T2a-1

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Chiral Forces 2

Plan: multiple topics addressing different aspects of chiral EFT forces

- (1) loose ends from yesterday
  - Story of Nijmegen fits with m_π as free parameter
    $\Rightarrow$ see (M2b-5) notes.
  - (Maybe) go back and step through (M2b-11) on calculating g_A.

(2) Nuclear Forces from chiral Lagrangians
  - method of unitary transformations

- (3) NDA, naturalness, and resonance saturation for LEC's fit with Weinberg counting

(4) Deltaful vs. deltaless chiral EFT

(5) Renormalization issues with Weinberg counting

(6) Building chiral Lagrangians

A lot of stuff $\Rightarrow$ partial detail to whet your appetite for more!

m26-5

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How can we really test both the one-pion and two-pion exchange interactions predicted from chiral symmetry are present in NN scattering?

- The Nijmegen group together with Bira van Kolck and Jim Friar (in various combinations) used the Nijmegen PWA methods to make tests (e.g. very careful fits with χ^2 taken very seriously) [see Rentmeester et al., Phys Rev C. 67, 044001 (2003) and references]
- Particularly convincing is when they let the mass of the pion be a free parameter in their χ-square minimization that fits the couplings to pp and np data.

- In the original work from 1993, they determined the $pp\pi^0$ coupling constant in each partial wave except for $^1S_0$ (by letting one float in that partial wave) and they agree at the $\pm 1\%$ level with each other and the extracted value from πN scattering.
  - Extracting the neutral and charged pion masses agreed with experiment within estimated one percent errors.

- Subsequent analyses in 1999 and 2003 looked with finer resolution, to look for the direct evidence of two-pion exchange physics.
  - Here they found that using the pion mass as a free parameter gave agreement with experiment at the 10 percent level.
  - The c_i couplings in the TPE fit from NN are consistent with those from πN scattering, although there are still sizable uncertainties in both these determinations. (We'll see the c_i's many times still!)

- Open question: can we do something similar for nuclear structure?
  - My dream: have m_π as a free parameter in an energy density functional with long-range chiral effects (next week) and determine from a fit (say along an isotope chain).
  - Do nuclei know about pions in their structure? The answer could be NO if low resolution!

later: discussion of naturalness and NDA in functionals maybe look at pictures first

MQb-11

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Let's consider the calculation for g_A.

What we want is the proton matrix element of the axial vector current:

$$\langle p' s' | A_\mu^{u-d} | p, s\rangle = \bar{u}(p' s')\left[\gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2M_N} G_p(q^2)\right] u(p,s)$$

$\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$

spinors for p,s or $p's'$

$\langle p' s' | p s\rangle = (2\pi)^3 2p^0 \delta(\vec{p}-\vec{p}')\delta_{ss'}$

When $q = p' - p \to 0$ then $g_A = G_A(0)$ at $p = p'$.

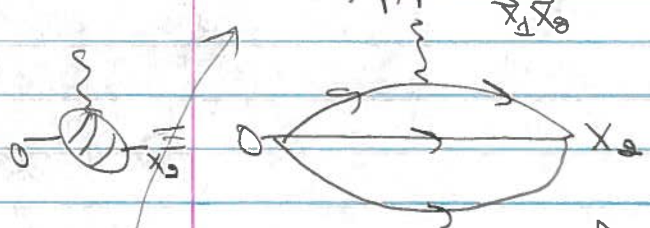
This is a 3pt function. In general (suppressing many indices):

$$G(t, \tau, \vec{p}, \vec{p}') = \sum_{\vec{x}_1 \vec{x}_2} e^{-i\vec{p}'\cdot(\vec{x}_2 - \vec{x}_1)} e^{-i\vec{p}\cdot\vec{x}_1} \langle 0 | N(\vec{x}_2, t) A_\mu^{ud}(\vec{x}_1, \tau) \bar{N}(0) | 0\rangle$$

annihilate state at final time t

insert operator at time τ

create state with quantum numbers of proton at $t=0$



set $\vec{p} = \vec{p}' = 0$ here $\Rightarrow$ just sum over $\vec{x}_1, \vec{x}_2$

- use $N(\vec{x}, t) = e^{\hat{H}t} e^{-i\hat{\vec{P}}\cdot\vec{x}} N(0) e^{i\hat{\vec{P}}\cdot\vec{x}} e^{-\hat{H}t}$
- insert $I = \sum_{B, p, s} |B, p, s\rangle\langle B, p, s|$ twice

$$G(t, \tau, \vec{p}, \vec{p}') = \sum_{BB'}\sum_{ss'} e^{-E_{B'}(\vec{p}')(t-\tau)} e^{-E_B(\vec{p})\tau} \langle 0 | N(0) | B' p' s'\rangle$$
$$\times \langle B' p' s' | A_\mu^{ud}(\vec{q}) | B, p, s\rangle \langle B, p, s | \bar{N}(0) | 0\rangle$$

what we want, for B = proton

$0 \ll \tau \ll t$

$\xrightarrow{\quad}$ with $\vec{p} = \vec{p}' = 0$

$$\sum_{ss'} \langle \text{proton } p'=0\ s' | A_\mu^{ud}(0) | \text{proton } p=0\ s\rangle e^{-E_p(t-\tau)} e^{-E_p\tau} \langle 0 | N(0) | \text{proton}\rangle$$
$$\times \langle \text{proton} | \bar{N}(0) | 0\rangle$$

calculate 2pt function (without A_μ^{ud} insertion) and divide!

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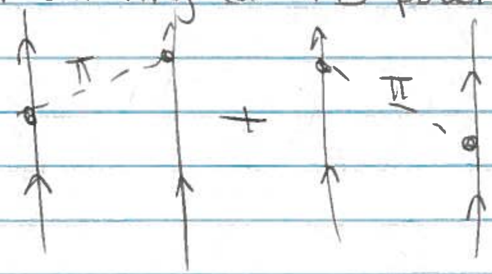
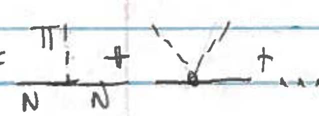
Nuclear forces from chiral Lagrangians (from Epelbaum lectures)

- There are several paths from the chiral Lagrangians we wrote down yesterday, which had both nucleon and pion fields, to a potential between nucleons (so no pion coordinates).

① S-matrix-based methods
- Robilotta, da Rocha '97, Kaiser et al. 97, 01, ..., Higa et al. 03, 04, ...
- Calculate the amplitude for 2-2 scattering in χPT (field theory) and require it match in perturbation theory to the Lippmann-Schwinger series
    - Two ways to calculate the same thing: field theory from $\mathcal{L}$ and with a potential ⇒ make them agree order by order

② Hamiltonian-based methods
- We've already used time-ordered perturbation theory for deriving the OPE potential:



$\mathcal{L}_{\pi N} \rightarrow \mathcal{H}_{\pi N} =$ 

- Weinberg '90, '91; Ordonez et al. '92, '94
    - leads to energy dependent potentials that are (at least) inconvenient for A>2 calculations
- The work by Epelbaum, Glöckle, and Meissner has used the so-called method of unitary transformations to decouple the π part of the Hamiltonian from the nucleon part.
    - We look at this further because the use of unitary transformations in analogous ways will be the topic of future lectures.

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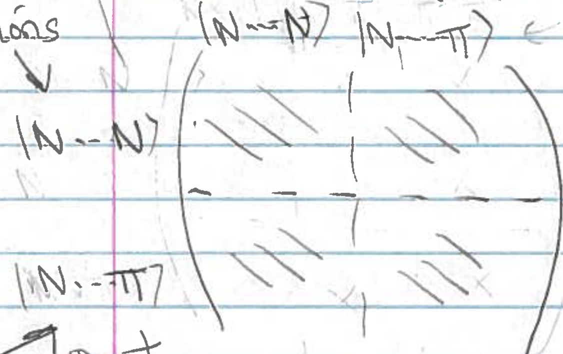
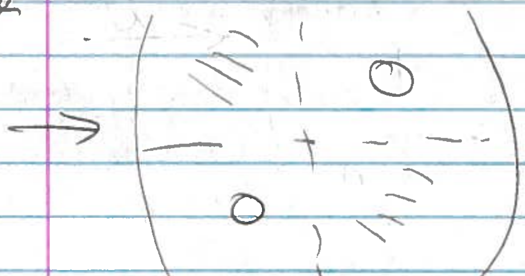
Schematically, we can view the Hamiltonian as a matrix where we have segregated

states with no pions



states with at least one pion

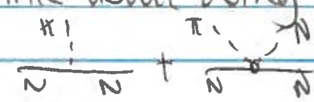
- we would like to find a unitary transformation such that the transformed Hamiltonian has no off-diagonal sectors connecting the N--N block



- By making these sectors non-zero, we have decoupled them.

- The result is an energy-independent internucleon potential with V_{NN}, V_{3N}, V_{4N}, ... parts.

- In practice it is easiest to think about doing this in second quantization. $H_{\pi N}$ =



$$H_{\pi N} = \sum_{\substack{\alpha\beta\ldots \\ \alpha'\beta'\ldots}} n^{\dagger}_{\alpha'} n_{\alpha}\left[h_{\alpha'\alpha} + h_{\alpha'\alpha\beta}(a^{\dagger}_{\beta}+a_{\beta}) + h_{\alpha'\alpha\beta\gamma}(a^{\dagger}_{\beta}a_{\gamma}+a^{\dagger}_{\beta}a_{\gamma}+a^{\dagger}_{\gamma}a_{\beta}+a_{\beta}a_{\gamma}) + \ldots\right] + \ldots$$

create, destroy nucleons

create, destroy pions

Now devise a unitary transformation that has the same form:

$$U = \sum n^{\dagger}_{\alpha'} n_{\alpha}\left[u_{\alpha'\alpha} + u_{\alpha'\alpha\beta}(a^{\dagger}_{\beta}+a_{\beta}) + u_{\alpha'\alpha\beta\gamma}(a^{\dagger}_{\beta}a^{\dagger}_{\gamma}+a^{\dagger}_{\beta}a_{\gamma}+a^{\dagger}_{\gamma}a_{\beta}+a_{\beta}a_{\gamma}) + \ldots\right] + \ldots$$

- We require the u coefficients to be such that $U^{\dagger}U = 1$, but there is still much freedom.

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So we require the off-diagonal matrix elements to be zero for the transformed Hamiltonian:

$$H_{nucl} = U^{+} H_{\pi N} U$$

with

$$\langle N_1 N_2 \cdots N_n \Pi_1 \Pi_2 \cdots \Pi_m | U^{+} H_{\pi N} U | N_1 N_2 \cdots N_n \rangle = 0 \quad \text{for } m \geq 1$$

Alternative description

Let η project on nucleus only subspace $|\phi\rangle$ of $|\Psi\rangle$ and let $|\psi\rangle$ be the rest of the space with projector λ

$$\Rightarrow |\phi\rangle = \eta|\Psi\rangle, \quad |\psi\rangle = \lambda|\Psi\rangle \qquad (\eta, \lambda \text{ are } P, Q \text{ in other contexts})$$

Find U such that

$$\hat{H} = U^{+} H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

Okubo says

$$U = \begin{pmatrix} \eta(1+A^{+}A)^{-1/2} & -A^{+}(1+AA^{+})^{-1/2} \\ A(1+A^{+}A)^{-1/2} & \lambda(1+AA^{+})^{-1/2} \end{pmatrix} \quad \text{with } A = \lambda A \eta$$

- A satisfies $\lambda(H - [A, H] - AHA)\eta = 0$
- This can be solved perturbatively

$$A = \sum_{n=1}^{\infty} g^{n} A^{(n)}$$

- see the Epelbaum review for details.

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NDA, naturalness, and resonance saturation

- We often speak of "naturalness" in discussing the size of low energy constants (LECs). The idea is that we expect a particular size based on dimension analysis combined with (or based on) physics considerations.
  - Recall from last week the case of a pionless EFT with a single momentum scale Λ_b in dim.reg (the breakdown scale → we call it that because an expansion in Q/Λ_b fails when $Q \gtrsim \Lambda_b$). [cf. Λ_c, a cutoff. Independent!! If $\Lambda_c < \Lambda_b$ then it is the scale that appears]
  - For a short-range potential (e.g., hard-sphere) with range R, then $\Lambda_b \sim 1/R$
  - We found that (again, in dimensional regularization)

← $2i$ derivatives

$$C_{2i} = a_i \frac{4\pi}{M} \frac{1}{(\Lambda_b)^{2i+1}} \qquad D_{2i} = b_i \frac{4\pi}{M} \frac{1}{\Lambda_b^{2i+4}}$$

with $a_i, b_i \sim O(1)$. [Often we say natural is $\frac{1}{3} \lesssim a_i, b_i < 3$.]
  - The $4\pi/M$ came from relating the T-matrix, described by the diagrams, to the scattering amplitude and the actual observables.
  - Then it is just dimensions, because $\Lambda_b \sim 1/R$ is all there is.
  - This is called naive dimensional analysis or NDA
    - not because it is foolish (after all, you have to be smart to figure out the 4π's)
    - but because it neglects other considerations that might cause a very different estimate (like shallow bound states or symmetries).

- Do we do the same thing for constants in the chiral Lagrangian?

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- Georgi and Manohar originally argued that a low-energy theory of QCD, that is to say, one describing the physics of hadrons below a scale characterizing spontaneous symmetry breaking, should be analyzed with two dimensional factors:
  - pion decay constant $f_\pi \simeq 100$ MeV
  - the chiral symmetry breaking scale $\Lambda_\chi \simeq 500\text{-}1000$ MeV
    $\Rightarrow$ typical mass of low-lying (non-Goldstone) bound states (e.g. $m_\rho \simeq 763$ MeV)

- How should we combine them? Georgi, in Generalized dimensional analysis (Phys. Lett B298 (1993) 187) says for each term in $\mathcal{L}_{EFT}$:
  1. include an overall factor of $f_\pi^2 \Lambda_\chi^2$
  2. include a factor of $1/f_\pi$ for each strongly interacting field
  3. add factors of Λ to get the dimension to 4
     (with $\hbar=1$, $S = \int d^4x\, \mathcal{L}$ is dimensionless, so $\mathcal{L} \sim [\Lambda]^4$)

- When applied to nucleon fields N and pion fields π:

$$\mathcal{L}_{\chi EFT} = C_{lmn} \left(\frac{N^\dagger \Gamma N}{f_\pi^2 \Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi}\right)^n f_\pi^2 \Lambda_\chi^2$$

- If we scale the Lagrangian this way, we expect the dimensionless constant C_{lmn} to be order unity.

- Georgi's argument is that f_π is a universal measure of the amplitude for producing a strongly interacting bound state, so each field gets such a factor. The only other thing happening is at the dimensional scale set by Λ.

- Let's try it out for some contact terms in the contact potential:

$$V_{cont} = C_S + C_T(\vec{\sigma}_1\cdot\vec{\sigma}_2) + C_1 \vec{q}^{\,2} + C_2 \vec{k}^2 + C_3(\vec{q}^{\,2} + C_4 \vec{k}^2)\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + i C_5 \frac{\vec{\sigma}_1+\vec{\sigma}_2}{2}\cdot(\vec{q}\times\vec{k})$$
$$+ C_6(\vec{q}\cdot\vec{\sigma}_1)(\vec{q}\cdot\vec{\sigma}_2) + C_7(\vec{k}\cdot\vec{\sigma}_1)(\vec{k}\cdot\vec{\sigma}_2)$$

$\vec{q} = \vec{p}\,' - \vec{p}$

$\vec{k} = (\vec{p} + \vec{p}\,')/2$

Following Epelbaum (2002) et al.

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Now we have to step back to the Lagrangian

LO: $-\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma_i N)(N^\dagger \sigma_i N)$

$\Rightarrow$ $l=2$ for both, $m=0$, $n=0$

$$\Rightarrow \frac{1}{2}C_S \sim c_{200}\frac{1}{(f_\pi^2\Lambda_\chi)^2}\cdot f_\pi^2\Lambda_\chi^2 = \frac{c_{200}}{f_\pi^2} \quad \text{or} \quad c_{200} \sim f_\pi^2 C_S$$

Same for C_T. If we try this with the Epelbaum et al. NNLO potential, we find (varying the cutoff from 500–600 MeV)

$$f_\pi^2 C_S = -1.081 \cdots -0.95 \qquad f_\pi^2 C_T = 0.002 \cdots 0.040$$

- C_S works!
- C_T oops! But wait. Unnaturally small can signal an (unrecognized) symmetry; Wigner proposed that SU(4) spin-isospin transformations:

$$\delta N = i\,\varepsilon_{\mu\nu}\,\sigma^\mu \tau^\nu N, \quad N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \mu,\nu = 0,1,2,3$$

with $\sigma^\mu = (1,\vec{\sigma})$, $\tau^\nu = (1,\vec{\tau})$, $\varepsilon_{\mu\nu}$ a group parameter, is an approximate symmetry of the strong interaction.

- C_S term invariant but C_T breaks $\Rightarrow C_T \approx 0$.
(that is, the C_T term would be absent if the symmetry were exact.

But about NLO? A typical term is C_1:

$$\Rightarrow -\frac{1}{2}C_1\left[(N^\dagger \partial_i N)^2 + ((\partial_i N^\dagger)N)^2\right] \Rightarrow l=2,\ m=0,\ n=2$$

$$\Rightarrow C_1 \sim c_{202}\frac{1}{(f_\pi^2\Lambda_\chi)^2}\frac{1}{(\Lambda_\chi)^2}f_\pi^2\Lambda_\chi^2 = \frac{c_{202}}{f_\pi^2\Lambda_\chi^2} \quad \text{and the same}$$

is true for all the other $C_1 - C_7$. $\Rightarrow$ look at $f_\pi^2\Lambda_\chi^2 C_i$ but we also should not be completely naive and account for the 4 from $\vec{q}^{\,2} = (\vec{p}-\vec{p}\,')^2$ and $\vec{k}^2 = (\vec{p}+\vec{p}\,')^2/4$.

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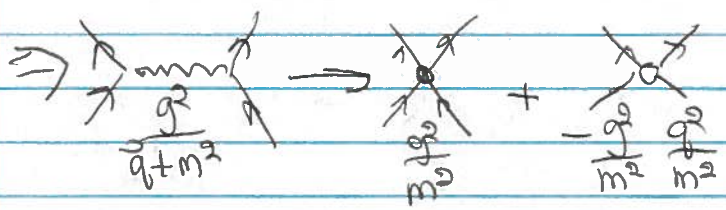
The results are

$$f_\pi^2 \Lambda_\chi^2 C_1 = 3.143 \cdots 2.665 \qquad 4 f_\pi^2 \Lambda_\chi^2 C_2 = 2.029 \cdots 2.251$$
$$f_\pi^2 \Lambda_\chi^2 C_3 = 0.403 \cdots 0.281 \qquad 4 f_\pi^2 \Lambda_\chi^2 C_4 = -0.364 \cdots -0.428$$
$$2 f_\pi^2 \Lambda_\chi^2 C_5 = 2.846 \cdots 3.410 \qquad f_\pi^2 \Lambda_\chi^2 C_6 = -0.728 \cdots 0.668$$
$$4 f_\pi^2 \Lambda_\chi^2 C_7 = -1.929 \cdots -1.681$$

$\Rightarrow$ pretty natural! So we can use this estimate for terms in higher order not calculated.

- But this doesn't tell us precise quantitative values or the sign of a coupling.
  - Let's use non-Goldstone boson exchange as a model for the short-distance physics unresolved in chiral EFT.
  - If this is correct, it should be encoded in the coefficients of contact terms:

$\Rightarrow$ 

$+ \ldots$ ← added up over all the mesons and projected on all the terms

← and spin, S_{12}, etc structure

- Let's try phenomenological boson exchange models $\Rightarrow$ tells us g's and m's. [See Epelbaum et al. (2002) for some subtleties]

$\star \Rightarrow$ see the slides.

Comments:

- The agreement is quite impressive!
- The assumption that most of the constants are given by virtual exchanges is called "resonance saturation."
- Does the agreement validate boson exchange or is that itself just an alternative way to parametrize short-distance physics?

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Deltaful vs. Deltaless EFT (based on H. Krebs)

- Our "standard" Deltaless EFT supposes that

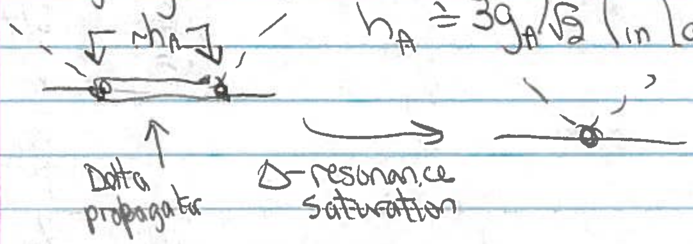
$$Q \sim m_\pi \ll \Delta \equiv m_\Delta - m_N \doteq 300 \text{ MeV}$$

- Hemmert, Holstein, Kambor 1998 proposed "small scale expansion":

$$Q \sim m_\pi \sim \Delta \ll \Lambda_\chi$$ to organize the expansion (decide what diagrams in each order)

- If we think of the LECs that encode the Delta contribution in going to Delta less:

$h_A \doteq 3g_A/\sqrt{2}$ (in Large N_c)



$\Rightarrow c_3 = -2c_4 = c_3(\not\Delta) - \frac{4h_A^2}{9\Delta m}$ ← unnaturally large

$$c_{2,3,4} \approx 2.8, -3.9, 2.9 \Rightarrow c_{2,3,4}(\not\Delta) \approx -0.3, -0.8, 1.3$$

- So we expect that including the Δ will lead to more natural LECs, better convergence, higher breakdown scale.

* ⇒ see pictures

- Achim will have more to say about 3NF and Δ's.

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Renormalization Issue

· If we consider iterating the leading order (LO) potential

$$V_{NN}^{(0)} = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_{OPE}^{(0)}$$

in Weinberg counting using the Lippmann-Schwinger (LS) equation:

$$T = V + V\frac{1}{E-H_0}T = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V + \ldots$$

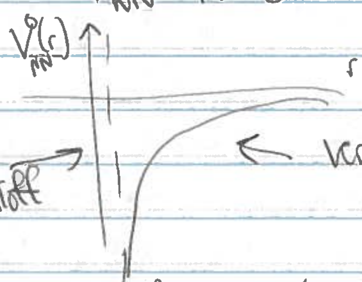
then we need to cutoff the potential, eg.

$$V(\vec{p}\,',\vec{p}) \longrightarrow e^{-\left(\frac{\vec{p}\,'^2}{\Lambda^2}\right)^n} V(\vec{p}\,',\vec{p})\, e^{-\left(\frac{\vec{p}^{\,2}}{\Lambda^2}\right)^n}$$

here nonlocal but we heard of a different local regulator by Alex.

· For an EFT, we ideally want the observable results to be insensitive to how we regulate and with what Λ we use.

· But strong cutoff dependence and divergences are found with $V_{NN}^{(0)}$ in $S=1$ channels where the tensor force is attractive.



← very singular $\sim \frac{1}{r^3}$ and worse when iterated

large Λ means cutoff moves to smaller r effectively → find spurious bound states,

· Nogga et al. found that S-wave can take $\Lambda \to \infty$ with stable results, but $^3P_0$, $^3P_2$, $^3D_2$ (attractive tensor) need counterterms even at LO to absorb Λ dependence.

· Practical solution is keep $\Lambda \sim \Lambda_{breakdown}$. On-going study for formal solution.

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Building chiral Lagrangians

- Here we follow the accessible but incomplete treatment in the review of Machleidt and Entem
  - includes references to details
  - more rigorous discussion with mathematical detail in E. Epelbaum's review

both linked under References

- $\mathcal{L}_{\chi EFT} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$

- Think in terms of building blocks, combined in all possible ways to generate $\mathcal{L}$'s. cf. take $1, \gamma^\mu, \sigma^{\mu\nu}, \ldots$ and making Lorentz invariant by contracting indices.

- For $\mathcal{L}_{\pi\pi}$ the building block is an SU(2) matrix U made up of the pion fields.
  - Perhaps easiest to start with adding a scalar meson σ
    $\Rightarrow (\vec{\pi}, \sigma) \equiv (\pi_1, \pi_2, \pi_3, \sigma)$ (real vector)
    with $SU(2)_L \times SU(2)_R$ chiral symmetry $\Rightarrow SO(4)$ rotation of this vector. A potential $V(\sigma^2 + \vec{\pi}^2)$: $\overset{SSB}{\Longrightarrow} \sigma^2 + \vec{\pi}^2 = f_\pi^2$ is minimum (pick $\sigma = f_\pi$ VEV)
  - But we don't want the σ as a low-energy degree of freedom $\Rightarrow$ take its mass to infinity (so sits in minimum)
    $\Rightarrow$ fixed constraint $\sigma^2 + \vec{\pi}^2 = f_\pi^2 \Rightarrow \sigma = \sqrt{f_\pi^2 - \vec{\pi}^2}$ eliminated
  - Under a vector transformation
    $\vec{\pi} \xrightarrow{\theta^V} \vec{\pi}' = \vec{\pi} + \vec{\theta}^V \times \vec{\pi}$ linear rotation (isospin rotation of π fields)
  - But axial vector is now non-linear from constraint
    $\vec{\pi} \xrightarrow{\theta^A} \vec{\pi}' = \vec{\pi} + \vec{\theta}^A \sqrt{f_\pi^2 - \vec{\pi}^2}$

· isospin mixes the π_i's
· chiral rotation mixes σ and $\vec{\pi}$'s

vector subgroup

- Write $U = (\sigma + i \vec{\tau} \cdot \vec{\pi}) / f_\pi \qquad \sigma = \sqrt{f_\pi^2 - \vec{\pi}^2}$ as 2×2 complex matrix

Check $U^\dagger U = \frac{1}{f_\pi^2}(\sigma - i\vec{\tau}\cdot\vec{\pi})(\sigma + i\vec{\tau}\cdot\vec{\pi}) = \frac{1}{f_\pi^2}(\sigma^2 + \vec{\pi}^2) = I_{2\times 2}$ ✓

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Under a global chiral transformation

$$U \to U' = g_L U g_R^\dagger \Rightarrow U^\dagger \to g_R U g_L^\dagger$$

$SU(2)_L$ $\quad$ $SU(2)_R$

$$\text{So } tr(U^\dagger U) \to tr(g_R U^\dagger \cancel{g_L^\dagger g_L} U g_R^\dagger) = tr(\cancel{g_R^\dagger g_R} U^\dagger U) = tr(U^\dagger U)$$

$\Rightarrow$ chiral invariant but $tr(U^\dagger U) \propto I$! $\Rightarrow \mathcal{L}^{(0)}_{\pi\pi}$ is trivial

$\Rightarrow$ need derivatives. Also need mass term

two derivatives or m_π^2

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4} tr\left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2 (U + U^\dagger)\right]$$

chosen to get kinetic and mass term correct

breaks chiral symmetry in same way that mass terms in QCD do.

- U can be parametrized many different ways, e.g.

$$U = e^{i\vec{\tau}\cdot\vec{\pi}/f_\pi} \qquad (\text{check } U^\dagger U = I \text{ trivially})$$

- Find the pions by expanding:

$$U = 1 + \frac{i}{f_\pi}\vec{\tau}\cdot\vec{\pi} - \frac{1}{2f_\pi^2}\vec{\pi}^2 - \frac{i\alpha}{f_\pi^3}(\vec{\tau}\cdot\vec{\pi})^3 + \frac{8\alpha-1}{8f_\pi^4}\vec{\pi}^4 + \ldots$$

- all powers of $\vec{\pi}$ field related $\Rightarrow$ chiral symmetry!
- α reflects different parametrizations, but related by redefinitions of $\vec{\pi}$: "field redefinitions" $\Rightarrow$ equivalence theorem say observables (S-matrix elements) are unchanged

- [basically a change of variables in path integral where you show that the Jacobian and new external source terms do not contribute to the S-matrix, see Furnstahl et al. for finite density!]

check that these work in exercise and find α for $e^{i\vec{\tau}\cdot\vec{\pi}/f_\pi}$

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$$\Rightarrow \mathcal{L}^{(2)}_{\pi\pi} = \frac{1}{2}\partial_\mu\pi\,\partial^\mu\pi - \frac{1}{2}m_\pi^2\pi^2 + \frac{1-4\alpha}{2f_\pi^2}(\vec{\pi}\cdot\partial_\mu\vec{\pi})(\vec{\pi}\cdot\partial^\mu\vec{\pi})$$

π-π scattering!

$$- \frac{\alpha}{f_\pi^2}\pi^2\partial_\mu\pi\cdot\partial^\mu\pi + \frac{8\alpha-1}{8f_\pi^2}m_\pi^2\pi^4 + \mathcal{O}(\pi^6)$$

- $\mathcal{L}^{(4)}_{\pi\pi} = \frac{l_1}{4}(\partial_\mu U\,\partial^\mu U)^2 + \ldots$ see Gasser-Leutwyler

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For πN, start with relativistic πN formulation [Gasser et al.] and then reduce to nonrel. with heavy baryon expansion (in $1/m$)

Building blocks are $\partial_\mu \rightarrow D_\mu$ (chiral covariant derivative) and u_μ $\Rightarrow$ combine in Lorentz contracted ways (with other symmetries obeyed)

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\left(i\gamma^\mu D_\mu - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi$$

$$u_\mu = i(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) = -\frac{1}{f_\pi}\vec{\tau}\cdot\partial_\mu\vec{\pi}$$

$$D_\mu = \partial_\mu + \Gamma_\mu\,, \quad \Gamma_\mu = \frac{1}{2}[\xi^\dagger, \partial_\mu\xi] \rightarrow \frac{i}{4f_\pi^2}\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi}) + \mathcal{O}(\pi^4)$$

and $\xi = \sqrt{U}$ (sometimes u) $\quad \xi = 1 + \frac{i}{2f_\pi}\vec{\tau}\cdot\vec{\pi} - \frac{1}{8f_\pi^2}\vec{\pi}^2 + \ldots$ [check $\xi\xi^\dagger = 1$ ✓]

$$\Rightarrow \mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m_N - \frac{1}{4f_\pi^2}\gamma^\mu\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi}) - \frac{g_A}{2f_\pi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi} + \ldots\right)\Psi$$

- Need to make nonrelativistic so loop expansion associated with EFT expansion (recall ν formula: $L\uparrow$, $\nu\uparrow$)
  - but $\partial_0\Psi \rightarrow m_N\Psi$ in numerator and $m_N/\Lambda_\chi \approx 1$ $\quad$ too fast $e^{-i(m+\ldots)t}$

- Use heavy baryon formalism $\rightarrow$ gives $\frac{1}{m}$ expansion

7/9/13 (12a-T4)

Nucleon momentum: $p^\mu = m v^\mu + \ell^\mu$ ← small residual momentum

↑ four velocity of nucleon (big lumbering thing with pions bouncing off)

$$\Rightarrow \Psi = e^{-imv\cdot x}(N + h)$$

fast time dependence ⇒ pull this out; large components; small components

$$N = e^{imv\cdot x} P_v^+ \Psi, \quad h = e^{imv\cdot x} P_v^- \Psi \qquad P_v^\pm = \frac{1 \pm \gamma_\mu v^\mu}{2} \qquad P_v^+ + P_v^- = 1$$

projection operators

· insert and use $v^\mu = (1, \vec{0})$. Kills e^{-imt}, leaves N upper only

$$\Rightarrow \mathcal{L}^{(1)}_{\pi N} = \bar{N}\left(i\gamma^\mu D_\mu + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)N + \ldots$$

$$\xrightarrow[\text{upper only}]{v^\mu = (1,\vec{0})} \bar{N}\left(iD_0 - \frac{g_A}{2}\vec{\sigma}\cdot\vec{u}\right)N$$

$$= \bar{N}\left(i\frac{\partial}{\partial t} - \frac{1}{4f_\pi^2}\vec{\tau}\cdot(\vec{\pi}\times\dot{\vec{\pi}}) - \frac{g_A}{2f_\pi}\vec{\tau}\cdot(\vec{\sigma}\cdot\vec{\nabla})\vec{\pi} + \begin{pmatrix}\text{3 or more}\\ \text{pions}\end{pmatrix}\right)$$

At dimension 2, write down all terms with covariant derivatives. After Heavy baryon reduction:

$$\mathcal{L}^{(2)}_{\pi N, ct} = N^\dagger\left[2c_1 m_\pi^2 (U + U^\dagger) + \left(c_2 - \frac{g_A^2}{8m_N}\right)u_0^2 + c_3 u_\mu u^\mu + \frac{i}{2}\left(c_4 + \frac{1}{4m_N}\right)\vec{u}\times\vec{u}\right]N$$

$$= N^\dagger\Big[4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\vec{\pi}^2 + \left(c_2 - \frac{g_A^2}{8m_N}\right)\frac{1}{f_\pi^2}\dot{\underset{\sim}{\pi}}\cdot\dot{\underset{\sim}{\pi}} + \frac{c_3}{f_\pi^2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi})$$

$$- \left(c_4 + \frac{1}{4m_N}\right)\frac{i}{2f_\pi^2}\varepsilon_{ijk}\varepsilon_{abc}\sigma^i\tau^a(\partial^j\pi^b)(\partial^k\pi^c)\Big]N + \ldots$$

· Get $\mathcal{L}_{NN}$ contact terms (no pions) from conventional constraints.