the production of milk, but none of the solutions he proposes do any good either. Finally, the physicist comes in and says, "First, we assume a spherical cow…"



$$\sum_{ij} \alpha_{ij} u_i v_j \rightarrow \vec{u} \cdot \vec{v} \qquad + \sum_{ij} \delta\alpha_{ij} u_i v_j$$

no, say, $u_1 v_2$

$$|\delta\alpha_{ij}| \ll 1$$

amenable to perturbation theory

Ingredients

- Relevant degrees of freedom

choose the coordinates that fit the problem

- All possible interactions

what is not forbidden is compulsory

- Symmetries

not everything is allowed

- Naturalness

't Hooft '79

After scales have been identified,

the remaining, dimensionless parameters are

$$\mathbf{O}(1)$$

unless suppressed by a symmetry

cow
non-sphericity…

Occam's razor:

simplest assumption, to be revised if necessary

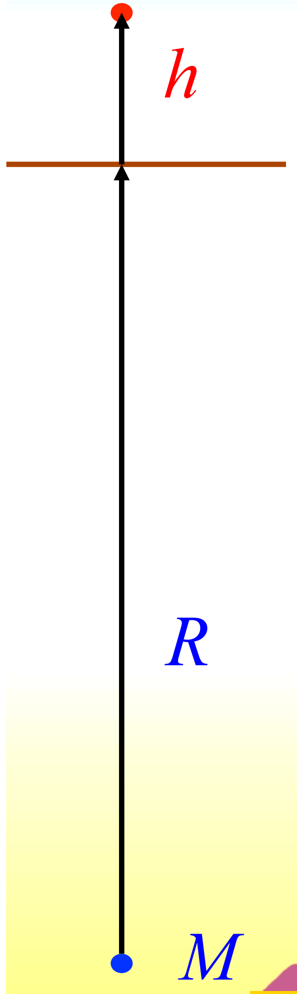
fine-tuning

➡ Expansion in powers of $\frac{E}{E_{und}}$

E: energy of probe

E_{und}: energy scale of underlying theory

A classical example: the flat Earth -- light object near surface of a large body



$$E \sim mgh \ll E_{und} \equiv mgR$$

d.o.f.: mass m

sym: $V_{eff}(h,x,y) = V_{eff}(h)$

$$V_{eff}(h) = m\sum_{i=0}^{\infty} g_i h^i = \text{const} + mg\left\{h + \eta h^2 + \ldots\right\}$$

parameters

(neglecting quantum corrections…)

naturalness: $\dfrac{m g_{i+1} h^{i+1}}{m g_i h^i} = \dfrac{E}{E_{und}} \times O(1) = \dfrac{h}{R} \times O(1) \Rightarrow g_{i+1} = O\left(\dfrac{g}{R^i}\right)$

$$V_{eff}(h) = -GMm\frac{1}{R+h} = m\left(\frac{GM}{R^2}\right)\sum_{i=0}^{\infty}\left(\frac{-1}{R}\right)^{i-1} h^i \Rightarrow g_{i+1} = (-1)^i \frac{g}{R^i}$$

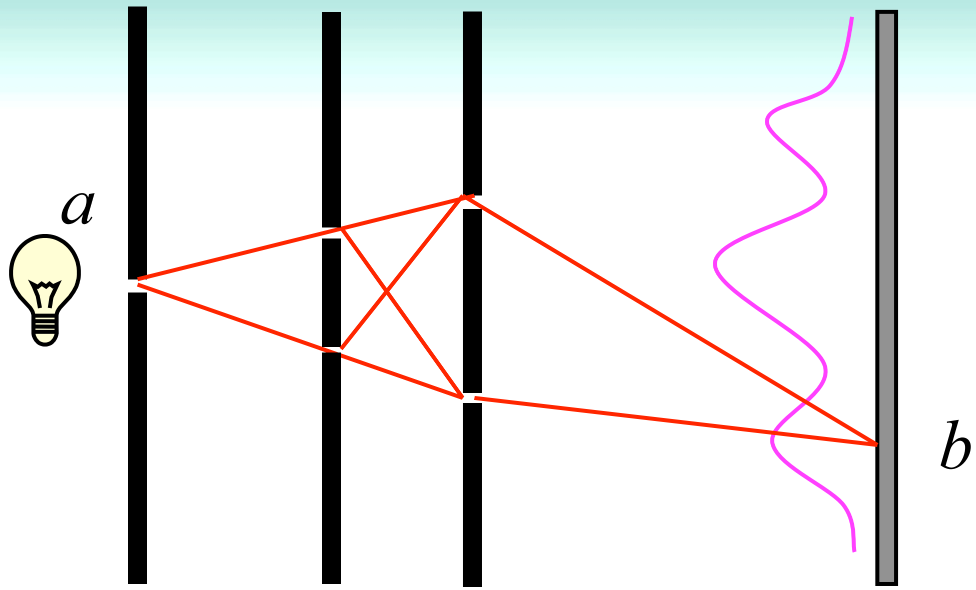
$h \ll R$

$\equiv g$

itself the first term in a low-energy EFT of general relativity…

Going a bit deeper...

A short path to quantum mechanics



$$P = \left|A_1 + A_2 + A_3 + A_4\right|^2$$

sum over
all paths

RULE

$$A_i \propto \exp\left(i\int_a^b dt\, \mathbf{L}(q(t))\right)$$

each path contributes a phase
given by the classical action

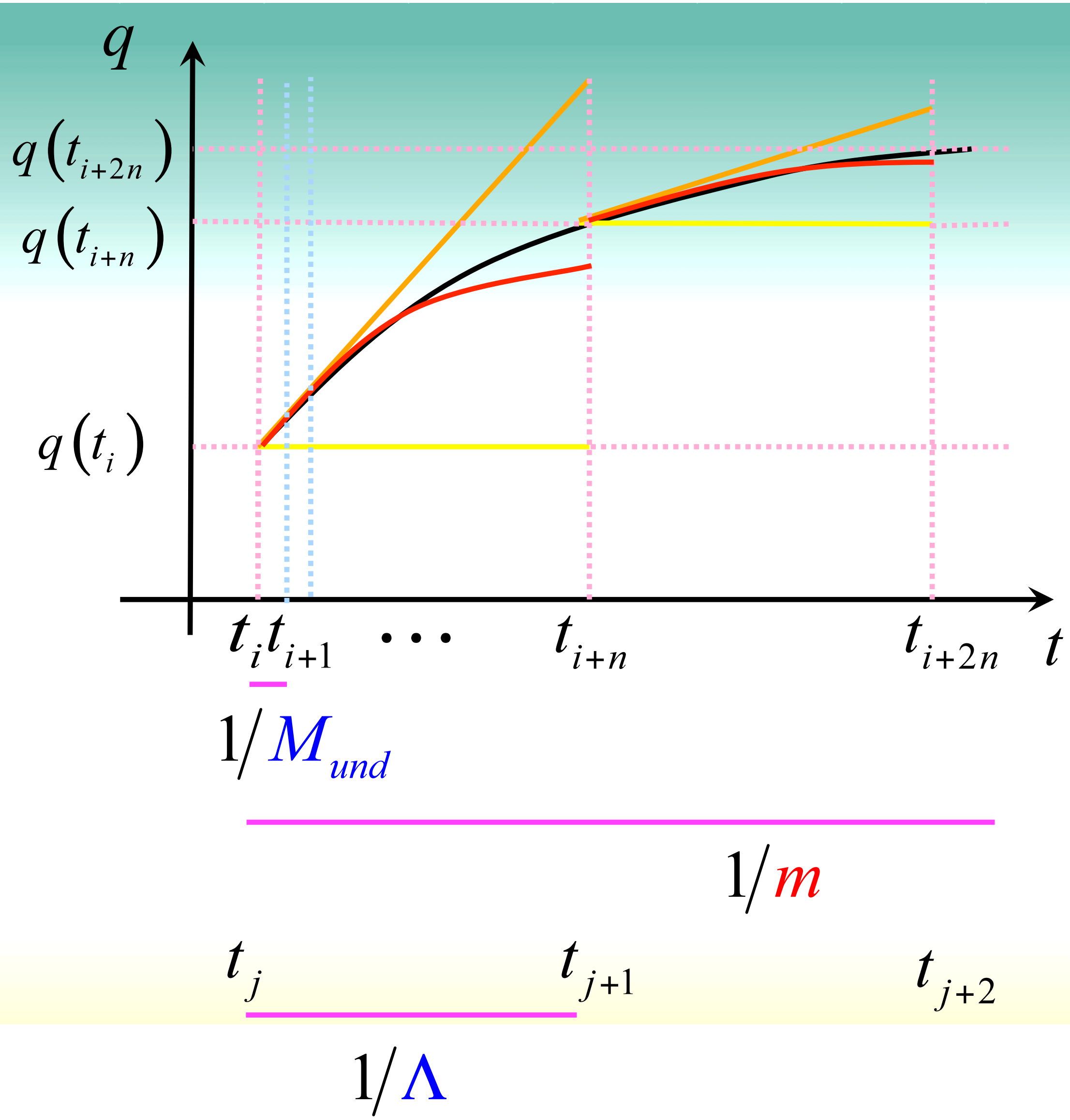
Path Integral Feynman '48

$$A = \int Dq \exp\left(i\int dt\, \mathbf{L}(q(t))\right)$$

$$\prod_i \int dq(t_i)$$

classical path $\delta\left(\int dt\, \mathbf{L}(q(t))\right) = 0$

EFFECTIVE THEORY



scale of fine-structure of dynamics

scale of variation of long-range dynamics

coarse-graining scale (cutoff)

$$\mathbf{L}\left(q\left(t_i\right)\right) \rightarrow \mathbf{L}\left(q\left(t_i\right) + \left.\frac{dq}{dt}\right|_{t_i}\left(t - t_i\right) + \frac{1}{2}\left.\frac{d^2 q}{dt^2}\right|_{t_i}\left(t - t_i\right)^2 + \ldots\right)$$

QM + special relativity: quantum field theory

$$q(t) \to \varphi(\vec{r},t) \equiv \varphi(x)$$

$$dt \to dt\, d^3 r \equiv d^4 x$$

$$\frac{d}{dt} \to \frac{\partial}{\partial x^\mu}$$

EFFECTIVE FIELD THEORIES

Partition function

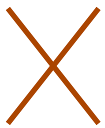
$$Z = \int D\varphi\ \exp\left(i\int d^4x\ \left\{\mathbf{L}_{\text{free}}(\varphi) + \mathbf{L}_{\text{int}}(\varphi)\right\}\right)$$

$$= \int D\varphi\ \left\{+ i\int d^4x\mathbf{L}_{\text{int}}(\varphi) + \left[i\int d^4x\mathbf{L}_{\text{int}}(\varphi)\right]^2 + \ldots\right\}\exp\left(i\int d^4x\mathbf{L}_{\text{free}}(\varphi)\right)$$

momentum space

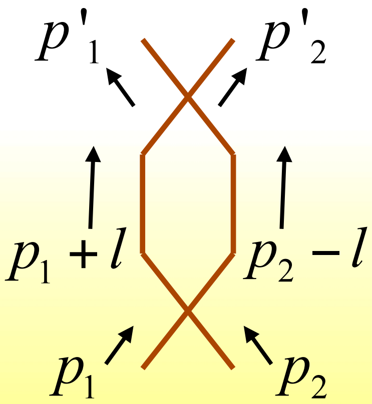
(skip many steps...)

$$\mathbf{L}_{\text{int}} = \frac{\lambda}{4}\varphi^4$$



$= i\lambda$

$$= \frac{i}{p^2 - m^2 + i\varepsilon}$$



$$= \int \frac{d^4 l}{(2\pi)^4}\ i\lambda\ \frac{i}{(p_1 + l)^2 - m^2 + i\varepsilon}\ \frac{i}{(p_2 - l)^2 - m^2 + i\varepsilon}\ i\lambda$$

$= \ldots$

but divergent from high-momentum region...

➡ needs a cutoff to separate high and low momenta

N.B. This is NOT tied to relativity, as we'll see explicitly in lecture 2

What is Effective?

Euler + Heisenberg '36
Weinberg '67 … '79
Wilson, early 70s
…

E

$\phi_H \; (Q > M)$

M

Λ

$\phi_L \; (Q < M)$

m

$$Z = \int D\phi_H \int D\phi_L \exp\left(i\int d^4x\, \mathbf{L}_{und}(\phi_H,\phi_L)\right)$$

$$\times \int D\varphi\, \delta\left(\varphi - f_\Lambda(\phi_L)\right)$$

$$= \int D\varphi \exp\left(i\int d^4x\, \mathbf{L}_{EFT}(\varphi)\right)$$

$$\mathbf{L}_{EFT} = \sum_{d=0}^{\infty} \sum_{i(d,n)} c_i(M,\Lambda)\, O_i\left((\partial, m)^d \varphi^n\right)$$

renormalization-group invariance $\quad \dfrac{\partial Z}{\partial \Lambda} = 0$

details of the underlying dynamics

local

$\phi_H : \Delta x \sim \dfrac{1}{Q} < \dfrac{1}{M}$

$\phi_L : \Delta x \sim \dfrac{1}{Q} > \dfrac{1}{M}$

underlying symmetries

characteristic external momentum

$$\begin{cases} T = T^{(\infty)}(Q) \sim N(M) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_i \tilde{c}_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right) \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

normalization

non-analytic, from loops

$$\nu = \nu(d, n, \ldots)$$

"power counting"

e.g. # loops L

For $Q \sim m$, truncate consistently with RG invariance

so as to allow systematic improvement (perturbation theory):

$$T = T^{(\bar{\nu})} + \mathbf{O}\left(N\left(\frac{Q}{M},\frac{Q}{\Lambda}\right)^{\bar{\nu}+1}\right) \qquad \frac{\partial T^{(\bar{\nu})}}{\partial \ln \Lambda} = \mathbf{O}\left(T^{(\bar{\nu})}\frac{Q}{\Lambda}\right)$$

Why is this useful?

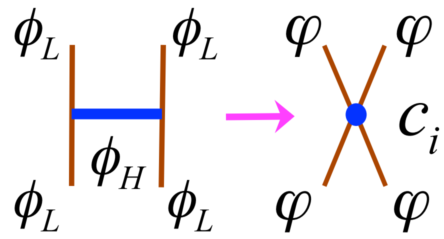
Because in general the appropriate degrees of freedom below M are not the same as above

φ

$\phi = (\phi_H, \phi_L)$

Examples:

- M is mass of physical particle -- virtual exchange in coefficients c_i (Appelquist-Carazzone decoupling theorem)



- M is scale associated with breaking of continuous symmetry -- appearance of massless Goldstone bosons or gauge-boson mass (Goldstone's theorem, Higgs mechanism)
- M is scale of confinement -- rearrangement of whole spectrum
- M is radius of Fermi surface -- BCS behavior

How can we do it? Two possibilities:

- know and can solve underlying theory --
  get c_i 's in terms of parameters in $\mathbf{L}_{und}(\phi_H, \phi_L)$
- know but cannot solve, or do not know, underlying theory --
  invoke Weinberg's "theorem": Weinberg '79

"The quantum field theory generated by the most general Lagrangian with some assumed symmetries will produce the most general S matrix incorporating quantum mechanics, Lorentz invariance, unitarity, cluster decomposition and those symmetries, with no further physical content."

Note: proven only for scalar field with Z_2 symmetry in E_4,
but no known counterexamples

Ball + Thorne '94

Bira's EFT Recipe

1. identify degrees of freedom and symmetries
2. construct most general Lagrangian
3. run the methods of field theory

what is not forbidden
is mandatory!

- compute Feynman diagrams with all momenta $Q < \Lambda$ ("regularization")
- relate $c_i(\Lambda), \Lambda$ to observables, which should be independent of Λ ("renormalization")

not a model form factor

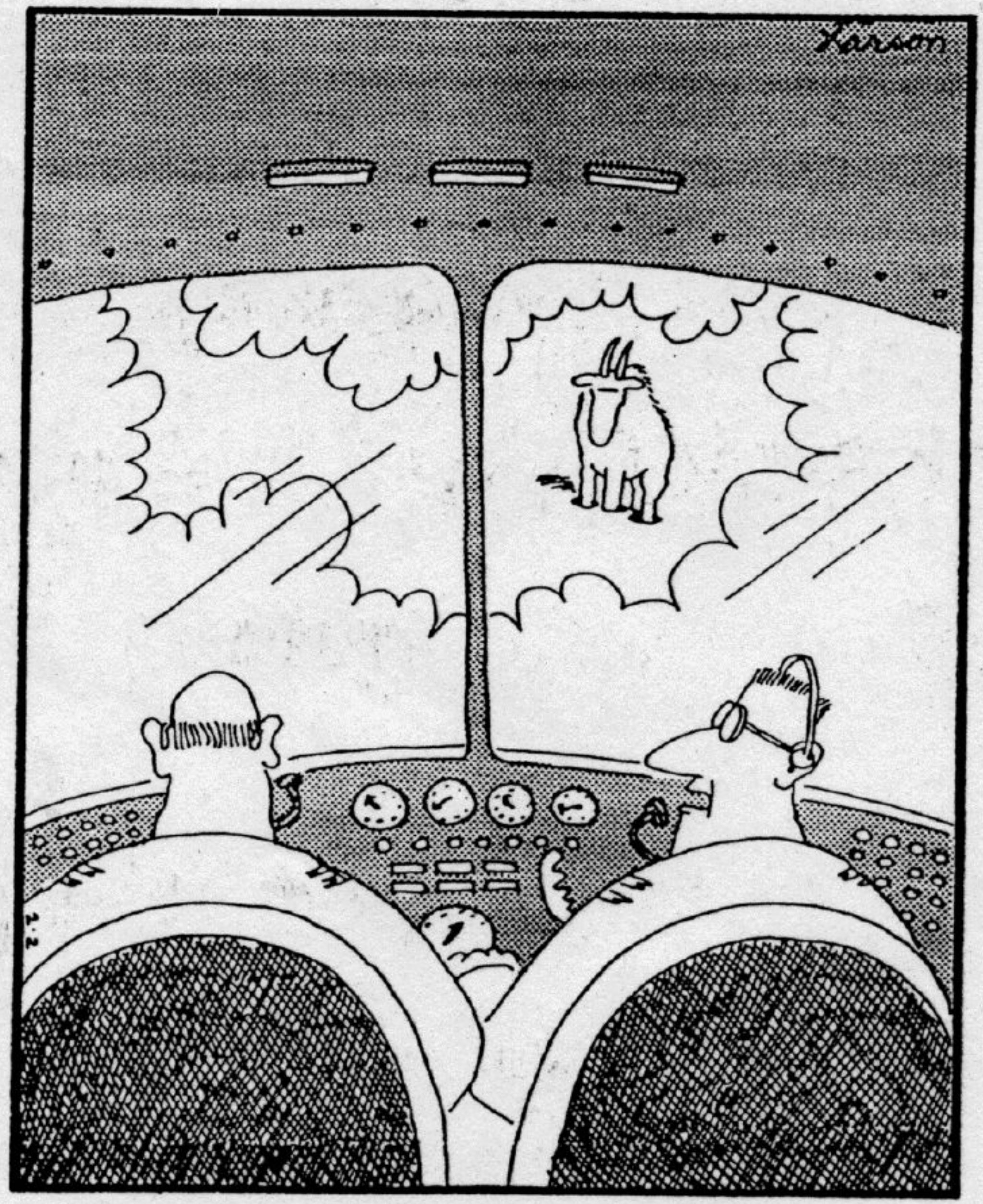
➡ controlled expansion in $\frac{Q}{M} \times \mathbf{O}(1)$

"naturalness": what else?
unless suppressed by symmetry…

contrast to models, which have fewer, but *ad hoc,* interactions;
useful in the identification of relevant degrees of freedom and symmetries,
but plagued with uncontrolled errors

A significant change in physicists' attitude towards what should be taken as a guiding principle in theory construction is taking place in recent years in the context of the development of EFT. For many years (…) renormalizability has been taken as a necessary requirement. Now, considering the fact that experiments can probe only a limited range of energies, it seems natural to take EFT as a general framework for analyzing experimental results.

T.Y. Cao, in *Renormalization, From Lorentz to Landau (and Beyond),* L.M. Brown (ed) 1993



"Say . . . What's a mountain goat doing way up here in a cloud bank?"

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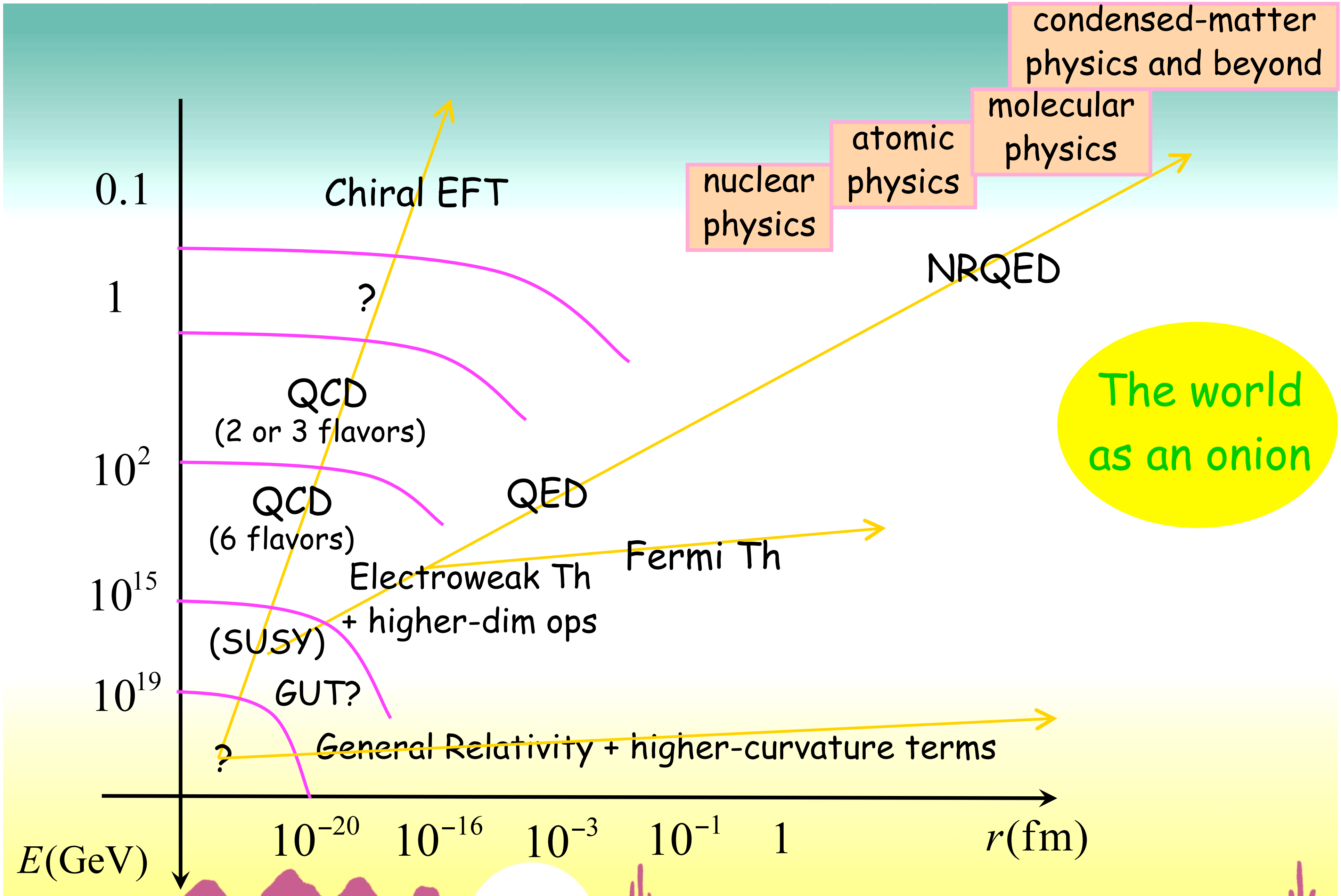
Time for a paradigm change, perhaps?


condensed-matter physics and beyond
molecular physics
atomic physics
nuclear physics
0.1
Chiral EFT
1
?
NRQED
QCD
(2 or 3 flavors)
The world as an onion
10^2
QCD
(6 flavors)
QED
Fermi Th
Electroweak Th
+ higher-dim ops
10^{15}
(SUSY)
10^{19}
GUT?
?
General Relativity + higher-curvature terms
10^{-20}
10^{-16}
10^{-3}
10^{-1}
1
r(fm)
E(GeV)

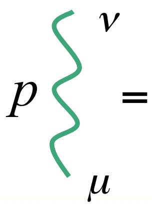
A quantum example: non-relativistic QED (NRQED)

- single fermion ψ of mass M, massless spin-1 boson A_μ
- Lorentz, parity, time-reversal, and U(1) gauge invariance

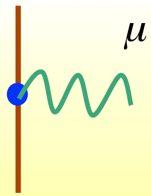
$$
\left[\begin{array}{l} D_\mu = \partial_\mu - ieA_\mu \\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \end{array}\right]
$$

$$
\mathbf{L}_{und} = \bar{\psi}\left(i\not{D} - M\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
$$

$$= \frac{i}{\not{p} - M + i\varepsilon}$$



$$= \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$

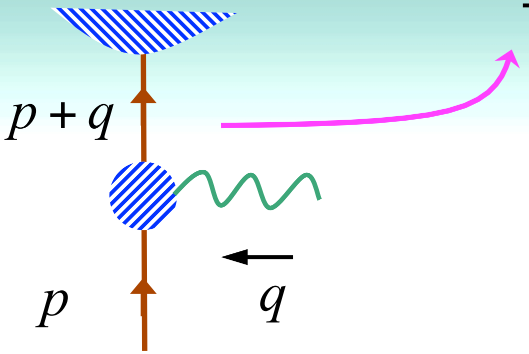


$$= ie\gamma_\mu$$

⇒ interactions $\propto e = \sqrt{4\pi\alpha} \sim \frac{1}{3}$ perturbation theory

How do E&M bound states arise?

$Q \ll M$



$$= \frac{i}{\not p + \not q - M + i\varepsilon} = \frac{i\left(p^0\gamma^0 - \vec{p}\cdot\vec{\gamma} + \not q + M\right)}{\left(p^0 + q^0\right)^2 - \left(\vec{p} + \vec{q}\right)^2 - M^2 + i\varepsilon}$$

$$= \frac{i\left(p^0\gamma^0 + M - \vec{p}\cdot\vec{\gamma} + \not q\right)}{2p^0q^0 + q^{02} - 2\vec{p}\cdot\vec{q} - \vec{q}^{\,2} + i\varepsilon}$$

$$= \frac{i}{q^0 + i\varepsilon}\,\frac{\left(1+\gamma^0\right)}{2} + \ldots$$

$$|\vec{p}| \sim |\vec{q}| = \mathbf{O}(Q)$$

$$q^0 = |\vec{q}| = \mathbf{O}(Q)$$

$$p^0 = \sqrt{\vec{p}^{\,2} + M^2} = M + \mathbf{O}\left(\frac{Q^2}{M}\right)$$

$$P_\pm \equiv \frac{1 \pm \gamma^0}{2} \qquad P_\pm P_\pm = P_\pm, \quad P_\pm P_\mp = 0$$

projector onto ± energy states

Georgi '90

"heavy-fermion formalism"

$$\Psi_\pm \equiv e^{iMt} P_\pm \psi \Leftrightarrow \psi = \left(P_+ + P_-\right)\psi = e^{-iMt}\left(\Psi_+ + \Psi_-\right)$$

particles:	annihilates	creates
antiparticles:	creates	annihilates

$$\mathbf{L}_{und} = \bar{\Psi}_+ iD_0 \Psi_+ - \bar{\Psi}_- i\vec{\gamma}\cdot\vec{D}\Psi_+ + \bar{\Psi}_+ i\vec{\gamma}\cdot\vec{D}\Psi_- - \bar{\Psi}_-\left(iD_0 + 2M\right)\Psi_- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

+ other, heavy d.o.f.s

$$Z = \int DA \int D\Psi_+ \int D\Psi_- \exp\left(i\int d^4x\, \mathbf{L}_{und}(\Psi_+, \Psi_-, A)\right) \times \int D\Psi\, \delta\left(\Psi - \Psi_+\right)$$

$$= \int DA \int D\Psi \, \exp\left(i\int d^4x\, \mathbf{L}_{EFT}(\Psi, A)\right)$$

complete square,
do Gaussian integral

$$\mathbf{L}_{EFT} = \bar{\Psi}\, iD_0 \Psi + \frac{1}{2M}\bar{\Psi}\vec{D}^2\Psi + \frac{e}{2M}\bar{\Psi}\sigma_i\Psi\,\varepsilon_{ijk}F^{jk} + \ldots - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \ldots$$

non-relativistic expansion

Pauli term

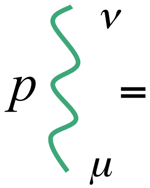
$$+ \frac{e\kappa}{2M}\bar{\Psi}\sigma_i\Psi\,\varepsilon_{ijk}F^{jk} + \ldots$$

anomalous magnetic moment
=O(1)

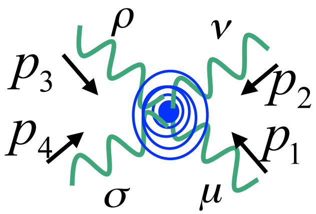
most general Lag with Y, A
invariant under U(1) gauge, parity, time-reversal,
and Lorentz transformations

$$\mathcal{L}_{EFT} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a}{M^4}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{b}{M^4}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2 + \ldots$$

Euler + Heisenberg '36



$$= \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$



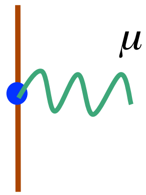
$$= \frac{i}{M^4}\left\{a\left[\eta_{\mu\rho}\eta_{\nu\sigma}\, p_1\cdot p_3\, p_2\cdot p_4 + \ldots\right] + b\left[\ldots\right]\right\}$$

$$+\bar{\Psi}\, i\, \mathbf{v}\cdot D\,\Psi + \frac{1}{2M}\bar{\Psi}\left(\left(\mathbf{v}\cdot D\right)^2 - D^2\right)\Psi + \frac{e}{M}(1+\kappa)\bar{\Psi}\, \mathbf{v}_\alpha S_\beta \Psi\, \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} + \ldots$$

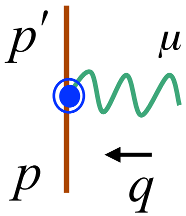
$$\mathbf{v} \equiv \left(1, \vec{0}\right)$$

$$S \equiv \left(0, \frac{\vec{\sigma}}{2}\right)$$

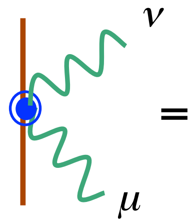
$$= \frac{i}{\mathbf{v}\cdot p + \frac{1}{2M}\left(p^2 - (\mathbf{v}\cdot p)^2\right) + \ldots + i\varepsilon}$$



$$= ie\,\mathbf{v}_\mu$$

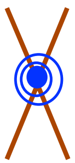


$$= \frac{e}{2M}\left\{i\left(p + p'\right)_\mu + 2(1+\kappa)\varepsilon_{\mu\nu\alpha\beta}\mathbf{v}^\gamma S^\alpha q^\beta\right\}$$



$$= i\frac{e^2}{M}\left(\eta_{\mu\nu} - \mathbf{v}_\mu \mathbf{v}_\nu\right)$$

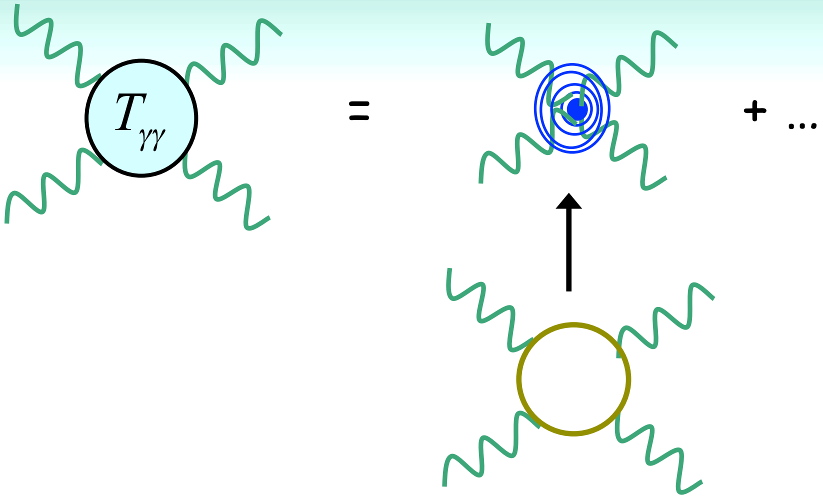
$$+\frac{\gamma_0^{(0)}}{M^2}\bar{\Psi}\Psi\,\bar{\Psi}\Psi + \frac{\gamma_0^{(1)}}{M^2}\bar{\Psi}S\Psi\cdot\bar{\Psi}S\Psi + \ldots$$



$$= \frac{i}{M^2}\left(\gamma_0^{(0)} + \gamma_0^{(1)} S_1\cdot S_2\right)$$

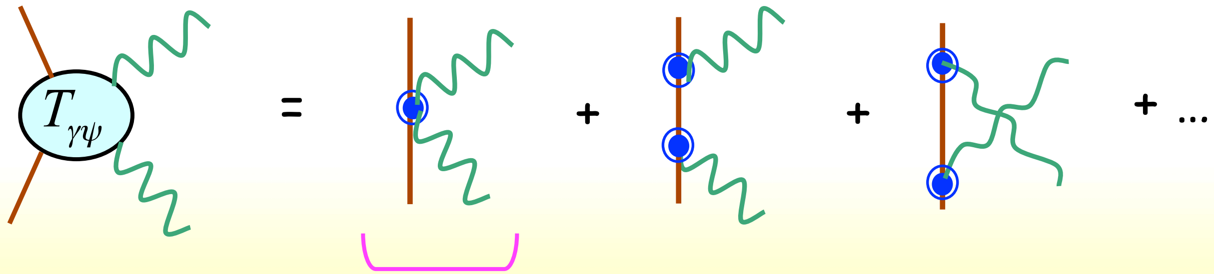
etc.

Various processes at low energies: e.g.



light-by-light scattering

no explicit
fermion-antifermion pair creation!



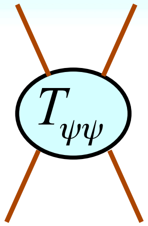
Compton scattering

Thompson
limit

no change in
heavy-fermion number!

Back to atomic bound states: the NRQED perspective

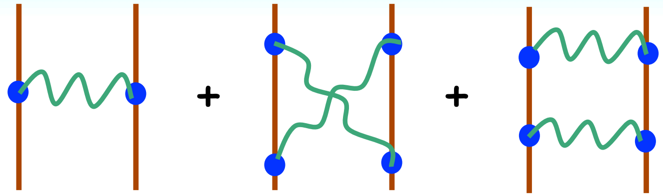
$\left(p'^0, \vec{p}\,'\right)$ $\left(p'^0, -\vec{p}\,'\right)$

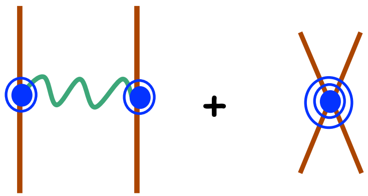


$\left(p^0, \vec{p}\right)$ CoM frame $\left(p^0, -\vec{p}\right)$

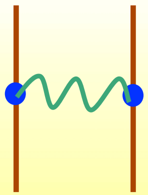
$|\vec{p}| \sim |\vec{p}\,'| = \mathbf{O}(Q)$

$p^0 \sim p'^0 = \mathbf{O}\left(\dfrac{Q^2}{M}\right)$

=  + ...

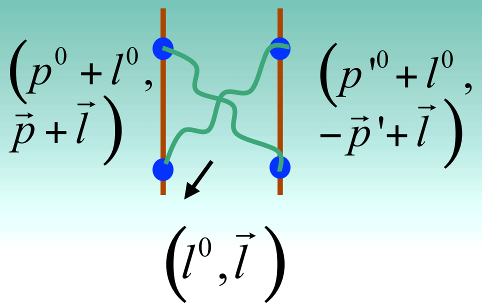
+  + ...

higher powers of $\dfrac{Q}{M}$

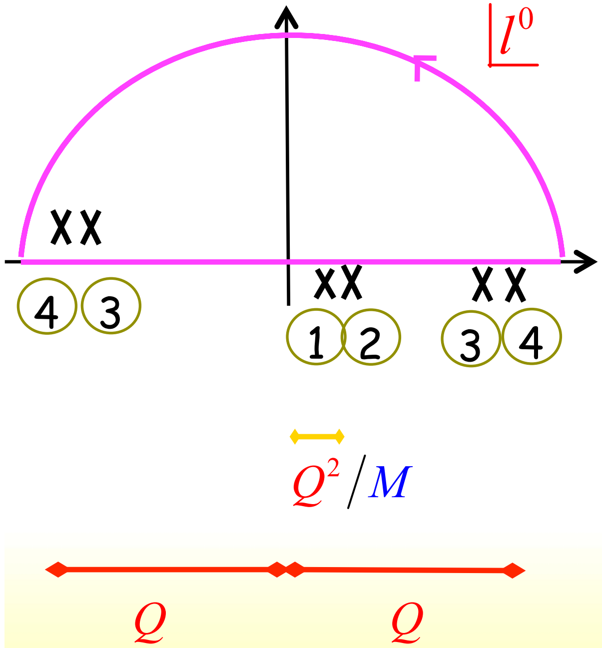


$$= \frac{i e^2}{(p-p')^2 + i\varepsilon} = \frac{-i e^2}{\left(p^0 - p'^0\right)^2 - (\vec{p}-\vec{p}\,')^2 + i\varepsilon} \simeq \frac{i e^2}{(\vec{p}-\vec{p}\,')^2 - i\varepsilon} \sim \frac{4\pi\alpha}{Q^2}$$

$$\rightarrow V(r) = \frac{\alpha}{r}$$



$$= e^4\int \frac{d^4l}{(2\pi)^4}\,\underbrace{\frac{1}{l^0+p^0-\frac{(\vec l+\vec p)^2}{2M}+i\varepsilon}}_{①}\,\underbrace{\frac{1}{l^0+p'^0-\frac{(\vec l-\vec p')^2}{2M}+i\varepsilon}}_{②}\,\underbrace{\frac{1}{\left(p^0-p'^0+l^0\right)^2-\left(\vec p-\vec p'+\vec l\right)^2+i\varepsilon}}_{③}\,\underbrace{\frac{1}{l^{02}-\vec l^{\,2}+i\varepsilon}}_{④}$$

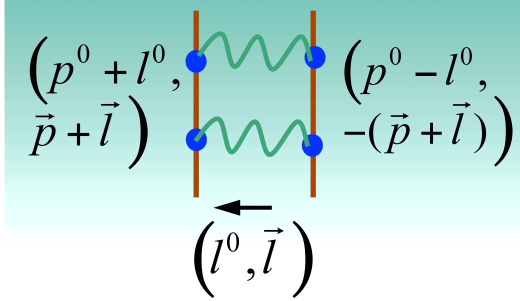


$$= i\,e^4\int \frac{d^3l}{(2\pi)^3}\,\frac{1}{|\vec l|-p^0+\frac{(\vec l+\vec p)^2}{2M}-i\varepsilon}\,\frac{1}{|\vec l|-p'^0+\frac{(\vec l-\vec p')^2}{2M}-i\varepsilon}\,\frac{1}{\left(p^0-p'^0-|\vec l|\right)^2-\left(\vec p-\vec p'+\vec l\right)^2+i\varepsilon}\,\frac{1}{2|\vec l|-i\varepsilon} \quad ④$$

$$+\ldots \quad ③$$

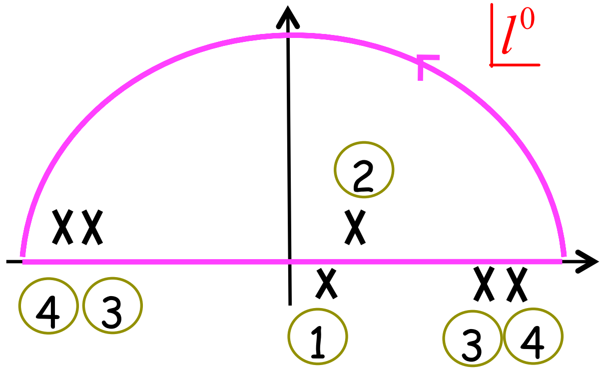
$$\sim e^4\,\frac{Q^3}{(4\pi)^2}\,\frac{1}{Q}\,\frac{1}{Q}\,\frac{1}{Q^2}\,\frac{1}{Q} \sim \underbrace{\frac{\alpha}{4\pi}}_{\ll 1}\,\frac{4\pi\alpha}{Q^2}$$

just as expected…



$$= e^4 \int \frac{d^4 l}{(2\pi)^4} \underbrace{\frac{1}{l^0 + p^0 - \frac{(\vec l + \vec p)^2}{2M} + i\varepsilon}}_{①} \underbrace{\frac{1}{-l^0 + p^0 - \frac{(\vec l + \vec p)^2}{2M} + i\varepsilon}}_{②}$$

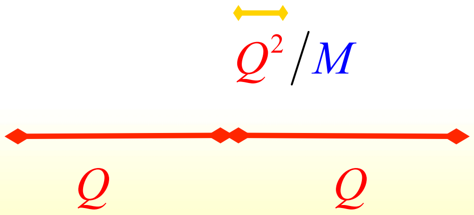
$$\underbrace{\frac{1}{\left(p^0 - p'^0 + l^0\right)^2 - \left(\vec p - \vec p' + \vec l\right)^2 + i\varepsilon}}_{③} \underbrace{\frac{1}{l^{02} - \vec l^2 + i\varepsilon}}_{④}$$



$$= i e^4 \int \frac{d^3 l}{(2\pi)^3} \frac{1}{-2p^0 + \frac{(\vec l + \vec p)^2}{M} - i\varepsilon} \frac{1}{\left(p^0 - \frac{(\vec l + \vec p)^2}{2M}\right)^2 - \vec l^2 + i\varepsilon}$$

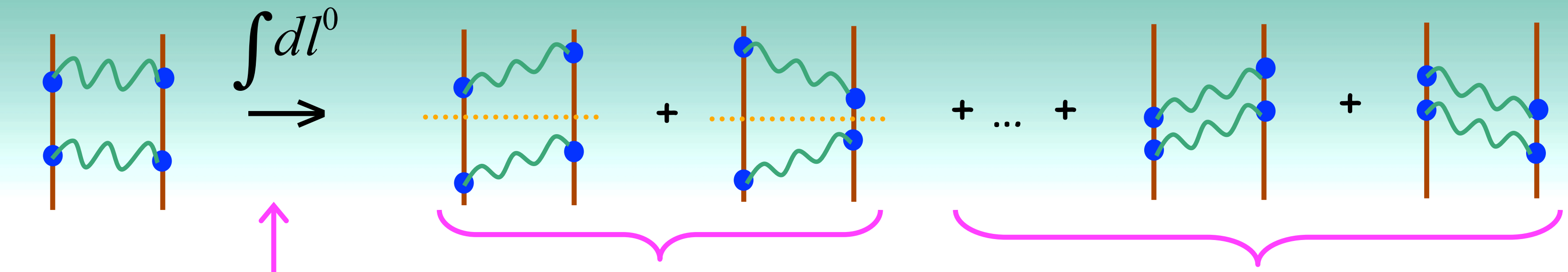
$$\frac{1}{\left(2p^0 - p'^0 - \frac{(\vec l + \vec p)^2}{2M}\right)^2 - \left(\vec p - \vec p' + \vec l\right)^2 + i\varepsilon} \quad ②$$

$$+ \ldots \quad ③④$$



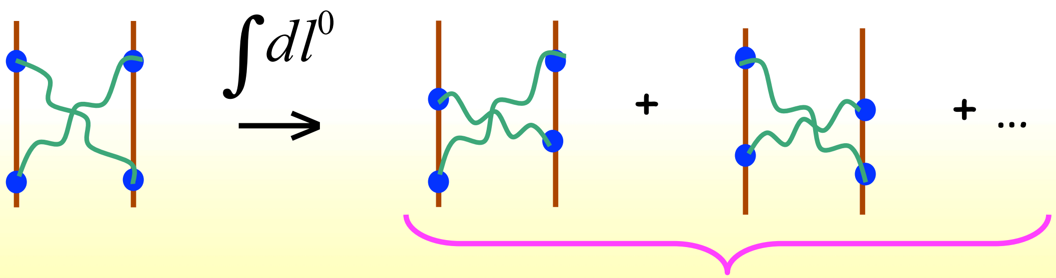
$$\sim (4\pi\alpha)^2 \frac{Q^3}{4\pi} \frac{M}{Q^2} \frac{1}{Q^2} \frac{1}{Q^2} + \frac{\alpha}{4\pi} \frac{4\pi\alpha}{Q^2} \sim \alpha \frac{4\pi\alpha}{Q^2} \left(\underbrace{\frac{M}{Q}}_{\gg 1} + \frac{1}{4\pi} \right)$$

infrared enhancement!



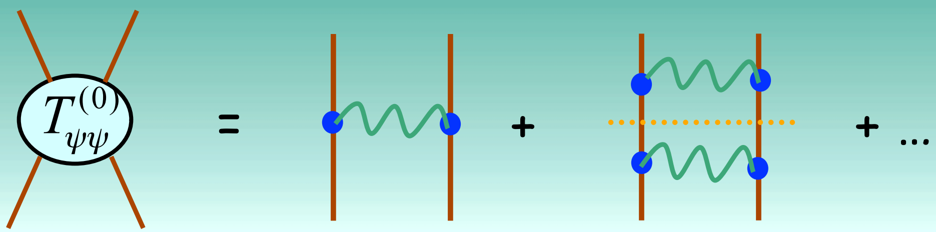
$$\propto \frac{1}{Q^2/M} \qquad\qquad \propto \frac{1}{Q}$$

$$\frac{1}{-l^0 - \frac{\vec{l}^{\,2}}{2M} + i\varepsilon} = 2\pi\delta\left(l^0\right) - \frac{1}{l^0 + \frac{\vec{l}^{\,2}}{2M} + i\varepsilon}$$



$$\propto \frac{1}{Q}$$

"time-ordered perturbation theory"

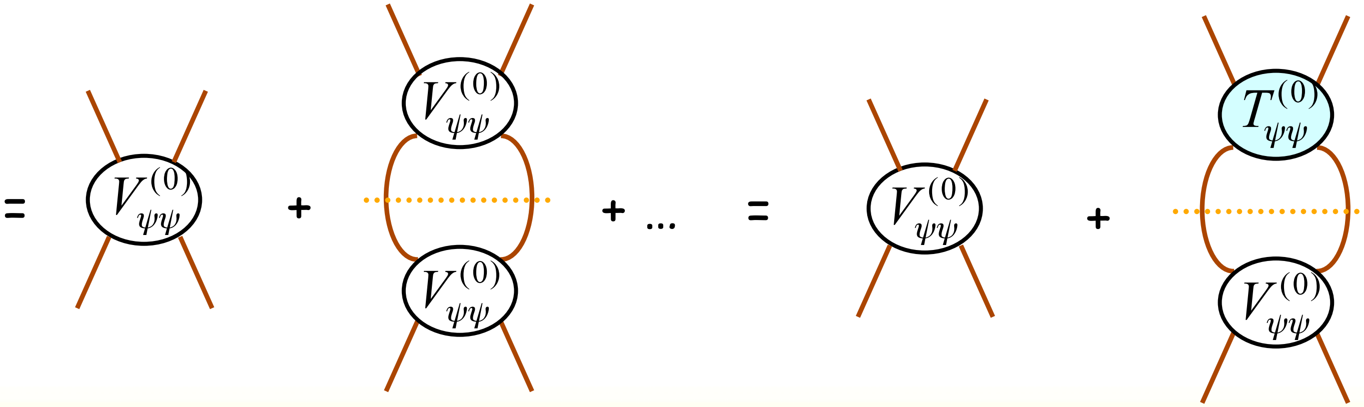


bound state at

$$\sim \frac{e^2}{Q^2}\left\{1+O\left(\alpha\frac{M}{Q}\right)+\ldots\right\}\sim\frac{e^2}{Q^2}\frac{1}{1-O\left(\alpha\frac{M}{Q}\right)}$$

$$Q\sim\alpha M$$

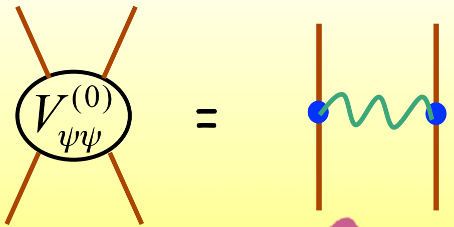
$$-E\sim\frac{Q^2}{M}\sim\alpha^2 M$$



Lippmann-Schwinger eq.

= Schroedinger eq.

$$\left(\frac{\hat{p}^2}{2M}+V^{(0)}_{\psi\psi}\right)\left|\psi^{(0)}\right\rangle = E^{(0)}\left|\psi^{(0)}\right\rangle$$

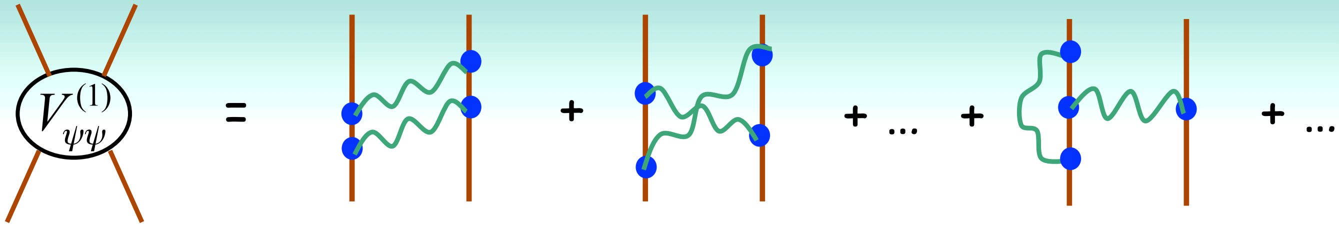


$$= O\left(\frac{e^2}{Q^2}\right)$$

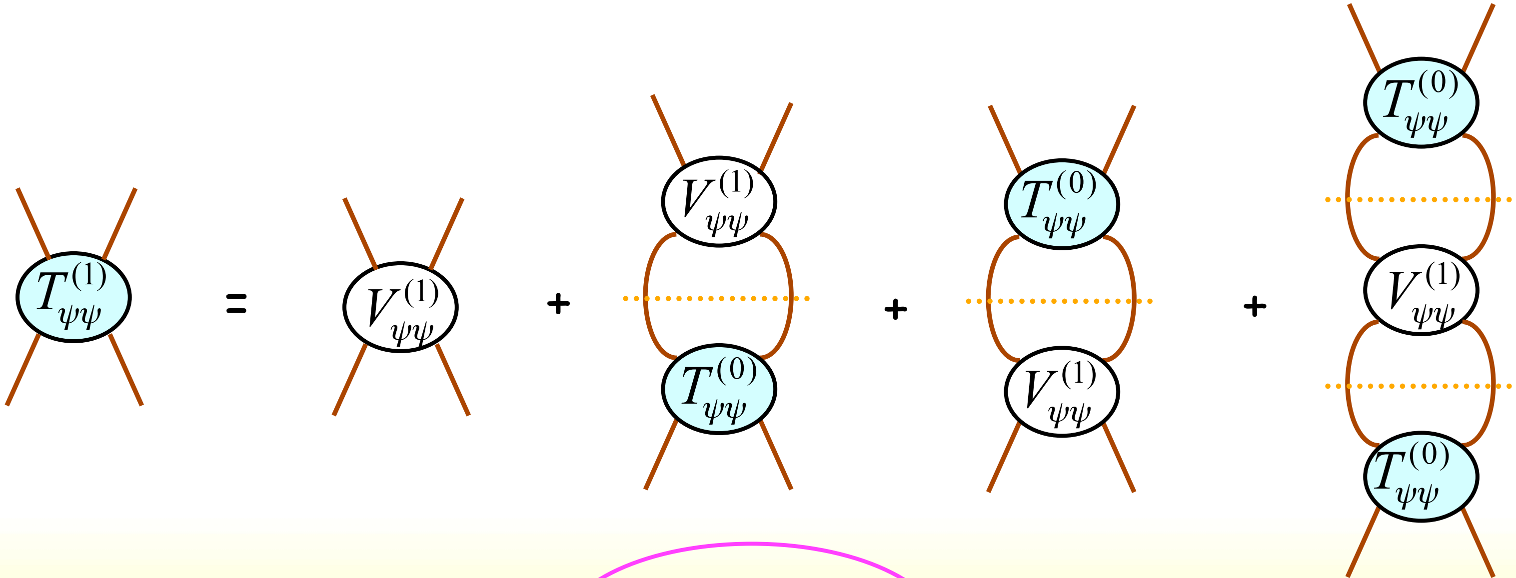
Coulomb potential

known results...

But more:



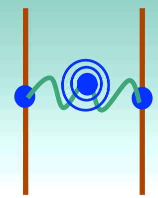
$$= O\left(\frac{\alpha}{4\pi}\,\frac{4\pi\alpha}{Q^2}\right)$$



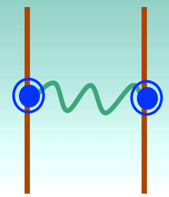
$$\Rightarrow\; E^{(1)} = E^{(0)} + \left\langle \psi^{(0)} \middle| V^{(1)}_{\psi\psi} \middle| \psi^{(0)} \right\rangle$$

$$= O\left(\frac{\alpha}{4\pi} E^{(0)}\right)$$

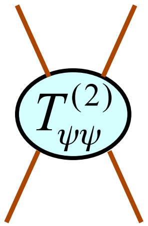
$=$



$+$



$$+ \ldots = O\left(\frac{Q^2}{M^2}\frac{4\pi\alpha}{Q^2}\right)$$



$= \quad \ldots$

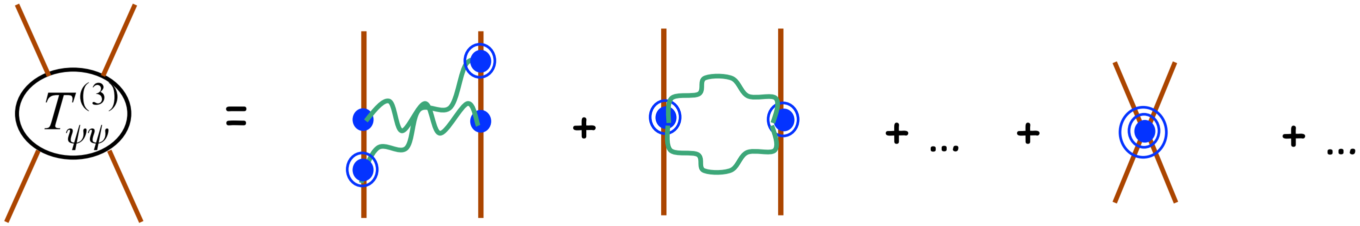
$$\Rightarrow \quad E^{(2)} = E^{(1)} + \left\langle \psi^{(0)} \middle| V^{(2)}_{\psi\psi} \middle| \psi^{(0)} \right\rangle + \ldots = O\left(\frac{Q^2}{M^2} E^{(0)}\right)$$

piece $\propto \vec{\mu}_1 \cdot \vec{\mu}_2 \int d^3\vec{r}\, \psi^{(0)*}(\vec{r})\delta^{(3)}(\vec{r})\psi^{(0)}(\vec{r}) = \vec{\mu}_1 \cdot \vec{\mu}_2 \left|\psi^{(0)}(0)\right|^2$

magnetic interaction

NOTE

starting at $T^{(3)}_{\psi\psi}$, sufficiently many derivatives appear at vertices so that loops bring positive powers of Λ, which need to be compensated by $\gamma_0^{(i)}(\Lambda)$ and higher-order "counterterms"



$$= O\left(\frac{\alpha}{4\pi}\frac{Q^2}{M^2}\frac{4\pi\alpha}{Q^2}\right)$$

$$\propto \frac{\alpha^2}{M^2}\ln\Lambda \qquad \leftrightarrow \qquad \gamma_0^{(i)} \propto \frac{\alpha^2}{M^2}\left(-\ln\Lambda + \text{constant}\right)$$

renormalization

to be determined by "matching" to QED (and/or from data)

Etc.

Example: g factor for electron bound in H-like atoms

$$g = 2(1+\kappa)$$

electron Larmor frequency — known — ion mass — measured

measured $$\frac{\omega_L}{\omega_c} = \frac{g}{2}\frac{|e|}{q}\frac{m_{\text{ion}}}{m}$$

trapped-ion cyclotron frequency — ion charge — electron mass

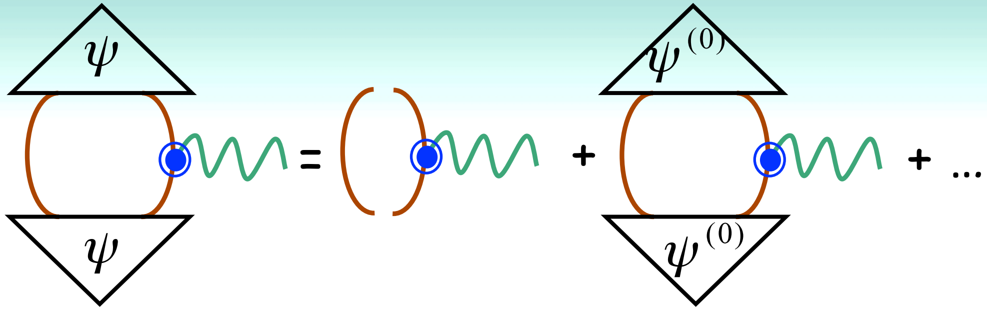


TABLE II. Individual contributions to the 1s bound-electron g factor, $1/\alpha$ from [12] is 137.035 999 11(46).

	$^{12}C^{5+}$	$^{16}O^{7+}$
Dirac value (point)	1.998 721 354 39(1)	1.997 726 003 06(2)
Finite nuclear size	0.000 000 000 41	0.000 000 001 55
Free QED, $\sim(\alpha/\pi)$	0.002 322 819 47(1)	0.002 322 819 47(1)
Binding SE, $\sim(\alpha/\pi)$	0.000 000 852 97	0.000 001 622 67(1)
Binding VP, $\sim(\alpha/\pi)$	−0.000 000 008 51	−0.000 000 026 37(1)
Free QED, $\sim(\alpha/\pi)^2\cdots(\alpha/\pi)^4$	−0.000 003 515 10	−0.000 003 515 10
Binding QED, $\sim(\alpha/\pi)^2(Z\alpha)^2$	−0.000 000 001 13	−0.000 000 002 01
Binding QED, $\sim(\alpha/\pi)^2(Z\alpha)^4$	0.000 000 000 41(11)	0.000 000 001 06(35)
Recoil	0.000 000 087 63	0.000 000 116 97
Total	2.001 041 590 52(11)	2.000 047 021 28(35)

Pachucki, Jentschura + Yerokhin '04

$$\left(u = \frac{m_{12C(gs)}}{12}\right)$$

$$m(^{12}C^{5+}) = 0.000\,548\,579\,909\,41\,(29)(3)\ \text{u},$$

$$m(^{16}O^{7+}) = 0.000\,548\,579\,909\,87\,(41)(10)\ \text{u},$$

Most precise determination of electron mass (expt)(th)

Summary

- Nuclear systems involve multiple scales but no obvious small coupling constant
- EFT is a general framework to deal with a multi-scale problem using the small ratio of scales as an expansion parameter
- Applied to low-energy QED, EFT reproduces well-known facts and also provides a systematic expansion for the potential, and thus for the scattering amplitude --
  NRQED is in fact the framework used in state-of-the-art QED bound-state calculations

Stay tuned:
next, how we can make nuclear physics as systematic as QED

Introduction to Effective Field Theories in QCD

U. van Kolck

University of Arizona

Supported in part by US DOE

Outline

- Effective Field Theories
- QCD at Low Energies
  - QCD and Chiral Symmetry
  - Chiral Nuclear EFT
  - Renormalization of Pion Exchange
  - Summary
- Towards Nuclear Structure

References:

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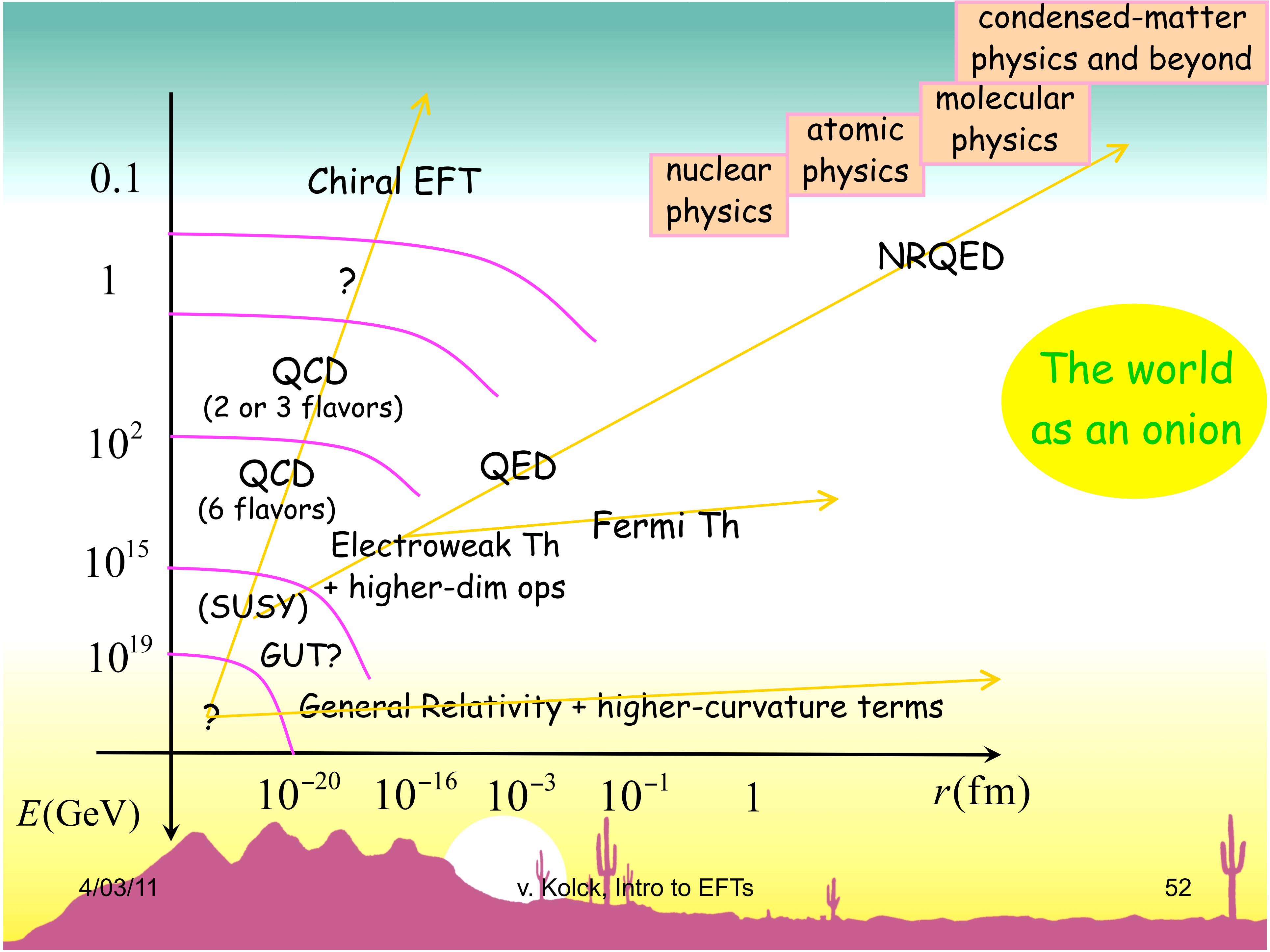

condensed-matter physics and beyond
molecular physics
atomic physics
nuclear physics
0.1
1
10^2
10^{15}
10^{19}
E(GeV)
Chiral EFT
?
QCD
(2 or 3 flavors)
QCD
(6 flavors)
NRQED
QED
Fermi Th
Electroweak Th
+ higher-dim ops
(SUSY)
GUT?
?
General Relativity + higher-curvature terms
10^{-20}
10^{-16}
10^{-3}
10^{-1}
1
r(fm)
The world as an onion

Game Plan

QCD

lattice

EFT

necessary to extrapolate to { large r / small m_π }

NCSM,
lattice, …

want model independence

Few-nucleon systems

Many-nucleon systems

Infinite-nucleon system

EFT at a few GeV= underlying theory for nuclear physics

d.o.f.s leptons: $l_f = \begin{pmatrix} l^+ \\ \nu \end{pmatrix}_f$ quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ photon: A_μ gluons: G^a_μ

symmetries: $SO(3,1)$ global, $U_{em}(1)$ gauge, $SU_c(3)$ gauge

$$\mathbf{L}_{und} = \sum_{f=1}^{3} \bar{l}_f \left(i\partial\!\!\!/ + eQ_l A\!\!\!/ - m_f \right) l_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \bar{q}\left(i\partial\!\!\!/ + eQ_q A\!\!\!/ + g_s G\!\!\!/ \right) q - \frac{1}{2} \mathrm{Tr}\left[G_{\mu\nu} G^{\mu\nu} \right]$$

$$- \frac{1}{2}(m_u + m_d)\bar{q}q - \frac{1}{2}(m_u - m_d)\bar{q}\tau_3 q$$

} QED + QCD

$$Q_l = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$Q_q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1 + 3\tau_3}{6}$$

$$+ \frac{m_u m_d}{m_u + m_d} \bar{\theta}\, \bar{q} i \gamma_5 q + \ldots$$

$\ldots$ → higher-dimension interactions: suppressed by larger masses — e.g. $G_F \propto 1/M^2_{W,Z}$

$\bar{\theta}$ term → unnaturally small T violation (strong CP problem) — $\bar{\theta} \lesssim 10^{-9}$

Focus on strong-interacting sector: four parameters

1) $m_u = m_d = 0, \; e = 0, \; \bar{\theta} = 0$ "chiral limit"

single, *dimensionless* parameter

$$\int d^4x \, \mathcal{L}_{QCD} = \int d^4x \left\{ \bar{q}\left(i\partial\!\!\!/ - g_s G\!\!\!/\right) q - \frac{1}{2} \mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu} \right\}$$

invariant under scale transformations

$$\begin{cases} x \to \lambda^{-1} x \\ q \to \lambda^{3/2} q \\ G \to \lambda G \end{cases}$$

but in

$$Z = \int DG \int D\bar{q} \int Dq \, \exp\left(i \int d^4x \, \mathcal{L}_{QCD}\right)$$

scale invariance

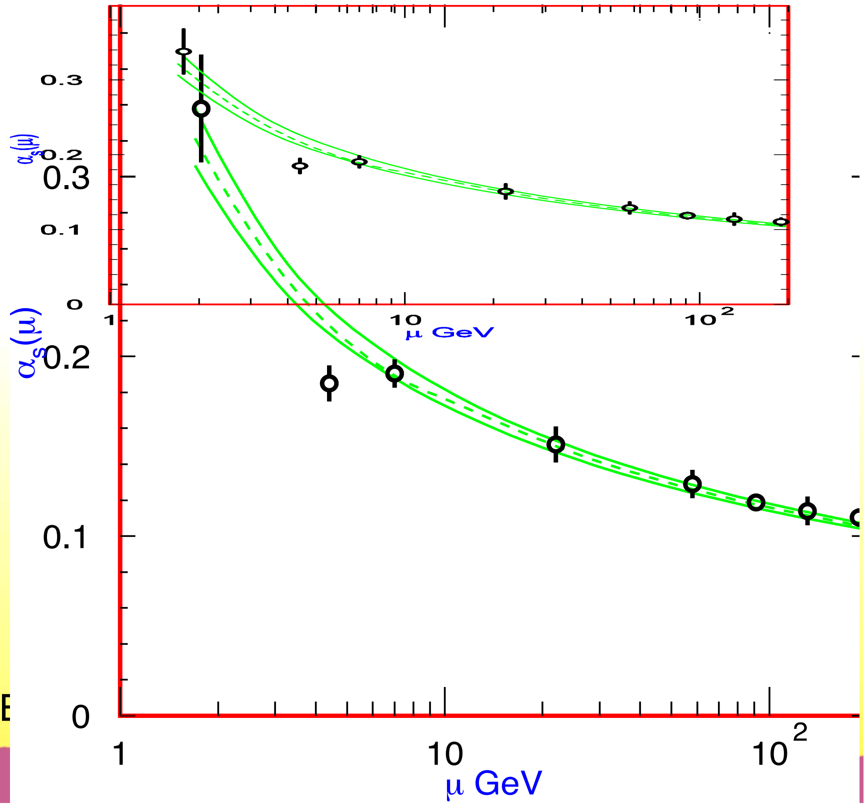
"anomalously broken"

by *dimensionful* regulator

⇨ coupling runs

$$\alpha_s\left(Q \sim 1\,\mathrm{GeV}\right) \sim 1$$

("dimensional transmutation")



Non-perturbative physics at $Q \sim 1\ \mathrm{GeV}$

Assumption 1: <u>confinement</u>

only colorless states ("hadrons") are asymptotic

Observation: (almost) all hadron masses $\gtrsim 1\ \mathrm{GeV}$

Assumption 2: <u>naturalness</u>

masses are determined by characteristic scale

⇒ $M_{QCD} \sim 1\ \mathrm{GeV}$

Observation: pion mass $m_\pi \simeq 140\ \mathrm{MeV} \ll M_{QCD}$

breakdown of naturalness? NO!

"spontaneous breaking" of chiral symmetry

Why is the pion special?

$$\mathcal{L}_{QCD} = \bar{q}_L \left(i\partial\!\!\!/ + g_s G\!\!\!/\right) q_L + \bar{q}_R \left(i\partial\!\!\!/ + g_s G\!\!\!/\right) q_R - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_L = \frac{1-\gamma_5}{2} q \qquad q_R = \frac{1+\gamma_5}{2} q$$

invariant under chiral symmetry

$$q_{L(R)} \to \exp\left(i\boldsymbol{\alpha}_{L(R)} \cdot \boldsymbol{\tau}\right) q_{L(R)} \qquad SU(2)_L \times SU(2)_R \sim SO(4)$$

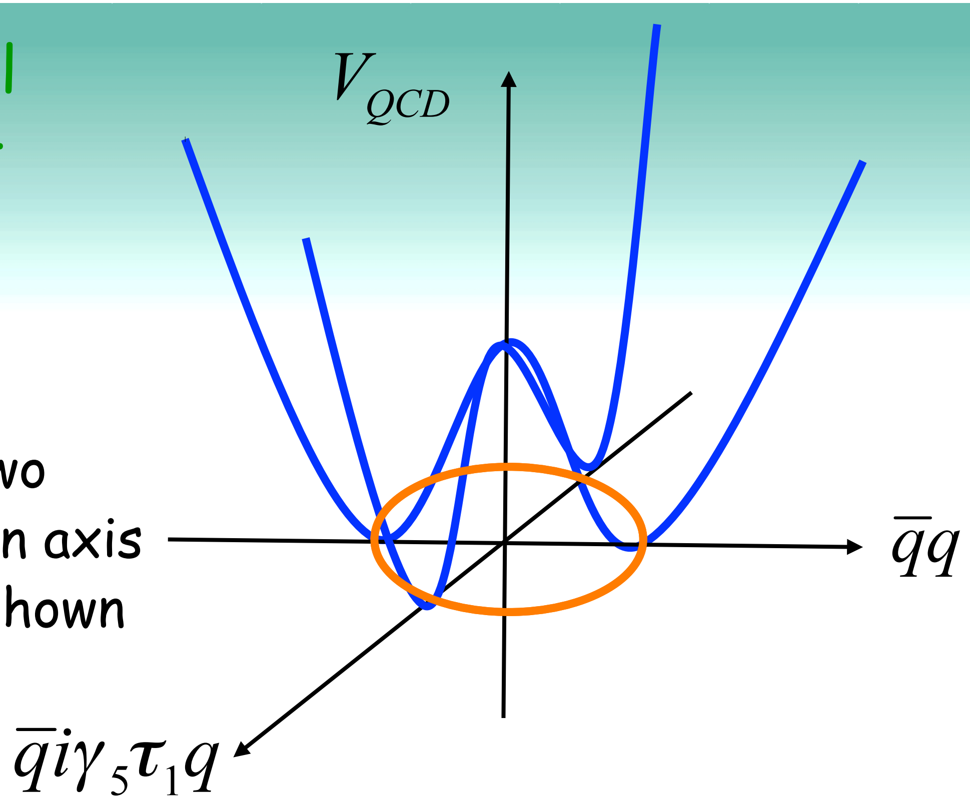
$$m_\sigma \gg m_\pi$$

$$m_{N_-} \gg m_{N_+}$$

broken by vacuum down to isospin

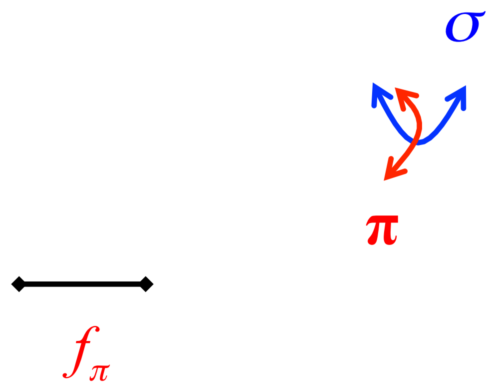
$$q \to \exp\left(i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}\right) q \qquad SU(2)_{L+R} \sim SO(3)$$

Chiral Limit



two isospin axis not shown

chiral circle



pion decay constant (in chiral limit)

$\mathbf{L}_{EFT}$ = piece invariant under $\boldsymbol{\pi} \to \boldsymbol{\pi} + \boldsymbol{\varepsilon}$ [function of $\partial_\mu \boldsymbol{\pi}$ on chiral circle]

$$\left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \ldots\right) \partial_\mu \boldsymbol{\pi}$$

2) $m_u \neq 0 \neq m_d, \; e = 0, \bar{\theta} = 0$

$$\mathcal{L}_{QCD} = \bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu} G_{\mu\nu}$$

v.K. '93

$$+\frac{1}{2}(m_u + m_d)\,\underbrace{\bar{q}q} + \frac{1}{2}(m_u - m_d)\,\underbrace{\bar{q}\tau_3 q} + \ldots$$

4th component of $SO(4)$ vector

$$S = \left(\bar{q}i\gamma_5 \boldsymbol{\tau} q, \bar{q}q\right)$$

3rd component of $SO(4)$ vector

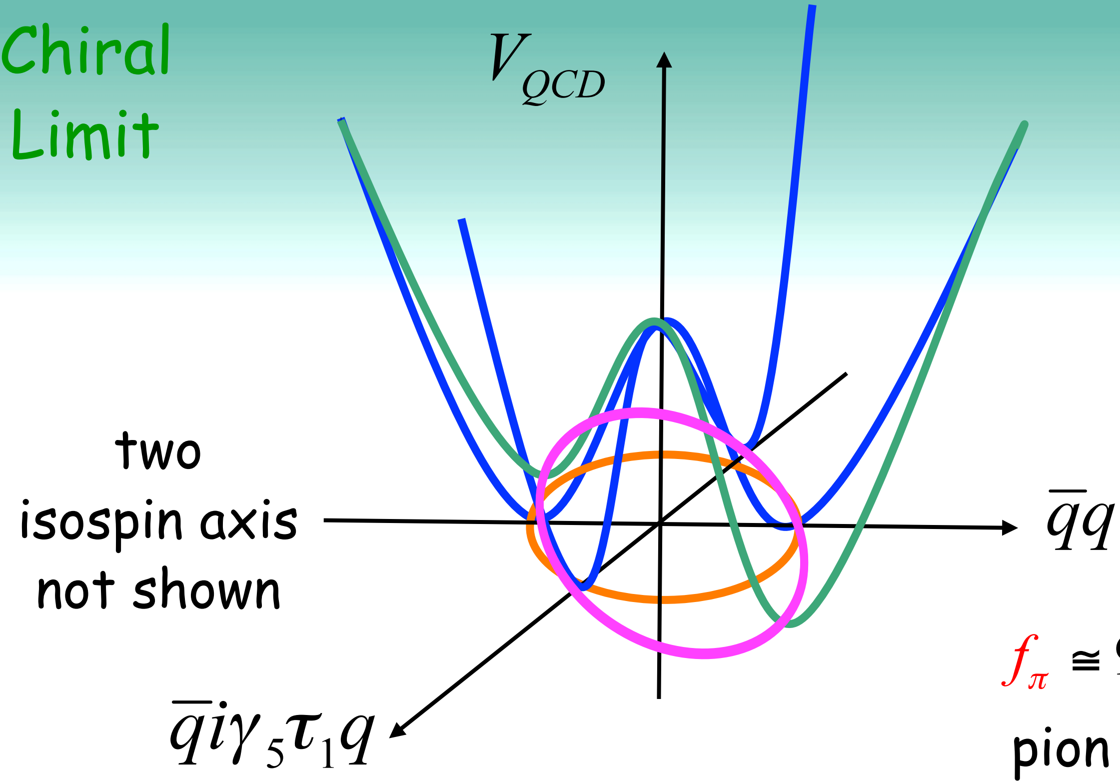
$$P = \left(\bar{q}\boldsymbol{\tau} q, \bar{q}i\gamma_5 q\right)$$

break

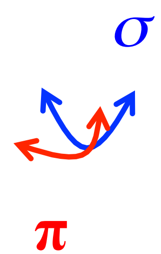
$SO(4) \quad \rightarrow SO(3) \quad\quad \rightarrow U(1)$

(explicit chiral-symmetry breaking) (isospin violation)

Chiral Limit



slightly-tilted chiral circle



$f_\pi \cong 92$ MeV

pion decay constant

$\mathbf{L}_{EFT}$ = piece invariant under $\boldsymbol{\pi} \to \boldsymbol{\pi} + \boldsymbol{\varepsilon}$ [function of $\partial_\mu \boldsymbol{\pi}$] $\quad \propto Q$

+ piece in $\bar{q}q$ direction [function of π explicitly] $\quad \propto (m_u + m_d)$

+ isospin breaking $\quad \propto (m_u - m_d)$

CHIRAL SYMMETRY ⇨ WEAK PION INTERACTIONS

3) $e \neq 0, \bar{\theta} = 0$

Two types of interactions:

- "soft" photons – explicit d.o.f. in the EFT

$$D_\mu = \partial_\mu - ieQ_q A_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- "hard" photons – "integrated out" of EFT

v.K. '93

$$\mathcal{L}_{und} = \ldots - e^2 \bar{q} Q_q \gamma_\mu q \, D^{\mu\nu}(\partial^2) \, \bar{q} Q_q \gamma_\nu q + \ldots$$

34 comp of antisymmetric tensor

$$F_\mu = \begin{pmatrix} \varepsilon_{ijk} \bar{q} i \gamma_\mu \gamma_5 \tau_k q & \bar{q} i \gamma_\mu \tau_j q \\ -\bar{q} i \gamma_\mu \tau_i q & 0 \end{pmatrix}$$

breaks $SO(4)$ (and $SO(3)$ in particular) $\rightarrow U(1)$

⇨ $\mathcal{L}_{EFT} =$ soft photons $\propto e$

+ further isospin breaking $\propto \alpha / 4\pi$

4) $\bar{\theta} \neq 0$

$$\mathcal{L}_{und} = \ldots + \frac{m_u m_d}{m_u + m_d}\bar{\theta}\,\underbrace{\bar{q} i\gamma_5 q} + \ldots$$

4th component of $SO(4)$ vector $P = \left(\bar{q}\boldsymbol{\tau}q, \bar{q}i\gamma_5 q\right)$

T violation linked to isospin violation: in EFT, combination is

$$-\frac{1}{2}\left(m_u - m_d\right)P_3 + \frac{m_u m_d}{m_u + m_d}\bar{\theta}\,P_4$$

Hockings, Mereghetti + v.K., '10

5) continue with higher-order operators,
e.g. T-violating quark EDM and color-EDM
P-violating four-quark operators

De Vries, Mereghetti, Timmermans + v.K., '10
...
Kaplan + Savage '96
Zhu, Maekawa, Holstein, Musolf + v.K. '02

Nuclear physics scales

"His scales are His pride", Book of Job

$\ln Q$

perturbative QCD

~1 GeV $M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$

brute force (lattice), ...?

hadronic th
with chiral symm

~100 MeV $M_{nuc} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

~30 MeV $\aleph \sim 1/a_{NN}$

this lecture

Q/M_{QCD}

Q/M_{nuc}

next lecture

no small coupling → expansion in ↑

Nuclear EFT

$Q \sim m_\pi \ll M_{QCD}$

pionful EFT

- d.o.f.s: nucleons, pions, deltas $\quad (m_\Delta - m_N \sim 2m_\pi)$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \boldsymbol{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} \left(\pi^+ + \pi^-\right)/\sqrt{2} \\ -i\left(\pi^+ - \pi^-\right)/\sqrt{2} \\ \pi^0 \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \\ \Delta^{-} \end{pmatrix}$$

- symmetries: Lorentz, ~~P~~, ~~T~~, ~~chiral~~

Weinberg '68
Callan, Coleman, Wess + Zumino '69

Non-linear realization of chiral symmetry

chiral invariants $\quad + S_4\text{'s},\ P_3\text{'s},\ F_{34}\text{'s}$

(chiral) covariant derivatives

pion $\quad \boldsymbol{D}_\mu \equiv \left(\frac{\partial_\mu \boldsymbol{\pi}}{f_\pi}\right)\left(1 - \frac{\boldsymbol{\pi}^2}{4f_\pi^2} + \ldots\right)$

fermions $\quad \mathcal{D}_\mu \equiv \left(\partial_\mu - \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{E}_\mu\right)$

$$\boldsymbol{E}_\mu \equiv \frac{\boldsymbol{\pi}}{f_\pi} \times \boldsymbol{D}_\mu$$

$$m_\pi^2 = O\left((m_u + m_d)M_{QCD}\right)$$

$$\Rightarrow m_u + m_d = O\left(\frac{m_\pi^2}{M_{QCD}}\right)$$

Schematically,

$$\mathcal{L}_{EFT} = \sum_{\{n,p,f\}} c_{\{n,p,f\}} \left(\frac{\boldsymbol{D}, \mathbf{D}, m_\Delta - m_N}{M_{QCD}} \right)^n \left(\frac{m_\pi^2}{M_{QCD}^2} \frac{\boldsymbol{\pi}^2}{f_\pi^2} \right)^{p/2} \left(\frac{\psi^+\psi}{f_\pi^2 M_{QCD}} \right)^{f/2} f_\pi^2 M_{QCD}^2$$

$c_{\{n,p,f\}}$:
- calculated from QCD: lattice, …
- fitted to data

$= \mathbf{O}(1)$ isospin conserving

$= \mathbf{O}\left(\varepsilon, \frac{\alpha}{4\pi}\right)$ isospin breaking

(NDA: naïve dimensional analysis)

$$= \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)} \qquad \Delta \equiv n + p + \frac{f}{2} - 2 \equiv d + \frac{f}{2} - 2 \geq 0$$

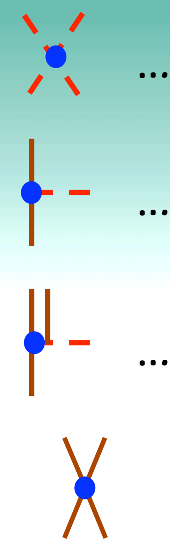
"chiral index" — chiral symmetry ($\geq$)

$$\mathcal{L}^{(0)} = \frac{1}{2}(\partial_\mu \boldsymbol{\pi})^2 \left(1 - \frac{\boldsymbol{\pi}^2}{2 f_\pi^2} + \ldots\right) - \frac{1}{2} m_\pi^2 \boldsymbol{\pi}^2 \left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \ldots\right)$$

$$+ N^+ \left[i\partial_0 - \frac{1}{4 f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) + \ldots \right] N + \frac{g_A}{2 f_\pi} N^+ \boldsymbol{\tau} \vec{\sigma} N \cdot (\vec{\nabla} \boldsymbol{\pi}) \left(1 - \frac{\boldsymbol{\pi}^2}{4 f_\pi^2} + \ldots\right)$$

$$+ \Delta^+ \left[i\partial_0 - (m_\Delta - m_N) + \ldots \right] \Delta + \ldots + \frac{h_A}{2 f_\pi} \left(N^+ \mathbf{T} \vec{S} \Delta + \text{H.c.}\right) \cdot \vec{\nabla} \boldsymbol{\pi} (1 + \ldots)$$

$$- C_S \left(N^+ N\right)^2 - C_T \left(N^+ \vec{\sigma} N\right)^2$$



$$\mathcal{L}^{(1)} = N^+ \left[\frac{1}{2 m_N} \left(\vec{\nabla} + \frac{1}{4 f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \vec{\nabla} \boldsymbol{\pi}) + \ldots \right)^2 + \frac{1}{2} (m_p - m_n) \left(\tau_3 - \frac{1}{2 f_\pi^2} \pi_3 \boldsymbol{\pi} \cdot \boldsymbol{\tau} + \ldots \right) \right] N$$

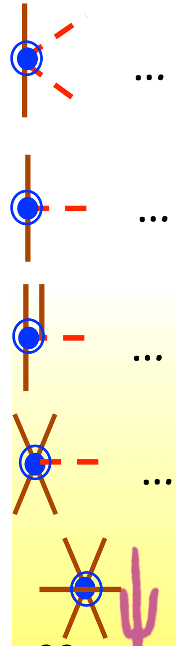
$$+ \frac{1}{f_\pi^2} N^+ \left[b_2 (\partial_0 \boldsymbol{\pi})^2 - b_3 (\vec{\nabla} \boldsymbol{\pi})^2 - 2 b_1 m_\pi^2 \boldsymbol{\pi}^2 + i b_4 \varepsilon_{ijk} \varepsilon_{abc} \sigma_k \tau_c (\partial_i \pi_b)(\partial_j \pi_c) \right] N + \ldots$$

$$- \frac{g_A}{4 m_N f_\pi} \left[i N^+ \boldsymbol{\tau} \vec{\sigma} \cdot \vec{\nabla} N + \text{H.c.} \right] \cdot (\partial_0 \boldsymbol{\pi}) (1 + \ldots)$$

$$- \frac{h_A}{4 m_N f_\pi} \left[i N^+ \mathbf{T} \vec{S} \cdot \vec{\nabla} N + \text{H.c.} \right] \cdot (\partial_0 \boldsymbol{\pi}) (1 + \ldots)$$

$$+ \frac{d}{f_\pi} N^+ N \, N^+ \boldsymbol{\tau} \vec{\sigma} N \cdot (\vec{\nabla} \boldsymbol{\pi}) (1 + \ldots)$$

$$- E \left(N^+ N\right)^3$$

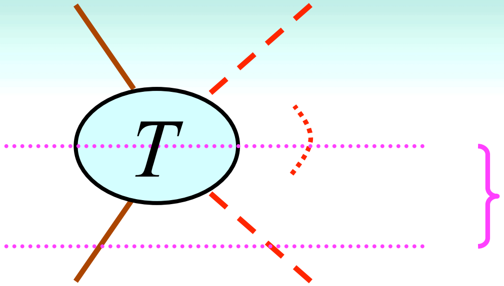


$$\mathcal{L}^{(2)} = \ldots$$

Form of pion interactions <u>determined</u> by chiral symmetry

A= 0, 1: chiral perturbation theory

Weinberg '79
Gasser + Leutwyler '84
…
Gasser, Sainio + Svarc '87
Jenkins + Manohar '91
…



$$\frac{1}{\Delta E} \sim \frac{1}{Q}$$

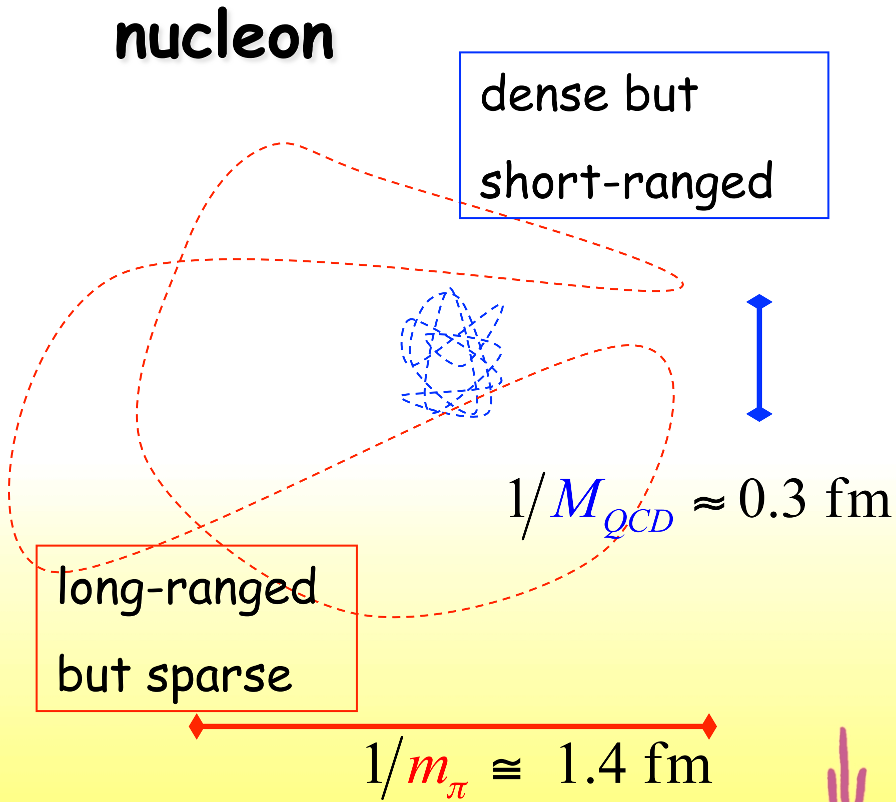
$$\sim \sum_{\nu} c_\nu \left(\frac{Q}{M_{QCD}}\right)^{\nu} F_\nu\left(\frac{Q}{m_\pi}\right)$$

$$\nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\min} = 2 - A$$

L: # loops V_i: # vertices of type *i*

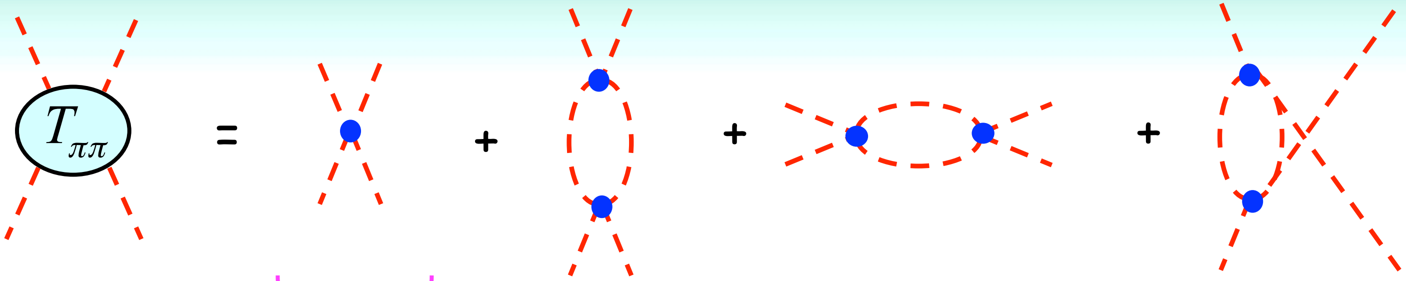
expansion in

$$\frac{Q}{M_{QCD}} \sim \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$$



Analogous to NRQED…

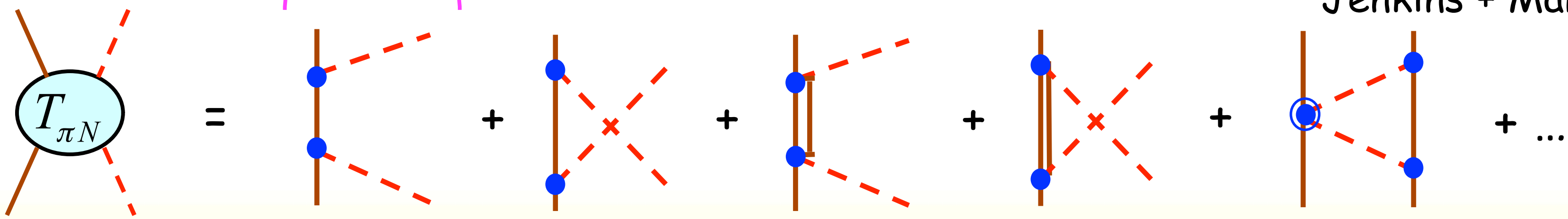
Weinberg '79
Gasser + Leutwyler '84
…



current algebra

Weinberg '66
…

Gasser, Sainio + Svarc '87
Bernard, Kaiser + Meissner '90
Jenkins + Manohar '91
…



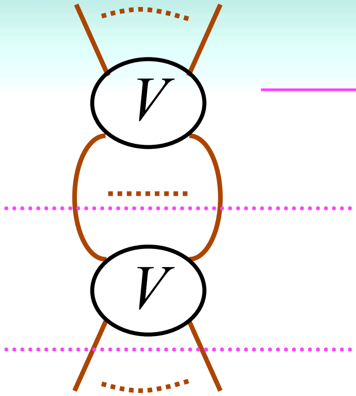
N.B. For $\left|E-(m_\Delta - m_N)\right| \lesssim \mathbf{O}\left(\frac{Q^3}{M_{QCD}^2}\right)$ a resummation is necessary

Phillips + Pascalutsa '02
Long + v.K. , '08

Etc.

A ≥ 2: resummed chiral perturbation theory

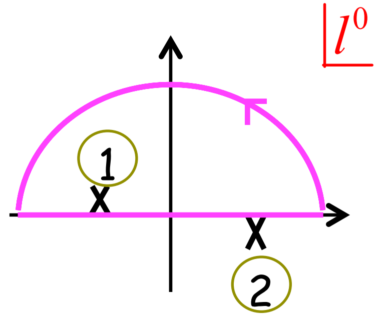
Weinberg '90, '91



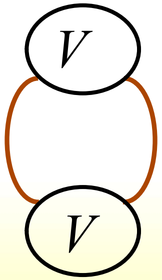
A-nucleon irreducible

$$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$$ infrared enhancement!

A-nucleon reducible



e.g.



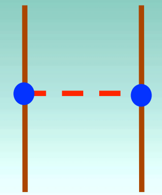
$$\simeq i\int \frac{d^4 l}{(2\pi)^4} V \frac{1}{l^0 + k^2/m_N - l^2/m_N - i\varepsilon} \frac{1}{-l^0 + k^2/m_N - l^2/m_N - i\varepsilon} V$$

(1) (2)

$$= \int \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V + \ldots \qquad \sim O\left(\frac{m_N Q}{4\pi} V^2\right)$$

$$E = \frac{k^2}{m_N}$$

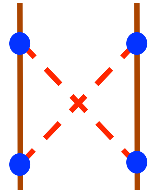
instead of $\frac{Q^2}{(4\pi)^2}$



$$\sim i\left(\frac{g_A}{2f_\pi}\right)^2\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\right)\frac{\left(S_{12}(\hat{q})+\vec{\sigma}_1\cdot\vec{\sigma}_2\right)}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\sim\frac{1}{f_\pi^2}$$

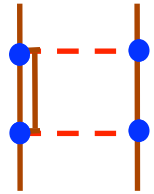
tensor force

$$S_{12}(\hat{q})=3\,\vec{\sigma}_1\cdot\hat{q}\;\vec{\sigma}_2\cdot\hat{q}-\vec{\sigma}_1\cdot\vec{\sigma}_2$$



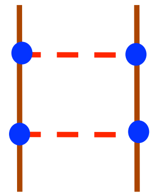
$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{(4\pi)^2}\frac{1}{Q}\frac{1}{Q}\frac{Q^2}{Q^2}\frac{Q^2}{Q}\sim\frac{1}{f_\pi^2}\frac{Q^2}{\underbrace{(4\pi f_\pi)^2}}$$

$$=O\left(\frac{Q^2}{M_{QCD}^2}\right)$$



$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{(4\pi)^2}\frac{1}{m_\Delta-m_N}\frac{1}{Q}\frac{Q^2}{Q^2}\frac{Q^2}{Q}\sim\frac{1}{f_\pi^2}\overbrace{\frac{Q^2}{(4\pi f_\pi)^2}}\frac{Q}{\underbrace{m_\Delta-m_N}}$$

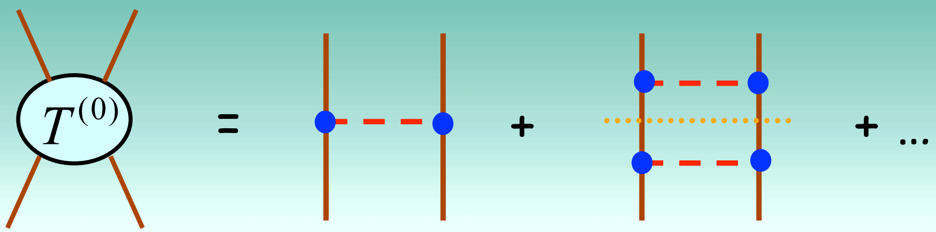
$$=O(1)$$



$$\sim\frac{1}{f_\pi^4}\frac{Q^3}{4\pi}\frac{m_N}{Q^2}\frac{Q^2}{Q^2}\frac{Q^2}{Q^2}\sim\frac{1}{f_\pi^2}\frac{m_N}{4\pi f_\pi}\frac{Q}{f_\pi}\sim\frac{1}{f_\pi^2}\frac{Q}{M_{NN}}$$

$$\equiv 1/M_{NN}$$

$=O(1)$ for $Q\sim M_{NN}$



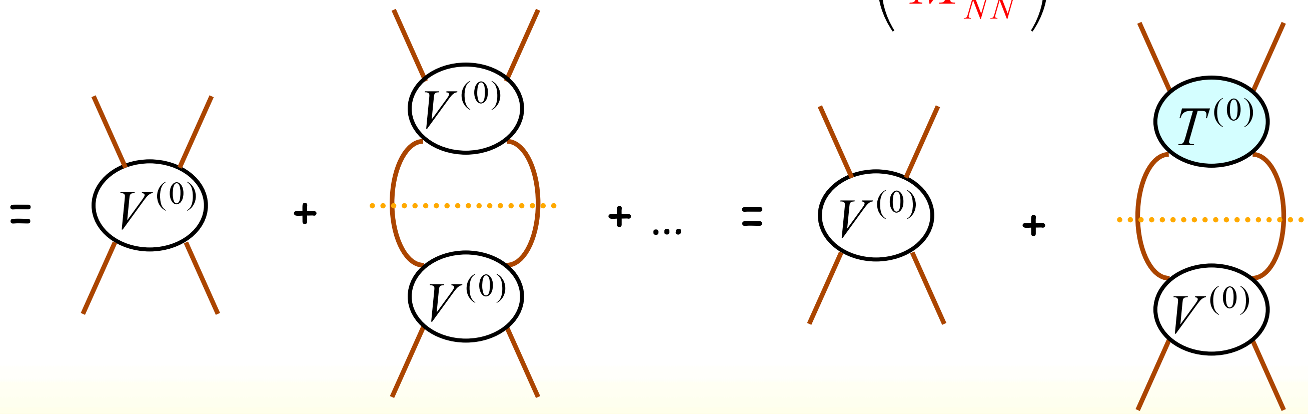
bound state at

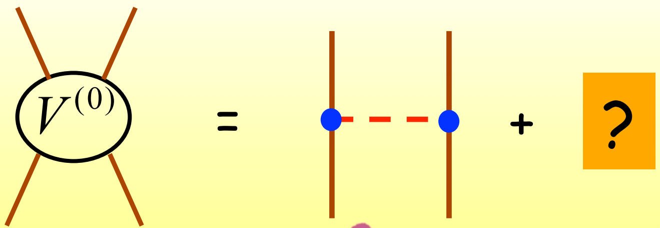
$Q \sim M_{NN}$ $\quad -E \sim \frac{Q^2}{m_N} \sim \frac{M_{NN}^2}{M_{QCD}}$

$$\sim \frac{1}{f_\pi^2}\left\{1+\mathrm{O}\left(\frac{Q}{M_{NN}}\right)+\ldots\right\} \sim \frac{1}{f_\pi^2}\frac{1}{1-\mathrm{O}\left(\frac{Q}{M_{NN}}\right)}$$

$$M_{nuc} = M_{NN} \sim \frac{4\pi f_\pi}{m_N} f_\pi \approx f_\pi$$

Nuclear scale arises naturally from chiral symmetry

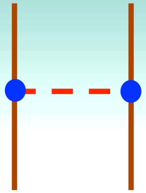




Is 1PE all there is in leading order?

That is, are observables cutoff independent with 1PE alone?

Issue: relative importance of pion exchange and short-range interactions



$$\sim i\left(\frac{g_A}{2f_\pi}\right)^2\left(\frac{\vec{q}^{\,2}}{\vec{q}^{\,2}+m_\pi^2}\right)\frac{\left(S_{12}(\hat{q})+\vec{\sigma}_1\cdot\vec{\sigma}_2\right)}{3}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\sim\frac{4\pi}{m_N M_{NN}}$$

$S=0$:

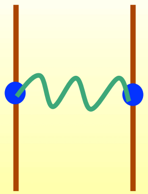
$$V(r)=\left(\frac{g_A}{2f_\pi}\right)^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(-\delta^{(3)}(\vec{r})+\frac{m_\pi^2}{4\pi r}e^{-m_\pi r}\right)$$

$S=1$:

$$V(r)=\left(\frac{g_A}{2f_\pi}\right)^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left\{\frac{1}{3}\left(\delta^{(3)}(r)-\frac{m_\pi^2}{4\pi r}e^{-m_\pi r}\right)+\frac{m_\pi^2}{4\pi r}\left(\frac{1}{(m_\pi r)^2}+\frac{1}{m_\pi r}+\frac{1}{3}\right)e^{-m_\pi r}\left\langle S_{12}(\hat{r})\right\rangle\right\}$$

$\langle S_{12}\rangle$	$j-1$	j	$j+1$
$j-1$	$-2\frac{j-1}{2j+1}$	0	$6\frac{\sqrt{j(j+1)}}{2j+1}$
j	0	2	0
$j+1$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2\frac{j+2}{2j+1}$

much more singular --and complicated!-- than

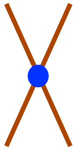


$$\sim\frac{i\,e^2}{(\vec{p}-\vec{p}\,')^2-i\varepsilon}\sim\frac{4\pi\alpha}{Q^2}\quad\rightarrow V(r)=\frac{\alpha}{r}$$

Weinberg '90, '91
Ordonez + v.K. '92
Ordonez, Ray + v.K. '96
...
Entem + Machleidt '03...
Epelbaum, Gloeckle + Meissner '04
...

Assume contact interactions are driven by short-range physics, and scale with M_{QCD} according to naïve dimensional analysis

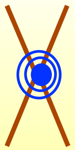
(W power counting)



$$\sim \underbrace{C_0^{(0)}}_{\equiv \frac{4\pi}{m_N M^{(0)}}} \frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2+1)}{4} - \underbrace{C_0^{(1)}}_{\equiv \frac{4\pi}{m_N M^{(1)}}} \frac{(\vec{\sigma}_1\cdot\vec{\sigma}_2-3)}{4}$$

$$\begin{cases} V(r) = \dfrac{4\pi}{m_N M^{(0)}}\delta^{(3)}(\vec{r}) & S=0 \\ V(r) = \dfrac{4\pi}{m_N M^{(1)}}\delta^{(3)}(\vec{r}) & S=1 \end{cases}$$

$M^{(i)} \sim M_{NN}$ ⇨ $C_0^{(i)}$ in LO



$$\sim \frac{4\pi}{m_N M_{NN}} \frac{Q^2}{M_{QCD}^2}$$ ⇨ in NNLO

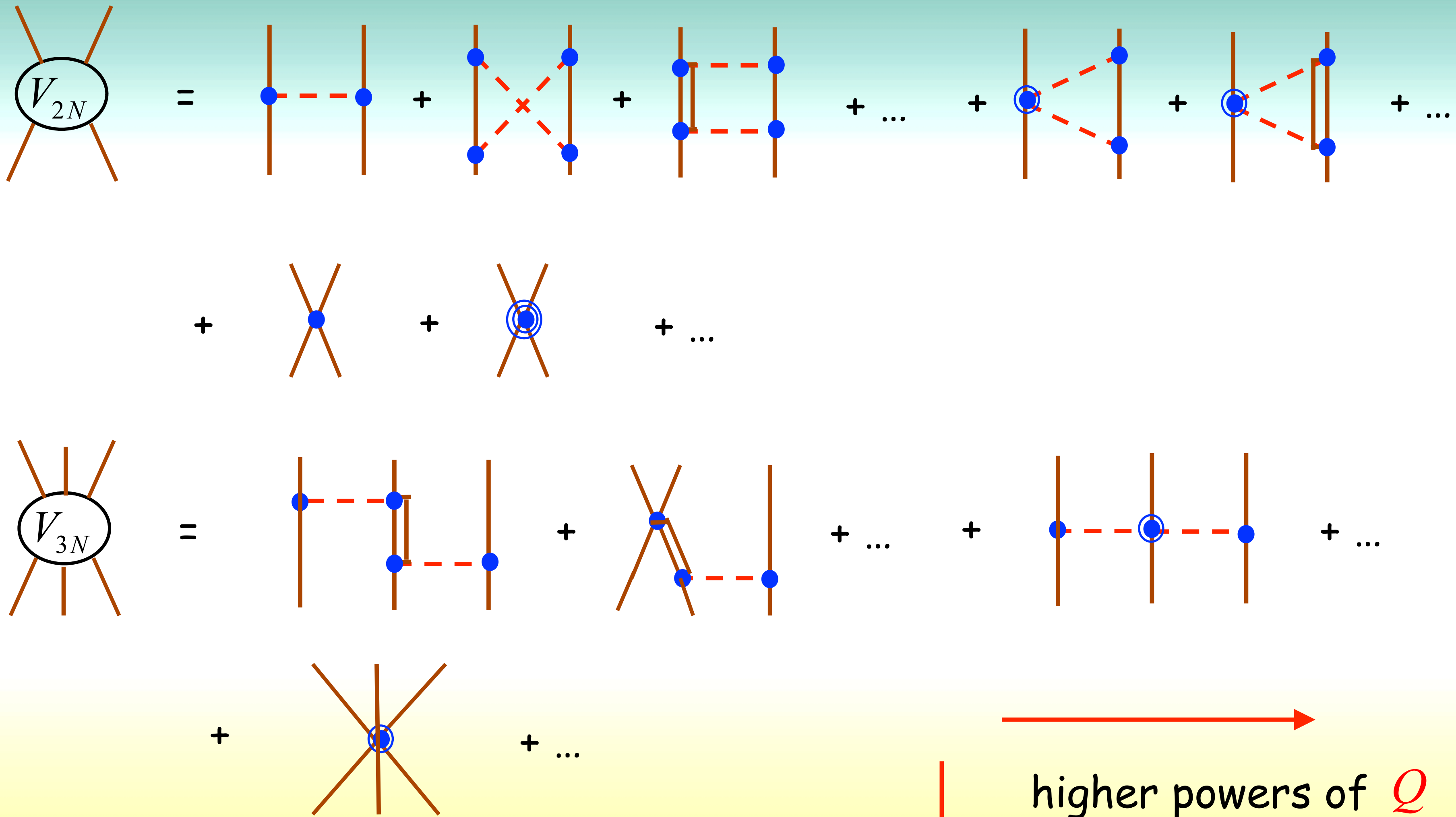
(NLO terms, linear in Q/M_{QCD}, break *P*, *T*)

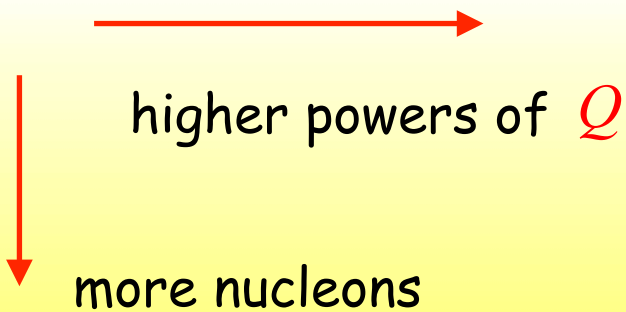
etc.

Ordonez + v.K. '92

v.K. '94

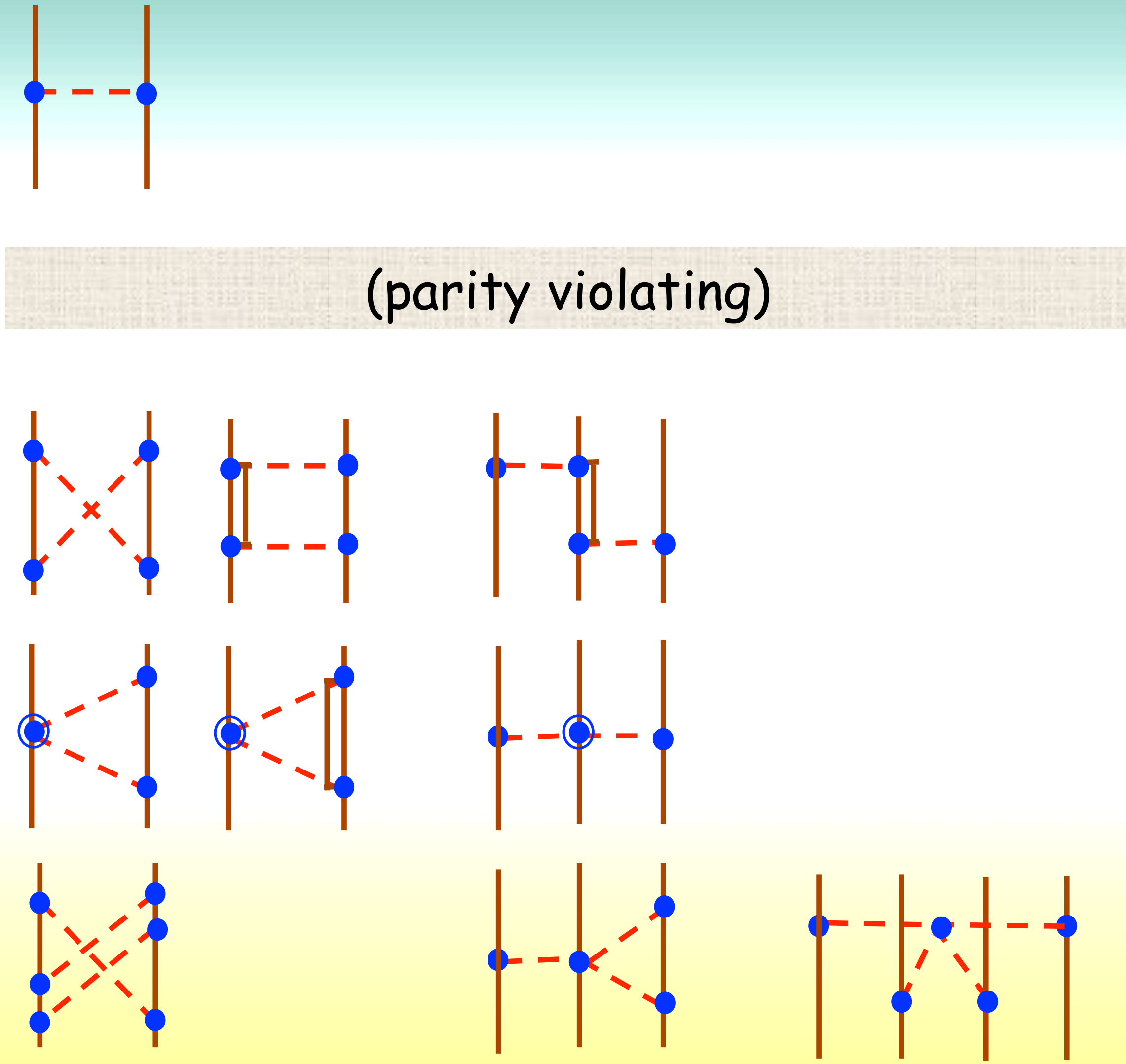
...





Etc.

		2-body	3-body	4-body	…
LO	$O\left(\frac{1}{f_\pi^2}\right)$				
in German NLO	$O\left(\frac{1}{f_\pi^2}\frac{Q}{M_{QCD}}\right)$				
NNLO	$O\left(\frac{1}{f_\pi^2}\frac{Q^2}{M_{QCD}^2}\right)$				
NNNLO	$O\left(\frac{1}{f_\pi^2}\frac{Q^3}{M_{QCD}^3}\right)$				
NNNNLO	$O\left(\frac{1}{f_\pi^2}\frac{Q^4}{M_{QCD}^4}\right)$				



ETC.

Hierarchies

many-body forces

$V_{2N} \gg V_{3N} \gg V_{4N} \gg \ldots$ A canon emerges! Weinberg '90, '91

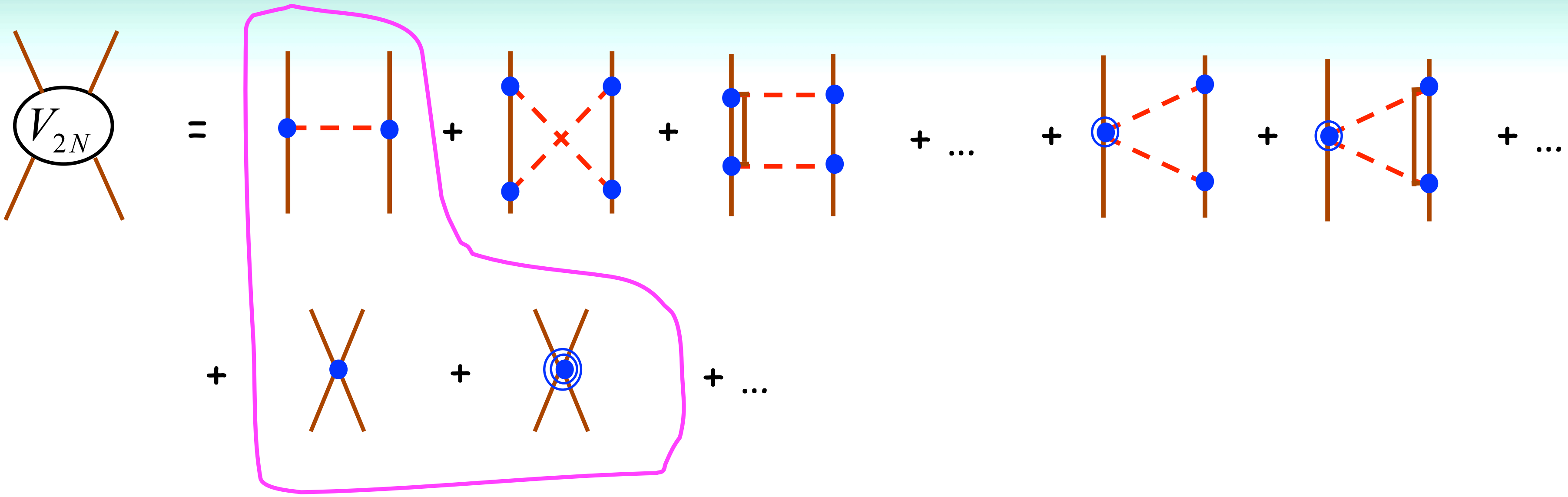
isospin-breaking forces

Similar explanation for

- $V_{IS} \gg V_{IV} \gg V_{CSB}$ v.K. '93
- $J_{1N} \gg J_{2N} \gg J_{3N} \gg \ldots$ Rho '92

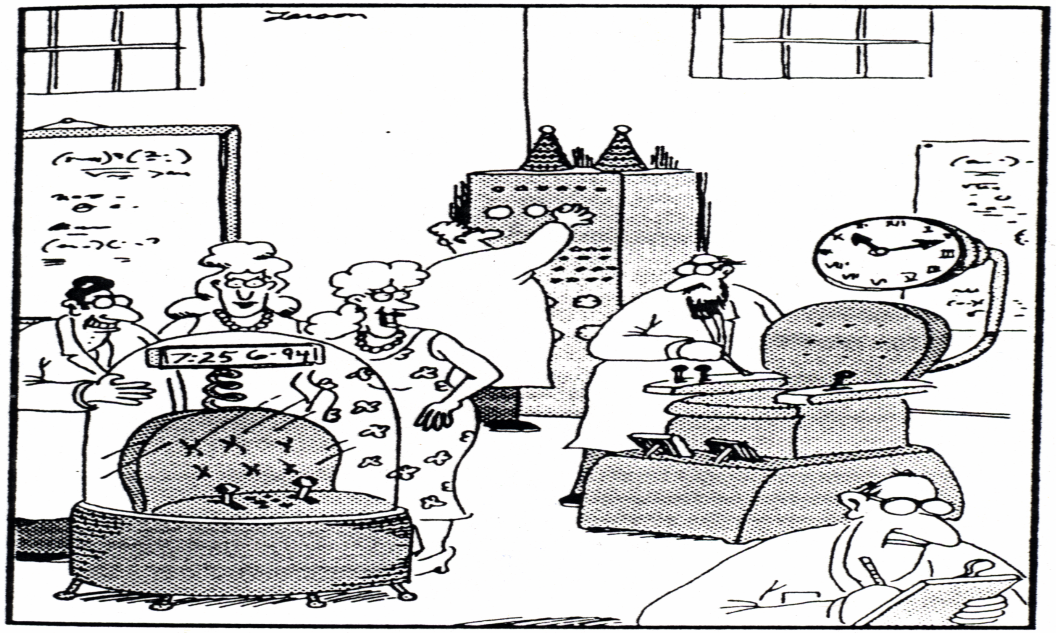
external currents

Ordonez + v.K. '92
v.K. '94
...

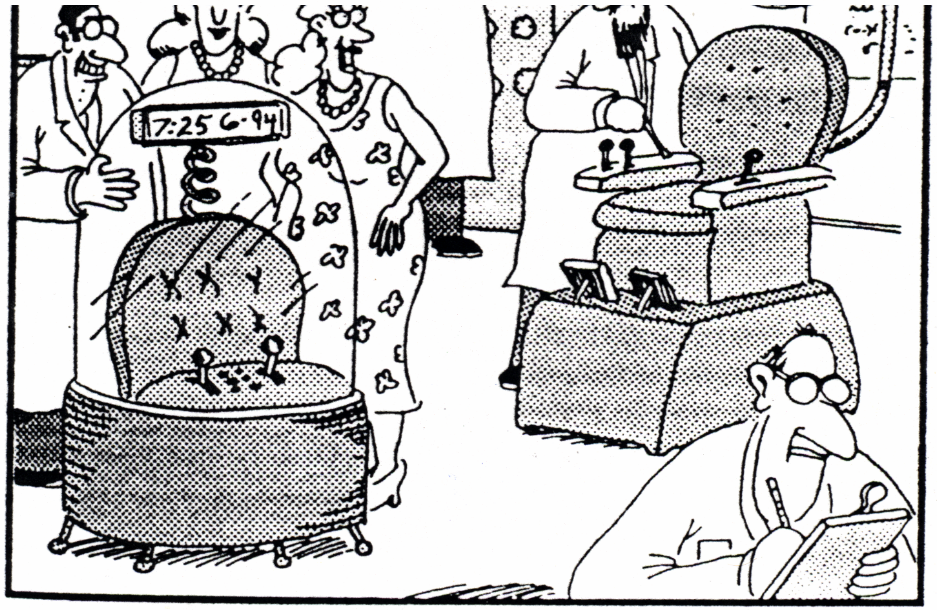


similar to phenomenological
potential models,
e.g. AV18 – (OPE)^2 + non-local terms

Stoks, Wiringa + Pieper '94



"Oh, Professor DeWitt! Have you seen Professor *Weinberg's* time machine? ... It's digital!"

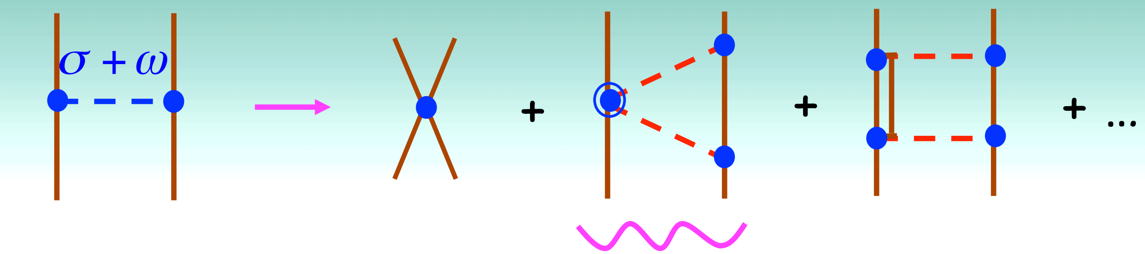


"Oh, Professor DeWitt! Have you seen Professor *Weinberg's* time machine? ... It's digital!"

But: NOT your usual potential!

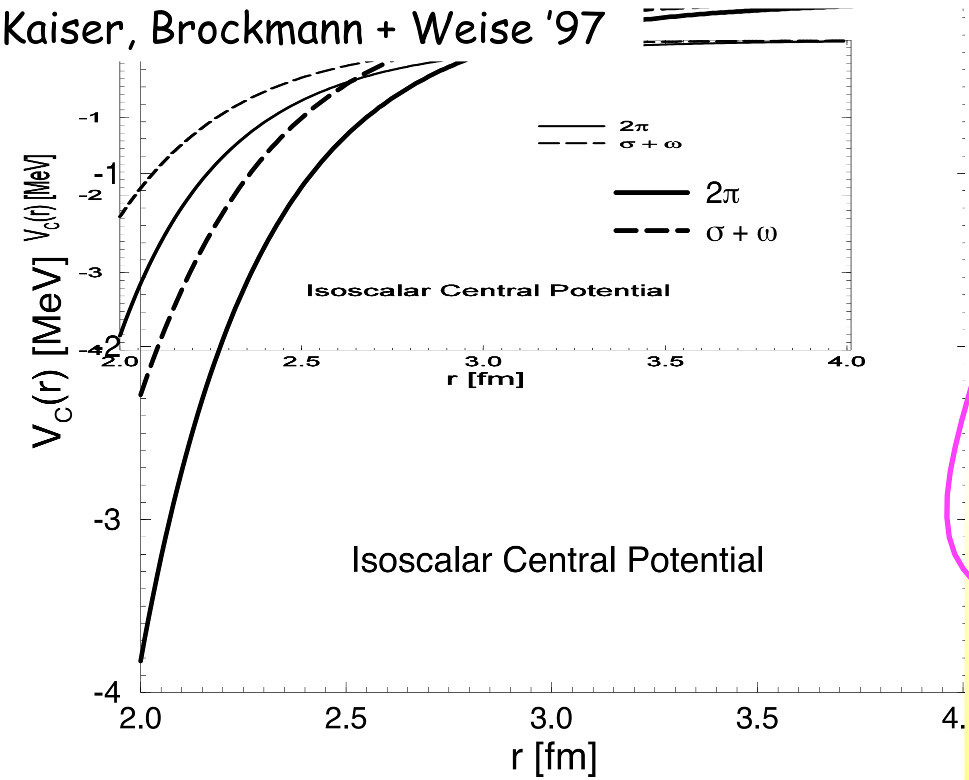
Ordonez + v.K. '92
(*cf.* Stony Brook TPE)

e.g.,



chiral v.d. Waals force $\sim \frac{1}{r^6}$ for $m_\pi^2 \to 0$

Kaiser, Brockmann + Weise '97



Rentmeester et al. '01, '03

Nijmegen PSA of 1951 *pp* data

long-range pot	# bc	χ^2_{min}
OPE	31	2026.2
OPE + TPE (*lo*)	28	1984.7
OPE + TPE (*nlo*)	23	1934.5
Nijm78	19	1968.7

parameters found consistent with πN data!

at least as good!

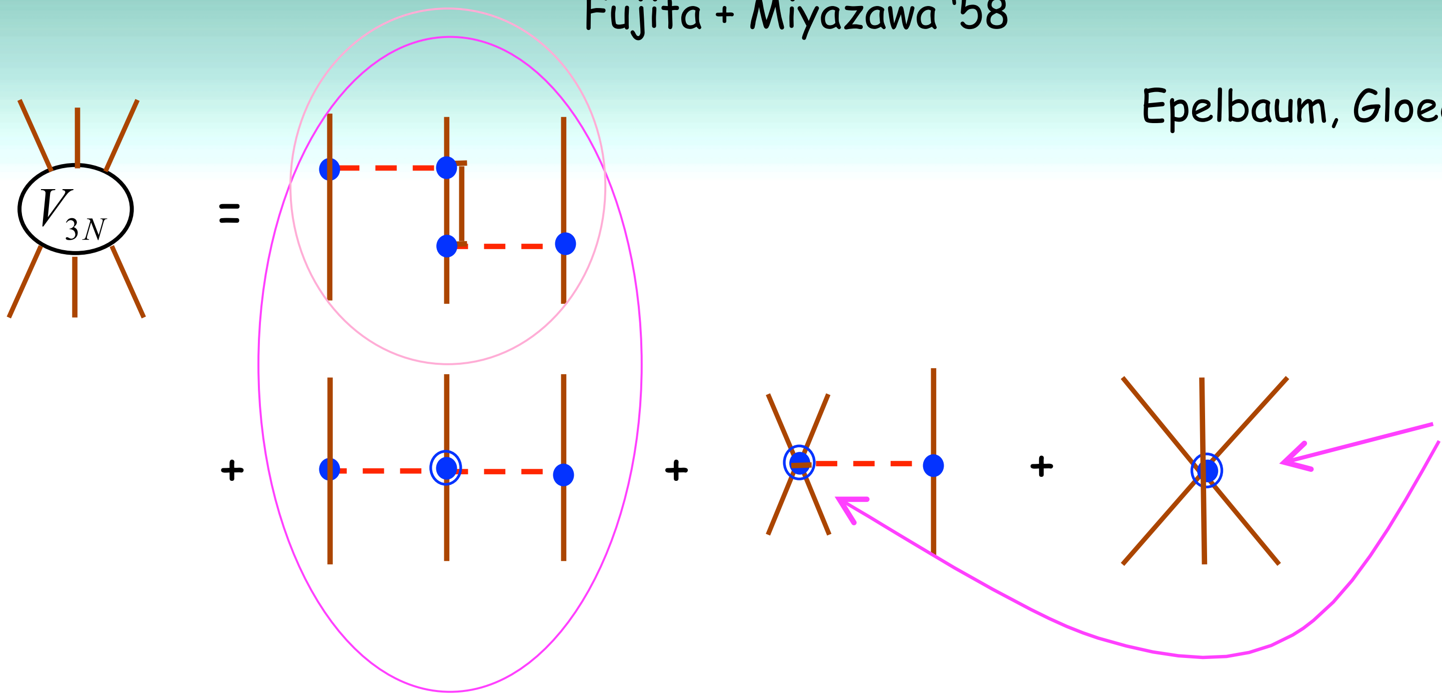
Similar results in other channels, *e.g.* spin-orbit force!

to EFTs

models with s, w, … might be misleading…

v.K. '94
Friar, Hueber + v.K. '99
Coon + Han '99
Epelbaum, Gloeckle+ Meissner '02
...

Fujita + Miyazawa '58



two unknown parameters

+ ...

Tucson-Melbourne pot with

Coon *et al.* '78

$$\begin{cases} a \to a - 2m_\pi^2 c \\ c \to 0 \end{cases}$$

TM' potential

$$\left(t_{\pi N}\left(\vec{q},\vec{q}\,'\right)\right)_{\alpha\beta} = \delta_{\alpha\beta}\left[a + b\vec{q}\cdot\vec{q}\,' + c\left(\vec{q}^{\,2} + \vec{q}\,'^{2}\right)\right] - d\varepsilon_{\alpha\beta\gamma}\tau_{3\gamma}\vec{\sigma}\cdot\vec{q}\times\vec{q}\,' + \ldots$$

Ordonez, Ray + v.K. '96
...
Epelbaum, Gloeckle + Meissner '02
Entem + Machleidt '03
...

Many successes of Weinberg's counting, *e.g.*,

✓ To NNNNLO (w/o deltas), fit to NN phase shifts comparable to those of "realistic" phenomenological potentials

Entem + Machleidt '03

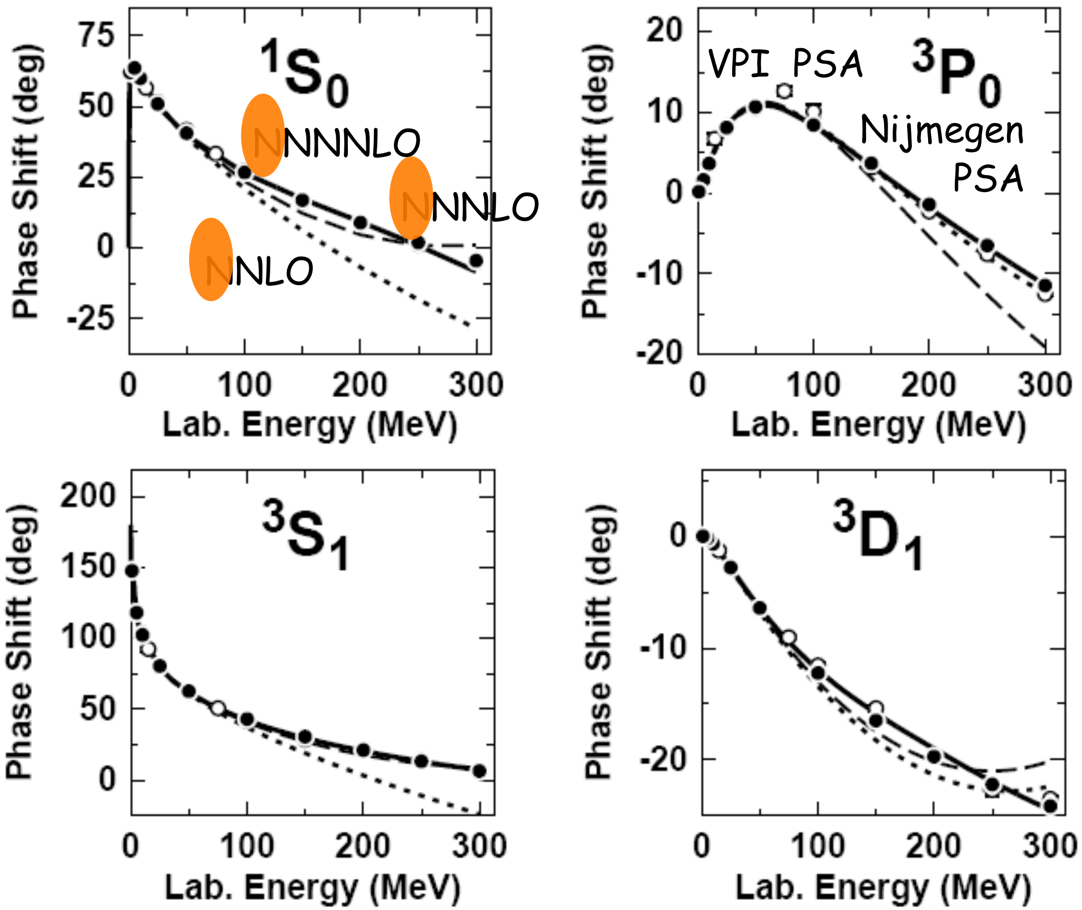


TABLE II. χ^2/datum for the reproduction of the 1999 *np* database [40] below 290 MeV by various *np* potentials.

Bin (MeV)	No. of data	N^3LO[a]	NNLO[b]	NLO[b]	AV18[c]
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04

TABLE V. Two- and three-nucleon bound-state properties. (Deuteron binding energy B_d; asymptotic *S* state A_S; asymptotic *D/S* state η; deuteron radius r_d; quadrupole moment *Q*; *D*-state probability P_D; triton binding energy B_t.)

	N^3LO[a]	CD-Bonn [10]	AV18 [22]	Empirical[b]
Deuteron				
B_d(MeV)	2.224575	2.224575	2.224575	2.224575(9)
A_S(fm$^{-1/2}$)	0.8843	0.8846	0.8850	0.8846(9)
η	0.0256	0.0256	0.0250	0.0256(4)
r_d(fm)	1.978[c]	1.970[c]	1.971[c]	1.97535(85)
Q(fm^2)	0.285[d]	0.280[d]	0.280[d]	0.2859(3)
P_D(%)	4.51	4.85	5.76	
Triton				
B_t(MeV)[e]	7.855	8.00	7.62	8.48

✓ With NNNNLO 2N and NNNLO 3N potentials (w/o deltas), good description of

- 3N observables and 4N binding energy
- levels of p-shell nuclei

Epelbaum et al. '02

Gueorguiev, Navratil, Nogga, Ormand + Vary '07

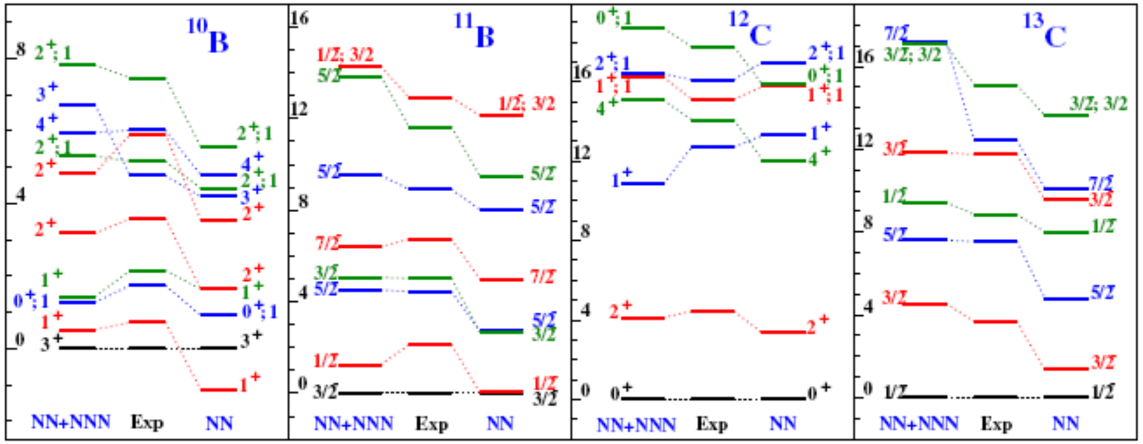


FIG. 4 (color online). States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin $T = 0$ or $1/2$, the isospin label is explicitly shown only for states with $T = 1$ or $3/2$. The excitation energy scales are in MeV.

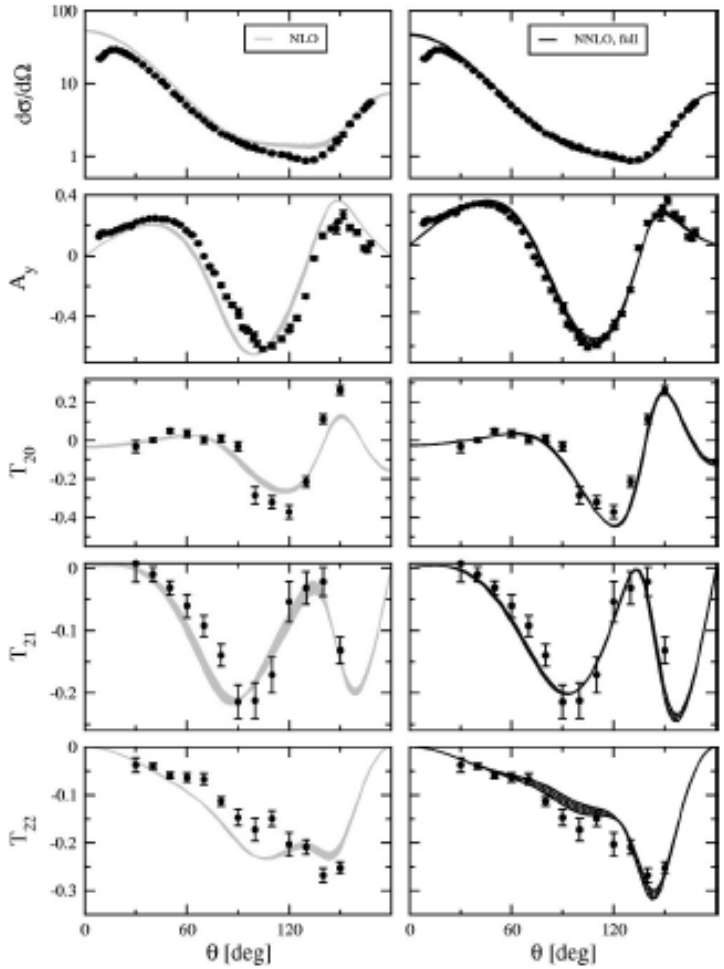


FIG. 6. nd elastic scattering observables at 65 MeV at NLO (left column) and NNLO (right column). The filled circles are pd data [63,69]. The bands correspond to the cutoff variation between 500 and 600 MeV. The unit of the cross section is mb/sr.

Many reactions:

$\gamma d \to d\gamma$

measured: Illinois '94, SAL '00, Lund '03

extracted nucleon polarizabilities: Beane, Malheiro, McGovern, Phillips + v.K. '04

$\gamma d \to d\pi^0$

threshold amplitude predicted: Beane, Bernard, Lee, Meissner + v.K. '97

confirmed: SAL '98, Mainz '01

$pp \to pp\pi^0$

$pp \to pn\pi^+$

$pp \to d\pi^+$

measured: IUCF '90-…, TRIUMF '91-…, Uppsala '95-…

S waves sensitive to high orders: Miller, Riska + v.K. '96

P waves converge, fix 3BF LEC: Hanhart, Miller + v.K. '00

$pn \to d\pi^0$

CSB asymmetry sign predicted: Miller, Niskanen + v.K. '00

confirmed: TRIUMF '03

$dd \to \alpha\pi^0$

measured: IUCF '03

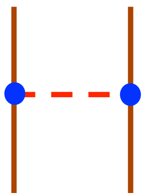
mechanisms surveyed: Fonseca, Gardestig, Hanhart, Horowitz, Miller, Niskanen, Nogga +v.K. '04 '06

+ **PARITY, TIME-REVERSAL VIOLATION**, *etc.*

BUT

Is Weinberg's power counting consistent?

No!



$$\sim \left(\frac{g_A}{2 f_\pi}\right)^2 \frac{m_\pi^3}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \frac{S_{12}(\hat{r})}{(m_\pi r)^3} + \ldots \right\} e^{-m_\pi r}$$

attractive in some channels

singular potential

not enough contact interactions for renormalization-group invariance even at LO

Nogga, Timmermans + v.K. '05
Pavon-Valderrama + Ruiz-Arriola '06

Problems!

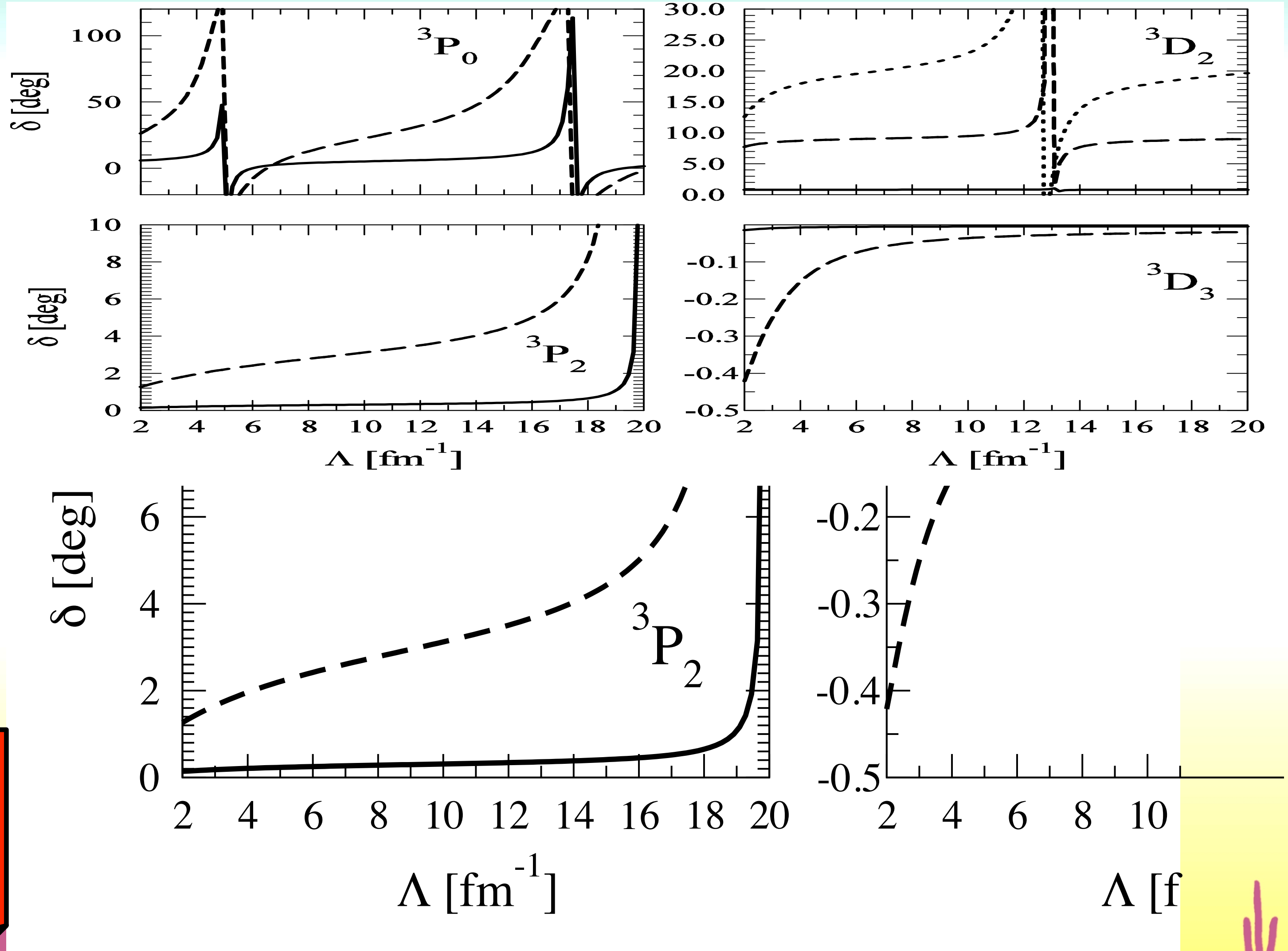
Attractive-tensor channels:

E (MeV)

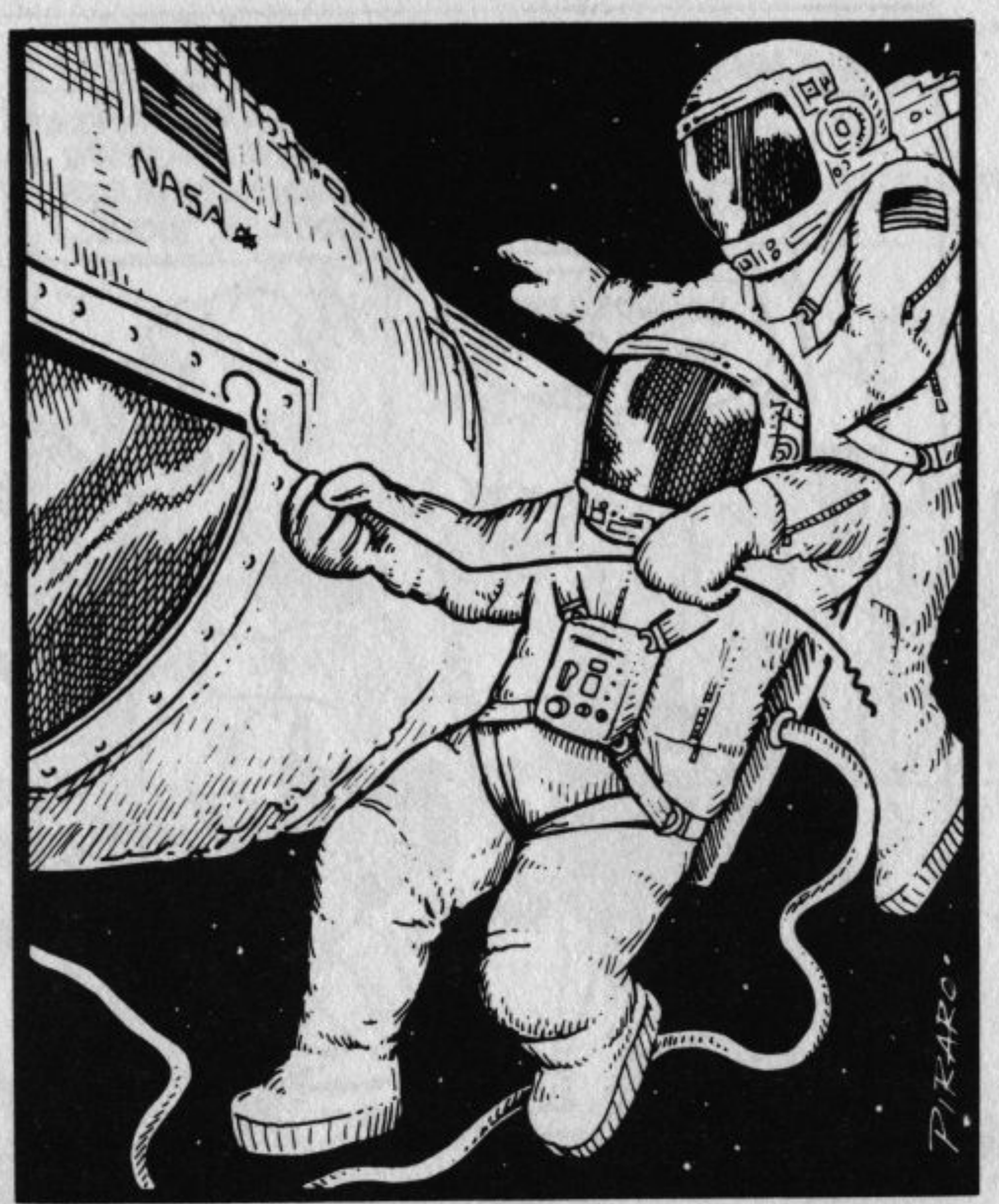
10 ———

50 - - -

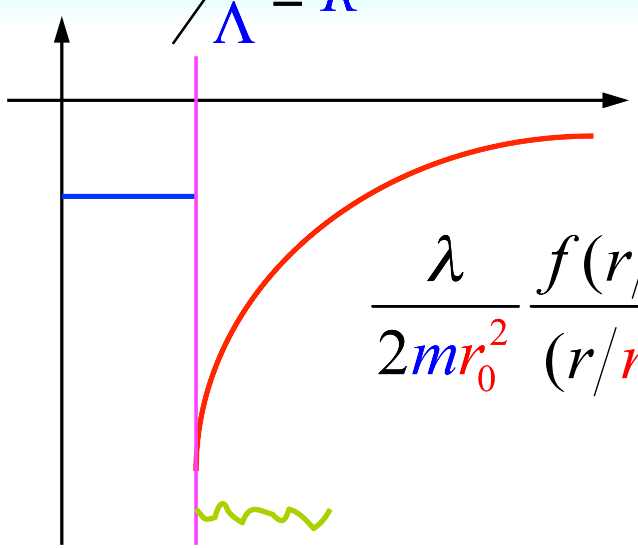
100 ....



incorrect renormalization…


NASA
PIRARO

Renormalization of the $1/r^n$ potential



$$C_0(\Lambda)\,\delta_\Lambda^{(3)}(\vec{r}) \rightarrow V_0(R)\,\theta\left(1-\frac{r}{R}\right)$$

$$\frac{\lambda}{2mr_0^2}\,\frac{f(r/r_0)}{(r/r_0)^n}$$

OPE:

$$\begin{cases} m = m_N/2 \\ r_0 = 1/m_\pi \\ \lambda = m_\pi/M_{NN} \\ f(r/r_0) = \exp(-r/r_0) \end{cases}$$

s wave $\quad \psi_n\left(r \sim R \ll r_0\right) \equiv \frac{u_n(r)}{r}$

matching

$$\sqrt{-2mR^2V_0}\,\cot\sqrt{-2mR^2V_0} = F_n\left(\lambda, r_0, R\right)$$

so that

$$\frac{\partial T_s}{\partial \ln R}\left(k \sim 1/r_0\right) = \mathrm{O}\left(T_s\,\frac{R}{r_0}\right)$$

$n \geq 2$

Beane, Bedaque, Childress, Kryjevski, McGuire + v.K. '02

Two **regular** solutions that oscillate!

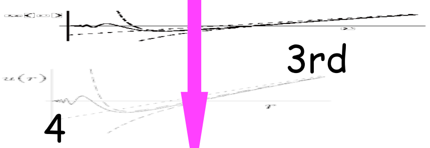
if no counterterm, will depend on cutoff R

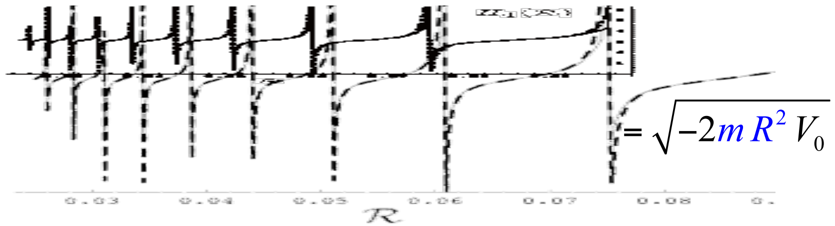
⇒ model dependence

$$u_n(r \ll r_0) = \left(\frac{\lambda}{(r/r_0)^n}\right)^{-\frac{1}{4}} \cos\left(\frac{\sqrt{\lambda}}{(n/2-1)(r/r_0)^{n/2-1}} + \delta_n\right) + \ldots$$

determined by low-energy data

$$F_n(\lambda, r_0, R) = \frac{n}{4} - \sqrt{\lambda}\left(\frac{R}{r_0}\right)^{1-n/2} \tan\left(\frac{\sqrt{\lambda}}{(n/2-1)(R/r_0)^{n/2-1}} + \delta_n\right) + \ldots$$





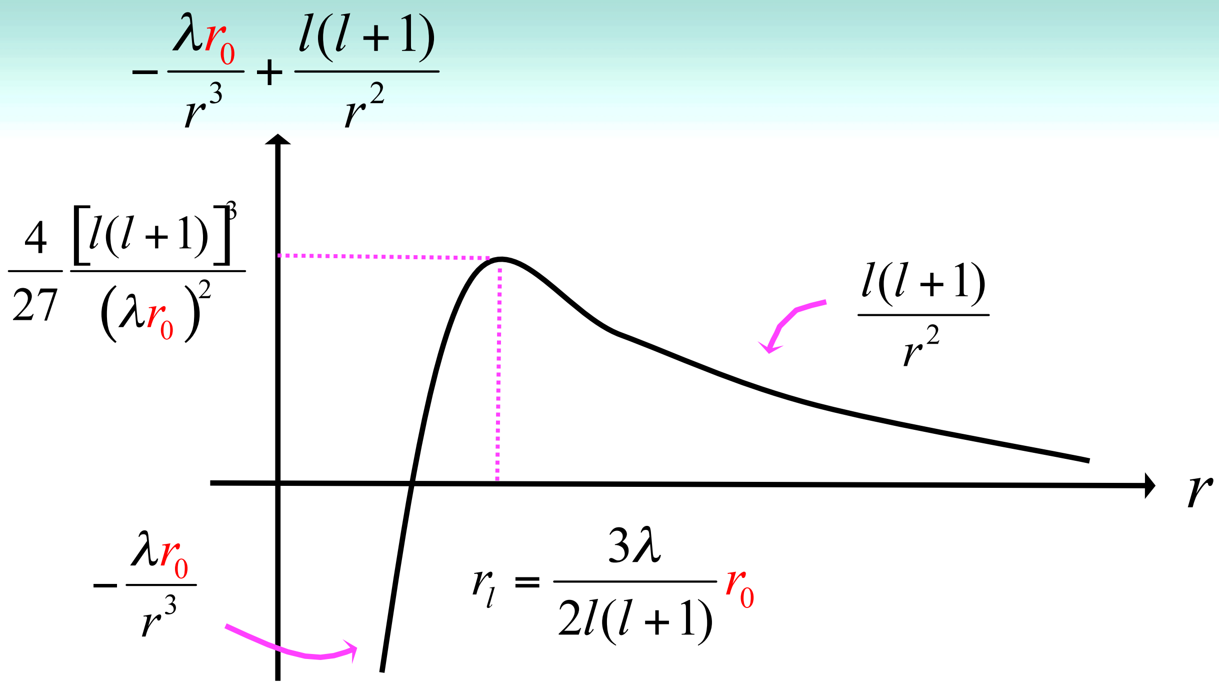
1st

2nd

exact vs perturbation th

limit-cycle-like behavior

Same is true in all channels where attractive singular potential is iterated



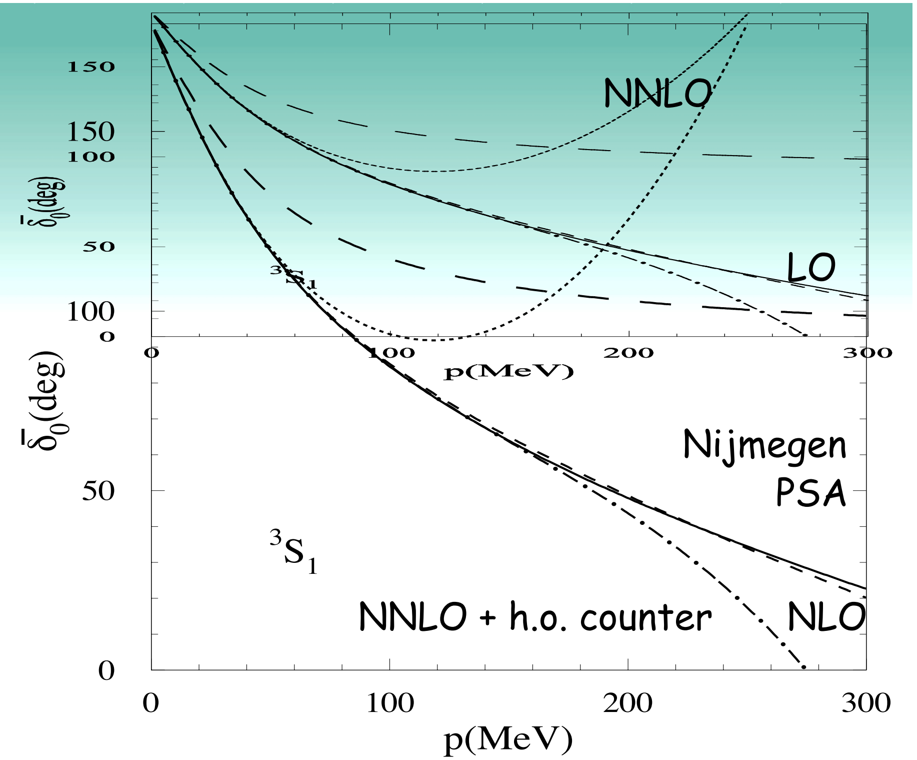
but $\quad r_l \sim \frac{1}{M} \ll r_0 \qquad$ for $\quad l(l+1) \gg \lambda$

⇒ singular potential only needs to be iterated in a few waves, where counterterms are needed

“Perturbative pions” $\lambda = \frac{m_\pi}{M_{NN}} \ll 1$

Kaplan, Savage + Wise ‘98

Fleming, Mehen + Stewart ’01



➡ $M_{NN} \sim f_\pi$ indeed

Non-peturbative pions

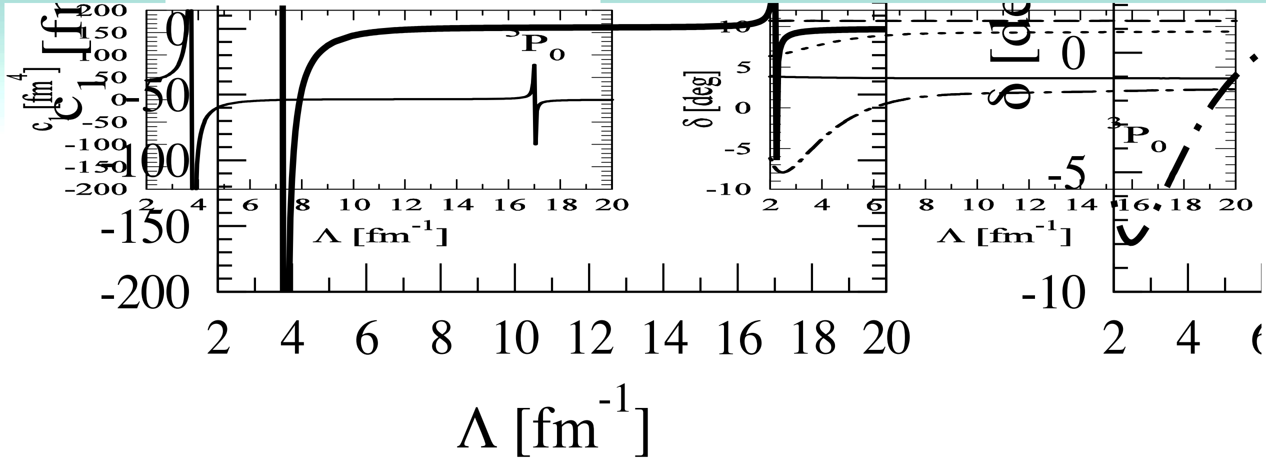
$$l(l+1) \lesssim \frac{3M_{QCD}}{2M_{NN}} \sim 5 \;\Rightarrow\; l \lesssim 2$$

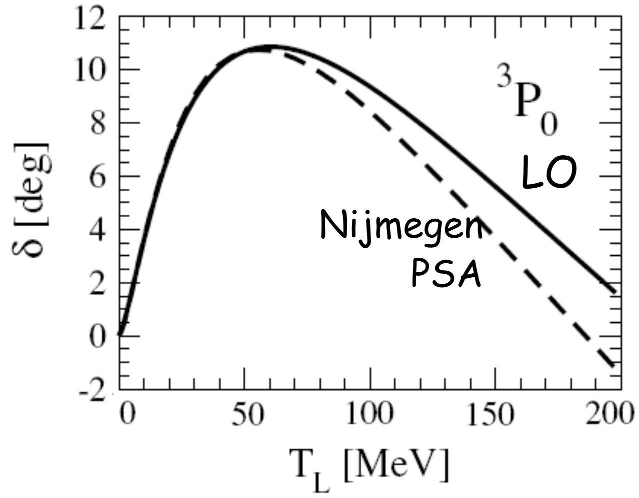
Beane, Bedaque, Savage + v.K. ‘02
Nogga, Timmermans + v.K. ‘05
Pavon Valderrama + Ruiz-Arriola ‘06
Birse, ‘06, ‘07
Long + v.K. ‘07
Pavon Valderrama ‘10

Add counterterms

Nogga, Timmermans + v.K. '05
(cf. Pavon Valderrama + Ruiz-Arriola, '06)

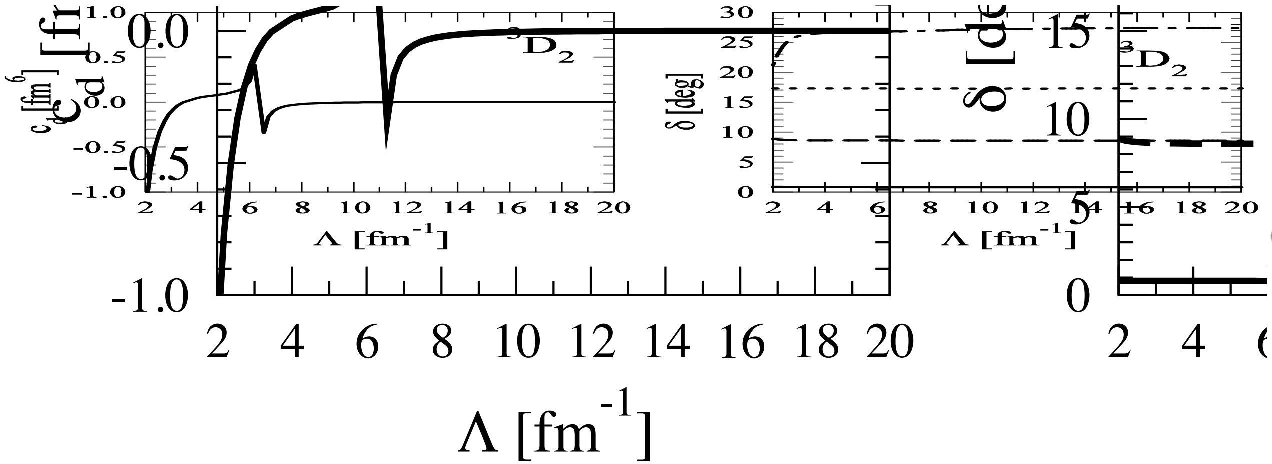
$$V_{l=1,j=0} = \frac{c_1}{(2\pi)^3}\, pp'$$

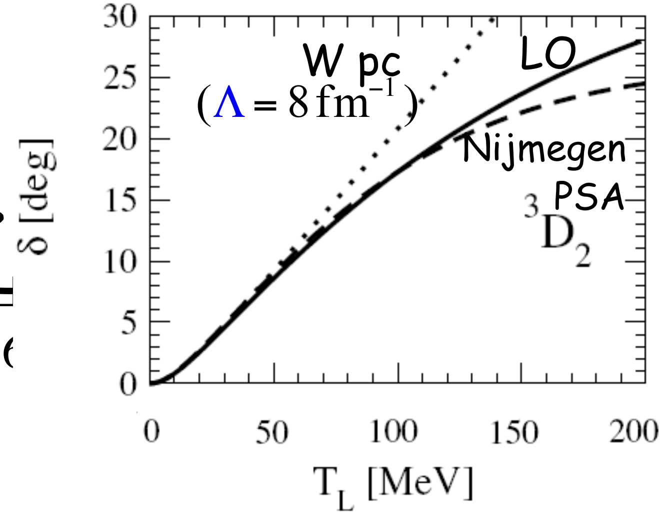




E (MeV)
10
50
100
190

$$V_{l=2,j=2} = \frac{c_d}{(2\pi)^3}\, p^2 p'^2$$





certain counterterms that in Weinberg's counting

were assumed suppressed by powers of $\frac{Q}{M_{QCD}}$

are in fact suppressed by powers of $\frac{Q}{lf_\pi}$

$\Downarrow$

short-range physics more important than assumed by Weinberg's;

most qualitative conclusions unchanged,

but quantitative results need improvement

ACTIVE RESEARCH AREA

Summary

- A low-energy EFT of QCD has been constructed and used to describe nuclear systems
- Chiral symmetry plays an important role, in particular setting the scale for nuclear bound states
- Nuclear physics canons emerge from chiral potential
- A new power counting has been formulated: more counterterms at each order relative to Weinberg's; expect even better description of observables

Stay tuned:
next, how to extend EFT to larger systems

Introduction to Effective Field Theories in QCD

U. van Kolck

University of Arizona

Supported in part by US DOE

Outline

- Effective Field Theories
- QCD at Low Energies
- Towards Nuclear Structure
  - Contact Nuclear EFT
  - Few-Body Systems
  - No-Core Shell Model
  - Halo/Cluster EFT
  - Conclusions and Outlook

References:

U. van Kolck,
Effective field theory of short-range forces,
Nucl.Phys.A645:273-302,1999, **nucl-th/9808007**

P.F. Bedaque, H.-W. Hammer, and U. van Kolck,
The three-boson system with short-range interactions,
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I. Stetcu, B.R. Barrett, and U. van Kolck,
No-core shell model in an effective-field-theory framework,
Phys.Lett.B653:358-362,2007, **nucl-th/0609023**

P.F. Bedaque, H.-W. Hammer, and U. van Kolck,
Narrow resonances in effective field theory,
Phys.Lett.B569:159-167,2003, **nucl-th/0304007**

Nuclear physics scales

"His scales are His pride", Book of Job

$\ln Q$

perturbative QCD

~1 GeV $M_{QCD} \sim m_N,\, m_\rho,\, 4\pi f_\pi,\, \ldots$

brute force (lattice), ...?

hadronic th with chiral symm

Chiral EFT

Q/M_{QCD}

~100 MeV $M_{nuc} \sim f_\pi,\, 1/r_{NN},\, m_\pi,\, \ldots$

~30 MeV $\aleph \sim 1/a_{NN}$

Q/M_{nuc}

this lecture

no small coupling → expansion in ↑

Lots of interesting nuclear physics at $E \sim 1\ \text{MeV}$
instead of $E \sim 10\ \text{MeV}$

within a few MeV of thresholds:

- many energy levels and resonances (cluster structures)
- most reactions of astrophysical interest

show universal features,
i.e. to a very good approximation are independent
of details of the short-range dynamics

bonus: same techniques can be used
for dilute atomic/molecular systems

- pionful EFT an overkill at lower energies!

cf. Bethe + Peierls '35

e.g. NN $\quad s_1$ channel:

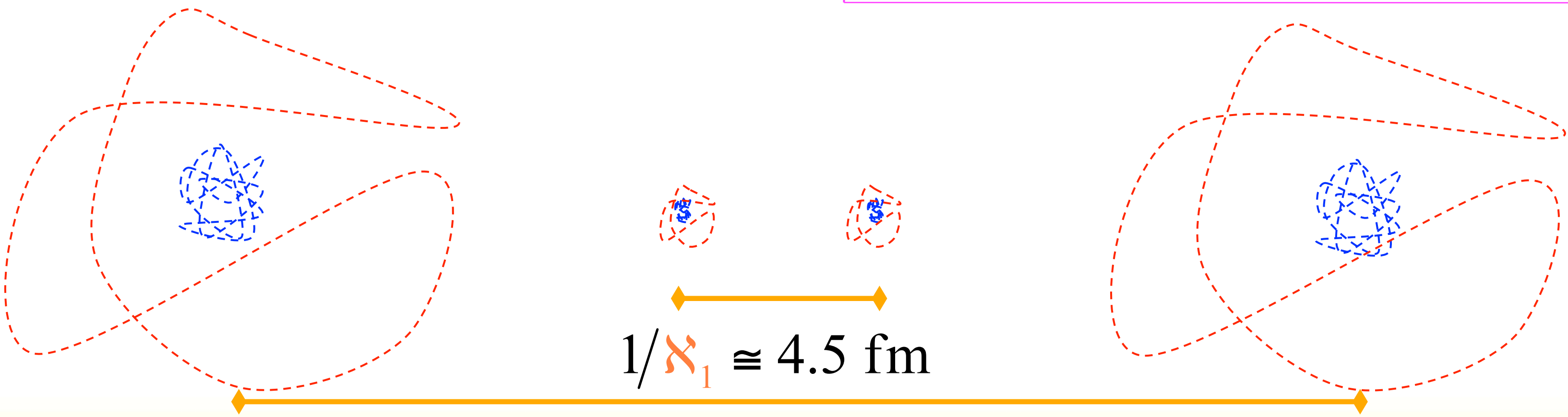
(real) bound state = deuteron

$$\aleph_1 \sim \sqrt{m_N B_d} \cong 45 \text{ MeV} < m_\pi$$

s_0 channel:

(virtual) bound state

$$\aleph_0 \sim \sqrt{m_N B_{d^*}} \cong 8 \text{ MeV} \ll m_\pi$$



multipole expansion of meson cloud:

contact interactions among local nucleon fields

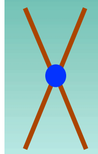
$Q \sim \aleph \ll M_{nuc}$

pionless EFT

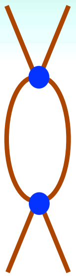
- d.o.f.: nucleons
- symmetries: Lorentz, P̸, T̸

$$\mathcal{L}_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + C_0\, N^{+}N\, N^{+}N$$
$$+ N^{+}\frac{\nabla^4}{8m_N^3}N + C_2\, N^{+}N\, N^{+}\nabla^2 N$$
$$+ C_2'\, N^{+}\vec{\nabla}N \cdot N^{+}\vec{\nabla}N + \ldots$$

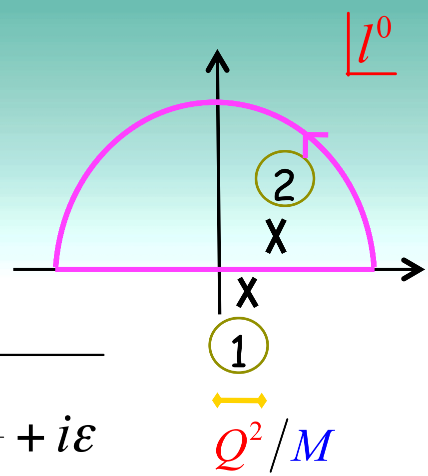
(omitting spin, isospin)



$\sim iC_0(\Lambda)$

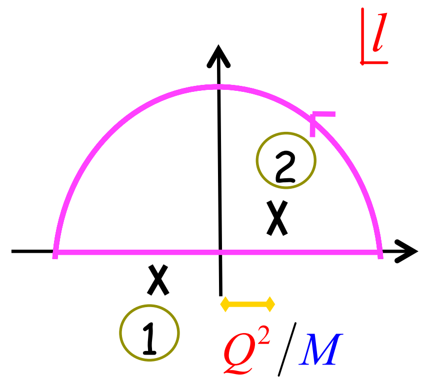


$$\sim C_0^2(\Lambda)\int \frac{d^4 l}{(2\pi)^4} \underbrace{\frac{1}{l^0 + p^0 - \frac{(\vec{l}+\vec{p})^2}{2m_N} + i\varepsilon}}_{①} \underbrace{\frac{1}{-l^0 + p^0 - \frac{(\vec{l}+\vec{p})^2}{2m_N} + i\varepsilon}}_{②}$$

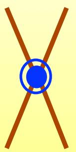


$$p^0 \equiv \frac{k^2}{2m_N} \qquad = -i\, m_N C_0^2(\Lambda)\int \frac{d^3 l}{(2\pi)^3} \frac{1}{\vec{l}^{\,2} - k^2 - i\varepsilon}$$

$$= -i\frac{m_N}{2\pi^2} C_0^2(\Lambda)\left\{\int_0^\Lambda dl + k^2 \int_0^\Lambda dl \underbrace{\frac{1}{l+k+i\varepsilon}}_{①} \underbrace{\frac{1}{l-k-i\varepsilon}}_{②}\right\}$$



$$= -i\, m_N C_0^2(\Lambda)\left\{\frac{1}{2\pi^2}\Lambda + i\frac{k}{4\pi} + \mathrm{O}\left(\frac{k^2}{4\pi\Lambda}\right)\right\} \equiv -i\, C_0^2(\Lambda) I_0(\Lambda)$$



$\sim i\, C_2(\Lambda)\, k^2$

absorbed in $C_0(\Lambda)$ | non-analytic in E | absorbed in $C_2(\Lambda)$

etc.

$$\left\{ \begin{array}{l} C_0(\Lambda) \to C_0^{(R)} \equiv C_0(\Lambda)\left\{1 - \frac{m_N \Lambda}{2\pi^2} C_0(\Lambda) + \ldots \right\} = \dfrac{C_0(\Lambda)}{1 + \frac{m_N \Lambda}{2\pi^2} C_0(\Lambda)} \\ C_2(\Lambda) \to C_2^{(R)} \equiv C_2(\Lambda) - \frac{m_N}{4\pi\Lambda} C_0^2(\Lambda) + \ldots \\ \ldots \end{array} \right.$$

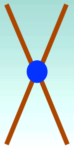
Naïve dimensional analysis

$$C_0^{(R)} \equiv \frac{4\pi}{m_N M_0} \qquad C_0^{(R)} \sim \frac{4\pi}{m_N M_{nuc}} \quad \Rightarrow \quad M_0 \sim M_{nuc}$$

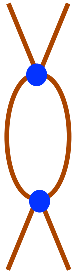
$$C_2^{(R)} \equiv \frac{4\pi}{m_N M_{nuc} M_2^2} \qquad C_2^{(R)} \sim \frac{m_N}{4\pi M_{nuc}} C_0^{(R)2} \quad \Rightarrow \quad M_2 \sim M_0$$

etc.

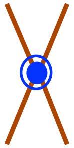
But in this case:



$$\rightarrow C_0^{(R)} \sim \frac{4\pi}{m_N M_0}$$



$$\rightarrow \frac{m_N Q}{4\pi} C_0^{(R)2} \sim \frac{4\pi}{m_N M_0} \frac{Q}{M_0} \quad \ll 1 \text{ for } Q \sim M_{nuc}$$



$$\rightarrow C_2^{(R)} Q^2 \sim \frac{4\pi}{m_N M_0} \frac{Q^2}{M_{nuc} M_0} \quad \ll 1 \text{ if } M_0 \sim M_{nuc}$$

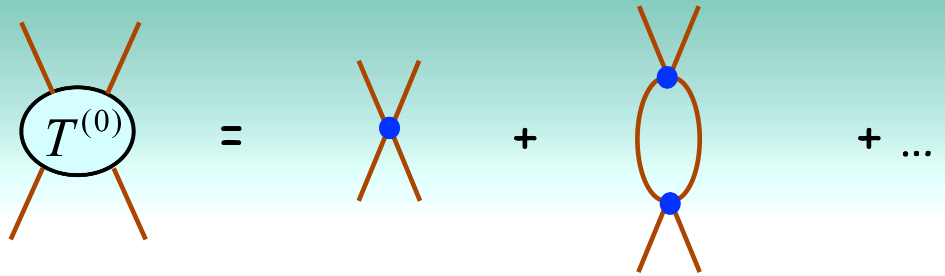
etc.

⇒ no b.s. at $Q \lesssim M_{nuc}$, no good: just perturbation theory

need one fine-tuning: $M_0 \equiv \aleph \ll M_{nuc}$

assume no other, *e.g.* still $M_2 \sim M_0$, *etc.*

v.K. '97 '99
Kaplan, Savage + Wise '98
Gegelia '98



$$= iC_0\left\{1 - C_0 I_0 + \left(C_0 I_0\right)^2 + \ldots\right\} = \frac{i}{\frac{1}{C_0} + I_0} = \frac{4\pi}{m_N}\frac{i}{\frac{4\pi}{m_N C_0(\Lambda)} + \frac{2\Lambda}{\pi} + ik + \mathrm{O}\left(\frac{k^2}{\Lambda}\right)}$$

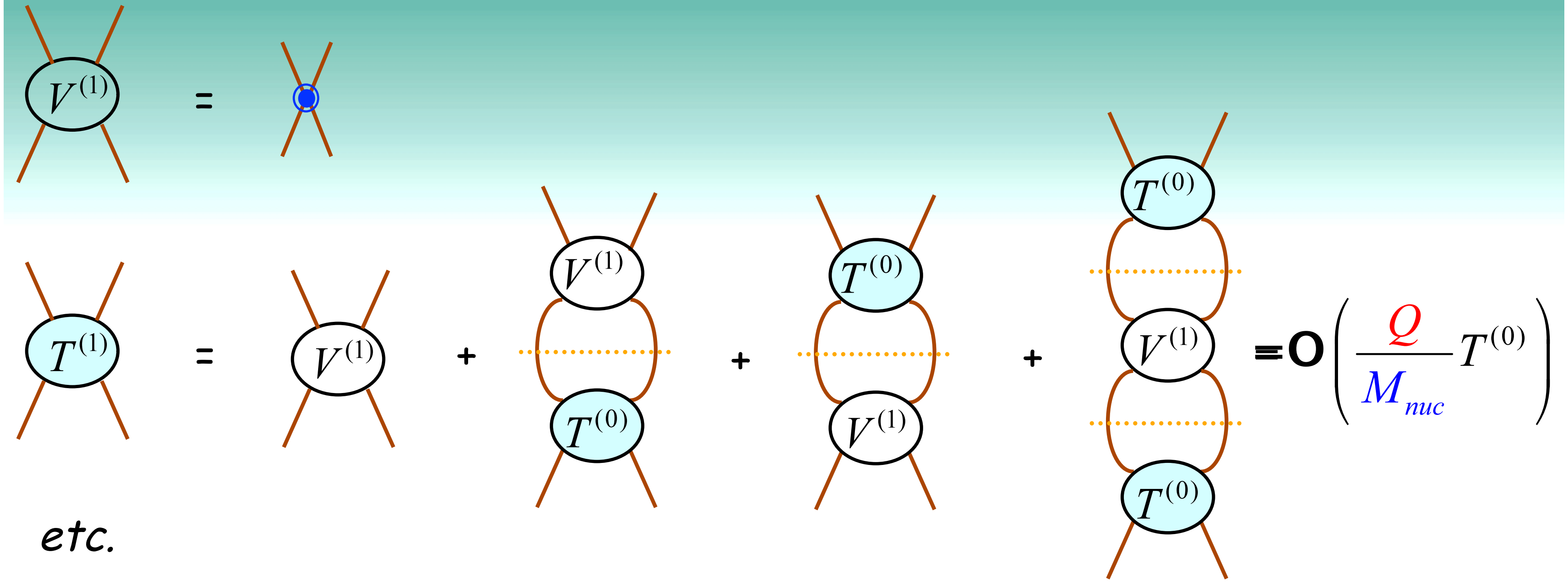
$k \sim \aleph$

$$\frac{4\pi}{m_N C_0(\Lambda)} + \frac{2\Lambda}{\pi} = \frac{4\pi}{m_N C_0^{(R)}} \equiv \aleph$$

$$= \frac{4\pi}{m_N}\frac{i}{\aleph + ik}\left[1 + \mathrm{O}\left(\frac{k}{\Lambda}, \frac{k^2}{\aleph\Lambda}\right)\right]$$

cf. effective range expansion
s wave

- scattering length $a_0 = 1/\aleph$
- bound state $k = i\aleph$ $\quad -E = \frac{\aleph^2}{2m_N}$



etc.

$$\Rightarrow \quad T_{NN} \sim \frac{4\pi}{m_N M_{nuc}} \left\{ \frac{M_{nuc}}{\aleph + iQ} + \left(\frac{Q}{\aleph + iQ} \right)^2 + \ldots \right\}$$

$\nu = -1$ $\quad$ $\nu = 0$

s wave:
- scattering length $a_0 \sim 1/\aleph$
- effective range $r_0 \sim 1/M_{nuc}$

p, other waves

Q

Example: square well $V(r) = -\frac{\alpha^2}{mR^2}\theta\left(1-\frac{r}{R}\right)$

$$\Rightarrow \quad T_{NN}(k) = -i\left[e^{-2ikR}\frac{\sqrt{\alpha^2+(kR)^2}\cot\sqrt{\alpha^2+(kR)^2}+ikr}{\sqrt{\alpha^2+(kR)^2}\cot\sqrt{\alpha^2+(kR)^2}-ikr}-1\right]$$

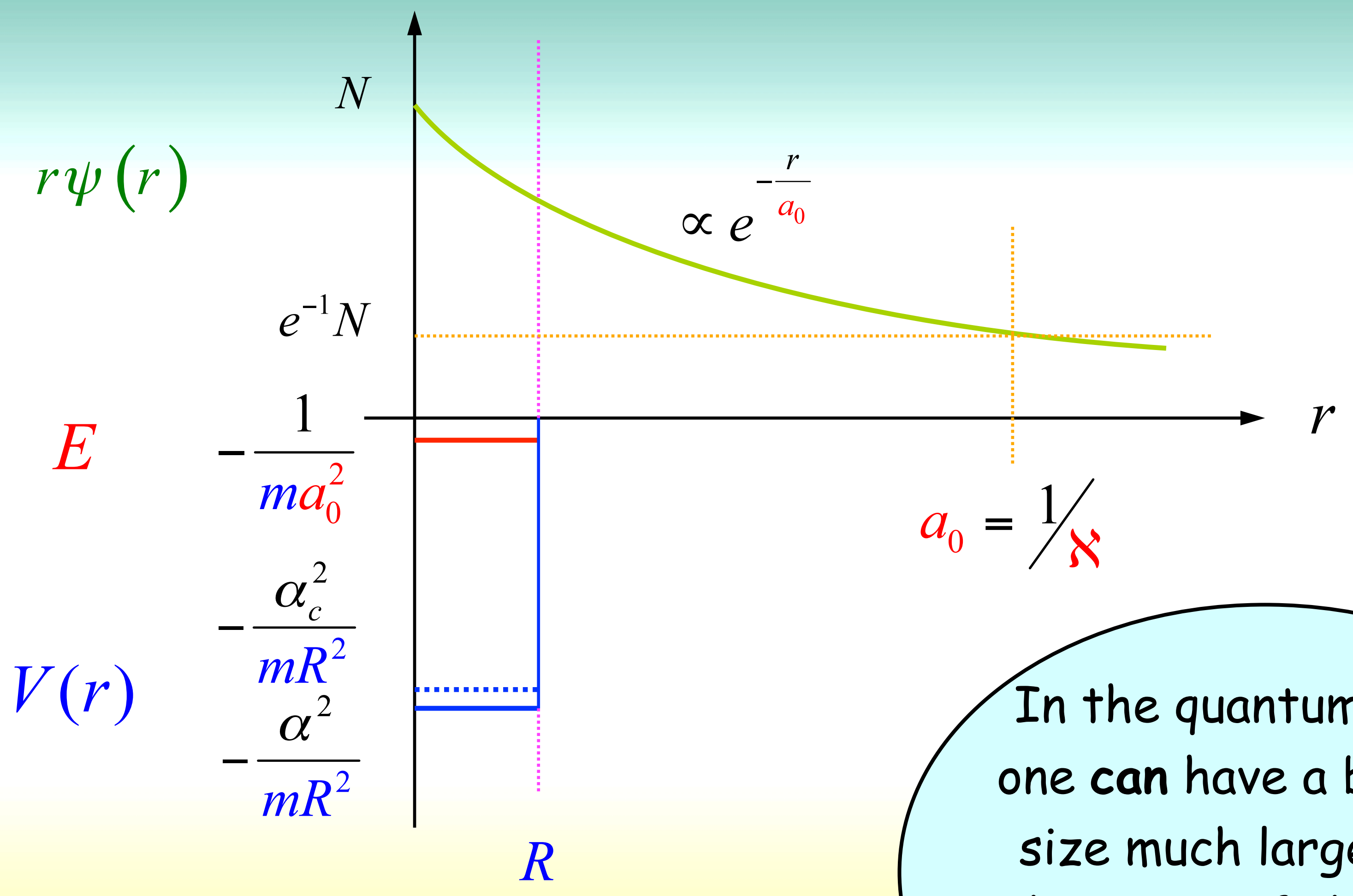
zero-energy poles when

$\alpha_c \equiv (2n+1)\pi/2$

	generic	fine-tuning
	$\alpha = O(1)$	$\lvert 1-\alpha/\alpha_c \rvert \ll 1$
$a_0 = R\left(1-\frac{\tan\alpha}{\alpha}\right)$	$a_0 \sim R$	$a_0 = -\frac{R}{\alpha_c^2}\left(1-\frac{\alpha}{\alpha_c}\right)^{-1}\{1+\ldots\} \sim \frac{1}{\aleph}$
$r_0 = R\left(1-\frac{R}{a_0\alpha^2}-\frac{R^2}{3a_0^2}\right)$	$r_0 \sim R$	$r_0 = R\{1+\ldots\} \sim R$

etc.

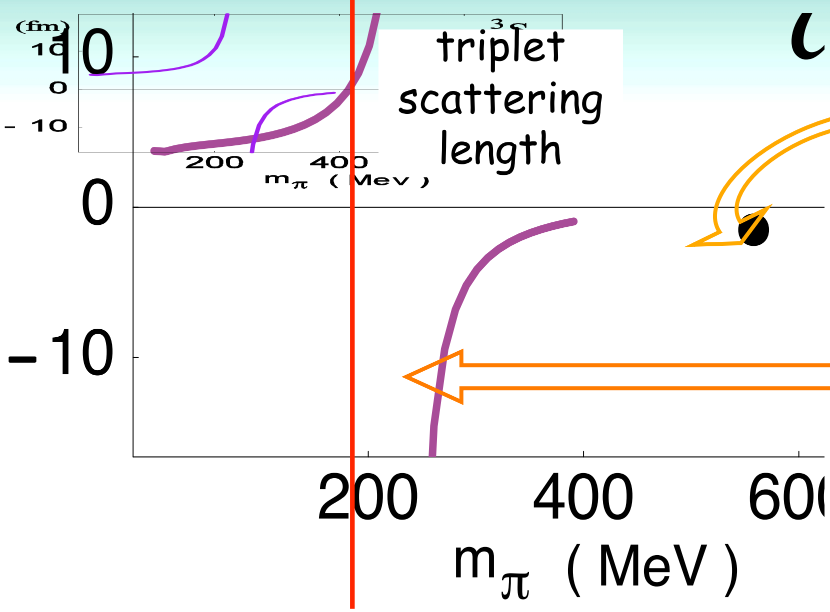
$$\aleph \equiv \frac{\lvert 1-\alpha/\alpha_c \rvert}{R} \ll \frac{1}{R}$$



In the quantum world,
one **can** have a b.s. with
size much larger than
the range of the force
provided
there is fine-tuning

Pion-mass dependence

unitarity limit
$|a_2| \to \infty$



$m_\pi^*\left(M_{QCD}\right)$

Lattice QCD: quenched

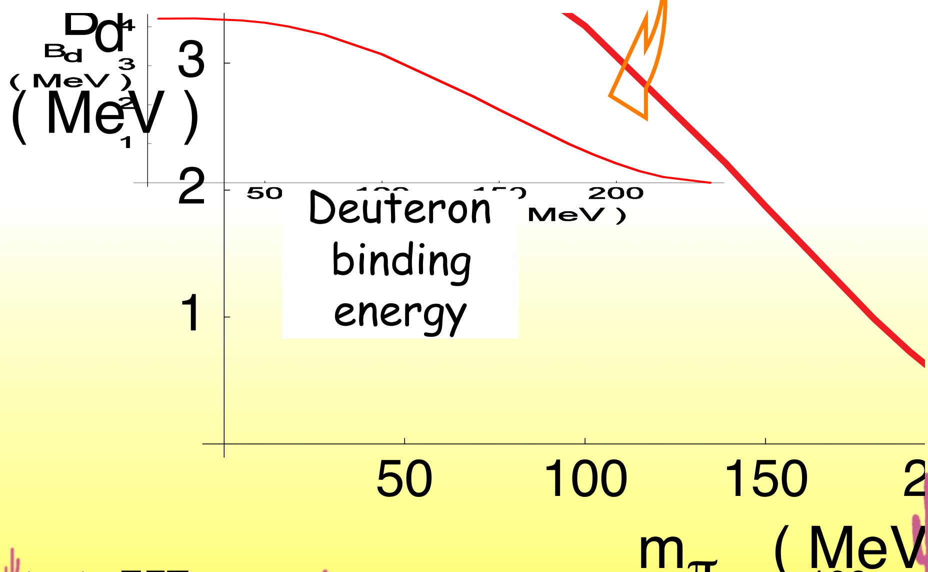
Fukugita et al. '95

cf. Beane, Bedaque, Orginos + Savage '06

Beane, Bedaque, Savage + v.K. '02

EFT: (incomplete) NLO

...

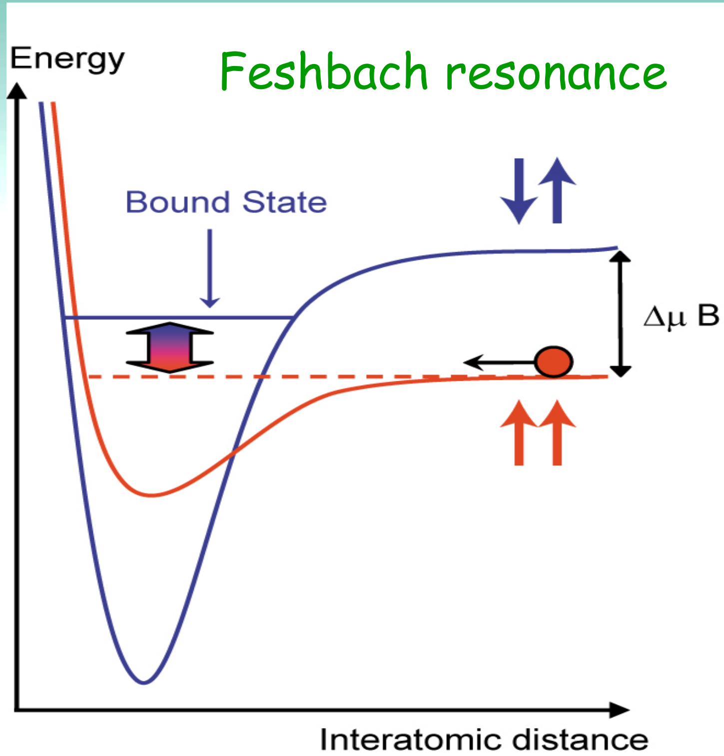


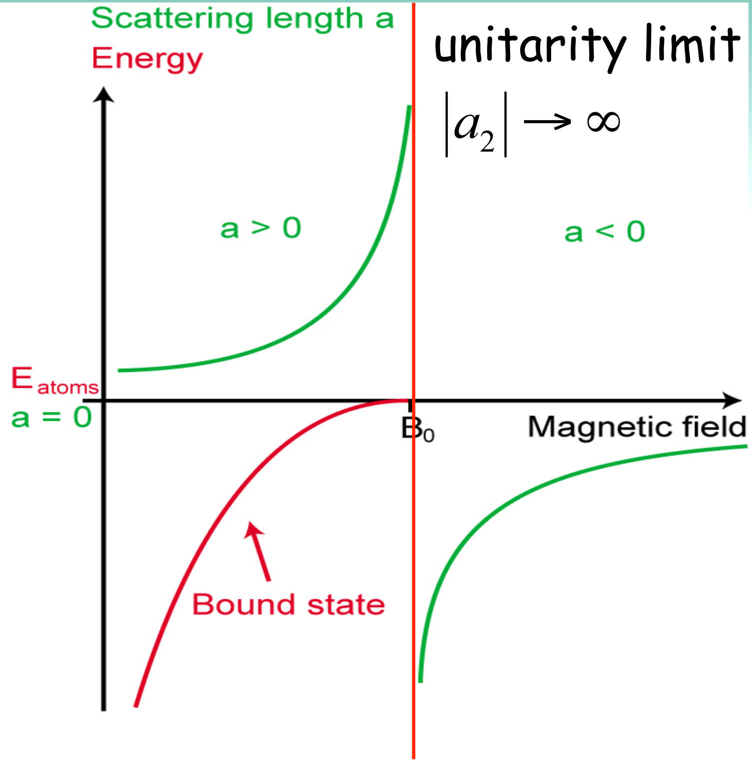
Large deuteron size because

$$m_\pi \sim m_\pi^*\left(M_{QCD}\right)$$

$$\aleph \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} M_{nuc}$$

Cf. trapped fermions





unitarity limit

$$|a_2| \to \infty$$

MIT group webpage

quark masses analog to magnetic field:
close to critical values

$$m_\pi^{*2} = \mathbf{O}\left(\left(m_u^* + m_d^*\right) M_{QCD}\right) \simeq (200\ \mathrm{MeV})^2$$

contact EFT can, and has been, used for atomic systems with large scatt lengths:

universality!

Alternative: auxiliary field

Kaplan '97
v.K. '99

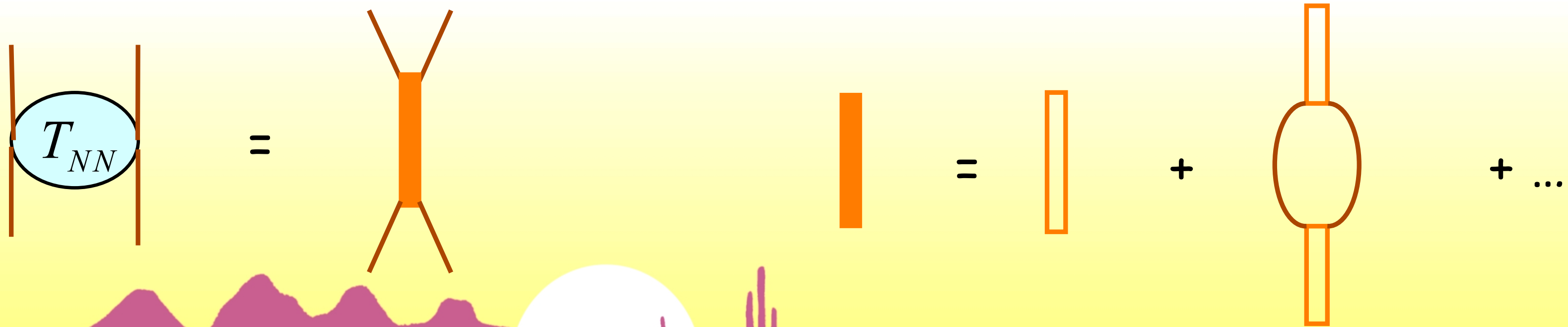
$$\mathcal{L}_{EFT} = N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + T^+(-\Delta)T + \frac{g}{\sqrt{2}}\left[T^+NN + N^+N^+T\right]$$

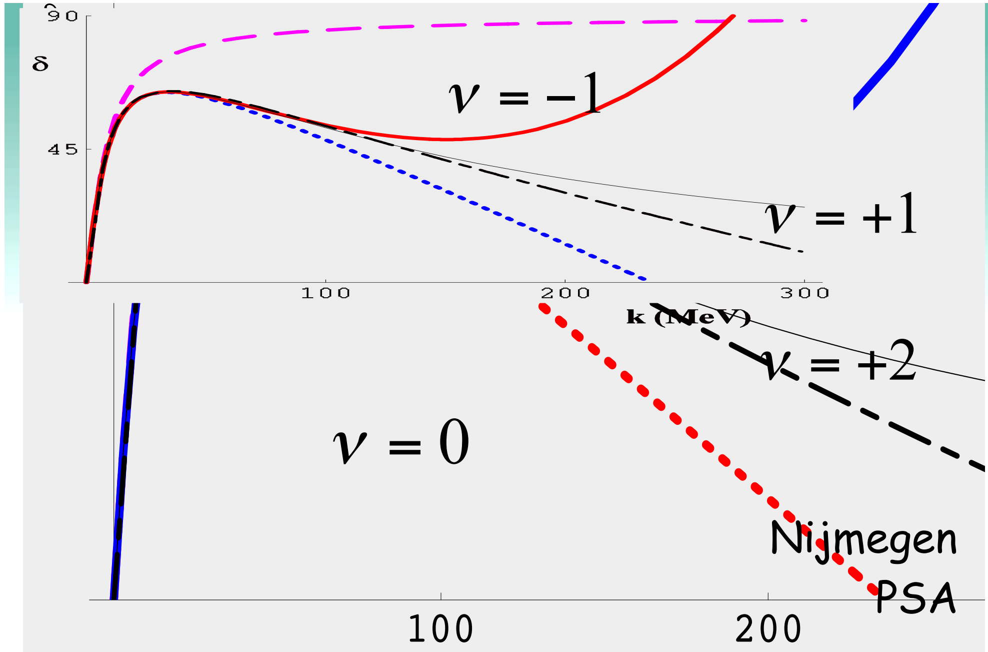
$$+ N^+\frac{\nabla^4}{8m_N^3}N + \sigma T^+\left(i\partial_0 + \frac{\nabla^2}{4m_N}\right)T + \ldots$$

sign

integrate out auxiliary field: same Lag as before with $C_0 = \frac{g^2}{\Delta}, \ldots$

$$\Delta \sim \aleph, \quad \frac{g^2}{4\pi} \sim \frac{1}{m_N}, \quad \ldots$$





Chen, Rupak + Savage '99

fitted $a_1 = 5.42\ \text{fm}\,(\text{exp})$

$r_1 = 1.75\ \text{fm}\,(\text{exp})$

predicted

$B_d = 1.91\ \text{MeV}\ (\nu = 0)$

$B_d = 2.22\ \text{MeV}\ (\text{exp})$

S_0

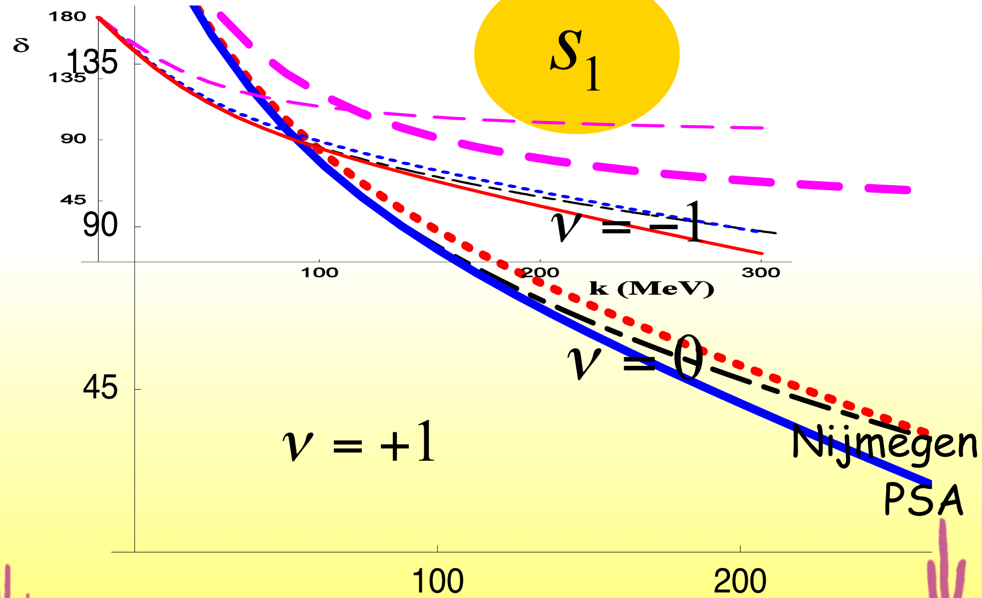
fitted $a_0 = -20.0\ \text{fm}\,(\text{exp})$

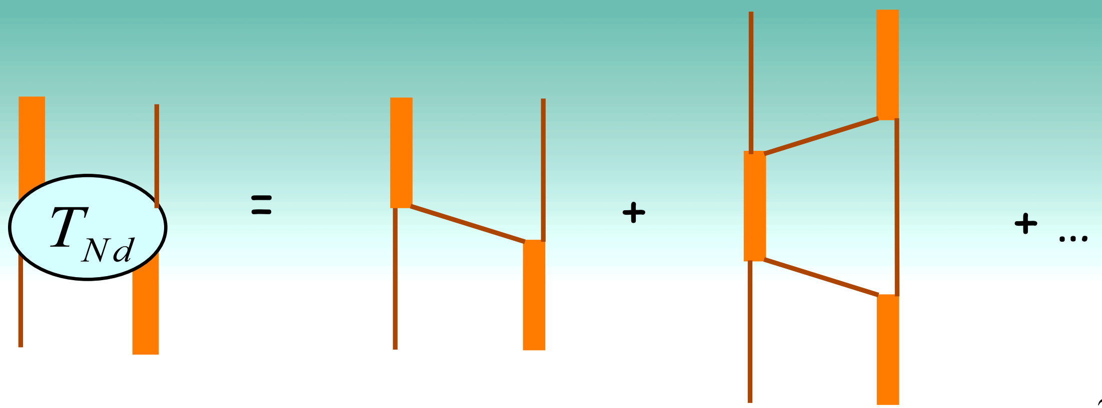
$r_0 = 2.78\ \text{fm}\,(\text{exp})$

predicted

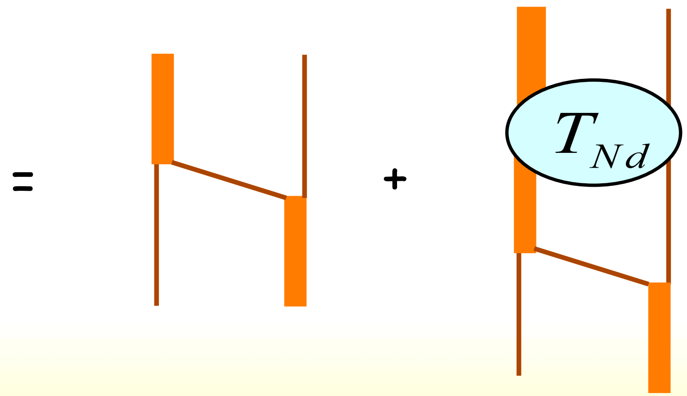
$B_{d^*} = 0.09\ \text{MeV}\ (\nu = 0)$

S_1



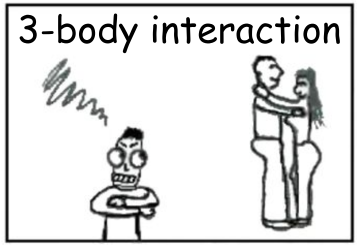


$$\sim \frac{g^2}{Q^2/m_N} \qquad \sim \frac{Q^3}{4\pi}\left(\frac{g^2}{Q^2/m_N}\right)^2 \qquad \sim \frac{g^2}{Q^2/m_N}\,\frac{Q}{\aleph}$$



$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3}\,\frac{K_{ONE}\,T_{Nd}}{D}$$

$$\mathcal{L}_{EFT} = \ldots + D_0\, N^+ N\, N^+ N\, N^+ N + \ldots$$



naïve dimensional analysis

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{M_{nuc}^3} \quad (\nu = +1)$$

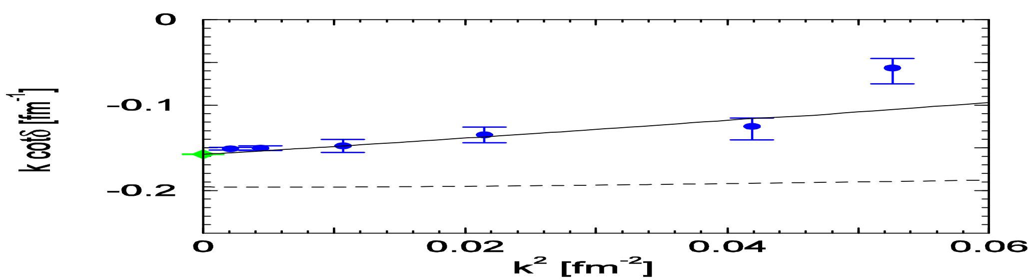
Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

$S_{3/2}$ no three-body force up to $\nu = +3$

$$\lambda \le \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg \aleph} \frac{1}{p^2}$$

$$\Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} 0$$



v.Oers + Seagrave '67

$\nu = +1$

Dilg et al. '71

predicted

$$a_{3/2} = 6.33 \pm 0.10\ \text{fm}\,(\nu = +1)$$

$$a_{3/2} = 6.35 \pm 0.02\ \text{fm}\,(\text{exp})$$

$\nu = -1$

QED-like precision!

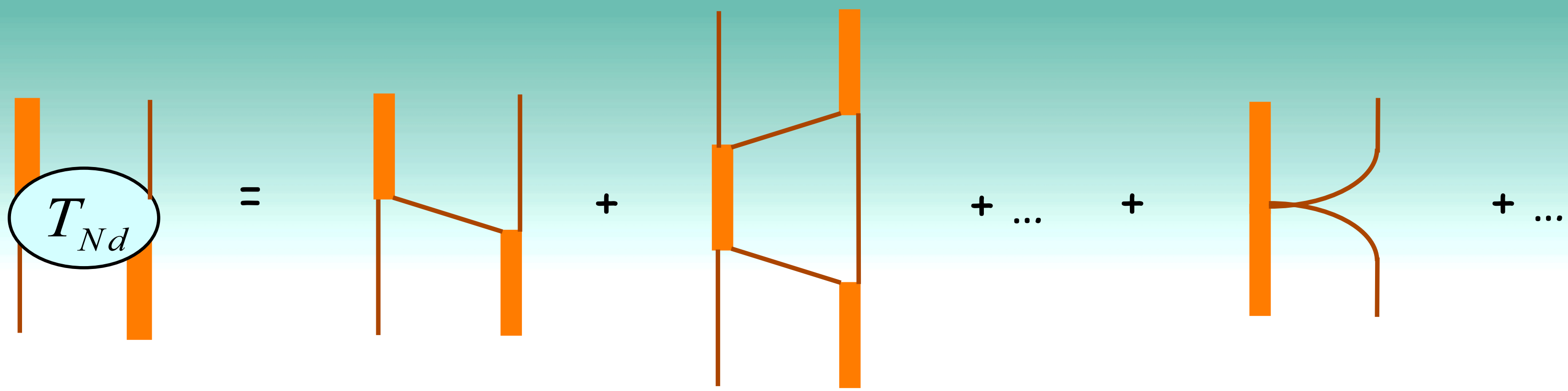
Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03

$S_{1/2}$

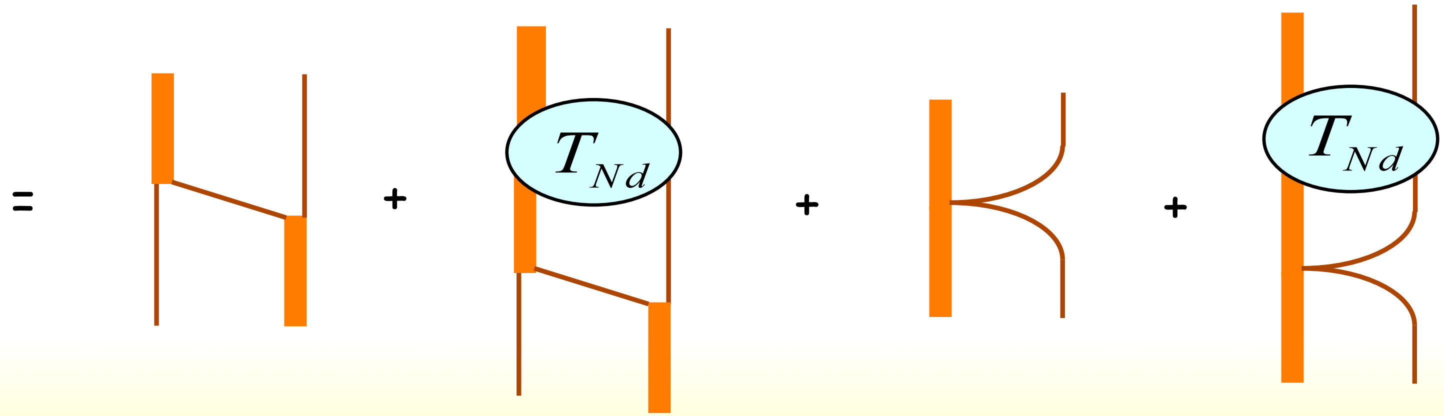
$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg ℵ} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta \right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim ℵ} 0, \text{ unless}$$

$$D_0 \sim \left(\frac{4\pi}{m_N} \right)^2 \frac{1}{ℵ^2 M_{nuc}} \quad (\nu = -1)$$



$$\sim \frac{g^2}{Q^2 / m_N} \qquad\qquad \sim \frac{4\pi}{\aleph^2}$$



$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} \frac{K_{ONE} T_{Nd}}{D} + K_{TBF} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} \frac{K_{TBF} T_{Nd}}{D}$$

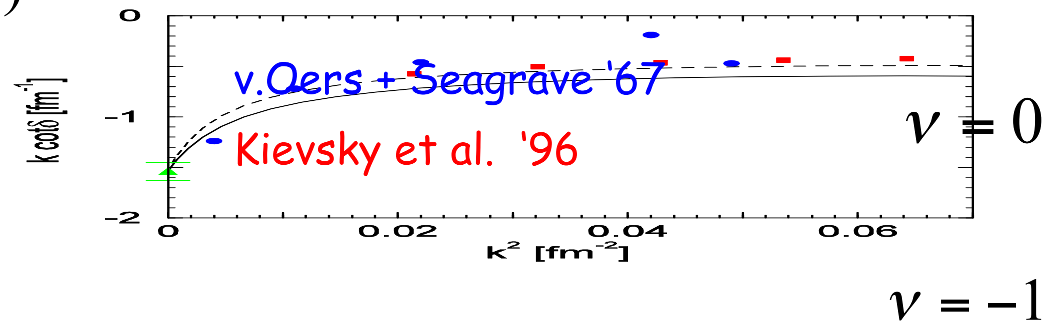
Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03
...

$S_{1/2}$ $\quad \lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$

$$T_{Nd} \xrightarrow{p \gg \aleph} A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} 0$$ unless

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

(limit cycle!)



fitted

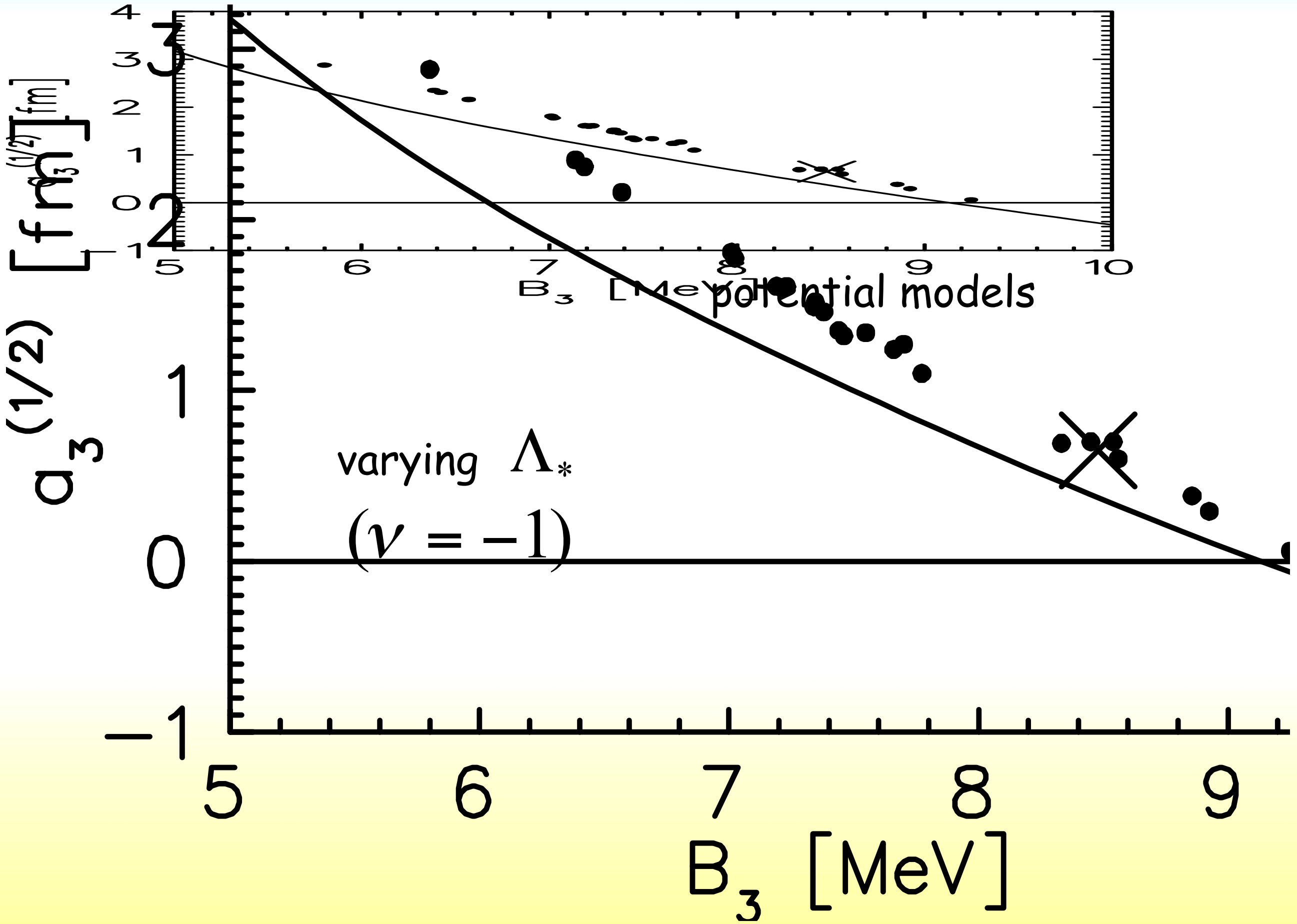
$$a_{1/2} = 0.65 \text{ fm (exp)}$$

predicted

$$B_t = 8.3 \text{ MeV } (\nu = 0)$$
$$B_t = 8.48 \text{ MeV (expt)}$$

Phillips line

Bedaque, Hammer + v.K. '99 '00

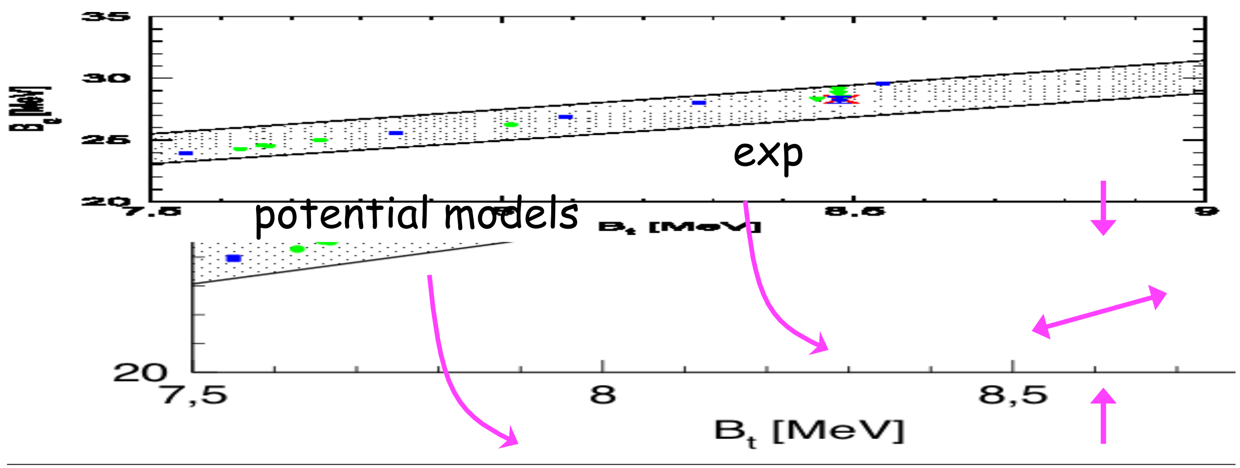


+ four-body bound state can be addressed similarly

⇨ no four-body force at $\nu = -1$

Hammer, Meissner + Platter '04

Tjon line



varying Λ_*

$(\nu = -1)$

Summary:

Expansion parameter $\frac{Q}{M_{nuc}} \sim \frac{\aleph}{M_{nuc}} \sim \frac{r_0}{a_0}$

- LO: two two-nucleon + one three-nucleon interactions

$$C_0^{(0)},\ C_0^{(1)},\ D_0$$

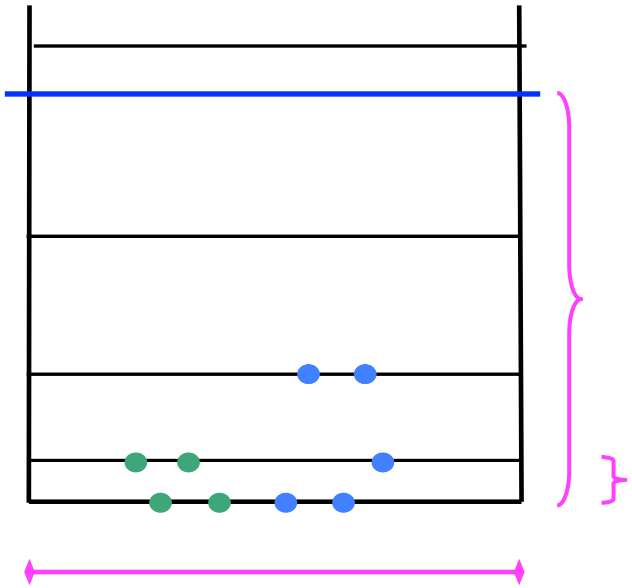
- NLO: two more two-nucleon interactions
- Etc.

~ larger nuclei?

As A grows,
given computational power limits
number of accessible one-nucleon states ⇒ IR cutoff $\lambda \ll Q$ in addition to UV cutoff $\Lambda \gg Q$

Finite Volume

Lattice Box

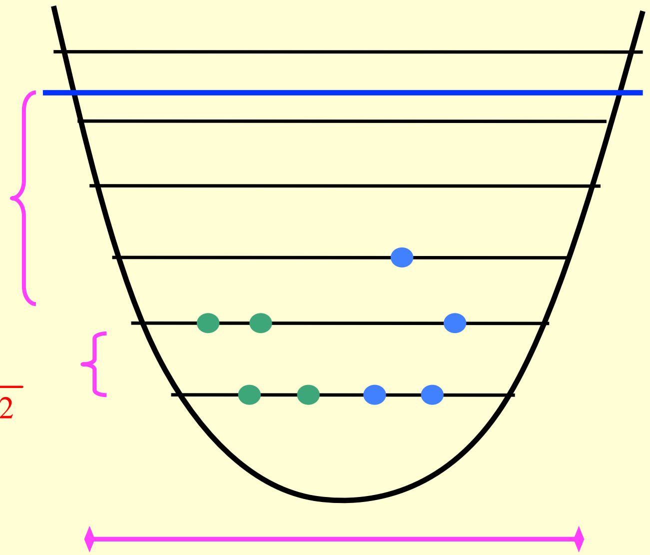


$$\frac{N^2\pi^2}{mL^2}$$

$$\frac{\pi^2}{mL^2}$$

$$L = Na$$

Harmonic Oscillator



$$\frac{N_{\max}}{mb^2}$$

$$\frac{1}{mb^2}$$

$$b = \frac{1}{\sqrt{m\omega}}$$

nuclear matter — Mueller, Seki, Koonin + v.K. '99

few nucleons — Lee *et al* '05 …

finite nuclei — Stetcu, Barrett + v.K. '07; Rotureau, Stetcu, Barrett + v.K., in preparation

few atoms — Stetcu, Barrett + v.K. '09; Rotureau, Stetcu, Barrett + v.K. '10

Two possible approaches

Lattice EFT	Harmonic EFT

- Use input EFT infinite-volume potential $(0, \Lambda_0)$;
  minimize regulator mismatch with $\Lambda \ll \Lambda_0$

Lee *et al* '05

...

Barrett, Vary + Zhang '93

...

"No-Core Shell Model"

- Define EFT directly within finite volume (λ, Λ) ;
  fit parameters to binding energies or to *E* given by

$$\sqrt{mE}\cot\delta(E) = \frac{1}{\pi L}\left[\sum_{\mathbf{n}}^{|\mathbf{n}|<N} \frac{1}{\mathbf{n}^2 - \frac{mEL^2}{4\pi^2}} - 4\pi N\right]$$

$$\sqrt{mE}\cot\delta(E) = -\frac{2}{b}\frac{\Gamma\left(\frac{3}{4} - \frac{Emb^2}{2}\right)}{\Gamma\left(\frac{1}{4} - \frac{Emb^2}{2}\right)}$$

cf. Fukuda + Newton '54

Luescher '91

Busch *et al.* '98

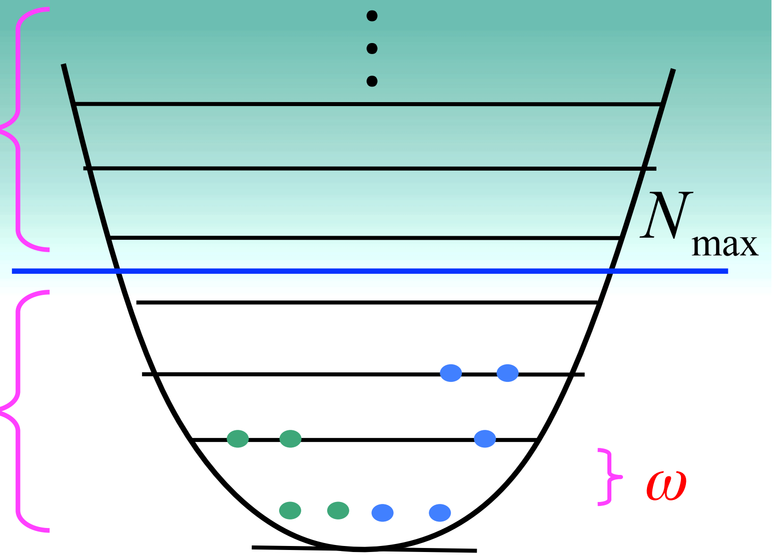
A-body problem: shell model

"excluded space"

$$Q = 1 - P$$

"model space"

$$P = \sum_{n,l}^{2n+l \le N_{\max}} |nl\rangle\langle nl|$$



What are the "effective interactions" in the model space?

traditional NCSM approach: Barrett, Vary + Zhang '93 ...

start with god-given (can be non-local!) potential, and run the RG in a harmonic-oscillator basis

e.g., chiral pot from last lecture

$$O_a \to P O_a^{\text{eff}} P = P O_a P + P H Q \frac{1}{E - Q H_2 Q} Q O_a P + \ldots$$

Feshbach projection

$$= O_a' + O_{a+1}' + \ldots + O_{A'}' + \ldots + O_A'$$

(the terms $+ \ldots + O_A'$ are struck out)

arbitrary truncation ("cluster approximation")

convergence:

$A' \to A$ for fixed P

$P \to 1$ for fixed A'

issues: systematic truncation error, consistent currents, *etc.*

EFT addresses just these issues!

EFT + NCSM

Stetcu, Barrett +v.K., '07
Stetcu, Barrett, Vary + v.K., '08
Rotureau, Stetcu, Barrett + v.K., in preparation

start with EFT in restricted space;
fit parameters in few-nucleon systems
for various ω and $N_{\max}\omega$;

cutoffs — IR, UV

$$\lambda = \sqrt{m_N \omega} \qquad \Lambda = \sqrt{m_N \left(N_{\max} + 3/2\right)\omega}$$

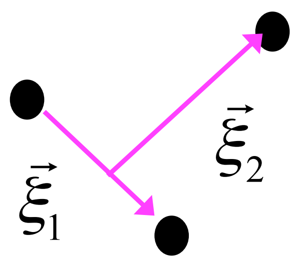
and
predict larger nuclei

strategies:
determine parameters from { light-nuclei binding energies / scattering phase shifts }

Basis

single particle $$\phi_{nl(s)j} = N_{nl}\, r^l\, L_n^{l+1/2}\left(\frac{m_N \omega r^2}{2}\right)\exp\left(-m_N\omega r^2/2\right)\left[Y_l(\hat{r})\otimes\chi_s\right]_j$$

$A \leq 4$: relative coordinates



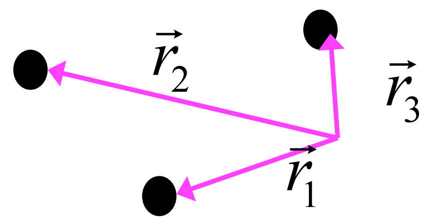
$$\psi\left(\vec{\xi}_1,\vec{\xi}_2\right) = \mathbf{A}\left[\phi_{nlj}\left(\vec{\xi}_1\right)\phi_{n'l'j'}\left(\vec{\xi}_2\right)\right]_{JI}$$

code `a la

Navratil, Kamuntavicius + Barrett '00

(reduced dimensions, but difficult antisymmetrization)

$A \geq 3$: Slater-determinant basis



$$\psi\left(\vec{r}_1,\vec{r}_2,\vec{r}_3\right) = \begin{vmatrix} \phi_{n_1l_1j_1}(\vec{r}_1) & \phi_{n_2l_2j_2}(\vec{r}_1) & \phi_{n_3l_3j_3}(\vec{r}_1) \\ \phi_{n_1l_1j_1}(\vec{r}_2) & \phi_{n_2l_2j_2}(\vec{r}_2) & \phi_{n_3l_3j_3}(\vec{r}_2) \\ \phi_{n_1l_1j_1}(\vec{r}_3) & \phi_{n_2l_2j_2}(\vec{r}_3) & \phi_{n_3l_3j_3}(\vec{r}_3) \end{vmatrix}$$

code: REDSTICK

Navratil + Ormand '03

LO Pionless EFT: ingredients

- matrix elements of 2-, 3-body delta-functions, *e.g.*

$$\langle n_1, l=0 | \delta(r) | n_2, l=0 \rangle \sim \left(\frac{n_1!\, n_2!}{\Gamma\left(n_1 + 3/2\right) \Gamma\left(n_2 + 3/2\right)} \right)^{\frac{1}{2}} L_{n_1}^{1/2}(0) L_{n_2}^{1/2}(0)$$

EFT PC effectively justifies (modified) cluster approximation

- parameters

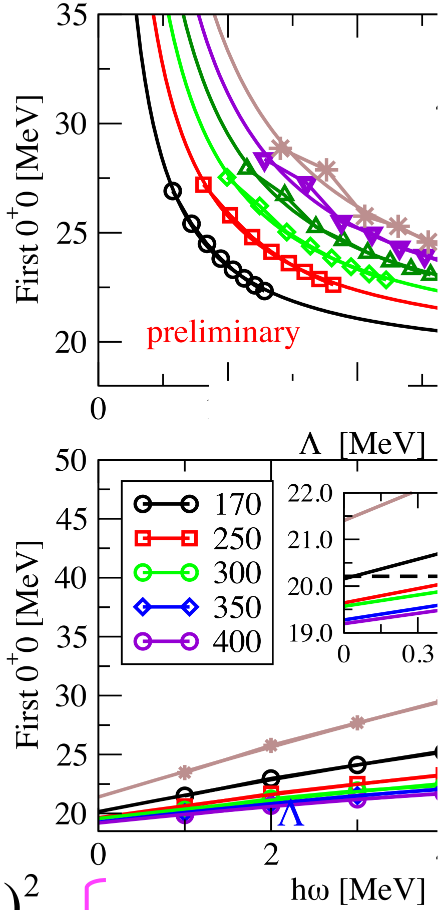
$$C_0^{(0)}(\Lambda),\ C_0^{(1)}(\Lambda),\ D_0(\Lambda)$$

fitted to d, t, a ground-state binding energies

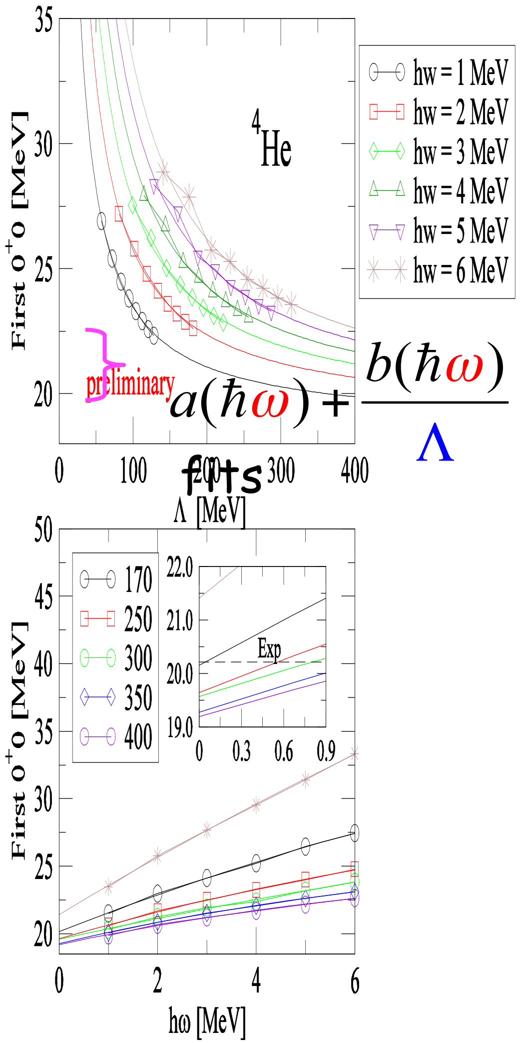
LO $(\nu = -1)$

Stetcu, Barrett +v.K., '07

$N_{\max} \leq 16$



$\alpha + \beta\,\hbar\omega + \gamma(\hbar\omega)^2$ fits



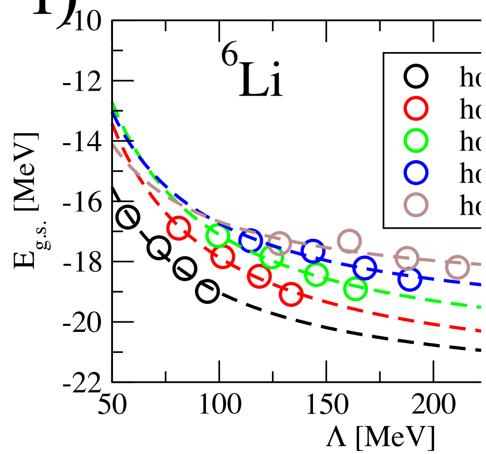
$a(\hbar\omega) + \dfrac{b(\hbar\omega)}{\Lambda}$ fits

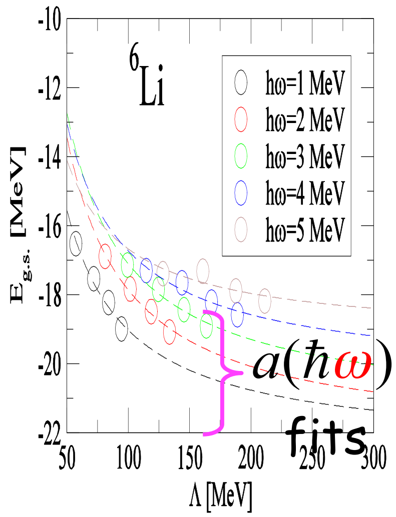
works within ~10% !

Stetcu, Barrett +v.K., '07

LO $(\nu = -1)$

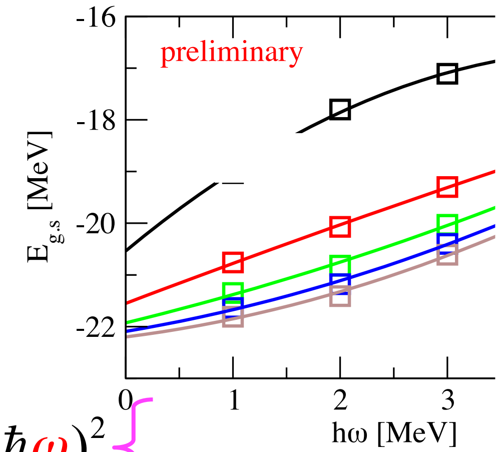
$N_{max} \leq 8$

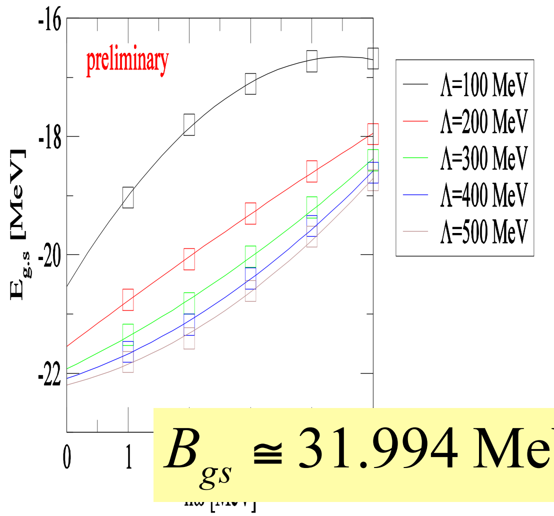




$$a(\hbar\omega) + \frac{b(\hbar\omega)}{\Lambda}$$

fits





$$\alpha + \beta\,\hbar\omega + \gamma(\hbar\omega)^2$$

fits

$B_{gs} \cong 31.994$ MeV (exp)

works within ~30%

NLO fitted to trap energies in progress

- many-body systems get complicated rapidly

+ (continuing) focus on simpler ***halo/cluster nuclei***

one or more loosely-bound nucleons around one or more cores

$$\aleph \equiv \sqrt{m_N E_p} \ll \sqrt{m_N E_c} \equiv M_c$$

(esp. near driplines)

E_p: particle separation energy

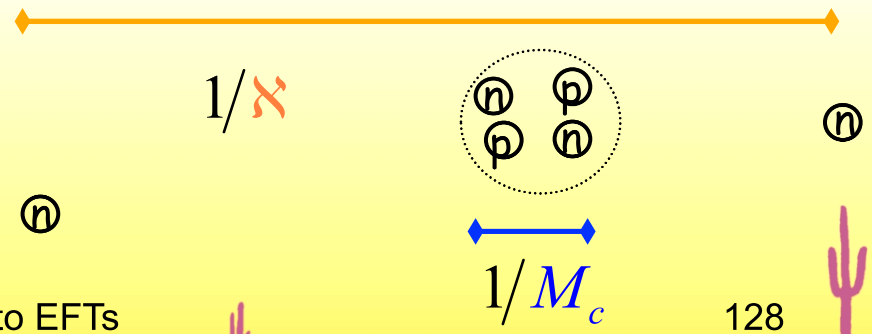
E_c: core excitation energy

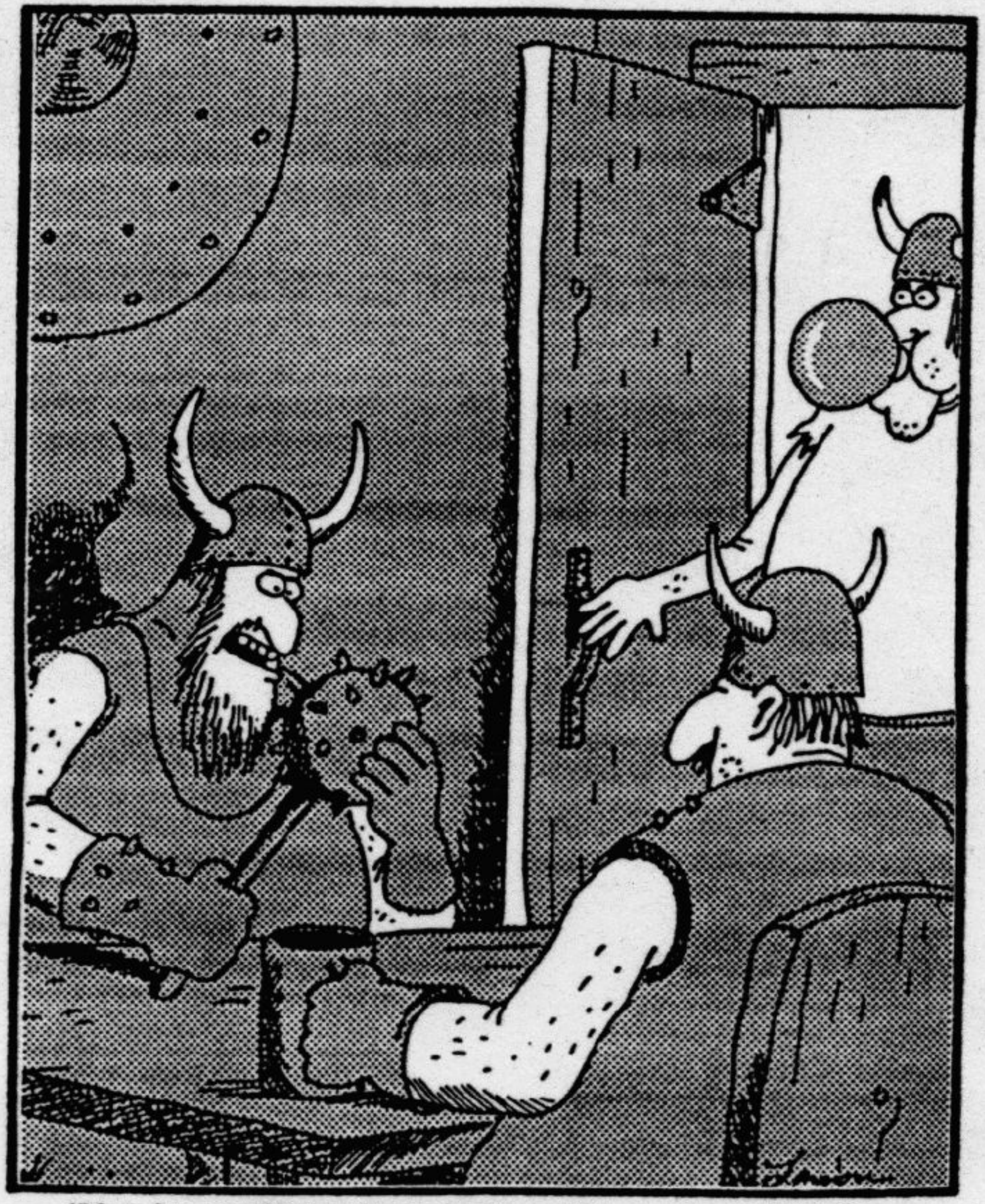
e.g.

^{4}He $B_{\alpha^*} \cong 8$ MeV, $B_\alpha \cong 28$ MeV $\Big\}$ $E_\alpha = B_\alpha - B_{\alpha^*} \cong 20$ MeV

"^{5}He" $p_{3/2}$ resonance at $E_n \sim 1$ MeV

^{6}He s_0 bound state at $E_{2n} \sim 1$ MeV





"You know, Bjorg, there's something about holding a good, solid mace in your hand—you just look for an excuse to smash something."

$Q \sim ℵ \ll M_c$

halo EFT

- degrees of freedom: nucleons, cores
- symmetries: Lorentz, ~~P~~, ~~T~~
- expansion in:

$$\frac{Q}{M_c} = \begin{cases} Q/m_N\,,\; Q/m_c & \text{non-relativistic} \\ Q/m_\pi\,, \cdots & \text{multipole} \end{cases}$$

simplest formulation: auxiliary fields for core + nucleon states

e.g.

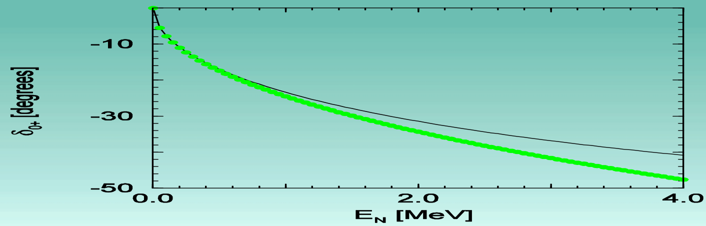
$$^4\text{He} \;\mapsto\; \text{scalar field}\,\varphi$$

$$^4\text{He} + \text{N} \quad \begin{cases} s_{1/2} \equiv 0+ \;\mapsto\; \text{spin-0 field}\, s \\ p_{1/2} \equiv 1- \;\mapsto\; \text{spin-1/2 field}\, T_1 \\ p_{3/2} \equiv 1+ \;\mapsto\; \text{spin-3/2 field}\, T_3 \\ \vdots \end{cases}$$

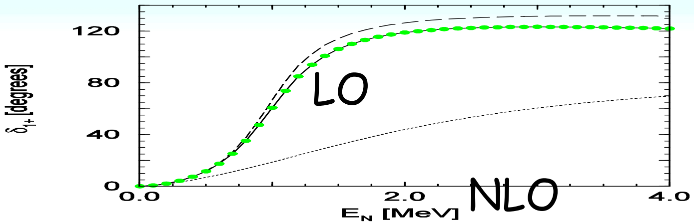
$N\alpha$

Bertulani, Hammer + v.K. '02

● PSA, Arndt et al. '73



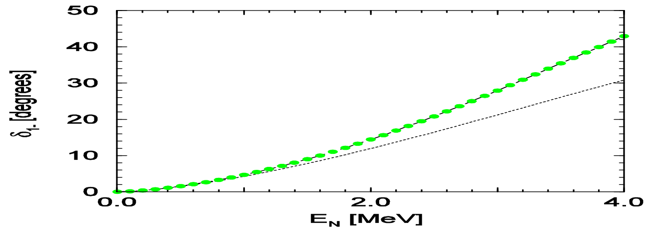
LO



NLO

NLO

scatt length only

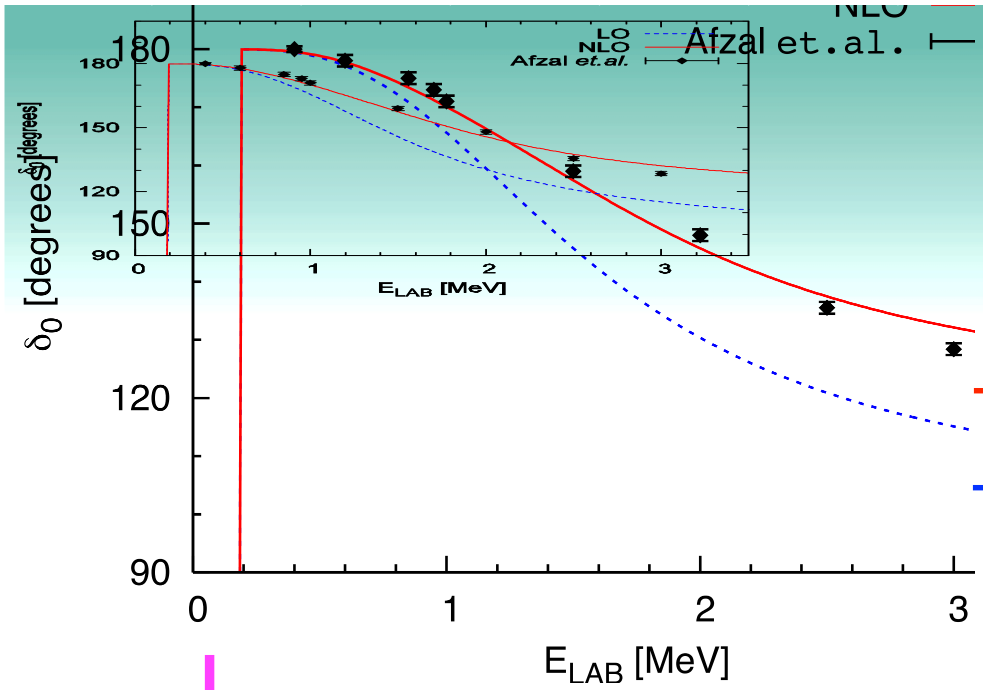


$$E_R \cong 0.80 \text{ MeV}$$

$$\Gamma(E_R) \cong 0.55 \text{ MeV}$$

NNNLO

LO, NLO, NNLO



Higa, Hammer + v.K. '08

$\alpha\alpha$

Extra fitting parameters

$\tilde{P}_0 = P_0 + \frac{1}{15k_C^3}$

none

Bohr radius

$$\frac{1}{k_C} \equiv \frac{1}{Z_\alpha^2 \alpha_{em} \mu} \simeq 3.6 \text{ fm}$$

$E_R = 92.07 \pm 0.03$ keV
$\Gamma(E_R) = 5.57 \pm 0.25$ eV

fitted with a_0 and $\tilde{r}_0 = r_0 - \frac{1}{3k_C}$

More fine-tuning!!!

	a_0 (10^3 fm)	r_0 (fm)	$\mathcal{P}_0$ (fm^3)
LO	-1.80	1.083	—
NLO	-1.92 ± 0.09	1.098 ± 0.005	-1.46 ± 0.08

fine-tuning of 1 in 1000!

$$|a_0| \sim M_c^2/\aleph^3 \qquad r_0 \sim 1/M_c$$

$$|a_0^{E\&M}| = \mathbf{O}(1/2k_C) \simeq 1.8 \text{ fm} \qquad \tilde{r}_0 = -0.13 \text{ fm}$$

fine-tuning of 1 in 10

What next

- Coulomb interaction in higher waves:

  e.g. $p + {}^4\mathrm{He} \rightarrow {}^4\mathrm{He} + p$ — Bertulani, Higa + v.K., in progress

  [cf. $p + p \rightarrow p + p$ — Kong + Ravndal '99]

- three-body bound states:

  e.g. 1) ${}^6\mathrm{He} = \mathrm{b.s.}\left({}^4\mathrm{He} + n + n\right)$ — Rotureau + v.K., in progress

  2) ${}^{12}\mathrm{C} = \mathrm{b.s.}\left({}^4\mathrm{He} + {}^4\mathrm{He} + {}^4\mathrm{He}\right)$

  [cf. ${}^3\mathrm{H} = \mathrm{b.s.}\left(p + n + n\right)$ — Bedaque, Hammer + v.K. '99]

- reactions:

  e.g. $p + {}^7\mathrm{Be} \rightarrow {}^8\mathrm{B} + \gamma$ — Higa + Rupak, in progress

  [cf. $p + n \rightarrow d + \gamma$ — Chen et al. '00]

Conclusion

EFT the framework to describe nuclei within the SM

- ✓ is consistent with symmetries
- ✓ incorporates hadronic physics
- ✓ has controlled expansion

many successes so far, but still much to do

grow to larger nuclei!

➢ new, systematic approach to physics near *drip* lines?