Lecture Three-body forces and halo nuclei

Consider spinless bosons $\rightarrow$ can easily generalize to nucleons $n\uparrow, n\downarrow, p\uparrow, p\downarrow$
and distinguishable particles

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi - C_0 (\psi^\dagger \psi)^2 - D_0 (\psi^\dagger \psi)^3 + \dots$$

S-wave 2-body scattering amplitude

$$f_2(k) = \frac{1}{-\frac{1}{a} - ik} \left[1 - \frac{r_e k^2/2}{-\frac{1}{a} - ik} + \dots \right] =$$

$\uparrow$

large $a > 0$
bound-state pole
at $k = i/a$

$E_2 = -\frac{1}{ma^2}$

$r_e \sim \frac{1}{M_{heavy}}$

Q: correction $\sim \frac{Q}{M_{heavy}}$

Naive dimensional analysis: 3-body interaction $\sim Q^3$

breaks down for large a because of Λ-divergence in 3-body system

Simplest 3-body process for 3 bosons: boson-dimer scattering

$\downarrow$

dimer

Boson-dimer scattering equation = Skorniakov-Ter-Martirosian equ.

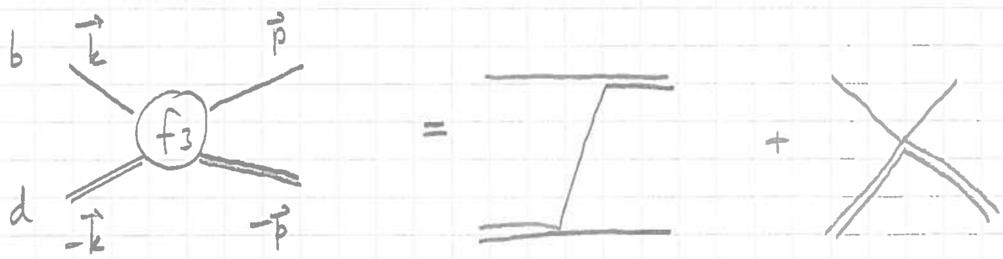
for $l = 0$ Q: which l?

$$f_3(k,p;E) = \frac{16}{3a} M(k,p;E) + \frac{4}{\pi} \int_0^\Lambda dq\, q^2 f_3(k,q;E) \frac{M(q,p;E)}{-\frac{1}{a} + \sqrt{3q^2/4 - mE - i\varepsilon}}$$

with inhomogeneous term

$$M(k,p;E) = \frac{1}{2kp} \ln \left(\frac{k^2 + kp + p^2 - mE}{k^2 - kp + p^2 - mE} \right) + \frac{H(\Lambda)}{\Lambda^2}$$

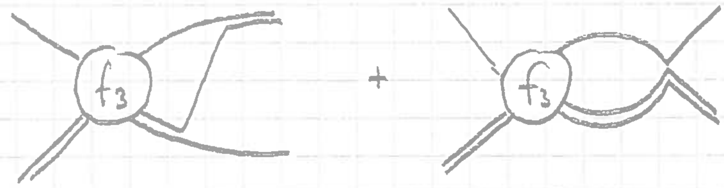
and $E = \frac{3k^2}{4m} - \frac{1}{ma^2}$ $\leftarrow$ Q?



on-shell

$$k\cot\delta = \frac{1}{f_3(k,k,E)} + ik$$

+



with $D_0(\Lambda) = -4m\, C_0(\Lambda)^2 \frac{H(\Lambda)}{\Lambda^2}$

For $H=0$ (no 3-body interaction): integral equation has no unique solution for $\Lambda \to \infty$

regularized integral equation has unique solution, but 3-body observables show strong dependence on Λ!

$\Rightarrow$ remove cutoff dependence by running coupling $D_0(\Lambda) \sim H(\Lambda)$

Bedaque, Hammer, van Kolck (1999) H(Λ) has limit cycle running

$$H(\Lambda) \approx \frac{\cos[s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0]}{\cos[s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0]}$$

with $s_0 \approx 1.00624$ is a transcendental number

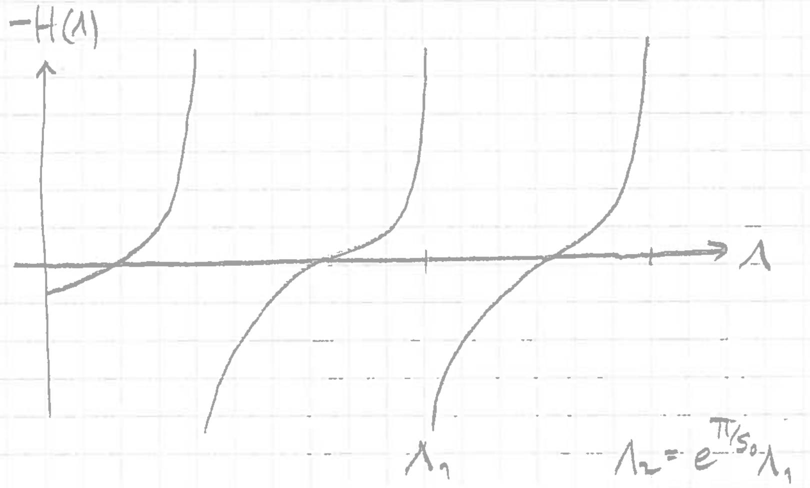
Λ_* dimensional 3-body parameter ($\rightarrow$ dimensional transmutation)
fix to reproduce one low-energy datum

$\Rightarrow$ discrete scale invariance by scaling factor in momentum $e^{\pi/s_0} \approx 22.7$

in energy $\sim e^{2\pi/s_0} \approx 515$

scale invariance in unitary limit is broken by Λ_*

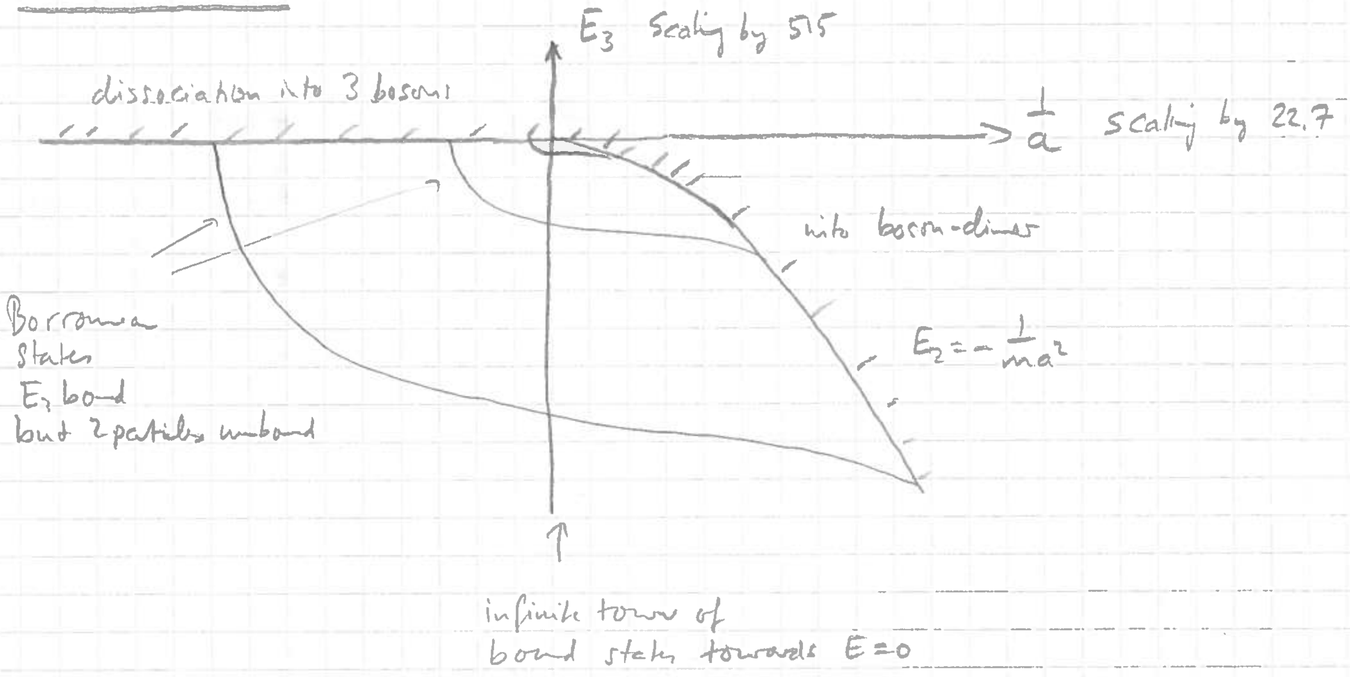
leaves 3-body observables unchanged



$\Rightarrow$ discrete scaling symmetry leads to the Efimov effect in 3-body system

$\rightarrow$ geometric spectrum of 3-body bound states

Efimov spectrum



fix Λ_* to one low-energy E_3