T1a. Nuclear Forces 1

a. Overview

The quest for an accurate description of the nuclear force spans eight decades, starting from Yukawa's meson theory in 1935. From the early days, pion exchange was understood to be critical and correctly described the longest-ranged part of the NN interaction. In the 1950's, attempts to include multi-pion exchange failed but these were eventually supplanted in the 1960's and 1970's

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Figure 7: Cartoons for quark-gluon and pion-exchange representations of the NN force.

The alternative was a detailed phenomenological parametrization of physics as in the picture on the right (based on boson exchange plus some additional phenomenology), and that became good enough to fit the relevant NN data to great accuracy ($\chi^2/\mathrm{dof} \approx 1$ up to the inelastic threshold) and do quantitative calculations in light nuclei. This was a pragmatic approach, but could also be defended as good physics. We understand the strong forces that bind nuclei as the *residual* colorless interactions between the quarks and gluons that comprise hadrons. By analogy, the van der Waals interactions between neutral atoms at low energies are usefully described by potentials rather than

in terms of the Coulomb interactions between constituent electrons and nuclei (and certainly not in terms of quantum electrodynamics). With the additional complication of confinement in QCD, maybe this *is* the way to go, at least at long distances (with lingering doubts about the short-distance part).

But now with ever more sophisticated lattice QCD calculations, should we plan to go back to the left picture? In fact, the plan we advocate is to exploit *both* pictures in Fig. 7, with the idea that both are correct at different levels of *resolution* and that we can use calculations on the left to inform the interactions on the right, which in most cases are better suited for nuclear structure and reactions. (Indeed, as we will see, for some applications the picture becomes even simpler, with no resolved exchanged particles at all.) Next week we'll explore the state-of-the-art of LQCD for nuclear forces and examine what is possible now and in the forseeable future. But we can proceed with the picture on the right in the absence of direct LQCD input within the framework of *effective theories* and use hadronic degrees of freedom (dofs) systematically (as opposed to a model) to do nuclear structure and reactions and astrophysics. Some scattered preview comments:

- We seek model independence and theory error estimates. It is not enough to do a single calculation and use the comparison to experiment as the sole judge. We need to know if we did as well as we should, and how we can improve to a desired accuracy (or *if* we can improve). Effective field theory (EFT) is a key tool for this program.

- Quark substructure *is* included through derivative expansions. Much more on this later, but think about a multipole expansion using point multipoles to systematically represent a charge distribution. Clearly substructure is included!

- In the future: Use lattice QCD to *match* via "low-energy constants" (LECs of an EFT of hadrons) for processes/systems where both are valid. In the meantime, match EFTs to experimental data to determine LECs.

- Our focus is on low-energy nuclear physics. We definitely need explicit quark dof's at higher densities (resolutions) where phase transitions happen or at at very high momentum transfers (at least for the interaction part; but more on this in the third week).

Today we examine what we can say in general about the NN interaction, survey what is currently available for NN forces, and begin our discussion of effective theories. There is sometimes a tendency for some EFT practitioners to be dismissive of phenomenological potentials but the latter have demonstrated successes and valuable insight. There are a lot of ways to think about potentials and we want to try to make sense of the connections between them. Otherwise nuclear physicists find themselves talking at cross purposes. An alternative is to be dogmatic, but this is not good physics. In the course of our survey we can keep in mind an important conceptual question: Is there a best Hamiltonian for low-energy nuclear physics?

b. Constraints on nuclear NN forces

We start with a conventional discussion of general symmetry constraints on the NN Hamiltonian as you would find in nuclear physics textbooks, having in mind that the idea of writing the most
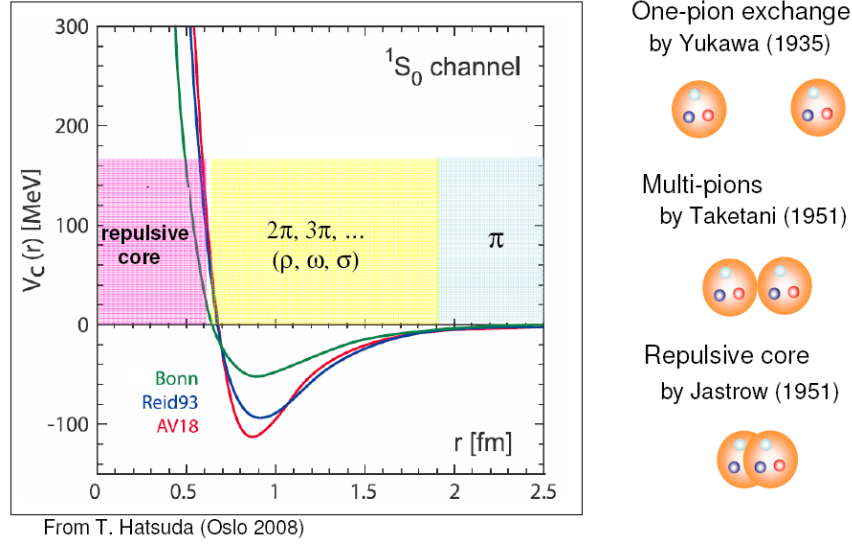
Figure 8: Several phenomenological potentials (S-wave part) and cartoons of the contributions (from T. Pappenbrock).

general possible Hamiltonian consistent with *QCD symmetries* will be a key one in our discourse. There is more than one way to proceed; we can see the same constraints arising from different perspectives. Here we consider the NN potential as an operator that acts in the spin, isospin, and position space Hilbert space. That means that our building blocks are the Pauli matrics for spin, $\boldsymbol{\sigma}_i$, and isospin, $\boldsymbol{\tau}_i$, multiplied by functions of the spatial coordinates.

So we can specify the potential operator $\widehat{V}$ by giving all of the matrix elements

$$\langle \mathbf{r}_1' s_1' t_1' \, \mathbf{r}_2' s_2' t_2' | \widehat{V} | \mathbf{r}_1 s_1 t_1 \, \mathbf{r}_2 s_2 t_2 \rangle \tag{1}$$

where $s_i = \pm 1/2$ and $t_i = \pm 1/2$ are spin and isospin projections. The bras and kets span the product spaces of the coordinate wave functions and the spin and isospin vector, so this is a sufficient basis (since it is complete). Suppressing spin and isospin for the moment, the action of $\widehat{V}$ on the coordinate basis is

$$\widehat{V} | \mathbf{r}_1 \mathbf{r}_2 \rangle = \int V(\mathbf{r}_1', \mathbf{r}_2', \mathbf{r}_1, \mathbf{r}_2) | \mathbf{r}_1' \mathbf{r}_2' \rangle d^3 r_1' \, d^3 r_2' \ . \tag{2}$$

The familiar local potential corresponds to the special

$$V(\mathbf{r}_1', \mathbf{r}_2', \mathbf{r}_1, \mathbf{r}_2) = V(\mathbf{r}_1, \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_1') \delta(\mathbf{r}_2 - \mathbf{r}_2') \quad \Longrightarrow \quad \widehat{V} | \mathbf{r}_1 \mathbf{r}_2 \rangle = V(\mathbf{r}_1, \mathbf{r}_2) | \mathbf{r}_1 \mathbf{r}_2 \rangle \ . \tag{3}$$

This evidently has no dependence on the velocities of the particles, but just their positions. It is not surprising then that we can translate the more general non-locality into a velocity (or momentum dependence). To do so, we first expand in a Taylor series the primed variables about the unprimed ones:

$$\begin{aligned} | \mathbf{r}_1' \mathbf{r}_2' \rangle &= | \mathbf{r}_1 \mathbf{r}_2 \rangle + [(\mathbf{r}_1' - \mathbf{r}_1) \cdot \boldsymbol{\nabla}_1 + (\mathbf{r}_2' - \mathbf{r}_2) \cdot \boldsymbol{\nabla}_2] | \mathbf{r}_1 \mathbf{r}_2 \rangle + \cdots \\ &= \, : \exp \left\{ (\mathbf{r}_1' - \mathbf{r}_1) \cdot \boldsymbol{\nabla}_1 + (\mathbf{r}_2' - \mathbf{r}_2) \cdot \boldsymbol{\nabla}_2] \right\} : | \mathbf{r}_1 \mathbf{r}_2 \rangle \ , \end{aligned} \tag{4}$$

where the "normal-ordering" notation $: \widehat{O} :$ means here that the derivatives be moved to act only to the right of the coordinates (and not on the coordinates). Thus,

$$\widehat{V}|\mathbf{r}_1\mathbf{r}_2\rangle = \int V(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{r}_1, \mathbf{r}_2) \exp\left\{ \frac{i}{\hbar}(\mathbf{r}'_1 - \mathbf{r}_1)\cdot\mathbf{p}_1 + \frac{i}{\hbar}(\mathbf{r}'_2 - \mathbf{r}_2)\cdot\mathbf{p}_2 \right\} |\mathbf{r}_1\mathbf{r}_2\rangle d^3r'_1\, d^3r'_2$$

$$= \widetilde{V}(\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2)|\mathbf{r}_1\mathbf{r}_2\rangle \ . \tag{5}$$

So the general form is built from these operators, generally restricted to a low order in momentum (quadratic), plus the allowed spin and isospin dependence.

But this is *too* general a formulation: if we allow arbitrary dependence on $\mathbf{r}_1, \mathbf{p}_1, \mathbf{r}_2, \mathbf{p}_2$ we will violate symmetries like spatial translation invariance. Similarly with possible structure for spin and isospin. For example, if we let σ_0 be the identity matrix, then since the σ_i, $i = 0, 1, 2, 3$ form a complete basis in the spin space of one particle, a general expression for $\widehat{V}$ is

$$\widehat{V} = \sum_{i,j=0}^{3} V_{ij}(\sigma_1)_i(\sigma_2)_j \ . \tag{6}$$

So there are 16 V_{ij} functions at this point. Similarly for isospin. But not every combination is consistent with the symmetries, so in fact there are fewer. What are they?

To decide what combinations are allowed, we recall that symmetry operations are cast as unitary transformations, which we indicate generically as

$$U = e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{G}} \tag{7}$$

where $U^\dagger U = UU^\dagger = I$ (the identity operator), $\boldsymbol{G}$ is a Hermitian generator of the transformation, and $\boldsymbol{\alpha}$ is the "displacement" of the transformation. (We write this as a dot product, but more generally it can be any rank tensor contraction.) The operation of U on a vector is

$$|\psi'_i\rangle = U|\psi_i\rangle \ . \tag{8}$$

If we have an operator $\widehat{O}$, then we have invariance under the symmetry if

$$\langle\psi'_i|\widehat{O}|\psi'_j\rangle = \langle\psi_i|U^\dagger\widehat{O}U|\psi_j\rangle = \langle\psi_i|\widehat{O}|\psi\rangle \tag{9}$$

from which it follows that

$$U^\dagger\widehat{O}U = \widehat{O} \quad \Longrightarrow \quad [\widehat{O}, U] = 0 \ . \tag{10}$$

From the expansion

$$e^{i\widehat{S}}\widehat{O}e^{-i\widehat{S}} = \widehat{O} + i[\widehat{S}, \widehat{O}] + \frac{i^2}{2!}[\widehat{S}, [\widehat{S}, \widehat{O}]] + \frac{i^3}{3!}[\widehat{S}, [\widehat{S}, [\widehat{S}, \widehat{O}]]] + \cdots \tag{11}$$

it then follows that $[\boldsymbol{G}, \widehat{O}] = 0$ is the condition we should check for symmetry consistency. [*Checking these is an exercise!*]

As an example to get oriented, we will claim that if isospin is a good symmetry, then the potential will take the form of $V = \alpha_1 + \beta_1\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ where α_1 and β_1 have all of the other spin and

space dependences. What does an isospin rotation look like and how does $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ stay invariant? The generator of isospin rotations is $\boldsymbol{T} = \frac{1}{2}(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)$. We then want to check whether $[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{T}] = 0$ or not. We can evaluate directly:

$$[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{T}]_j = \frac{1}{2}[\tau_{1i}\tau_{2i}, \tau_{1j} + \tau_{2j}] = \frac{1}{2}[\tau_{1i}, \tau_{1j}]\tau_{2i} + \frac{1}{2}\tau_{1i}[\tau_{2i}, \tau_{2j}] \ . \tag{12}$$

Now use $[\tau_i, \tau_j] = 2i\epsilon_{ijk}\tau_k$ to find

$$[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{T}]_j = i\epsilon_{ijk}\tau_{1k}\tau_{2i} + i\epsilon_{kji}\tau_{1i}\tau_{2k} = 0 \ . \tag{13}$$

by the antisymmetry of ϵ_{ijk}.

b.1 Operator structure of NN forces

Let us step through the constraints on $\widetilde{V}(\mathbf{r}_1, \mathbf{p}_1, \boldsymbol{\sigma}_1, \boldsymbol{\tau}_1, \mathbf{r}_2, \mathbf{p}_2, \boldsymbol{\sigma}_2, \boldsymbol{\tau}_2)$ based on continuous space-time and discrete symmetries (but not yet chiral symmetry). Generically when we use $\widehat{V}(1, 2)$, the 1 and 2 refer to all of the indices.

1. $\widehat{H} = \widehat{T} + \widehat{V}$ is hermitian and so is $\widehat{T}$, therefore $\widehat{V}$ is hermitian.

2. For identical particles, invariance under interchange of coordinates: $V(1, 2) = V(2, 1)$. This is connected to the symmetry of the two-particle wave function $|1\,2\rangle$.

3. Translational invariance in space. The unitary transformation for translations by $\boldsymbol{a}$ is

$$U = e^{-i\boldsymbol{a}\cdot\boldsymbol{P}} \ , \tag{14}$$

   where $\boldsymbol{P}$ is the total center-of-mass momentum (the generator of translations). It results in

$$\mathbf{r}'_i = \mathbf{r}_i - \mathbf{a} \ , \quad \mathbf{k}'_i = \mathbf{k} \ , \quad \boldsymbol{\sigma}'_i = \boldsymbol{\sigma}_i \ , \quad \boldsymbol{\tau}'_i = \boldsymbol{\tau}_i \ . \tag{15}$$

   Therefore we require

$$[\boldsymbol{P}, \widehat{V}] = 0 \ . \tag{16}$$

   In the full coordinate-space form (suppressing spin and isospin),

$$\langle \mathbf{r}_1\,\mathbf{r}_2 | \widehat{V} | \mathbf{r}'_1\,\mathbf{r}'_2 \rangle = \langle \mathbf{r}_1 - \mathbf{a}\,\mathbf{r}_2 - \mathbf{a} | \widehat{V} | \mathbf{r}'_1 - \mathbf{a}\,\mathbf{r}'_2 - \mathbf{a} \rangle \implies \langle \mathbf{r}_1 - \mathbf{r}_2 | \widehat{V} | \mathbf{r}'_1 - \mathbf{r}'_2 \rangle = \langle \mathbf{r} | \widehat{V} | \mathbf{r}' \rangle \ . \tag{17}$$

   For $\widetilde{V}$ this means

$$V(1, 2) = \widetilde{V}(\mathbf{r}, \mathbf{p}_1, \boldsymbol{\sigma}_1, \boldsymbol{\tau}_1, \mathbf{p}_2, \boldsymbol{\sigma}_2, \boldsymbol{\tau}_2) \ . \tag{18}$$

4. Galilean invariance (more generally Lorentz invariance). Nuclear low-lying states have $|\mathbf{p}| \sim 200\,\text{MeV}$, so $p/m \approx 0.2$ and $(v/c)^2 < 0.1$. This is small but not *negligible*, which suggests we should treat it as a $1/m$ expansion. Galilean invariance says that the physics looks the same from a moving frame, so $\mathbf{r}'_i = \mathbf{r}$, $\mathbf{p}'_i = \mathbf{p} - m_i\boldsymbol{u}$, $\boldsymbol{\sigma}'_i = \boldsymbol{\sigma}_i$, $\boldsymbol{\tau}'_i = \boldsymbol{\tau}_i$. The unitary operator is (total mass M and center-of-mass position operator $\boldsymbol{R}$):

$$U = e^{-iM\boldsymbol{u}\cdot\boldsymbol{R}} \ , \quad \text{where} \quad M = \sum_i m_i \quad \text{and} \quad \boldsymbol{R} = \frac{1}{M}\sum_i m_i\mathbf{r}_i \ . \tag{19}$$

   This results in

$$V(1, 2) = \widetilde{V}(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}_1, \boldsymbol{\tau}_1, \boldsymbol{\sigma}_2, \boldsymbol{\tau}_2) \ . \tag{20}$$

5. Rotational invariance requires $[\boldsymbol{J}, V] = 0$, where is the *total* angular momentum, i.e., $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. There are three independent scalars from $\mathbf{r}$ and $\mathbf{p}$, namely $\mathbf{r}^2$, $\mathbf{p}^2$, and $\mathbf{r} \cdot \mathbf{p} + \mathbf{p} \cdot \mathbf{r}$. The latter must appear quadratically because of time-reversal invariance (below), and it is more convenient to use $\boldsymbol{L}^2 = (\mathbf{r} \times \mathbf{p})^2$. The only vector to combine with a linear appearance of $\boldsymbol{S}$ is $\boldsymbol{L}$ to make a spin-orbit interaction. There are only limited choices (see Okubo and Marshak, Ann. Phys. **4**, 166 (1968) for full details).

6. Parity P is space reflection, which has the effect:

$$\mathbf{r}'_i = -\mathbf{r}_i , \quad \mathbf{k}'_i = -\mathbf{k} , \quad \boldsymbol{\sigma}'_i = \boldsymbol{\sigma}_i , \quad \boldsymbol{\tau}'_i = \boldsymbol{\tau}_i . \tag{21}$$

If we perform two space reflections we are back where we started, so $P^2 = 1$ and the eigenvalues are ± 1. Parity is conserved by the strong interaction but violated by the weak interactions; however, the parity-violating matrix elements are small (of order $\sim 0.1\,\text{eV}$). This violation was first observed in the beta decay of polarized ^{60}Co by Wu et al. in 1957, where the polarization of the nuclear spin defines a direction and a preferential emission of electrons in the opposite direction of the spin was observed.

7. Time reversal T has the effect:

$$\mathbf{r}'_i = \mathbf{r}_i , \quad \mathbf{k}'_i = -\mathbf{k} , \quad \boldsymbol{\sigma}'_i = -\boldsymbol{\sigma}_i , \quad \boldsymbol{\tau}'_i = \boldsymbol{\tau}_i . \tag{22}$$

T is violated in the standard model [from indirect CP violation in K^0 decay]. C exchanges particles and antiparticles. There are active direct searches for T violation by looking for permanent dipole moments in neutrons, nuclei, and atoms.

8. Baryon and lepton number conservation. B violated by weak interactions. L would be violated if neutrinos are Majorana, which means they are their own antiparticles.

9. Isospin charge symmetry. $p \leftrightarrow n$ and
   Charge independence. Scattering length for $^1\text{S}_0$:

$$a_{nn} \approx (a_{pp} - \text{Coulomb}) \approx -18\,\text{fm} \tag{23}$$

which is isospin charge symmetry. But $a_{np} \approx -23.7\,\text{fm}$, which shows that charge independence breaking is stronger. Both are broken by Coulomb and other e/m plus $m_u \neq m_d$ effects.

In the end, the constraints lead to

$$V_{NN} = V_1(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2) + V_\tau(\mathbf{r}, \mathbf{p}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2)) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tag{24}$$

where we have explicitly applied translational and rotational invariance to write these in terms of $\mathbf{r}$ and $\mathbf{p}$ (remember that these are operators). We further classify V_1 and V_τ into central (scalar), vector (spin-orbit), and tensor, with spin structures of rank 0,1,2:

- **central parts:**
$$V_1(\mathbf{r}, \mathbf{p}) + V_\sigma(\mathbf{r}, \mathbf{p}) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 , \tag{25}$$

- **vector parts:**
$$V_{LS}(\mathbf{r}, \mathbf{p}) \boldsymbol{L} \cdot S , \tag{26}$$

- **tensor parts:**

$$V_T(\mathbf{r}, \mathbf{p}) S_{12}(\widehat{\mathbf{r}}) \tag{27}$$

with tensor operator

$$S_{12}(\widehat{\mathbf{r}}) \equiv \widehat{\mathbf{r}} \cdot \boldsymbol{\sigma_1} \widehat{\mathbf{r}} \cdot \boldsymbol{\sigma_2} - \frac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \tag{28}$$

[*Quick question: Why do we subtract off the 2nd term?*]

The full operator form in coordinate space is:

$$\left\{ 1_{\text{spin}},\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,\, S_{12}(\widehat{\mathbf{r}}),\, S_{12}(\widehat{\mathbf{p}}),\, L \cdot S,\, (L \cdot S)^2 \right\} \times \left\{ \mathbf{1}_{\text{isospin}},\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right\}\ , \tag{29}$$

times scalar operator-like functions of r^2, p^2, and L^2 (rather than $\mathbf{r} \cdot \mathbf{p}$). In momentum space, the full operator form is:

$$\left\{ 1_{\text{spin}},\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,\, S_{12}(\widehat{\mathbf{q}}),\, S_{12}(\widehat{\mathbf{k}}),\, i\mathbf{S} \cdot (\mathbf{q} \times \mathbf{k}),\, \boldsymbol{\sigma_1} \cdot (\mathbf{q} \times \mathbf{k})\, \boldsymbol{\sigma_2} \cdot (\mathbf{q} \times \mathbf{k}) \right\} \times \left\{ \mathbf{1}_{\text{isospin}},\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right\}\ , \tag{30}$$

where $\mathbf{q} \equiv \mathbf{p}' - \mathbf{p}$ and $\mathbf{k} \equiv (\mathbf{p}' + \mathbf{p})/2$, times scalar functions of p^2, p'^2, and $\mathbf{p} \cdot \mathbf{p}'$.

b.2 Aside: Henley and Miller's classification of isospin structure

For completeness, here we consider charge independence, charge symmetry breaking and the like. We will come back to these later. In the following, the α_i and β_i are position-spin operators. There are four classes, all but class I has isospin breaking.

- Suppose we have isospin invariance. This falls under Class I:

$$V_I = \alpha_I + \beta_I \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\ . \tag{31}$$

- Now suppose we have charge symmetry but charge independence is not good (CIB or charge independence breaking). The charge symmetry transformation is

$$P_{\text{CS}} = e^{i\pi T_2} \quad \text{where} \quad \boldsymbol{T} = \frac{1}{2}(\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)\ . \tag{32}$$

This class of potential has the form

$$V_{II} = \alpha_{II} \tau_{1z} \tau_{2z}\ . \tag{33}$$

- If charge symmetry is broken but there is no NN isospin mixing we have Class III:

$$V_{III} = \alpha_{III}(\tau_{1z} + \tau_{2z})\ . \tag{34}$$

With this force there are no transitions between $T = 0$ and $T = 1$ two-nucleon states.

- Class IV takes the form:

$$V_{IV} = \alpha_{IV}(\tau_1^3 - \tau_2^3) + \beta_{IV}[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3\ , \tag{35}$$

where β_{IV} is a time-reversal odd transformation operator. This class of forces breaks charge symmetry and does cause isospin mixing in the NN system.

b.3 Accidental symmetries

The Wigner symmetry is an approximate $SU(2) \times SU(2) = SO(4)$ symmetry reflected in the fact that the $^1S_0$ and $^3S_1$ scattering lengths are both large. The idea is that we can rotate both spin and isospin and mix them up like a 4-component object rather than two separate 2-component transformations.

c. Laundry list of interactions on the market

There are three classes of potentials consistent with the symmetry restrictions we just discussed that are still actively used for nuclear structure calculations. (Many others introduced over the years have been largely abandoned.)

- *i)* High precision phenomenological NN potentials. These include meson exchange potentials such as CD-Bonn (see http://nn-online.org/code/nn/ for fortran codes for the Nijmegen potentials), the AV18 potential, and inverse scattering potentials.
- *ii)* Chiral EFT NN potentials. We will consider these in detail later this week and next week.
- *iii)* "Toy" NN potentials.

We'll briefly consider here some examples of the first and third.

c.1 Boson exchange

Boson (or meson) exchange models are formulated in terms of simple exchanges of mesons corresponding to the allowed Lorentz character of the exchanges. The masses are those of real mesons, but generally the couplings are free parameters. There is also a high-momentum cutoff with each exchange. We can think of generating a general nonrelativistic interaction by considering all possible exchanges of bosons with different spins and isospins. In the non-relativistic limit, this becomes a sum of Yukawa potentials, e.g.,

$$\mathcal{V}(1,2) = \left(\frac{-g_s^2}{4\pi}\right) \frac{e^{-m_s r_{12}}}{r_{12}} + \gamma_1^\mu \gamma_{2\mu} \left(\frac{g_\omega^2}{4\pi}\right) \frac{e^{-m_\omega r_{12}}}{r_{12}} + \gamma_1^5 \gamma_2^5 \, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\frac{g_\pi^2}{4\pi}\right) \frac{e^{-m_\pi r_{12}}}{r_{12}} + \cdots, \quad (36)$$

where $r_{12} \equiv |\mathbf{r}_1 - \mathbf{r}_2|$. But this is still meant to be evaluated between Dirac four-component spinors. If one makes a non-relativistic reduction of the simple covariant form of scalars and pseudovectors, etc. using an on-shell Dirac spinor U:

$$U(\mathbf{p}) \propto \begin{pmatrix} \chi \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E_p + m} \chi \end{pmatrix} \quad (37)$$

with χ a two-component spinor, you find many terms, including spin-orbit and tensor terms. We'll come back to this explicitly when we discuss one-pion exchange in two more days.

c.2 Argonne V18

The Argonne v_{18} potential is one of the most widely used phenomenological NN potentials. It was designed specifically for use in quantum Monte Carlo applications, which needed an operator basis (as opposed to a table of matrix elements in a basis, such as partial wave momentum or harmonic oscillator), with each operator multiplied by a radial function. It consists of $V_{\text{EM}} + V_\pi + V_{\text{short range}}$, with all functions cut off at small r. We'll save V_{EM} for the next lecture and focus on the strong-interaction part. We've already seen the central part in the $^1S_0$ channel in Fig. 8. The longest ranged part is one-pion exchange (which we'll see in almost every potential),

$$V_\pi(\mathbf{r}) \propto f^2(\boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2}) \left[(3\boldsymbol{\sigma_1} \cdot \widehat{\mathbf{r}}\, \boldsymbol{\sigma_2} \cdot \widehat{\mathbf{r}} - \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2})(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}) + \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \right] \frac{e^{-m_\pi r}}{r} \,, \qquad (38)$$

where $f^2 = 0.075$. In practice the expression includes also terms reflecting the difference in neutral and charged pion masses (called charge-dependent or CD terms). The one-pion exchange potential is multiplied by short-range cutoff functions.

The operator structure of the first 14 terms in the shorter-range potential (and also V_π) has central, spin-spin, non-central tensor and spin-orbit parts:

$$\{1, \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}, S_{12}, \mathbf{L} \cdot \mathbf{S}, \mathbf{L}^2, \mathbf{L}^2 \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}, (\mathbf{L} \cdot \mathbf{S})^2\} \otimes \{1, \boldsymbol{\tau_1} \cdot \boldsymbol{\tau_2}\} \,. \qquad (39)$$

There are intermediate-range (two-pion-exchange range) and short-range parts, with 40 parameters to be fit. These are also multiplied by short-range cutoff functions. The fit was to the Nijmegen NN scattering data base from 0 to 350 MeV and the deuteron binding energy, with a χ^2/data point close to 1.

Some additional comments:

- See Phys. Rev. C **51**, 38 (1995) for the full details.

- The cutoff function for the OPE part is a Gaussian:

$$1 - e^{-cr^2} \qquad (40)$$

  with $c = 2.1 \, \text{fm}^{-2}$ and this function appears squared for the tensor part. The cutoff function for the shorter-range parts is a Woods-Saxon form:

$$W(r) = \left[1 + e^{(r-r_0)/a} \right]^{-1} \qquad (41)$$

  with $r_0 = 0.5 \, \text{fm}$ and $a = 0.2 \, \text{fm}$. Attempts to make the cutoff "softer" apparently resulted in poorer fits.

- AV18 contains a two-pion *range* piece, but it is not chirally symmetric.

- The last four of the terms in AV18 are CD and CSB terms, which are small but needed to get χ^2/dof ≈ 1 for np, nn, *and* pp scattering. The potential fit without these terms is called AV14.

- The nonrelativistic kinetic energy used with the AV18 potential takes the form (for the ith nucleon):

$$K_i = K_i^{CI} + K_i^{CSB} \equiv -\frac{\hbar^2}{4}\left(\frac{1}{m_p} + \frac{1}{m_n}\right)\nabla_i^2 - \frac{\hbar^2}{4}\left(\frac{1}{m_p} - \frac{1}{m_n}\right)\tau_{zi}\nabla_i^2 ,\qquad (42)$$

  which has charge-independent (CI) and charge-symmetry-breaking (CSB) components (here τ_i is the isospin of nucleon i. The CSB part arises because of different proton and neutron masses.

- You may hear about AV8, AV8', AV6 and so on. These use more limited sets of operators. They are used for tests or to evaluate the importance of different components. AV8 has the first 8 terms in Eq. (39), which are most important to reproduce the S and P phase shifts. AV8' omits some electromagnetic terms. AV6' deletes the spin-orbit terms (and adjusts the potential to preserve the deuteron binding energy).
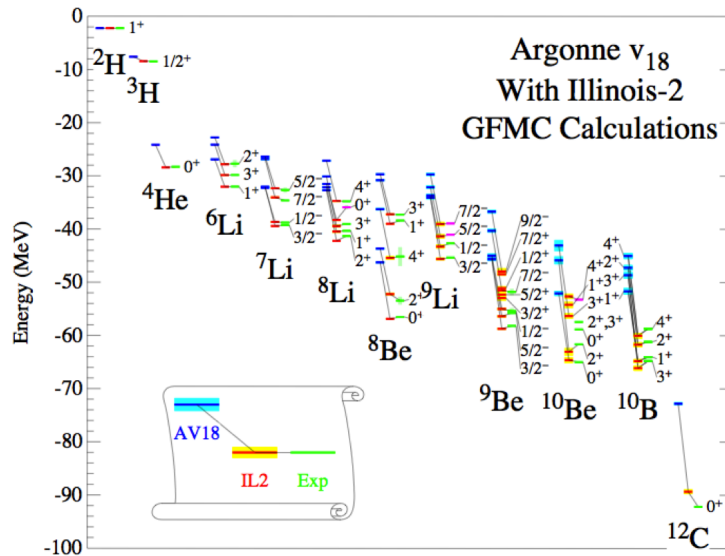
Figure 9: Energy spectra in light nuclei with AV18 compared to experiment (and an added three-body force). Note: Steve Pieper and Bob Wiringa won the APS Bonner prize for this and related work. We'll come back to this later; the point now is that three-body forces are needed for a quantitative (and in some cases qualitative) reproduction of the spectra.

Figure 9 is a preview that a two-body NN interaction such as AV18, although near perfect for NN phase shifts, will be inadequate for precision (say 1%) calculations of energies and level splittings in light nuclei (but how well does JISP16 do?). This implies three-nucleon forces are needed.

d. Structure of a local three-nucleon force

We will have much more to say about three-nucleon forces, but for now we just note the most general structure for a local three-nucleon force as it appeared recently in a paper by Krebs, Gasparyan,

and Epelbaum (arXiv:1302.2872). The building blocks are $\boldsymbol{\tau}_1$, $\boldsymbol{\tau}_2$, $\boldsymbol{\tau}_3$, $\boldsymbol{\sigma}_1$, $\boldsymbol{\sigma}_2$, $\boldsymbol{\sigma}_3$, $\mathbf{r}_{12}$, and $\mathbf{r}_{23}$. The constraints are locality, isospin symmetry, parity and time-reversal invariance. This leads to 22 structures (plus permutations) of the form

me. This equation has the form of Eq. (5.39) with

$$\sum_{i=1}^{22} \mathcal{G}_i F_i(r_{12}, r_{23}, r_{31}) , \tag{43}$$

$$^{-1}(N_i) + \sum_{j,k=1}^{2} \frac{1}{18}\mathcal{D}_{jk}(P)P^{-1}(L_{jk}^i) \Bigg\} \text{ where a few of the } \mathcal{G}\text{s are} \tag{5.42}$$

$$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3 \tag{44}$$

$$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \tag{45}$$

LONG-RANGE TAIL OF THE 3NF $\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \boldsymbol{\sigma}_1 \cdot \widehat{\mathbf{r}}_{23} \, \boldsymbol{\sigma}_2 \cdot \widehat{\mathbf{r}}_{23} \, \boldsymbol{\sigma}_3 \cdot (\widehat{\mathbf{r}}_{12} \times \widehat{\mathbf{r}}_{23})$ (46)

The F_i functions are derivable in chiral perturbation theory, with the long-range terms being
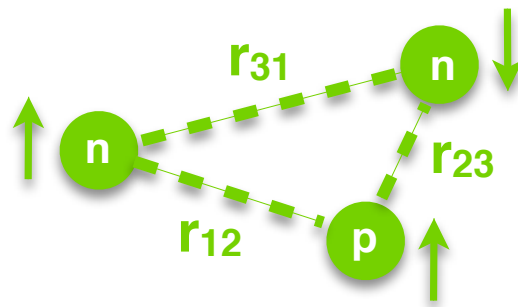ess the convergence of the feel predictions for the long-
e previous section can also be applied to operators in
ving basis of 22 operators:

Figure 10: Three-body force coordinates.

$$\boldsymbol{\tau}_1, \ \boldsymbol{\tau}_2, \ \boldsymbol{\tau}_3, \ \vec{\sigma}_1, \ \vec{\sigma}_2, \ \vec{\sigma}_3, \ \vec{r}_{12}, \ \vec{r}_{23}, \ \vec{r}_{31} \tag{1}$$

d.1 Inverse scattering potentials

$$V_{2N}^{(0)} \equiv -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2, \tag{2}$$

These potentials are derived directly from the scattering data and possible other input. The JISP potentials are most extensively used, particularly in combination with the no-core shell model (NCSM). It is a two-body potential based on a separable expansion that uses additional few-body experimental data (binding energies) to fix the two-body off-shell behavior. As a result, it does a good job of reproducing energies of light nuclei without a three-body force. See Ref. ?? for more

$$T(\vec{p}', \vec{p}) = V_{2N}^{(0)}(\vec{p}', \vec{p}) + \int \frac{d^3k}{(2\pi)^3} V_{2N}^{(0)}(\vec{p}', \vec{k}) \frac{m_N}{p^2 - k^2 + i\epsilon} T(\vec{k}, \vec{p}) \tag{3}$$

details and examples of how well it works. We'll discuss the implications of this success further next week.

$$V_{3N} = \sum_{i=1}^{22} \mathcal{G}_i \, F_i(r_{12}, r_{23}, r_{31}) \ + \ 5 \text{ perm.} \tag{1}$$

d.2 Toy potentials

$$T(\vec{p}', \vec{p}) = V_{2N}^{(0)}(\vec{p}', \vec{p}) + \frac{m_N^2}{2}\int \frac{d^3k}{(2\pi)^3} \frac{V_{2N}^{(0)}(\vec{p}', \vec{k}) \, T(\vec{k}, \vec{p})}{(k^2 + m_N^2)(E - \sqrt{k^2 + m_N^2} + i\epsilon)}, \tag{4}$$

$$\boldsymbol{\tau}_1, \ \boldsymbol{\tau}_2, \ \boldsymbol{\tau}_3, \ \vec{\sigma}_1, \ \vec{\sigma}_2, \ \vec{\sigma}_3, \ \vec{r}_{12}, \ \vec{r}_{23}, \ \vec{r}_{31} \tag{2}$$

By "toy" we mean model potentials that are not high precision and lack the full spin and isospin dependence we know is need for fully quantitative calculations of nuclei. They are useful for test calculations. Some of these are

$$V_{2N}^{(0)} = -\left(\frac{g_A}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2}\right)^2 + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2, \tag{3}$$

nucleon i with respect to nucleon j. The 3NF is a linear
y scalar functions $\mathcal{F}_i(r_{12}, r_{23}, r_{31})$. These functions have
tial energy between three static nucleons projected onto
contributions from the long-range and intermediate-
n terms of the chiral expansion. In order to explore the

$$T(\vec{p}', \vec{p}) = V_{2N}^{(0)}(\vec{p}', \vec{p}) + \int \frac{d^3k}{(2\pi)^3} V_{2N}^{(0)}(\vec{p}', \vec{k}) \frac{m_N}{p^2 - k^2 + i\epsilon} T(\vec{k}, \vec{p}) \tag{4}$$

$(7.6 \pm 3.1) \times 10^{-3} M_\pi^{-3}$

- The Minnesota model is a purely central potential that is a sum of three Gaussian terms of different ranges and strengths, adjusted to roughly reproduced some specific properties of (for example) the alpha particle. An example of recent uses is for comparative studies of neutron drops calculated with various ab initio methods *and* with a microscopic density functional approach (we'll talk about this later).

- The Malfliet-Tjon potential is another purely central potential that is the sum of two Yukawa potential. Various parameter sets exist that reproduce properties such as the binding energy of the deuteron.

- It is also sometimes useful to consider *one-dimensional* potentials for proof-of-principle calculation. An example is the sum of two Gaussians introduced by Negele et al. with scaled parameters chosen to mimic features of three-dimensional nuclear matter.

e. Motivation and background for EFT

e.1 Problems with phenomenological potentials

The best potential models (such as AV18) can describe with $\chi^2/\text{dof} \approx 1$ all of the NN data (about 6000 points) below the pion production threshold. [What energy is this? Exercise!] So what more do we need? Some limitations of these potentials:

- They usually have a very strong repulsive short-range part that requires special (non-systematic) treatment in (some types of) many-body calculations of nuclear structure.

- It is difficult to estimate the theoretical error in a calculation and the range of applicability (i.e., where should it fail?).

- Three-nucleon forces (3NF) are largely included as under-constrained and non-systematic models. How can we define *consistent* 3NF's and operators (e.g., including-meson exchange currents)?

- Models are largely unconnected to QCD (e.g., chiral symmetry is only respected in part). They don't connect NN and other strongly interacting processes (e.g., $\pi\pi$ and πN). Lattice QCD will be able to predict NN, 3N observables for high pion masses. How can we extrapolate to physical pion masses?

To address these limitations, we seek a systematic alternative: effective (field) theory. We'll finish this lecture with an introduction to the philosophy and principles.

e.2 Effective theories: Appeal to authority :)

A recommended (though in some respects dated) reference is H. Georgi, Ann. Rev. Nucl. Part. Sci. **43**, 209 (1993). [This reference also includes an interesting discussion of how dimensional

regularization modifies ultraviolet physics, which is obvious for the more physical case of an energy/momentum cutoff but not for dimensional regularization.] Some relevant quotes about effective theories:

- *One of the most astonishing things about the world in which we live is that there seems to be interesting physics at all scales.*

- *To do physics amid this remarkable richness, it is convenient to be able to isolate a set of phenomena from all the rest, so that we can describe it without having to understand everything. Fortunately, this is often possible. We can divide up the parameter space of the world into different regions, in each of which there is a different appropriate description of the important physics. Such an appropriate description of the important physics is an "effective theory."*

- *The common idea is that if there are parameters that are very large or very small compared to the physical quantities (with the same dimension) that we are interested in, we may get a simpler approximate description of the physics by setting the small parameters to zero and the large parameters to infinity. Then the finite effects of the parameters can be included as small perturbations about this simple approximate starting point.*

Any time there is a hierarchy of (separated) energy scales, think EFT! What do we mean when we say that we exploit the "separation of scales"? Fast and slow, Born-Oppenheimer. Move to Effective theories section.

Examples from our physics experiences and this program of such effective theories (you give more!):

- non-relativistic quantum (or classical) mechanics: $c \to \infty$;

- size of a charge distribution $\to 0$ (multipole expansion);

- mass of the proton in a hydrogen atom $\to \infty$;

- asphericity of a cow $\to 0$;

- number of colors in QCD $\to \infty$;

- chiral effective *field* theory (EFT): $m_\pi \to 0$, $M_N \to \infty$.

You add some!

e.3 Principles of low-energy effective theories

Summary of the basic principles:

- If system is probed at low energies, fine details are not resolved.

- So we can use low-energy variables for low-energy processes (as they can be easier, more efficient, ...).
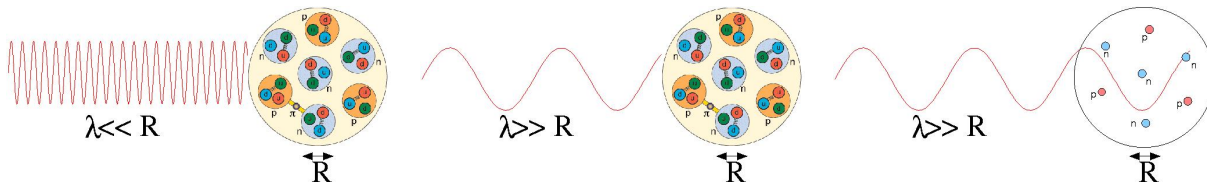
Figure 11: Left: high-resolution, with wavelength of probe short compared to characteristic size of probed structure. Middle: low-resolution probe doesn't resolve the details (because of diffracton). Right: low-energy theory takes advantage and replaces short-distance with long-distance degrees of freedom.

- The short-distance structure can be *replaced* by something simpler (and wrong at short distances!) without distorting low-energy observables. It is important that being wrong at short distances ("incorrect UV behavior") doesn't matter but also cautions us that when it works for long distances, we should not conclude that we know about the UV.

- This is systematically achieved by the effective field theory.

There are some basic physics principles underlying any low-energy effective model or theory. A high-energy, short-wavelength probes sees details. E.g., electron scattering at Jefferson Lab resolves the quark substructure of protons and neutrons in a nucleus. But at lower energies, details are not resolved, and one can replace short distance structure, as in a multipole expansion of a complicated charge or current distribution. So it is not necessary to do full QCD. It is not obvious that this will work in quantum mechanics as it does for pixels or point dots or the classical multipole expansion, because *virtual* states can have high energies that are not, in reality, simple. Renormalization theory says it can be done! (More tomorrow!) It doesn't say that we are *insensitive* to all short-distance details, only that there effects at low energies can be accounted for in a simple way. Effective *field* theory is a systematic approach to carrying out this program using a local Lagrangian framework.