review running coupling, jets, QCD phase diagram 1

Lecture scattering theory 2

Theme: What NN phase shifts tell us about NN forces?

<u>NN scattering</u> orbital angular momentum $\vec{l}$ $l = 0, 1, 2, \ldots$ Q: Which of these are conserved?

spin conserved $\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ $S = 0, 1$

total angular momentum conserved (rotational symmetry) $\vec{J} = \vec{l} + \vec{S}$ $J = |l-S|, \ldots, l+S$

isospin $\vec{T} = \frac{1}{2}(\vec{\tau}_1 + \vec{\tau}_2)$

$$\Rightarrow J = \begin{cases} l, & S = 0 \\ |l-1|, l, l+1, & S = 1 \end{cases}$$

<u>allowed partial waves</u> for NN scattering: Pauli principle for fermions

even $l = 0, 2, 4, \ldots$ $\longrightarrow$ $S = 0$ $\longrightarrow$ $T = 1$
spatial wf. sym. $\times$ antisym. $\times$ sym $=$ antisym
$\longrightarrow$ $S = 1$ $\longrightarrow$ $T = 0$

odd $l = 1, 3, 5, \ldots$ $\longrightarrow$ $S = 0$ $\longrightarrow$ $T = 0$
antisym
$\longrightarrow$ $S = 1$ $\longrightarrow$ $T = 1$

isospin is determined by antisymmetry

need only l, S, J $\longrightarrow$ T is specified

use spectroscopic notation $^{2S+1}l_J$ $l = 0, 1, 2, 3, 4, \ldots$ low l dominate at low energies
$S, P, D, F, G, \ldots$

Q: Write down the few lowest.

$^1S_0$ $^3S_1$ $^1P_1$ $^3P_0$ $^3P_1$ $^3P_2$ $^1D_2$ $^3D_1$ $^3D_2$ $^3D_3$ $^1F_3$ $^3F_2$ $^3F_3$ $^3F_4$

same S, J conserved, but l can change due to interactions

$\longrightarrow$ <u>coupled channel</u> for $S=1$, given $J \longrightarrow |l-1|, l+1$

Q: Why does $J = l$ not mix? $\longrightarrow$ parity

Coupled channels

potential V, Hamiltonian H in partial-wave basis
scattering amplitude f, S- and T-matrix
are given by 2×2 matrix for S, J

$$\begin{array}{c|cc} & J_<=|l-1| & J_>=l+1 \\ \hline J_< & \cdot & \cdot \\ J_> & \cdot & \cdot \end{array} \qquad \text{e.g.} \qquad \begin{array}{c|cc} & {}^3S_1 & {}^3D_1 \\ \hline {}^3S_1 & \cdot & \cdot \\ {}^3D_1 & \cdot & \cdot \end{array}$$

channels are coupled due to the tensor force $\Rightarrow$ Thursday

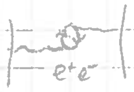
Conventional choice to parametrize S-matrix in terms of "bar" phase shifts $\delta_{J_<}$ $\delta_{J_>}$ and mixing angle ε_J

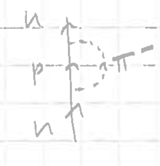
(other convention "eigen" p.s. and ε)

$$S\text{-matrix} = \begin{pmatrix} e^{2i\delta_{J_<}} \cos 2\varepsilon_J & i\, e^{i(\delta_{J_>}+\delta_{J_<})} \sin 2\varepsilon_J \\ i\, e^{i(\delta_{J_<}+\delta_{J_>})} \sin 2\varepsilon_J & e^{2i\delta_{J_>}} \cos 2\varepsilon_J \end{pmatrix}$$

Q: $\varepsilon_J = 0$ reduces to uncoupled S-matrix $= e^{2i\delta}$

We will consider strong interactions, but to compare with experimental cross sections one needs to include electromagnetic interactions (long-range!)
(see Phys. Rev. C 51, 38 (1995) $\rightarrow$ Argonne v_{18} potential)

$V_{em}(pp)$ = one- and two-photon exchange Coulomb terms
Darwin-Foldy correction
vacuum polarization 
magnetic moment interaction

$V_{em}(np)$ = Coulomb term due to neutron charge distribution 
MM interaction

$V_{em}(nn) = V_{MM}$

attractive / repulsive

NN partial-wave analysis: Nijmegen nn-online.org/NN

Q for S-waves + ε_1 → bound state?

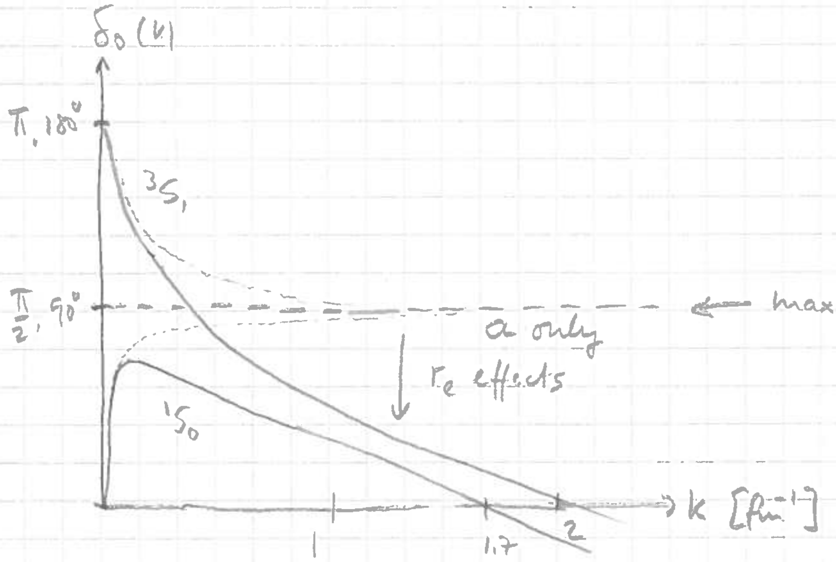
both S-waves have large scattering a: $^1S_0$ $a_{nn} \approx (a_{pp} - \text{em effects}) \approx -18$ fm

large compared to range ~ fm

$a_{np} = -23.7$ fm → almost bound / resonance

$^3S_1$ $a_{np} = +5.4$ fm → bound deuteron

mixing angle $\varepsilon_1 < 5^\circ$ small for $E_{lab} \lesssim 300$ MeV



What are maximally strong interactions? Consider S-waves

unitary limit of cross section $\frac{d\sigma}{d\Omega} \leq \frac{1}{k^2}$

Q: for which phase shifts $\Rightarrow \delta = \frac{\pi}{2}$ for all k

$$\frac{d\sigma}{d\Omega} = \frac{1}{(k\cot\delta)^2 + k^2}$$

$\Leftrightarrow \frac{1}{a} = 0$ and all $r_e, \ldots = 0$

NN interactions in S-waves strong at low energies → large a

but weaker for higher energies due to effective range effects

S-waves are attractive at low energies, repulsive at higher energies

→ so local core or repulsive mom. dependence

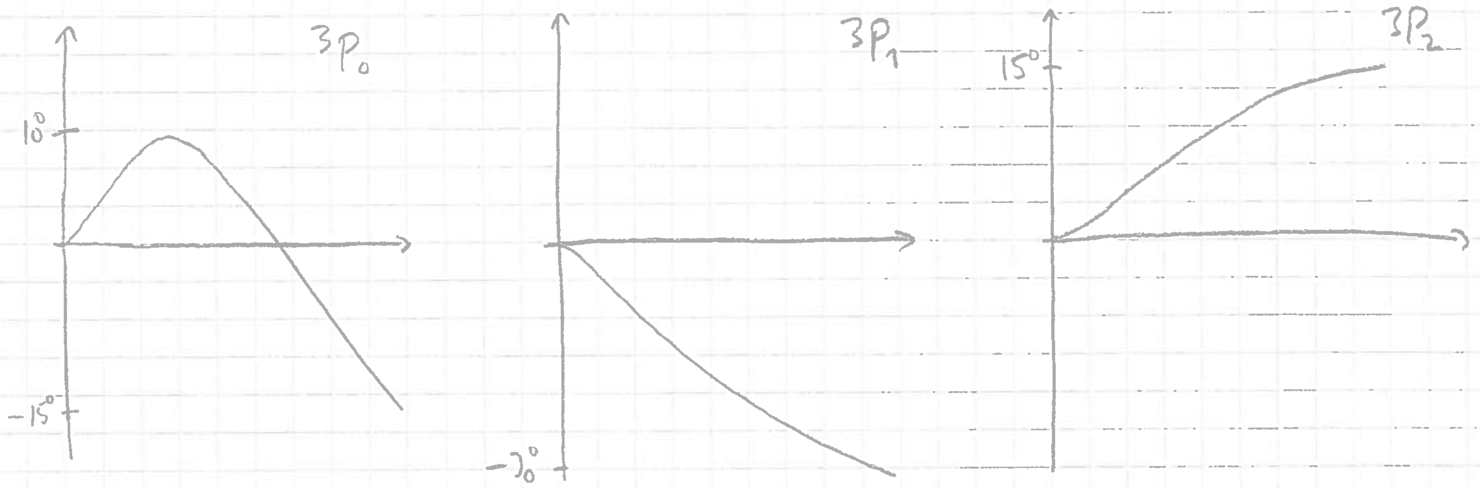
large scattering lengths in both spin channels lead to approximate symmetry at low energies

$\Rightarrow$ approximate $SU(2)_{\text{isospin}} \times SU(2)_{\text{spin}} = SU(4)$ symmetry = Wigner symmetry

Q: broken in nuclei due to spin-orbit splitting (breaks symmetry of spin ↑ and ↓)

Triplet P-waves and insights to nuclear forces

$l=1,\ S=1 \Rightarrow J=0,1,2 \longrightarrow$ nn-online.org $\longrightarrow$ Q: attractive/repulsive?



central interactions $V_{\mathbb{1}} + V_{\sigma_1\cdot\sigma_2}\, \vec{\sigma}_1\cdot\vec{\sigma}_2$ give identical contributions to ${}^3P_{0,1,2}$

Q: Why?

central part of ${}^3P$ waves = average of ${}^3P$ weighted by $2J+1$

$$= \overline{\delta}_{l=1}(k) = \frac{\sum_J (2J+1)\, \delta^J_{l=1}(k)}{\sum_J (2J+1)}$$

HW: Calculate $\overline{\delta}_{l=1} < 5°$ for $E_{lab} < 150\,\mathrm{MeV}$ small!

$\Rightarrow$ central ${}^3P$ interactions are small

Q: What can contribute to the splitting of the ${}^3P$ waves?

$\Rightarrow$ spin-orbit force $\sim \vec{l}\cdot\vec{S} = \frac{1}{2}(\vec{J}^2 - \vec{l}^2 - \vec{S}^2) \longrightarrow \frac{1}{2}\left(J(J+1) - S(S+1) - l(l+1)\right)$

$$= \begin{cases} -2 & {}^3P_0 \\ -1 & {}^3P_1 \\ 1 & {}^3P_2 \end{cases}$$

$\Rightarrow$ need $V_{\vec{l}\vec{S}}\, \vec{l}\cdot\vec{S}$ with attractive spin-orbit interaction $V_{\vec{l}\vec{S}} < 0$.

$\rightarrow$ ${}^3P_2$ attractive o.k. + ${}^3P_1$ repulsive o.k. but ${}^3P_2$ only works for high energies and only the sign

$\Rightarrow$ There must be additional contributions to nuclear forces beyond central and $\vec{l}\cdot\vec{S}$ interactions

$\rightarrow$ tensor interactions $S_{12}(\hat{r}) = \vec{\sigma}_1 \cdot \hat{r}_1 \; \vec{\sigma}_2 \cdot \hat{r}_2 - \frac{1}{3}\vec{\sigma}_1 \cdot \vec{\sigma}_2$

couples spin and space $\rightarrow$ couples $J_<, J_>$ partial waves

Q: Why doesn't $\vec{l}\cdot\vec{S}$ do this?

$\Rightarrow$ Can learn a lot from phase shifts about nuclear forces!

Scales in nuclear forces

momentum scales Q

i) $\left|\frac{1}{a}\right| \approx \frac{1}{1-20\,\text{fm}} \approx - 10 - 40\,\text{MeV}$

or $\frac{1}{5\,\text{fm}}$

ii) $m_\pi \approx 140\,\text{MeV}$

$\frac{1}{r_e} \approx \frac{1}{2.7\,\text{fm}} \sim m_\pi$

iii) $m_\Delta - m_N \sim 2 m_\pi$

iv) $m_{heavy = \rho, \omega, \ldots} \sim 1\,\text{GeV}$

$\Rightarrow$ Separation of scales