W2a-1

7/10/2013

Wednesday Renormalization Group 1

Agenda:

① Revisit "Building chiral Lagrangians" from T2a

② Warm-up: Some colloquium style slides "Atomic Nuclei at Low Resolution"

③ Similarity renormalization group for NN
$\Rightarrow$ basics (on the board)

④ Revisit the rest of the slides

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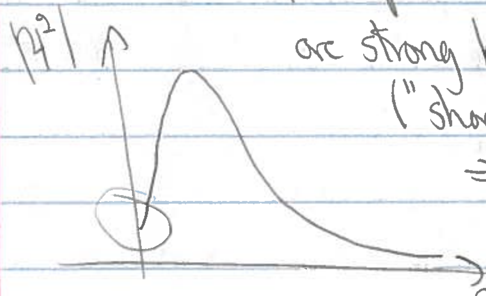
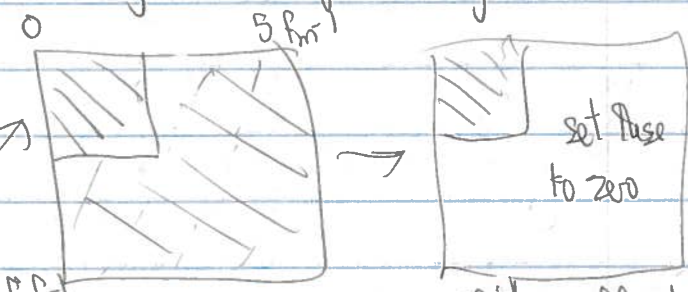
- Recap of important points from the slides

- Nuclei would be at low resolution based on Fermi momentum in large nucleus.
  - recall from exercises $\rho = \frac{2}{3\pi^2} k_f^3$ (for protons and neutrons)
    $\Rightarrow k_f = (3\pi^2 \rho/2)^{1/3}$ (see next page)
    and density of heavy nuclei about constant $\Rightarrow k_f \approx 1.1 - 1.3\ \mathrm{fm}^{-1}$
  - So typical relative momentum of $\approx 1\ \mathrm{fm}^{-1}$ (≈ 200 MeV) in a large nucleus
  - But if the potential has a repulsive core, then there are strong high momentum components ("short-range correlations")



$\Rightarrow$ • slow-down convergence of many-body nuclei.
• e.g. matrices get too big.

- low pass filter fails even for low energy.



Why? Because of quantum mechanics. $T = V + V \frac{1}{E - H_0} V + \ldots$

If strong off-diagonal coupling $\Rightarrow$ can't ignore.

$$\Rightarrow \langle k | T | k \rangle = \langle k | V | k \rangle + \frac{2}{\pi} \int k'^2 dk' \frac{\langle k | V | k' \rangle \langle k' | V | k \rangle}{(k^2 - k'^2)/m}$$

low momentum

Solution? Unitary transformation to decouple! Use RG to do it.

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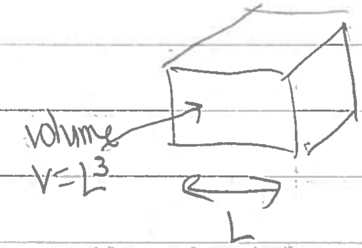
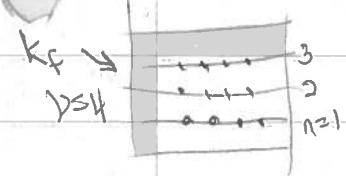
[Aside: deriving $\rho = \frac{2}{3\pi^2} k_F^3$ ...

- For a non-interacting Fermi gas, imagine putting N particles in a box of side L $\Rightarrow \rho = \frac{N}{L^3}$



- Let ν be the spin-isospin degeneracy:
  $\nu = 2$ for neutrons only (spin up, spin down)
  $\nu = 4$ for symmetry matter $(\uparrow, \downarrow, n, p)$

- Apply periodic boundary conditions (pbc) $\Rightarrow$ discrete momentum levels:



Then $N = \sum_n^{n(k_F)} \nu \cdot 1$ ← ν in each state

But pbc: $e^{ik_n(x+L)} = e^{ik_n L}$ in each dimension

$\Rightarrow k_n L = 2\pi n \quad n = 1, 2, 3, \ldots$ are allowed

$\Rightarrow n = \frac{k L}{2\pi}$ or $\Delta n = 1 = \frac{L}{2\pi} \Delta k$ in each dimension

in large V limit $\Rightarrow \sum_n \rightarrow \left(\frac{L}{2\pi}\right)^3 \int d^3k = \frac{V}{(2\pi)^3} \int d^3k$

$\Rightarrow N = \frac{V}{(2\pi)^3} \int^{k_F} d^3k \, \nu$

↖ volume of sphere in large V limit

or $\boxed{\frac{N}{V} = \rho = \frac{1}{8\pi^3} \cdot \frac{4}{3}\pi k_F^3 \cdot \nu = \frac{\nu k_F^3}{6\pi^2}}$

]

7/10/2013 (6Qa-3)

RG is a good thing to

i) make problems more perturbative (how do you measure this?)
- convergent vs. nonconvergent but also the rate?
- exercise: Weinberg eigenvalue as diagnostic of convergence

$$T = V + V G_0 V + V G_0 V G_0 V + \ldots$$

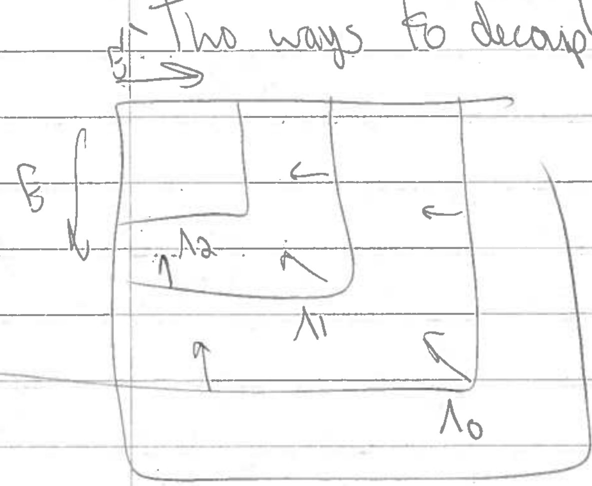
look at eigenvectors of this operator (try it!)

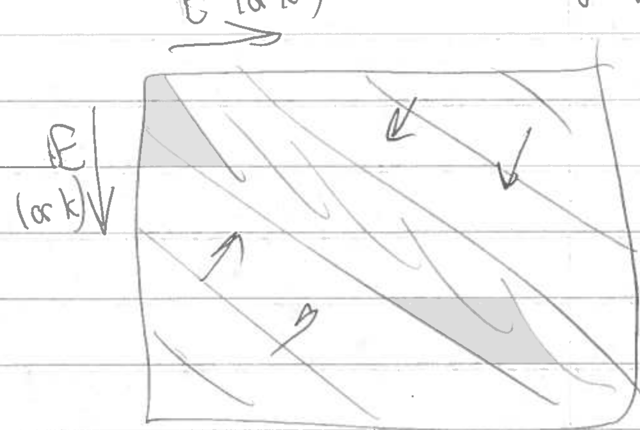
ii) Revealing universal characteristics by filtering out model dependent short-distance details

iii) simplify nuclear structure/reactions calculations
- quantum chemistry methods works with nuclear interactions – coupled cluster, configuration interaction, ...
- note: chiral EFT potentials are already "soft"

no coupling to very high momentum

Two ways to decouple





lower cutoff step by step in Λ and demand something invariant, eg. $\frac{d}{d\Lambda} T(k,k';k^2)$

drive the Hamiltonian toward the diagonal with "flow equation" [Wegner; Glazek/Wilson 1990s)

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- Historical perspective on flow equations and SRG
  - In the early 1970's, Ken Wilson and Franz Wegner both made important contributions to understanding critical phenomena and the renormalization group (RG)
  - Then 20 years later in the early 1990's they independently innovate again
    - unitary RG flow to make many-particle Hamiltonians increasingly energy diagonal
    - Glazek and Wilson "Renormalization of Hamiltonians" in 1993 ⇒ SRG for QCD on the light front.
    - Wegner "Flow equations for Hamiltonians" (1994) ⇒ application to condensed matter problems
      - S. Kehrein, "Flow equation approach to many-particle systems
      - Dissipative quantum systems to correlated electron physics to non-equilibrium problems to ...
  - Particularly well suited for low energy nuclear physics
    - only applied since 2007
    - technically simpler and more versatile than other methods

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SRG Flow Equation

- We wish to transform an initial Hamiltonian

$$H_0 = \underset{\text{kinetic energy}}{T} + \underset{\text{potential energy}}{V}$$

which you should imagine are stored as a matrix in some basis. We transform with a unitary transformation

$$U_s^\dagger U_s = U_s U_s^\dagger = 1$$

$$\Rightarrow \quad H_s = U_s H U_s^\dagger \equiv T + V_s \leftarrow \text{define } V_s \equiv H_s - T$$

so that the kinetic energy is always the same.

- s is called the "flow parameter" — it is just a label for where we are in the evolution of H_s.
   - For the RG, we imagine changing s a little, so we can differentiate H_s:

$$\frac{dH_s}{ds} = \frac{dU_s}{ds} H U_s^\dagger + U_s H \frac{dU_s^\dagger}{ds} \quad (H_0 = H_{s=0} \text{ doesn't depend on } s)$$

$$U_s^\dagger U_s \qquad\qquad U_s^\dagger U_s$$

$$= \underbrace{\frac{dU_s}{ds} U_s^\dagger}_{\equiv \eta_s} \underbrace{U_s H U_s^\dagger}_{H_s} + \underbrace{U_s H U_s^\dagger}_{H_s} \underbrace{U_s \frac{dU_s^\dagger}{ds}}_{\eta_s^\dagger} \leftarrow \text{claim: } \eta_s^\dagger = -\eta_s \text{ [exercise]}$$

$$= [\eta_s, H_s] \,.$$ η_s is anti-Hermitian $\Rightarrow$ can choose $\eta_s = [G_s, H_s]$ with G_s Hermitian

$$\Rightarrow \boxed{\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s]}$$ This is the SRG flow equation.

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- The pattern of the flow is determined by $\hat{G}_s$.

- Most of the nuclear applications have used $\hat{G}_s = \hat{T}_{rel}$, the relative kinetic energy.

$$\Rightarrow \frac{d\hat{V}_s}{ds} = [[\hat{T}_{rel}, \hat{H}_s], \hat{H}_s]$$

- Evaluate in a partial wave momentum basis
  $\Rightarrow |k\,l\,m\rangle \equiv |k\rangle$ with l implicit
  - Completeness:

$$1 = \frac{2}{\pi}\int_0^\infty |q\rangle q^2 dq \langle q|$$

with $\hbar^2/m = 1$ units,

  - $\hat{T}_{rel}|k\rangle = \frac{\hbar^2}{m}k^2|k\rangle \rightarrow k^2|k\rangle$
  - $\langle k|\frac{d\hat{V}_s}{ds}|k'\rangle \equiv \frac{d}{ds}V_s(k,k')$
  - $\hat{H}_s = \hat{T}_{rel} + \hat{V}_s$

- This is all you need to derive [exercise]

$$\frac{dV_s}{ds}(k,k') = -(k^2-k'^2)^2 V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq\, (k^2+k'^2-2q^2) V_s(k,q) V_s(q,k')$$

[Hint: Write out the double commutator and take $\langle k|\ \ |k'\rangle$ matrix elements.
$[[T_{rel}, T_{rel}+V_s], T_{rel}+V_s]$ and insert complete sets of states as needed.
Eg. $[[T_{rel}, V_s], T_{rel}] = T_{rel}V_sT_{rel} - T_{rel}T_{rel}V_s - V_sT_{rel}T_{rel} + T_{rel}V_sT_{rel}$
and $\langle k|\underset{\text{act left}}{T_{rel}}V_s\underset{\text{act right}}{T_{rel}}|k'\rangle = k^2\langle k|V_s|k'\rangle k'^2$]

- This is an equation that is easy to discretize and solve as a set of coupled, first-order differential equations

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If we consider the two terms on the right side of the partial wave equation, then we find that the first one dominates for off-diagonal momenta.

$$\Rightarrow \frac{dV_s}{ds}(k,k') \approx -(k^2-k'^2)V_s(k,k') \quad \text{for } k \gg k' \text{ or } k' \gg k$$

But now the equations are all decoupled and the solution is [in the exercises]

$$V_s(k,k') = V_{s=0}(k,k')\, e^{-s(k^2-k'^2)^2}$$

so off-diagonal matrix elements are driven to zero as s increases, and they go to zero faster the further off diagonal they are.

- Note that s has dimensions of $[L]^4$ [so fm^4 here]. We define $\lambda = s^{-1/4}$, which has dimensions of fm^{-1}, a momentum.
- λ^2 gives roughly the width in k^2 of V_s [exercise]

* Go back to slides and see k^2 width and universality.
  - also teaser for 3-body forces and block diagonalization.
    - If we decouple high momentum states, we expect to get 3-body (and higher body) forces
      $\Rightarrow$ eliminating degrees of freedom