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m3b-1

m3b: many-body perturbation theory

Plan for m3b with goal of making literature accessible (not becoming expert in many-body theory — that's in other courses)

(1) Recap of ab initio many-body methods with emphasis on how forces enter as inputs
- Go through F2 a 5-7 [after looking at some slides] ← you make questions as consumer!
- Distinguish two classes of methods wrt inputs
  - (A) Those that use free-space fit Hamiltonians as a given and then does the best possible many-body calculation. So an EFT expansion on input but also approximation errors. (inputs like Entem/Machleidt or EGM potentials)
  - (B) Those methods that build forces as part of the many-body approach.

(2) Revisit natural pionless EFT at finite density to see how perturbation theory in the medium works.
- diagrams and Feynman rules
- renormalization in free space renormalizes at finite density
- power counting and perturbative expansion → low density expansion

why it is a different story with modern forces!

(3) Traditional G-matrix and hole-line expansion MBPT
- brief discussion of diagrams — Goldstone vs. Feynman
- power counting with hard and soft interaction ⇒ take away: very different
- pictures comparing G-matrix and T-matrix
  - change resolution in free space ⇒ MBPT can work, DFT feasible

shades with lessers ⇒

look at a few slides first

F2c-5

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Classification of many-body methods

"ab initio" → starting from microscopic 2+3-body forces

"configuration interaction" → traditional shell model → diagonalize a semi-empirical Hamiltonian within a restricted space. New: MBPT with low-momentum interaction (Achim)
(also refers to NCSM!)

"density functional theory" (DFT) ⇒ Skyrme, Gogny, relativistic mean field energy density functionals (EDF)

These lectures: How developments in nuclear forces impact calculations in each of these categories.

* • ab initio now part of all three categories

Friday → • put EDF's in more general context

Laundry list of ab initio methods:

in operator form, eg. $V_6(r)\,\vec{\sigma}_1\cdot\vec{\sigma}_2$ ⇒ AIS invented for this

① continuum Monte Carlo (QMC)

- GFMC; A=1-12, many observables, needs local potential (AV18+3NF)
- AFDMC (Alex Gi's lectures); established for neutron matter, nuclear matter and larger nuclei? (sample spaceand spin/isospin), new N²LO local potential ⇒ chiral EFT with QMC?

② configuration interaction: matrix diagonalization in a basis (often HO) (separable center-of-mass)

- NCSM: no core shell model → all nucleons active ("no core")
- NCFC: no core full configuration → see Maris, Vary for definitions
- * needs soft interaction for adequate convergence
  - chiral EFT → Lee Suzuki unitary transformation or SRG/Vlowk
  - JISP potentials (some of separable terms — use off-shell freedom to reduce 3NF contributions)

soft ← inverse scattering potential

inputs are harmonic oscillator matrix elements of NN interaction

F2a-6

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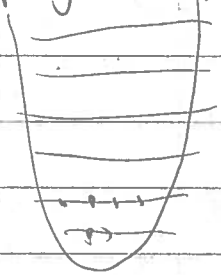
- limited by matrix size ⇒ grows very rapidly with A, N_{max}

^{4}He $N_{max}=4 \Rightarrow 10^3$ dimension $(10^3 \times 10^3)$
$N_{max}=8 \Rightarrow 4\times10^4$
$N_{max}=14 \Rightarrow \sim 3\times10^6$



^{12}C $N_{max}=4 \Rightarrow 10^6$
$N_{max}=8 \Rightarrow 7\times10^8$

- Feasible to do several billion size matrix (i.e. $10^9 \times 10^9$)
- How is this possible at all? Sparse matrices ⇒ mostly 0's.
  - Use Lanczos method:

    If $H|\psi_k\rangle = E_k|\psi_k\rangle$, then $H^m|\psi\rangle = \sum_k C_k E_k^m |\psi_k\rangle$ ($|\psi\rangle$ arbitrary)

    ⇒ largest eigenvalue $|E_k|$ in magnitude dominates
  - Use $(H-\sigma I)^m$ with $\sigma>0$ large so $|E_0-\sigma|$ is largest
    ⇒ project out lowest eigenvalues and vectors. Maybe 20 at a billions!

(Iterate m m orthogonalizing as you go)

How can we push higher?

- Importance Truncated NCSM ⇒ don't use all the basis states
  - determine relevant basis states before diagonalizing using first-order perturbation theory.
  - reference state $|\psi_{ref}\rangle$ (from small N_{max} diagonalization), calculate

$$x_\nu = \frac{-\langle \Phi_\nu | H_{int} | \psi_{ref}\rangle}{E_\nu - E_{ref}}$$ for all $|\Phi_\nu\rangle$ basis states

⇒ keep for $|x_\nu| \geq x_{min}$

* • Perturbation theory must be reliable ⇒ enabled by soft interactions
  - R. Roth et al. using SRG evolved interactions

- CI applies to any nucleus, any observable (need operators!)

E2a-7

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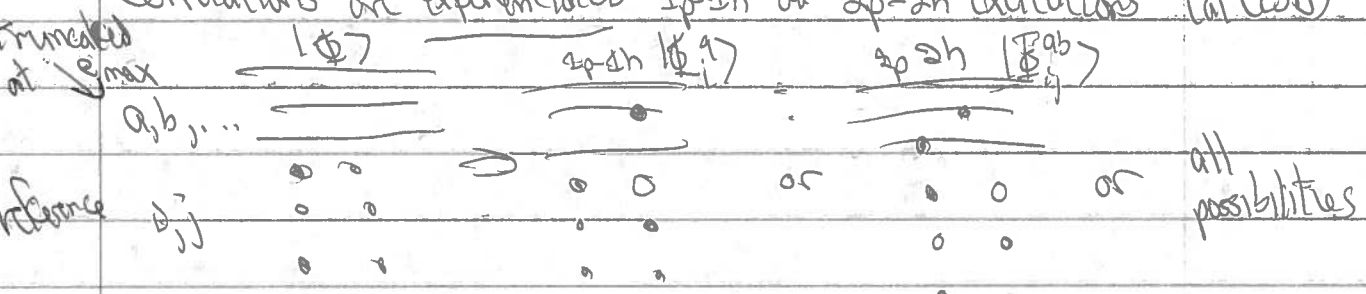
③ Coupled Cluster – efficient, resummation of many-body PT (infinite)

wf ansatz: $|\Psi\rangle = e^{T}|\Phi\rangle$ ← reference state

for: $T = T_1 + T_2 + \ldots$ ⇐ defines the truncation (eg. CCSD)

$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$, $T_2 = \sum_{ijab} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i$, ...

Correlations are exponentiated 1p-1h and 2p-2h excitations (at CCSD)



* determine t_i^a, t_{ij}^{ab}, ... coefficients by requiring a similarity transformed Hamiltonian to decouple 1p-1h, 2p-2h

$\bar{H} = e^{T} H e^{T} \Rightarrow E = \langle\Phi|\bar{H}|\Phi\rangle$

$0 = \langle\Phi_i^a|\bar{H}|\Phi\rangle$ } CCSD

$0 = \langle\Phi_{ij}^{ab}|\bar{H}|\Phi\rangle$ }

- "triples" means $a^\dagger a^\dagger a^\dagger aaa$
- good scaling properties with A, size of space
- many on-going developments → ORNL/UT and Darmstadt groups
- uses chiral EFT forces as input (soft)
  - SRG would work well, but more development
  - in oscillator matrix element form

inputs ⇒

④ Self-consistent Green's function } recent developments using soft interactions ⇒ see INT program talks for March

- Solve Dyson's equations non-perturbatively
- diagrammatic resummation (self-energy + 3,3 body)

⑤ In-medium SRG ⇒ SRG evolution with a reference state (cf. CC)

⑥ Lattice EFT

⑦ Other: MBPT (Manbuy), shell model EFT ← fit EFT interaction within shell model

④ and ⑤ ⇒ take Hamiltonian inputs

See slides

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Comments on
⑥ lattice EFT
⑦ Shell model EFT

⑥ lattice EFT

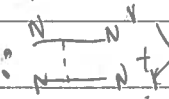
- new many-body method
- discretized space time $V = L_s^3 \times L_t$ (space direction, time direction)
  - nucleons are point-like fields at lattice sites
  - discrete version of chiral potential
    - pion exchanges
    - contact interaction

(doesn't take chiral EFT potentials as input like ⑤–⑤)

- lattice cutoff is $\Lambda = \frac{\pi}{a}$ and a most typically about 2fm
  $\Rightarrow \Lambda = \frac{3}{2\,\text{fm}} \cdot 200\ \text{MeV-fm} \simeq 300\ \text{MeV}$.
  $\Rightarrow$ a very soft potential $\Rightarrow$ is $\frac{Q}{\Lambda}$ expansion adequate?
- Expection value of operators

Slater determinant of A nucleons (alpha clusters or shell model wavefunctions)

$$\langle \Psi_A | \mathcal{O} | \Psi_A \rangle = \lim_{\text{large } t} \frac{\langle \Psi_A | e^{-Ht/2} \mathcal{O} e^{-Ht/2} | \Psi_A \rangle}{\langle \Psi_A | e^{-Ht} | \Psi_A \rangle}$$

- H is not full $H \Rightarrow$ leading order LO: 
- higher order potentials included in $\mathcal{O}$ perturbatively: (eg, TPE)
- path integral is over pion field and auxiliary fields
- Why not the nucleons? As fermions these are Grassman fields. Do this integral if Gaussian. But $\mathcal{X}$ is $(N^\dagger N)^2$, which is not Gaussian.

nucleon propagator in pion and auxiliary background fields

$\Rightarrow$ Write $\exp\left(-\frac{C}{2}(N^\dagger N)^2\right) = \int_{-\infty}^{\infty} ds\, e^{-\frac{1}{2}s^2 + \sqrt{C} s (N^\dagger N)}$ ← now quadratic in N → path integral over s

$\Rightarrow \mathcal{X}(N^\dagger N)^2 \rightarrow$ ⟩ $sN^\dagger N$ where s is a background field

N auxiliary fields
$sN^\dagger N$, $\vec{i}s_s \cdot N^\dagger \vec{\sigma} N$, ...

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use ^{3}H binding and weak axial current for c_D, c_E

- See the literature for details. How are constants fit? ← spherical wall method on lattice
- Pattern of results ($L = 11.8$ fm) in MeV

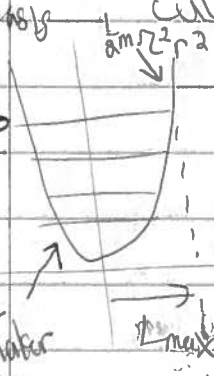
nucleus	^{4}He	^{8}Be	^{12}C	^{16}O	( ) ↑ statistical error only
LO [Q^0]	−28.0(3)	−57(2)	−96(2)	−144(4)	
NLO [Q^2]	−24.9(5)	−47(2)	−77(3)	−116(6)	
NNLO [Q^3]	−28.3(6)	−55(2)	−92(3)	−135(6)	
experiment	−28.30	−56.5	−92.2	−127.6	

- Impressive NNLO results – what questions should you ask?
  - Why this pattern LO → NLO → NNLO; it doesn't seem convergent.
    - Claim: there is a rearrangement between NLO and LO that makes this pattern.
    - But then how do we power count?
  - Where is the cutoff variation?

⇒ very promising, highly complementary approach. Look for results with more cutoffs to validate convergence.

(7) Shell model EFT – Stetcu et al (2006, 2007)

- Harmonic oscillator basis – truncated to N_{max} – provides both high momentum (UV) and long distance (IR) cutoffs. Oscillator length $b = \sqrt{\hbar/m\Omega}$ (ultraviolet, infrared ↑)



$\Delta_{UV} \approx \sqrt{2(N+3/2)}\,\hbar/b$ ⇒ regulate → fit constants in shell model

$L_{max} \approx \sqrt{2(N+3/2)}\,b$ ⇒ need to take $L_{max} \to \infty$ limit: $\hbar\Omega \to 0$ extrapolation (or $b \to \infty$)

- fit parameters within this space to some observables, then calculate observables of fitting EM or EGM potentials with a given momentum cutoff. } in most extrapolate to remove finite basis artifacts

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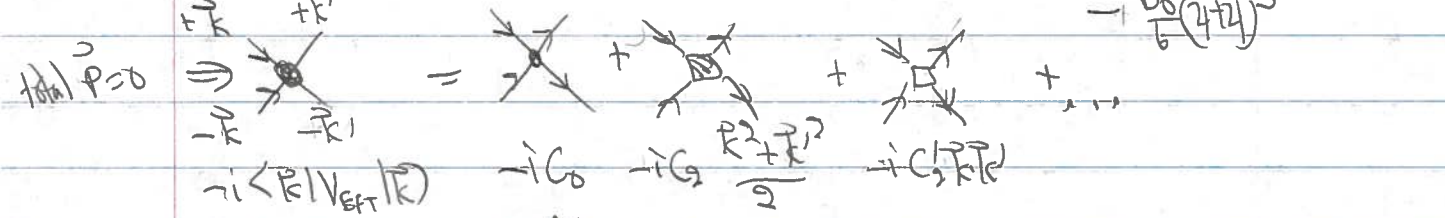
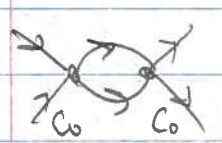
Pionless natural EFT at finite density (uniform system)

- Recall from first week scattering calculated with EFT:

$$\mathcal{L}_{EFT} = \psi^\dagger\left[i\frac{\partial}{\partial t} + \frac{\vec\nabla^2}{2m}\right]\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 + \frac{C_2}{16}\left[(\psi\psi)^\dagger(\psi\vec\nabla^2\psi) + h.c.\right] + \frac{C_2'}{8}(\psi\vec\nabla\psi)^\dagger\cdot(\psi\vec\nabla\psi) - \frac{D_0}{6}(\psi^\dagger\psi)^3$$



$-i\langle\vec k'|V_{EFT}|\vec k\rangle$



$$\Rightarrow \int^{\Lambda_c}\frac{d^3q}{(2\pi)^3}\frac{1}{k^2-q^2+i\varepsilon} \to \frac{\Lambda_c}{2\pi^2} - \frac{ik}{4\pi} + O\left(\frac{k^2}{\Lambda_c}\right)$$ with cutoff Λ_c

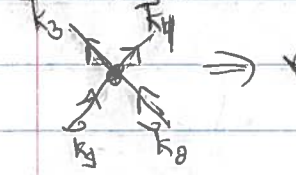
changes in Λ_c absorbed into $C_0^{(4)}(\Lambda_c)$

or $$\Rightarrow \int\frac{d^Dq}{(2\pi)^3}\frac{1}{k^2-q^2+i\varepsilon} \xrightarrow{D\to 3} -\frac{ik}{4\pi}$$ with dimensional regularization and MS

- So now apply at $T=0$ to a system of N fermions in a box of volume $V=L^3$ with spin-isospin degeneracy ν.
  - Sum over the Fermi sea to find the non-interacting density and energy density

$$\frac{N}{V} = \frac{\nu}{V}\sum_{\vec k}^{k_F} 1 = \frac{\nu}{V}\frac{V}{(2\pi)^3}\int^{k_F} d^3k = \frac{\nu k_F^3}{6\pi^2} \equiv \rho, \quad \frac{E^{(0)}}{V} = \frac{\nu}{V}\sum_{\vec k}^{k_F}\frac{k^2}{2m} = \frac{3}{5}\frac{k_F^2}{2m}\rho$$ [exercise]

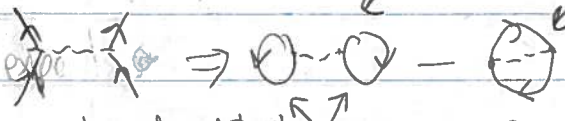
- Find the interacting energy by summing over the Fermi sea



$$\Rightarrow \mathcal{E}_{LO} = \frac{C_0}{2}\nu(\nu-1)\left(\sum_{\vec k}^{k_F}1\right)^2 = \frac{C_0}{2}\left(1-\frac{1}{\nu}\right)\rho^2$$

Hartree, Fock; trace of spin-isospin on fermion lines

Resolve ψ_i exp



$\Rightarrow$ one sum over ν with minus sign $\Rightarrow \propto\nu$

two independent spin sums over $\nu \Rightarrow \nu^2$

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Aside: Standard Hartree-Fock discussion for local $V(\vec{x},\vec{y}) \to V(\vec{x}-\vec{y})$

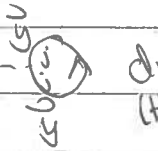
(not relative coordinates here)

- Best Slater determinant in a variational sense

$|\Psi_{HF}\rangle = \det\{\phi_i(\vec{x}), i=1\cdots A\}$ $\quad \vec{x}=(\vec{r},\sigma,\tau)$

$$\langle \Psi_{HF}|\hat{H}|\Psi_{HF}\rangle = \sum_{i=1}^{A}\frac{1}{2m}\int d\vec{x}\,\nabla\phi_i^*\cdot\nabla\phi_i$$

$$+\frac{1}{2}\sum_{i,j=1}^{A}\int d\vec{x}\int d\vec{y}\,|\phi_i(\vec{x})|^2 V(\vec{x},\vec{y})|\phi_j(\vec{y})|^2$$



direct (Hartree)

(local potential)

$$-\frac{1}{2}\sum_{i,j=1}^{A}\int d\vec{x}\int d\vec{y}\;\phi_i^*(\vec{x})\phi_i(\vec{y})V(\vec{x},\vec{y})\phi_j^*(\vec{y})\phi_j(\vec{x})$$

exchange (Fock)

- Determine the ϕ_i by varying with fixed normalization

$$\frac{\delta}{\delta\phi_i^*(\vec{x})}\left(\langle\Psi_{HF}|\hat{H}|\Psi_{HF}\rangle - \sum_{j=1}^{A}\varepsilon_j\int d\vec{y}\,|\phi_j(\vec{y})|^2\right)=0$$

$\Rightarrow$ standard Hartree-Fock equation

- If $V(\vec{x},\vec{y}) = C_0\,\delta^3(\vec{x}-\vec{y})$, and $\phi_i(\vec{x}) \to \frac{1}{\sqrt{V}}e^{i\vec{k}\cdot\vec{x}}\eta_\alpha$,
  we recover our alternative results, [exercise]
  - zero range

- More generally (still with local): $\left\{-\frac{\hbar^2}{2m}\nabla^2 + \Gamma_H(\vec{x})\right\}\phi_i(\vec{x}) + \int d\vec{y}\,\Gamma(\vec{x},\vec{y})\phi_i(\vec{y}) = \varepsilon_i\phi_i(\vec{x})$

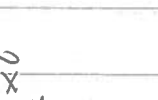
(non-local potential)

Solve self-consistently (potentials depend on ϕ_i's)

where

$$\Gamma_H(\vec{x}) = \int d\vec{y}\,V(\vec{x},\vec{y})\sum_{j=1}^{A}|\phi_j(\vec{y})|^2 = \int d\vec{y}\,V(\vec{x},\vec{y})\rho(\vec{y})$$

$$\Gamma_F(\vec{x},\vec{y}) \equiv -V(\vec{x},\vec{y})\sum_{j=1}^{A}\phi_j^*(\vec{y})\phi_j(\vec{x}) = -V(\vec{x},\vec{y})\rho(\vec{x},\vec{y})$$



- more complicated with non-local potential

volume $\Omega \equiv V$

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- the 2nd quantized form for the Hamiltonian (from Friday) is

$$\hat{H} = \sum_{\vec{k}\alpha} \frac{k^2}{2m} a^\dagger_{\vec{k}\alpha} a_{\vec{k}\alpha} + \frac{1}{2}\frac{C_0}{\Omega} \sum_{\vec{k}_1\alpha_1, \vec{k}_2\alpha_2} \sum_{\vec{k}_3\alpha_3, \vec{k}_4\alpha_4} a^\dagger_{\vec{k}_3\alpha_1} a^\dagger_{\vec{k}_4\alpha_2} a_{\vec{k}_4\alpha_4} a_{\vec{k}_3\alpha_3} \underbrace{\delta_{\alpha_1\alpha_3}}_{\text{set } \alpha_3=\alpha_1} \underbrace{\delta_{\alpha_2\alpha_4}}_{\text{set } \alpha_4=\alpha_2} \delta_{\vec{k}_1+\vec{k}_2,\, \vec{k}_3+\vec{k}_4}$$

$\Rightarrow E^{(1)} = \langle F|\hat{H}_1|F\rangle =$ direct + exchange

- we can do these contractions or switch to normal-ordering wrt $|F\rangle$ [exercise: check that you get the same result]

direct: $\langle F| a^\dagger_{\vec{k}_3\alpha_1} a^\dagger_{\vec{k}_2\alpha_2} a_{\vec{k}_4\alpha_2} a_{\vec{k}_3\alpha_1} |F\rangle \, \delta_{\vec{k}_1+\vec{k}_2,\, \vec{k}_3+\vec{k}_4}$

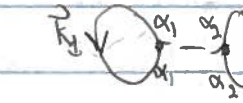
$$\theta(k_f - |\vec{k}_2|)\delta_{\vec{k}_2\vec{k}_4} \underbrace{\delta_{\alpha_2\alpha_3}}_{\nu} \delta_{\vec{k}_1\vec{k}_3} \underbrace{\delta_{\alpha_1\alpha_1}}_{\nu} \times \theta(k_f - |\vec{k}_1|)\, \delta_{\vec{k}_1+\vec{k}_2,\, \underbrace{\vec{k}_3+\vec{k}_4}_{\vec{k}_1+\vec{k}_2}}$$

↙ so no restriction

$$\Rightarrow E^{(1)}_{\text{direct}} = \frac{1}{2}\frac{C_0}{\Omega}\left(\frac{\Omega}{(2\pi)^3}\int d^3k_1\, \nu\, \theta(k_f - |\vec{k}_1|)\right)\left(\frac{\Omega}{(2\pi)^3}\int d^3k_2\, \nu\, \theta(k_f - |\vec{k}_2|)\right) = \left(\frac{1}{2}C_0\rho^2\right)\Omega$$

$\rho\Omega = A$

energy density $\Rightarrow \mathcal{E}^{(1)} = \frac{E^{(1)}_{\text{direct}}}{\Omega} = \frac{1}{2}C_0\rho^2 \propto k_f^6$



exchange $\langle F| a^\dagger_{\vec{k}_1\alpha_1} a^\dagger_{\vec{k}_2\alpha_2} a_{\vec{k}_3\alpha_2} a_{\vec{k}_4\alpha_1} |F\rangle \, \delta_{\vec{k}_1+\vec{k}_2,\, \vec{k}_3+\vec{k}_4}$

↖ 1 extra minus sign from anticommutator

$$-\theta(k_f - |\vec{k}_1|)\delta_{\vec{k}_1\vec{k}_3}\delta_{\alpha_1\alpha_2}\, \theta(k_f - |\vec{k}_2|)\delta_{\vec{k}_2\vec{k}_4}\delta_{\alpha_1\alpha_2}\, \delta_{\vec{k}_1+\vec{k}_2,\, \underbrace{\vec{k}_3+\vec{k}_4}_{\vec{k}_2+\vec{k}_4}}$$

$\sum_{\alpha_1\alpha_2} \delta_{\alpha_1\alpha_2}\delta_{\alpha_1\alpha_2} = \sum_{\alpha_1} \delta_{\alpha_1\alpha_1} = \nu$

one less power of ν ↘

$$= \left(-\frac{1}{2}\frac{1}{\nu}C_0\rho^2\right)\Omega \Rightarrow \mathcal{E}_{LO} = \mathcal{E}^{(1)}_{\text{direct}} + \mathcal{E}^{(1)}_{\text{exchange}} = \frac{C_0}{2}\left(1 - \frac{1}{\nu}\right)\rho^2$$

Why does this vanish when $\nu = 1$?

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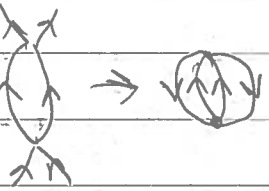
derived in grand quantization or from path integrals

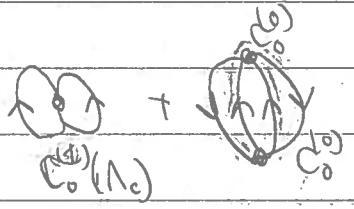
For higher order, much easier to use the Feynman rules, here for Feynman diagrams in momentum space. We'll do this in a moment; first anticipate the divergence in the next order.

linear divergence →  ← the rules will restrict the intermediate state momenta in ↕ to be above k_F while in free space there is no limit.

- But the Pauli blocking is at low k: 0 to k_F is excluded (IR physics) while the sensitivity to UV is near Λ_c.
- We can see this explicitly by writing the divergent part as

$$\int_{k_F}^{\Lambda_c} \frac{d^3q}{(2\pi)^3}\frac{-1}{q^2} = \int_0^{\Lambda_c}\frac{d^3q}{(2\pi)^3}\left(\frac{-1}{q^2}\right) - \int_0^{k_F}\frac{d^3q}{(2\pi)^3}\left(\frac{-1}{q^2}\right)$$

but this is what  cancels

$\Rightarrow$  has no linear divergence!

Moral: Finite density is IR physics, so counterterms for UV physics work as in free space, to remove sensitivity to cutoff.

$\Rightarrow$ We can do the UV renormalization in free space and then finite density is automatically renormalized.

energy/particle $\frac{E}{N} = \frac{E}{V}\cdot\frac{V}{N} = \frac{\mathcal{E}}{\rho}$ (energy/volume; energy density)

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As an example of finite-T rules, but only for uniform system

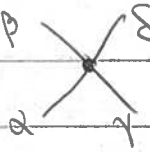
Feynman rules for energy density at $T=0$: n^{th} order of $\mathcal{E}-\mathcal{E}_0$

- Draw all distinct fully connected diagrams with n vertices.

① each line is assigned a non-relativistic four momentum $\tilde{k} \equiv (k_0, \vec{k})$ (frequency) and four-momentum is conserved at each vertex. Internal lines get the factor

$$iG_0(\tilde{k})_{\alpha\beta} = i\delta_{\alpha\beta}\left(\frac{\theta(k_f-|\vec{k}|)}{k_0-\omega_k+i\varepsilon} + \frac{\theta(|\vec{k}|-k_f)}{k_0-\omega_k-i\varepsilon}\right) \qquad \omega_k \equiv \frac{k^2}{2m}$$

② The vertex lines have spin (and isospin more generally) indices



$\rightarrow (\delta_{\alpha\delta}\delta_{\beta\gamma} + \delta_{\alpha\gamma}\delta_{\beta\delta})$ [if spin independent; otherwise there are δ's]

Get minus sign for exchange elsewhere

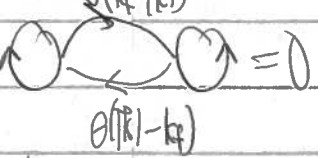
③ Do spin summations and $\delta_{\alpha\alpha} \Rightarrow -\nu$ for each closed fermion loop

④ Integrate $\int \frac{d^4k}{(2\pi)^3}$ over independent momenta $(d^4\tilde{k}_i = dk_{i0}\, d^3\vec{k}_i)$

- For $\oint_{k_0}$ assign $e^{ik_0 0^+}$ ("tadpoles")
- after frequency integral → back to time-independent results, but all time orderings.

⑤ multiply by a symmetry factor $i/S\prod_{l=0}^{l_{max}}(l!)^{n_l}$ where S is the number of vertex permutations and n is the number of equivalent l-tuples of lines.

- anomalous diagrams, with $\theta(|\vec{k}|-k_f)\,\theta(k_f-|\vec{k}|)=0$:



$= 0$ (only at $T=0$)

- Power counting rules when using dimensional regularization with MS

1. for every propagator line: $\frac{m}{k_f^2}$
2. for every loop integration: $(k_f^2/m)\,k_f^3 = k_f^5/m$
3. for every n-body vertex with $2i$ derivatives: $k_f^{2i}/m^{2i+3n-5}$

} very clean, if dimensional regularization used

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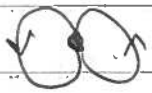
- A diagram with V_{2i}^{n} n-body vertices scales as k_F^{β} where

$$\beta = 5 + \sum_{n=2}^{\infty} \sum_{i=0}^{\infty} (3n + 2i - 5) V_{2i}^{n}$$

(V_{2i}^{n}: n-body vertex; $2i$: derivatives)

(same as in free space with E = # of external lines = 0!)

-  $\Rightarrow V_0^2 = 1 \Rightarrow \beta = 5 + (3\cdot 2 + 2\cdot 0 - 5)\cdot 1 = 6$
$\Rightarrow \mathcal{O}(k_F^6)$

-  $\Rightarrow V_0^2 = 2 \Rightarrow \beta = 5 + (3\cdot 2 + 2\cdot 0 - 5)\cdot 2 = 7 \Rightarrow \mathcal{O}(k_F^7)$

$$\mathcal{E} = \left(\rho \frac{k_F^3}{2m}\right)(\nu - 1)\frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a_0)^2$$

(k_F^5; pure geometric factor)

Exercise questions:

- Why does the formula for β ensure that k_F^{β} has always at least $\beta = 6$?
- Why does switching a vertex for one with more derivatives always increase β?
- Why does increasing the number of internal lines increase β?
- Check the claim on slide 34 of the β_F for each of the diagrams shown.

⇒ a perturbative expansion in $\frac{k_F}{\Lambda_b}$ ← not Λ_c! where $a_0, r_0 \sim \frac{1}{\Lambda_b}$

(Λ_b: breakdown scale)

- Does this formalism apply to nuclear systems?
  - dilute density: scattering length physics critical: natural or unnatural?
  - higher density: what about pions? Friday: looks like Skyrme EDF!

M3b-10

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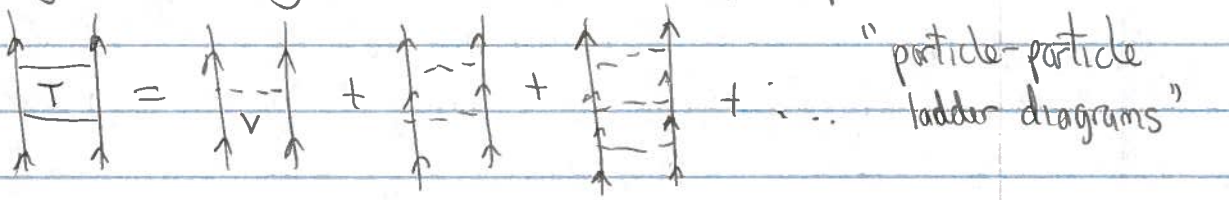
- Reminder: softened potentials at finite density
  - Weinberg eigenvalue analysis applied to the T-matrix Lippmann-Schwinger equation
    $\hat{T}(E) = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V + \ldots$
    showed increased convergence (smaller eigenvalue) when SRG λ or $V_{low\,k}$ Λ is reduced.
  - At finite density, the intermediate states are Pauli blocked $\Rightarrow$ changes the convergence even more $\Rightarrow$ perturbation theory in particle-particle ladders works.

- Diagrammatically, the Lippmann-Schwinger equation is



"particle-particle ladder diagrams"

  - That's all there is in free space. At finite density many more diagrams are possible. (Why?)
  - The question of MBPT is how to power count these diagrams:
    - What is an organizational principle that allows for systematic calculations?
    - How does the power counting depend on the resolution of the interaction?

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m3b-11

Traditional BBG power counting: G-matrix and hole-line expansion

Plan: Highlight the important features that depend on the interaction and how it changes with low-resolution interactions.

- For (much) more details, see the review by Day (RMP, 1978) on the original BBG theory and references therein and the many-body book by Negele and Orland.

- BBG → Brueckner-Bethe-Goldstone
  - Developed to deal with potentials with strongly repulsive cores
- Basics:
  - Write the Hamiltonian as $\hat{H} = \hat{H}_0 + \hat{H}_1$

    where

    $$\hat{H}_0 = \hat{T} + \hat{U} \quad \text{and} \quad \hat{H}_1 = \hat{V} - \hat{U}$$

    with $\hat{U}$ a single-particle potential to be specified.
  - Great freedom to choose $\hat{U}$
    - $\hat{H}_0|\Phi_0\rangle = E_0|\Phi_0\rangle$ is the reference state (in a finite system where $\rho(x)$)
    - In DFT, need freedom to make the density of $|\Phi_0\rangle$ the same as the full, exact density, order-by-order in an expansion (Kohn-Sham reference system has same density as exact system)
    - In conventional BBG, freedom is needed to enhance convergence, so not available for DFT

- Restatement of time-independent perturbation theory for ground-state energy E:

$$E = E_0 + \langle \Phi_0 | \hat{H}_1 \sum_{n=0}^{\infty} \left(\frac{1}{E_0 - \hat{H}_0} \hat{H}_1 \right)^n | \Phi_0 \rangle_{\text{connected}}$$

($E_0 = \langle \Phi_0 | \hat{H}_0 | \Phi_0 \rangle$)

looks like perturbation theory!

cf. Feynman diagrams
⇒ Feynman perturbation theory has time (or frequency) integrals. Do those and we get time-ordered Goldstone diagrams

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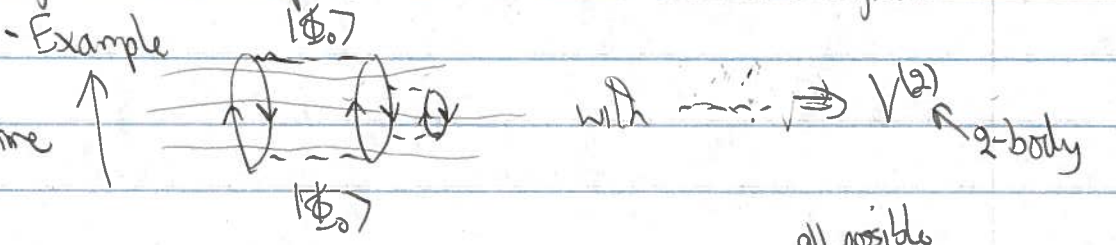
- Diagrammatic expansion called Goldstone diagrams
  - Example



- start with $|\Phi_0\rangle$ and apply $|\hat{H}_1\rangle$, which creates all possible particles and holes
  - For $V^{(2)}$, this is two particles and 2 holes
  - $\frac{1}{E_0 - \hat{H}_0}$ propagates the state ← particle lines up ↑ $\varepsilon > \varepsilon_F$, hole lines down ↓ $\varepsilon < \varepsilon_F$

schematic, see refs for details

"Connected" means $|\Phi_0\rangle$ is not an intermediate state
- ⇒ put in $\hat{P} = 1 - |\Phi_0\rangle\langle\Phi_0|$ projector

$$\Rightarrow E - E_0 = \sum_{\text{connected diagrams}} \frac{(-1)^{n_h + n_p}}{2^{n_\ell}} \prod \frac{1}{-\left(\sum_a \varepsilon_a - \sum_h \varepsilon_h\right)} \prod \langle ij|\hat{V}_{NN}|kl\rangle$$

(product) (anti-symmetrized)

n_e = # energy denom.
n_ℓ = # of closed loops
n_h = # of hole lines

single particle energies according to $\hat{H}_0$

ε_a: hole energies; ε_h: particle energies ⇐ sum of particle − hole energies

- The details are not so important to us as the basic organization and the consequences for a diagrammatic expansion.

For potentials with repulsive cores:

- What happens if you try to apply this in an expansion in the number of times $\hat{H}_1$ acts? Two infinite-order resummations needed:
  1. Successive particle-particle ladders within a series of diagrams are all the same size ⇒ sum into G matrices
  2. Expansion in # of G matrices is still not perturbative: only adding an independent hole line to a diagram makes it smaller
     ⇒ sum all diagrams with a given number of hole lines (infinite!)

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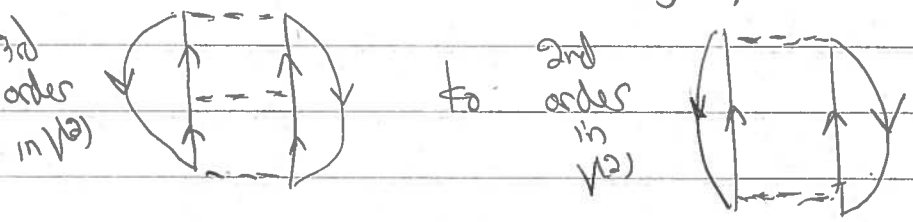
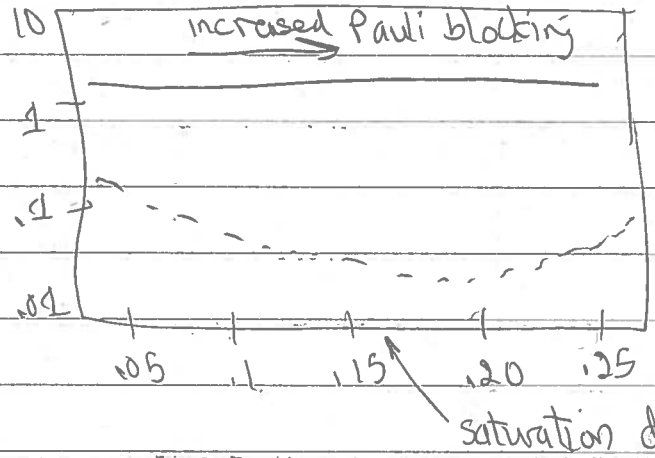
Let's start with numerical study of R_e, ratio of



Is it bigger or smaller? (And 4th to 3rd, 5th to 4th, etc.)

(see slides later)

[log plot]

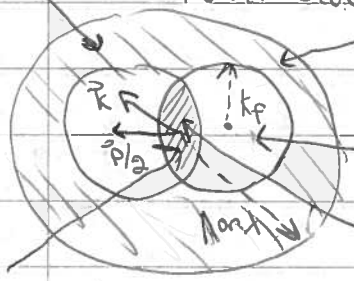


- Uses Hartree-Fock (HF) continuous single-particle spectrum
- contributions from higher-orders for AV18 have similar ratio

- N^{3}LO picture is much more favorable (less than unity) but still a dramatic reduction when evolved to $V_{low\,k}$ $\Lambda = 2\,\text{fm}^{-1}$ or SRG $\lambda = 2\,\text{fm}$

com and relative momenta of pair

- Why does decoupling high and low momentum lead to smaller contributions?
- Pauli blocking and weaker interaction in relevant phase space for S-waves



available phase space for in-medium NN scattering

- For large Λ and strong repulsive $V^{(2)}$, contribution is from large region and part excluded by Pauli blocking is small
  ⇒ density independent.
- For low Λ, there is a smaller and smaller region as density increases, and matrix elements are weaker there ⇒

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- So SRG or $V_{low\,k}$ at $\lambda, \Lambda \lesssim 2\,fm^{-1}$ is perturbative, each ladder rung added makes the diagram smaller
  - exactly consistent with Weinberg eigenvalues.

- For hard potentials, must add up the rungs, as we did in free space to form the T-matrix

$$\frac{1}{E-H_0} \text{ (free space)} \longrightarrow \frac{Q_P}{E'-H_0} \text{ (in-medium)}$$

(Q_P ← Pauli-blocking operator)

- It has been said in the literature that $V_{low\,k}$ interaction is just like a G-matrix,
  - But only true at low momentum (under certain assumptions for the treatment of the single-particle energies)
  - * but there is still a lot of off-diagonal strength in the G-matrix and this makes the energy still non-perturbative in the G-matrix while perturbative in the $V_{low\,k}$ or V_{SRG} low-momentum potential

(see pictures)

- Hole-line expansion: power counting for the G-matrix
  - an analysis shows that the size of a diagram with conventional potentials doesn't relate to how many particle lines there are, but how many (independent hole lines)
  - see slides for examples of 4th order to third-order diagram, where one 4th order has another particle line while the other adds a hole line ~○ ← extra hole line

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- The estimate for the ratio of 4th to 3rd order when a particle line is added goes like the defect wave function at the origin. (in coordinate space) [see B. Day, RMP (1967)]
  - This is the two-particle relative wf compared to non-interacting relative plane waves.
  - This defect is almost complete for highly repulsive cores but largely gone for low-momentum $^1S_0$ and greatly reduced for $^3S_1$,

see graphs ⇒

⇒ adding an interaction doesn't reduce the diagram in the hard case (so resum all) but does in the soft case.

- When a hole line is added, the relevant expansion parameter is the excluded volume at short-range to the average volume occupied by a particle (proportional to $1/\rho$).
  - This is the so-called "wound-integral" κ
  - It is less than one ⇒ expansion for even hard potentials.
  - Still better for soft potentials (and no resummation)

- Bottom line: For soft potentials, adding another potential line reduces the size of the diagrams (caveat: in all cases considered so far)

⇒ real perturbation theory may work. ⇐ tested for neutron matter so far with QMC comparison ✓

* • Regular hole-line expansion also needed to choose $\hat{U}$ to cancel diagrams to enhance convergence,
  - With low-momentum potentials we are free to choose it to make Kohn-Sham DFT work.

m36-76

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MBPT in finite nuclei (teaser)

- Here we'll just briefly mention two applications of MBPT in finite nuclei with some slides.
  - Achim will have more to say later this week on the shell model applications.

- High-order Rayleigh-Schrödinger MBPT in nuclei (Roth et al.)
  - SRG-evolved two-body interactions based on an initial N^{3}LO interaction
  - Perturbation series diverges even for very soft potentials
  - A simple resummation with Padé approximants results in stable energies in agreement with exact NCSM calculations in the same harmonic oscillator model space.

X ⇒ see slide

- Direct use of perturbative methods in microscopic valence-shell calculations
  - a small number of nucleons outside a closed-shell core interact via an appropriate effective interaction treated in MBPT
    - includes non-perturbative transformation to remove the energy dependence of the MBPT effective Hamiltonian
  - One application: impact of 3NF on location of neutron dripline
  - mass prediction for calcium isotopes contradicted existing masses but validated by new, high precision measurements