TNb-1

7/4/2013

<u>Overview</u>: We've introduced many threads in the first few days. All have connections — underlying themes — or else are foundations for upcoming topics.

- It will become more evident as we proceed
- So please be patient even if it seems incoherent at times!
- We need to develop in parallel so we can discuss impact

- In this lecture:
  - Follow up on three-body forces and aspects of the EFT example of pionless EFT. ⇒ tie these together
  - Continue with implications of the more complex features of the NN force (like tensor, spin-orbit) and in the simplest bound system: the deuteron.

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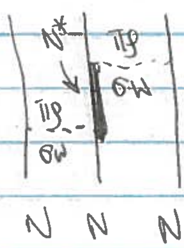
- Continuation of 3-body force introduction...

- We discussed the examples of the Earth-Moon-Sun system and the interaction of neutral atoms as places where the low-energy theory has three-body forces.
  - The general feature is that three-body forces arise from the elimination of degrees of freedom
  - if we included positions & individual masses or interactions between all electrons, then two-body only.
  - eliminating these variables (degrees of freedom) in favor of collective coordinates (center-of mass position) required three-body forces.

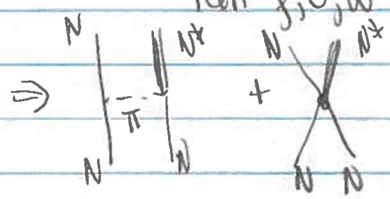
- So what about the nuclear case?



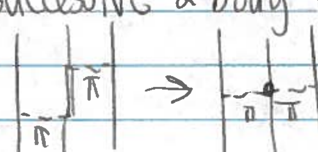
- In this diagram two nucleons exchange a boson, maybe a pion, maybe a heavier meson, exciting an N* (excited state of nucleon) for a brief period.
- N* could mean a Δ, could mean something else.

- Suppose our theory had both N's and N*'s explicitly and pions, but no ρ, σ, ω (treated as heavy)
  - Then ρ, σ, ω → expanded as contacts plus derivatives of contacts ← "derivative expansion"

  $$\left[\text{cf. } \frac{1}{(\vec{k}-\vec{k}')^2+\mu^2} \to C_0 + C_2(\vec{k}^2+\vec{k}'^2) + C_2'\, \vec{k}\cdot\vec{k}' + \ldots\right]$$

⇒ 

would be diagrams included in the low-energy theory, but no 3-body, because , etc. is two successive 2-body interactions.

- But now, if we eliminate N* ⇒ 

⇒ This is a 3-body force

- 3-body if can't be broken into successive two-body interactions

Th1b-3

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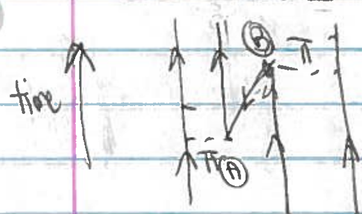
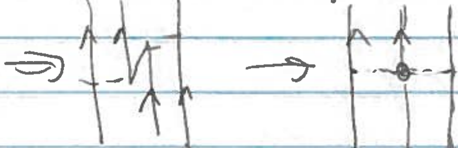
- When is it good to replace the Δ^* excitation? When we don't resolve that it was excited.
  - By the uncertainty principle, if we excite by ΔE a virtual state, it can last for $\Delta t = \hbar/\Delta E$, which is short if ΔE is large $\Rightarrow$ endpoint are close enough so they are not <u>resolved</u> $\Rightarrow$ replace by contact • and derivatives
  - So this is a danger if $M_\Delta - M_N \approx 300$ MeV, then it will break down much sooner than for energy differences $\approx$ 500-1000 MeV (such as heavier meson exchanges),
    - We will keep coming back to this!
    - Expansion parameter $Q/(m_\Delta - m_N)$ may be smaller than we want!

- How about a process like:



- So the idea is that a nucleon emits a pion that becomes a nucleon-<u>anti-nucleon</u> pair at Ⓐ. The anti-nucleon annhilates with a 2^{nd} nucleon at Ⓑ emitting a pion absorbed by a 3^{rd} nucleon.

Class: • In the previous case, we had $\Delta E \gtrsim M_\Delta - M_N$. What is it here?
- Initially $3 \times m_N$ + kinetic, in the middle an extra $2 m_N$ $\Rightarrow \Delta E \gtrsim 2 m_N$, which is large $\Rightarrow \Delta t$ is small

$\Rightarrow$  is a 3-body force that is a good approximation,

<u>Important</u>:  or  may be good or bad, or incomplete models — maybe it requires quarks and gluons to describe. As long as

 contains <u>all</u> allowed (by symmetries) vertices, then we don't care — we will be <u>model independent</u> with our EFT!

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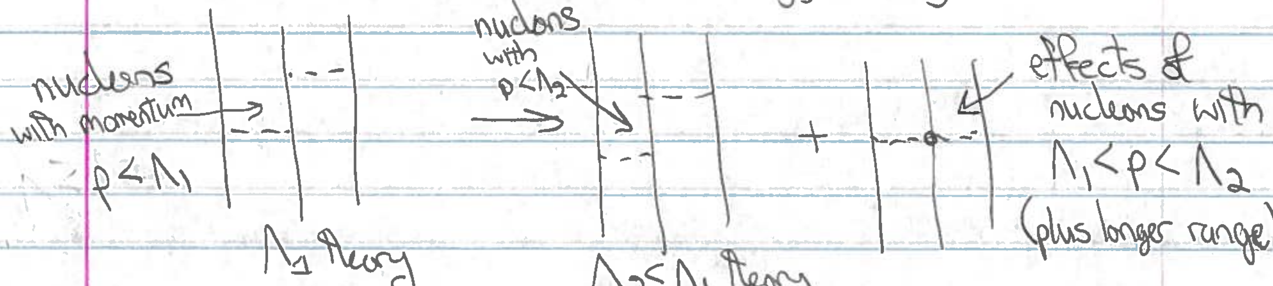
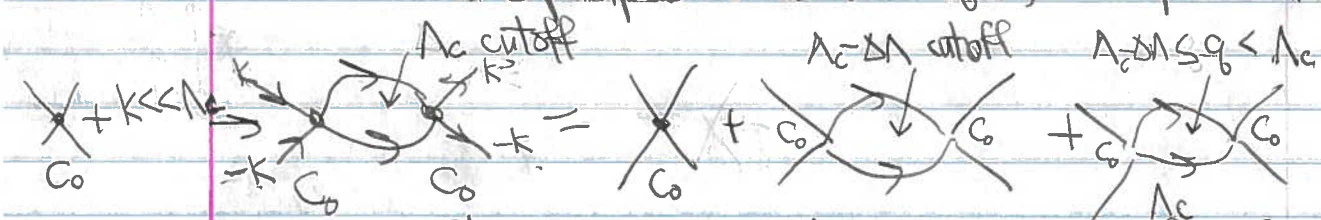
- Moral: whether we have a 3-body force or not and how large a contribution depends on our choice of degrees of freedom.
  - But this includes where our cutoff eliminates <u>nucleons</u> from our low-energy theory



$\Rightarrow$ even with just nucleons, two-body interactions become 3-body if we eliminate degrees of freedom (in this case by lowering Λ_c)

- This is the same principle as we had with $C_0(\Lambda)$ in the pionless theory:



$$\Rightarrow C_0(\Lambda)^2 \frac{m}{2\pi^2} \underbrace{\int_0^{\Lambda_c} dq \frac{q^2}{k^2 - q^2 + i\varepsilon}} \longrightarrow \underbrace{\int_0^{\Lambda_c - \Delta\Lambda}} + C_0(\Lambda)^2 \frac{m}{2\pi^2} \int_{\Lambda_c - \Delta\Lambda}^{\Lambda_c} dq \frac{q^2}{k^2 - q^2 + i\varepsilon}$$

Convince yourself! same as uncertainty principle argument

But $k^2 \ll q^2$ in 2^{nd} integral because $k \ll \Lambda_c - \Delta\Lambda \Rightarrow \frac{1}{k^2 - q^2 + i\varepsilon} \rightarrow \frac{-1}{q^2}\left(1 + \frac{k^2}{q^2} + \ldots\right)$ small

$$\Rightarrow \int_{\Lambda_c - \Delta\Lambda}^{\Lambda_c} dq \frac{q^2}{k^2 - q^2 + i\varepsilon} \rightarrow \int_{\Lambda_c - \Delta\Lambda}^{\Lambda_c} dq \frac{-q^2}{q^2} + k^2 \int_{\Lambda_c - \Delta\Lambda}^{\Lambda_c} dq \frac{-q^2}{q^4} + \ldots = -\Delta\Lambda\left(1 + \mathcal{O}\left(\frac{k^2}{\Lambda_c^2}\right)\right)$$

$\Rightarrow$ the contribution for $\Lambda_c - \Delta\Lambda_c < q < \Lambda_c$ looks mostly like a constant:

What if different regulator? Just factors!

$\Rightarrow$ Change C_0 to compensate $\Delta C_0 = C_0(\Lambda)^2 \frac{m}{2\pi^2}(-\Delta\Lambda)$ or $\frac{d}{d\Lambda} C_0(\Lambda) = \frac{m}{2\pi^2}\left(C_0(\Lambda)\right)^2$ (dC_0; $d\Lambda < 0$)

which is the RG equation from Achim's lecture! (sign <u>does</u> work!)

(sign does work: if $d\Lambda < 0$, then C_0 decreases ✓)

Th1b-5

7/4/2013

- How do we *estimate* truncation errors?
  - cf. Lepage plots
  - need to know how big the coefficients are.
- For the natural ($a_0 \sim \frac{1}{\Lambda} \sim R$) pionless theory ($\Lambda \approx m_\pi$) "breakdown scale"

$$iT(k,\cos\theta) \quad -iC_0 \quad -\frac{m}{4\pi}C_0^2 k \quad i\left(\frac{m}{4\pi}\right)^2 C_0^3 k^2 \quad -iC_2 k^2 \quad -iC_2' k^2\cos\theta$$

reproduces: $= -\frac{4\pi a_0}{m}\left[1 - ia_0 k + (a_0 r_0/2 - a_0^2)k^2\right] - \frac{4\pi a_p^3}{m}k^2\cos\theta + \mathcal{O}\left(\frac{k^3}{\Lambda^3}\right)$

$$C_0 = \frac{4\pi}{m}a_0 \sim \frac{4\pi}{m}\frac{1}{\Lambda}, \quad C_2 = \frac{4\pi}{m}\frac{a_0^2 r_0}{2} \sim \frac{4\pi}{m}\frac{1}{2}\frac{1}{\Lambda^3}, \quad C_2' = \frac{4\pi}{m}a_p^3 \sim \frac{4\pi}{m}\frac{1}{\Lambda^3}, \ldots$$

- __Power counting__: diagrams contribute k^ν where $\nu = 5 - \frac{3}{2}E + \sum_{n=2}^{\infty}\sum_{i=0}^{\infty}(2i+3n-5)V_{2i}^n$

where
- $2i$ = # of derivatives
- $n = 2$ for 2-body term, 3 for 3-body term, ..
- E = # external lines

[every extra k gets a $\frac{1}{\Lambda}$] $\Rightarrow C_{2i} \sim \frac{4\pi}{m}\left(\frac{1}{\Lambda}\right)^{2i+1}, \quad D_{2i} \sim \frac{4\pi}{m}\left(\frac{1}{\Lambda}\right)^{2i+4}$

$\leftarrow$ 3-body part!

$\Rightarrow$ estimates of how big coefficients are
- $\rightarrow$ just dimensional analysis with Λ as momentum or $\frac{1}{\text{length}}$
- "naive" $\rightarrow$ only Λ and natural: remaining coefficient is close to 1.

- So even if we didn't determine C_2 and , we can estimate the contribution as $\frac{4\pi}{m}\frac{1}{\Lambda^3}k^2$ (since $2i=2$ derivatives).

$\Rightarrow$ truncation error + we know that it will __fail__ for $k \sim \Lambda$

7/4/2013

- In our pionless example we didn't mention spin, because we said the interaction was spin independent, but this doesn't mean spin doesn't play a role.
  - Because the wave function must be antisymmetric, if the spin part is symmetric $|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$, or $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, then the wave function of two neutrons (isospin symmetric, $I=1$) would vanish for separation $\vec{x}=0$ (space antisymmetric) so the matrix element of $C_0\delta^3(\vec{x})$ vanishes: $\langle\uparrow\uparrow|C_0\delta^3(\vec{x})|\uparrow\uparrow\rangle = 0$
  - So we really need to keep track of the spins as well on the legs of 

- What about including spin at leading order $\nearrow Q^0$ in the pionless EFT.
  - Consider neutrons only. A general term consistent with symmetries is

$$V_{LO} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

so we might expect <u>two</u> S-wave scattering lengths.

in the lagrangian

$$\mathcal{L}_{LO} = \dots - \tfrac{1}{2}C_S(\psi^\dagger\psi)^2 - \tfrac{1}{2}C_T(\psi^\dagger\vec{\sigma}\psi)^2$$

$$= -\tfrac{1}{2}C_S(\psi_i^\dagger\psi_i)(\psi_j^\dagger\psi_j) - \tfrac{1}{2}C_T(\psi_i^\dagger\sigma^a_{ij}\psi_j)(\psi_{i'}^\dagger\sigma^a_{i'j'}\psi_{j'})$$

spin component $i, j, i', j' = 1, 2$

$a = 1, 2, 3$ $\leftarrow$ which Pauli matrix

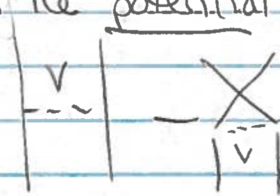
- If you did the Fierz rearrangement exercise, you would have found that these two terms are not independent
  $\Rightarrow$ there is only one combination, again because of antisymmetry and contact interactions.

- In Achim's W1b notes, there is a nice alternative way to show this, which I'll repeat here.

Th1b-7

7/4/2013

- We can include antisymmetry in the potential by including the exchange term (with the appropriate minus sign)



$$\Rightarrow V_{\text{antisym}} = (1 - \hat{P}_{12})V$$

where $\hat{P}_{12}$ is the exchange operator $\hat{P}_{12} = \hat{P}_{\vec{k} \leftrightarrow \vec{k}'} \hat{P}_{\text{spin}}$

and $\hat{P}_{\text{spin}} = \frac{1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2}{2}$ (if you've never seen this, act with $\hat{P}_{\text{spin}}$ on spin wf's to verify)

$$\Rightarrow V_{\text{antisym}} = (1 - \hat{P}_{\text{spin}})(C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2) \quad (\text{no } \vec{k}, \vec{k}'!)$$

$\boxed{\text{use } (\vec{\sigma}_1 \cdot \vec{\sigma}_2)^2 = 3 - 2\vec{\sigma}_1 \cdot \vec{\sigma}_2}$

$$= \frac{1}{2}\left[C_S - 3C_T + (3C_T - C_S)\vec{\sigma}_1 \cdot \vec{\sigma}_2\right] = \begin{cases} 0 & S=1 \quad \text{as before from Pauli} \\ 2(C_S - 3C_T), & S=0 \end{cases}$$

- So any choices of C_S, C_T for which $C_S - 3C_T$ is the same will give the same result $\Rightarrow$ only one independent constant.
- Your choice, e.g. $C_T = 0$. So what we had with $C_S = C_0$ was actually general at LO for neutrons only! Also, one scattering length.

- Now with spin and isospin: $\mathbb{1}$, $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, $\vec{\tau}_1 \cdot \vec{\tau}_2$, $\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \vec{\tau}_1 \cdot \vec{\tau}_2$, antisymmetry says only 2 independent. Choose which! (Fierz ambiguity)

Conventional choice: $V_{NN}^{LO} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$ — see Alex tomorrow!

At NLO $\mathcal{O}(Q^2)$, 14 possible, but only 7 linearly independent. Usual choice:

$$V_{NN}^{NLO} = C_2 \tfrac{1}{2}(\vec{k}^2 + \vec{k}'^2) + C_2' \vec{k} \cdot \vec{k}' + C_2^S \tfrac{1}{2}(\vec{k}^2 + \vec{k}'^2)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_2'^S \vec{k} \cdot \vec{k}' \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ i C_2^{LS} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{k} \times \vec{k}') \rightarrow \text{spin-orbit interaction (Exercise)}$$

$$+ C_2^T \vec{\sigma}_1 \cdot (\vec{k}' - \vec{k}) \vec{\sigma}_2 \cdot (\vec{k}' - \vec{k}) + C_2'^T \vec{\sigma}_1 \cdot (\vec{k}' + \vec{k}) \vec{\sigma}_2 \cdot (\vec{k}' + \vec{k}) \rightarrow \text{lead to tensor interaction}$$

7/4/2013

- Where does the tensor interaction in pionless EFT come from?
  - One source is the pion, and the pion tensor interaction has important effects on nuclear structure.

- Let's do a quick derivation of the one-pion exchange potential starting from the interacting Hamiltonian density

$$\mathcal{H}_{\pi N} = \frac{g_A}{2F_\pi} N^\dagger \vec{\sigma}\cdot(\vec{\nabla}\underset{\sim}{\pi}\cdot\underset{\sim}{\tau})N \quad \text{(lots of hidden indices!)}$$

$\leftarrow$ 2×2 matrix in isospin

2×2 matrix in spin

$g_A \approx 1.27$

$F_\pi \approx 92.4$ MeV

- In 2nd quantized form

create nucleon with $\vec{k}', m_s', m_t'$

$$\hat{H}_{int} = -i\frac{g_A}{2F_\pi}\int\frac{d^3k\, d^3k'}{(2\pi)^6}\, b^\dagger(\vec{k}', m_s', m_t')\, b(\vec{k}, m_s, m_t)\, \vec{\sigma}_{m_s' m_s}\cdot\vec{q}\,(\tau)^i_{m_t' m_t}$$

$\leftarrow$ T=1, isotriplet

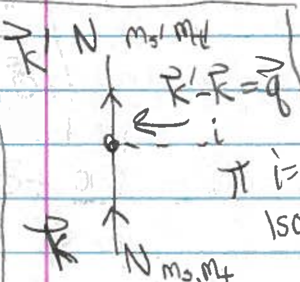
$$\times \frac{1}{\sqrt{2\omega_q}}\left(a_i^\dagger(-\vec{q}) + a_i(\vec{q})\right)\Big|_{\vec{q}=\vec{k}'-\vec{k}}$$

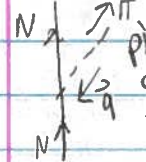
sum over possible momenta

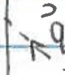
pseudoscalar, Goldstone boson (derivative

create pion in isospin state i with $-\vec{q}$

$\omega_q = \sqrt{m_\pi^2+q^2}$ normalization factor for pion

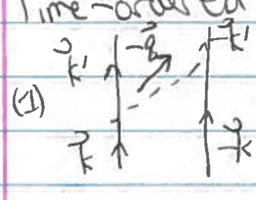






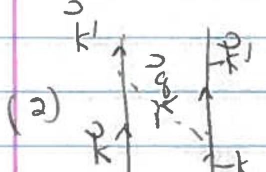
Time-ordered perturbation theory (2nd order)

$\vec{q}=\vec{k}'-\vec{k}$



$$\langle\vec{k}'|V^{(1)}_{OPE}|\vec{k}\rangle = \sum_{n=1\,\text{pion}}\frac{\langle\vec{k}'|\hat{H}_{int}|n\rangle\langle n|\hat{H}_{int}|\vec{k}\rangle}{E_i - E_n} \quad \leftarrow = -\sqrt{q^2+m_\pi^2} = -\omega_q$$

$$= -\frac{1}{\omega_q}\left(-i\frac{g_A}{2F_\pi}\right)^2 \vec{\sigma}_1\cdot\vec{q}\,\frac{1}{2\omega_q}\,\vec{\sigma}_2\cdot(-\vec{q})\;\vec{\tau}_1\cdot\vec{\tau}_2$$



$\leftarrow$ same with $\vec{\sigma}_1\cdot(-\vec{q})\;\vec{\sigma}_2\cdot\vec{q}$

Th1b-9

7/14/2013

Putting it together, class! so this is a local potential (any particle exchange at long distance)

$$V_{OPE}(\vec{k}',\vec{k}) = V_{OPE}(\vec{q} = \vec{k}' - \vec{k})$$

(1) + (2)

$$= -\frac{g_A^2}{(2F_\pi)^2}\,\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}\;\frac{2}{2\omega_q^2}\;\vec{\tau}_1\cdot\vec{\tau}_2$$

$$= -\frac{g_A^2}{4F_\pi^2}\,\frac{\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+m_\pi^2}\;\vec{\tau}_1\cdot\vec{\tau}_2$$

In the exercises for today, you carry out the Fourier transform showing $\int \frac{d^3q}{(2\pi)^3}\frac{1}{q^2+m_\pi^2}e^{i\vec{q}\cdot\vec{r}} = \frac{1}{4\pi}\frac{e^{-m_\pi r}}{r}$ (standard, but remind yourself)

and then evaluating the derivatives in $\vec{\sigma}\cdot\vec{q}$. The bottom line is

$$V_{OPE}(\vec{r}) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2\left[3\,T(r)S_{12}(\hat{r}) + Y(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2\right]$$

where $T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right)$ singular $\frac{1}{r^3}$!

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

and $S_{12}(\hat{r}) = \left[(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\right]$.

Th1b-10

7/4/2013

Let's talk a bit about the impact of the tensor force on NN bound states.

- nn and pp have no bound states
- np has one shallow bound state (large scattering length!)
  - $\Rightarrow$ deuteron $T=0$
- What's special about $T=0$?

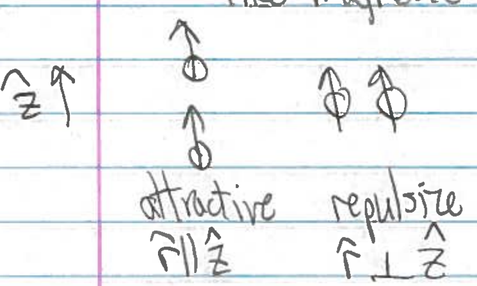
- Deuteron properties: (measured!)
  - binding energy $-2.224575(9)$ MeV small!
    - $\Rightarrow$ rms radius $\sqrt{\langle r^2 \rangle} \doteq 1.9752(1)$ fm large! (large tail)
  - $J^P = 1^+$ (angular momentum 1, parity +)
  - isospin $T=0$, $M_T=0$ (np)
  - electric quadrupole moment $Q_d \doteq 0.2859(3)\, e\, \text{fm}^2 > 0$

- Two spin-1/2 nucleons $\Rightarrow S=0,1$ $J-1 < l < J+1 \Rightarrow l = 0,1,2$
  - parity + $\Rightarrow l=0,2 \Rightarrow$ space symmetric so $S=1$
  - So $^3S_1$, $^3D_1$ possible
  - Expect $^3S_1$ energetically but $Q_d \neq 0 \Rightarrow l=2$ admixture
    - $\Rightarrow$ tensor force mixes $^3S_1 - {}^3D_1$ (recall T1b)
  - Attractive tensor $\Rightarrow$ extra binding (cf. nn)

- Is attractive tensor in $T=0$ consistent with $Q_d > 0$?

see slides

- like magnetic dipole-dipole



$\hat{Q} = e(3z^2 - r^2) = er^2(3\cos^2\theta - 1) \Rightarrow Q_d > 0 \Rightarrow \langle z^2 \rangle > \langle x^2 \rangle, \langle y^2 \rangle$

- like spins ($M_S = \pm 1$) prefer to be oriented head to tail
  - $\Rightarrow$ prolate $\Rightarrow Q_d > 0$

[note $Q_d = \langle \psi_D, M=J | \hat{Q} | \psi_d, M=J \rangle$]

- unlike spins ($M_S = 0$) prefer side-by-side configuration

- Wave functions: S-wave $u(r)$, D-wave $w(r)$ parts

$u(r) \xrightarrow[r\to\infty]{} A_S e^{-\gamma r}$ $\gamma = \sqrt{2E_d}$ binding momentum

$w(r) \xrightarrow[r\to\infty]{} \eta_d A_S \left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2}\right) e^{-\gamma r}$ η_d and A_S asymptotic normalizations

measurable