7/8/2013

Overview: Chiral symmetry in NN scattering and QCD 2.

First: looking for chiral symmetry in **peripheral** NN
partial waves at LO and NLO
- chiral symmetry predictions
- extracting parameters from phase shifts

Next: Recent lattice calculations of some basic inputs
to this comparison, g_A and f_π, ⇒ LECs!
- with some discussion of lattice QCD calculations

Finally: NN interactions from lattice QCD
- HALQCD collaboration approach today
- NPLQCD " " on Wednesday

7/18/2013

Several years ago there was a special issue in the Journal of Physics G organized by Jacek Dobaczewski dedicated to Open Problems in Nuclear Structure Theory (OPeNST). Achim and I contributed two essays:

i) "How should one formulate, extract, and interpret 'non-observables' for nuclei"
- This is about scale and scheme dependence in quantities extracted from experimental data
- we'll come back to this several times

ii) "Is chiral symmetry manifested in nuclear structure?"
- That is, where do you see the effects of spontaneously broken QCD in experimental observables.
- We've already discussed the direct consequence for nuclear forces — most clearly the pion as (approximate) Goldstone bosons leading to the "one-pion-exchange (OPE) potential
  - long range, "derivative coupling"
  - characteristic tensor structure, which we've already seen impacting the deuteron.
- May seem moot because chiral EFT-based potentials seem to be doing well in describing nuclear structure (we'll discuss in detail the current status!)

But we want to ask not whether theories or models consistent with chiral symmetry are sufficient to describe data, but whether the experimental data says this is necessary.

counter examples {
- pionless EFT describes very low energy scattering and up to ^{4}He
- energy-density functionals are very successful without explicit pions

- Here: review the "smoking guns" in NN scattering.

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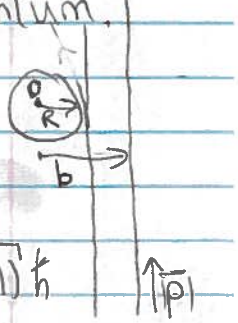
- What are the other smoking guns? [Review]
  - spectrum of mesons with $m_\pi^2 \ll m_\rho^2$
  - parity doublets (e.g. ρ and a_1 vector/axial vector mesons) are significantly split in mass
  - physics of low-energy π-π and π-N scattering is correctly described by chiral perturbation theory (e.g. predictions for $\pi\pi$ scattering lengths)

- Where can we isolate chiral symmetry in NN scattering?
  - Is it in the long-range or short-range part of the interaction? Long range because pions are approximate Goldstone bosons ⇒ smallest mass and longest range. Short-range includes lots of non-chiral physics.
  - Candidate: large scattering lengths? [Exercise]
    - No because comes from combination of long and short distance in the S-waves.
  - Plan: isolate long-range interaction effects by going to large relative (orbital) angular momentum.
    - Recall classical scattering:

$$L = b \cdot p \quad \text{(with respect to origin O)}$$

where p is the momentum and b is the impact parameter.



  - Quantum argument: $\delta_l = 0$ if $pR \ll \sqrt{l(l+1)}\,\hbar$
    - A quick exercise will be to use this to estimate the range of the repulsive "core" of a local NN interaction by noting that $^1S_0$ changes sign at $E_{lab} = 2p_{rel}^2/m \doteq 260$ MeV while $^1D_2$ stays positive. [Exercise]

Not a quantitative approach, But qualitative picture is ok.

⇒ for large L, a scattered nucleon will only feel the longest range one-pion exchange OPE and then 2π.

$$\mathcal{L}^{(0)}(\pi\text{-only}) = \underbrace{\tfrac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \tfrac{1}{2}m_\pi^2\vec{\pi}^2}_{\Rightarrow\ \text{pion propagator}} + N^\dagger\Big[i\partial_0 - \frac{g_A}{2f_\pi}\vec{\tau}\,\underset{\wedge}{\sigma}\cdot\underset{\wedge}{\nabla}\vec{\pi} - \frac{1}{4f_\pi^2}\vec{\tau}\cdot\vec{\pi}\times\dot{\vec{\pi}}\Big]N$$

m26-4

$$\mathcal{L}^{(1)}(\pi\text{-only}) = N^\dagger\Big[4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\vec{\pi}^2 + \frac{c_2}{f_\pi^2}\dot{\vec{\pi}}^2 + \frac{c_3}{f_\pi^2}(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - \frac{c_4}{2f_\pi^2}\epsilon_{ijk}\epsilon_{abc}\sigma_i\tau_a(\nabla_j\pi_b)(\nabla_k\pi_c)\Big]N$$

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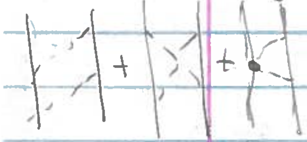
Kaiser, Brockmann, Weise [Nucl. Phys. A625, 758 (1997)] investigated this question: where is the NN interaction governed by chiral symmetry alone?

- 1π exchange range is long range



- ⇐ two pion exchange determined by low-energy πN scattering (plus iterated OPE )

Plan: calculate in perturbation theory up to $T_{Lab} = 280$ MeV (which is the $NN\pi^0$ threshold — why are fits done in the inelastic region? Because the phase shift is mostly elastic until higher energies. Usually 330-350 MeV taken for non-chiral)

- Inputs are

$\mathcal{L}^{(0)}$:
- pion decay constant $f_\pi = 92.4$ MeV [warning: f_π vs. F_π, 92 vs. 130 MeV ⇐ PDG! vs. 180 MeV for f_π. Factors of $\sqrt{2}$ and 2]
- nucleon axial charge coupling $g_A \doteq 1.27$ [they used Goldberger-Treiman relation $g_A = g_{\pi N} f_\pi / m_N$]

$\mathcal{L}^{(1)}$:
- Coupling constants c_1, c_2, c_3, c_4 fit from πN scattering
- masses for pion $m_\pi \doteq 138$ MeV and nucleon $M \doteq 939$ MeV

and just chiral symmetry! Nothing adjustable in the NN system!!

* Look at the pictures of their calculated phase shifts compared to empirical phase shift analysis (PSA).
- Why do they have different signs? (remember $(\vec{\tau}_1\cdot\vec{\tau}_2)$ and S_{12} and $\vec{\sigma}_1\cdot\vec{\sigma}_2$ factors.)

Try estimating what the net sign will be!

- high partial waves at low-energy ⇒ only OPE seen and then 2π exchange. For G, H, I ⇒ mostly quantitative for energies up to 100 MeV, F pretty good.
  - D wave shows deviations at lower energies

⇒ Try OPE in an exercise with Mathematica notebooks.

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How can we really test both the one-pion and two-pion exchange interactions predicted from chiral symmetry are present in NN scattering?

- The Nijmegen group together with Bira van Kolck and Jim Friar (in various combinations) used the Nijmegen PWA methods to make tests (e.g., very careful fits with χ^2 taken <u>very</u> seriously) [see Rentmeester et al., Phys Rev C. <u>67</u>, 044001 (2003) and references]
- Particularly convincing is when they let the mass of the pion be a free parameter in their χ-square minimization that fits the couplings to pp and np data.

- In the original work from 1993, they determined the $pp\pi^0$ coupling constant in each partial wave except for $^1S_0$ (by letting one float in that partial wave) and they agree at the ±1% level with each other and the extracted value from πN scattering.
  - Extracting the neutral and charged pion masses agreed with experiment within estimated one percent errors.

- Subsequent analyses in 1999 and 2003 looked with finer resolution, to look for the direct evidence of two-pion exchange physics,
  - Here they found that using the pion mass as a free parameter gave agreement with experiment at the 10 percent level.
  - The c_i couplings in the TPE fit from NN are <u>consistent</u> with those from πN scattering, although there are still sizable uncertainties in both these determinations. (We'll see the c_i's many times still!)

- Open question: can we do something similar for nuclear structure?
  - My dream: have m_π as a free parameter in an energy density functional with long-range chiral effects (next week) and determine from a fit (say along an isotope chain).
  - Do nuclei know about pions in their structure? The answer could be <u>NO</u> if low resolution!

later: discussion of naturalness and NDA in functionals

For reference: define f_π from the matrix element between the pion and vacuum of the axial current $A_\mu = \bar{u}\gamma_\mu\gamma_5 d$:

$\langle 0|A_\mu(0)|\pi^-(p)\rangle = i p_\mu f_\pi \sqrt{2}$ ← sometimes not there so 93 MeV vs. 130 MeV (PDG)

with $\langle \pi^-(p')|\pi^-(p)\rangle = 2p^0(2\pi)^3\delta^3(\vec{p}\,'-\vec{p})$ | Also f_π vs. F_π and sometimes 180 MeV!

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M26-6

Recent lattice QCD calculations of basics inputs to leading order $\mathcal{L}_{EFT}$: namely g_A and f_π.

- A recent paper posted in February, arXiv:1302.2233 "Nucleon axial charge and pion decay constant from two-flavor lattice QCD."
  - Let's take a look at the result and think a bit about the calculation.

- There is a lot to say about lattice QCD
  ⇒ last year at the INT there was a 3-week school on "Lattice QCD for Nuclear Physics"
  - videos and slides available online (through INT page)
  - for the present discussion the lectures by Anna Hasenfratz, Sinead Ryan, James Zanotti are particularly useful
- Obviously we can't cover these details in any depth
  ⇒ we'll do a low-resolution discussion of LQCD
  - recall that we can replace the fine details by something simpler (i.e. my explanations)
  - it's ok as long as you don't probe deeply!

- Let's start with what we would like to calculate
  ⇒ an imaginary time correlator of two operators constructed from quarks evaluated at two different times: $\langle X(t)\bar{X}(0)\rangle \equiv \langle 0|X(t)\bar{X}(0)|0\rangle$
  ← destruction ← creation ← vacuum →
  - we consider color singlet operators, so $\bar{X}$ will have a probability of creating a hadron with the same quantum numbers
  - For example, for the ρ meson we could take $\mathcal{O}_\rho = \bar{d}\gamma_\mu u$, $\mathcal{O}_\pi = \bar{d}\gamma_5 u$ for the pion, or $\mathcal{O}_N = \varepsilon^{abc}(u^a C\gamma_5 d^b)u^c$ for the proton.
    all indices contracted
    ← not obvious, but please accept for now!
    ↑ complex conjugation
  - not unique!!!! Also: smearing. How would you choose which to use? Maximize overlap.

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The time dependence of $X(t)$ is $e^{\hat{H}t} X(0) e^{-\hat{H}t}$ (no i's because we're in imaginary time (Euclidean)) [cf. Heisenberg picture]

$$\Rightarrow C(t) \equiv \langle 0 | X(t) \bar{X}(0) | 0 \rangle = \langle 0 | \overset{e^{-0_i t}}{\downarrow} X(0) e^{-\hat{H}t} \bar{X}(0) | 0 \rangle$$

insert complete set of states with the right quantum numbers

$$\longrightarrow = \sum_n \langle 0 | X(0) | E_n \rangle e^{-E_n t} \langle E_n | \bar{X}(0) | 0 \rangle$$

$\Rightarrow$ usual projection

$$\xrightarrow[t\to\infty]{} \langle 0 | X(0) | E_0 \rangle e^{-E_0 t} \langle E_0 | \bar{X}(0) | 0 \rangle$$

So if we look at a large enough time then

$$E_0 \doteq \lim_{t\to\infty} -\frac{d \ln C(t)}{dt} \longrightarrow \text{mass of lightest particle if at rest.}$$

↖ can't take $t\to\infty \Rightarrow$ noise, so look for plateau

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To do this in QCD, we can write it in path integral form

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi \, \mathcal{O} \, e^{-S_{QCD}[A_\mu, \bar{\psi}, \psi]} \qquad \mathcal{O} \text{ is an operator}$$

where Z is the same path integral without the $\mathcal{O}$ as S_{QCD} is the action

$$S_{QCD} = \int d^4x \left[\tfrac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(x) \gamma_\mu D_\mu \psi(x) \right] \qquad \text{Euclidean!}$$

color →

Lorentz indices →

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \qquad \underbrace{D_\mu = \partial_\mu - igA^a_\mu t^a}_{\text{covariant derivative so locally gauge invariant}}$$

- There are many complications we are leaving out, but this will do for our resolution.

- Very symbolic at this point $\Rightarrow$ we define everything in practice by <u>discretizing</u>.

M26-8

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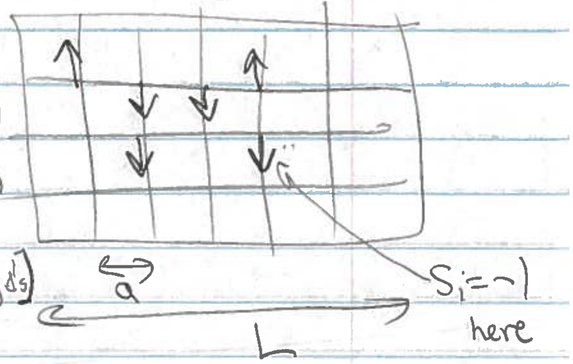
If you're unfamiliar with path integrals, think of this as a partition function for the Ising model, say in 2D.

- We have a lattice of size L (and spacing a for later comparison!)
- A "configuration" is a specification of all the spins ⇒ call one of them C_α [could store as series of n_{sites} 0's, 1's]
- There are a finite but large # N of them: $\alpha = 1, N$ at finite L.



Then there is a Hamiltonian, e.g. nearest neighbor and

$$\mathcal{H}(C_\alpha) = -J \sum_{\langle ij \rangle} S_i S_j \qquad S_i = \pm 1 \text{ for spin on } i^{th} \text{ site}, \quad i,j = 1 \ldots n_{sites}$$

Then if we want the expectation value of a quantity like the magnetization $m(C_\alpha) = \frac{1}{n_{sites}} \sum_i S_i$

$$\Rightarrow \langle m \rangle = \frac{\sum_{\alpha=1}^{N} m(C_\alpha)\, e^{-\beta \mathcal{H}(C_\alpha)}}{Z \longrightarrow \sum_{\alpha=1}^{N} e^{-\beta \mathcal{H}(C_\alpha)}}$$

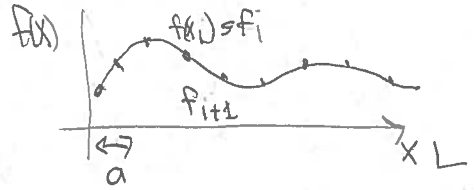
⟵ identify as the probability of configuration C_α

- If we can find a set of characteristic $\{C_\alpha\}$'s $\alpha = 1, \ldots n \ll N$ such that they follow the probability distribution: $\frac{e^{-\beta \mathcal{H}(C_\alpha)}}{Z}$ (we "sample" C_α according to this distribution)

$$\Rightarrow \langle m \rangle \doteq \frac{1}{n} \sum_{\alpha=1}^{n} m(C_\alpha)$$

[e.g. find them by Metropolis algorithm]

So we want to do an analogous thing, by putting QCD on a lattice!



mab9

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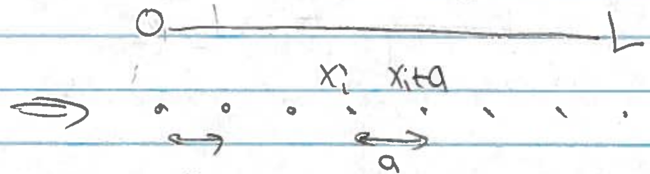
Let's think of discretizing a function f(x) on $x_i = 0, a, 2a, \ldots$



e.g., a derivative

$$\left.\frac{df(x)}{dx}\right|_{x=x_i} \doteq \frac{f(x_i+a)-f(x_i)}{a} \equiv \frac{f_{i+1}-f_i}{a} = \frac{df}{dx} + \mathcal{O}(a)$$

"order a" error

$$\left[\text{check } \frac{f(x+a)-f(x)}{a} = \frac{f(x) + a\frac{df}{dx} + \frac{1}{2}a^2\frac{d^2f}{dx^2} + \ldots - f(x)}{a} = \frac{df}{dx} + a\left.\frac{d^2f}{dx^2}\right|_{x=x_i}\right]$$

$\mathcal{O}(a)$

Note that we can improve the discretization by

$$\left.\frac{df}{dx}\right|_{x=x_i} \doteq \frac{f(x_i+a)-f(x_i-a)}{2a} = \frac{df}{dx} + \mathcal{O}(a^2) \quad \text{[you show it!]}$$

- the analog is done in LQCD all the time!

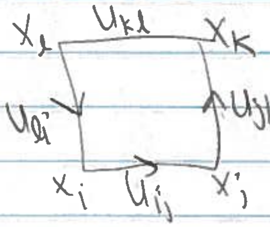
In QCD, quark fields live on the lattice points and the gluon field is replaced by a link variable

Ken Wilson! breaking Lorentz symmetry is ok; preserve gauge symmetry!

$$\Rightarrow \quad A_\mu(x) \longrightarrow U_{n,\mu} = e^{-iagA^b_{n,\mu}t^b} \qquad SU(3) \text{ matrices}$$

so that we have gauge invariant quantities: $\bar{\psi}_n \gamma_\mu U_{n,\mu} \psi_{n+\mu}$, even when discrete and we can take the product around a square (a "plaquette")

as a gauge-invariant construct



$$\Rightarrow \operatorname{tr} U_{ij}U_{jk}U_{kl}U_{li} \rightarrow \operatorname{tr} e^{ia^2F_{\mu\nu}} + \mathcal{O}(a^3)$$

at center of plaquette

and the action is built from these.

No more details here, but hopefully plausible!

$$D_j\psi(x) = \frac{1}{a}\left(U_j(x)\psi(x+\hat{j}) - U_j^\dagger(x-\hat{j})\psi(x-\hat{j})\right)$$

covariant derivative

M26-10

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Finally, we do the (Grassman) fermion integral analytically, (since it is a Gaussian integral) to get

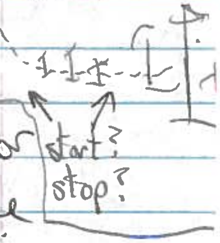
becomes a product of integrals ↙ over each link variable

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\int \mathcal{D}U\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}\,\mathcal{O}\,e^{-S_{QCD}} \rightarrow \frac{1}{Z}\int \mathcal{D}U\,(\det M)\,\mathcal{O}\,e^{-S_G}$$

so that in the end, goal is to generate gauge configurations, which are specifications of the U fields on the lattice.
⇒ a collection of SU(3) elements (matrices), for each link.

To get a real world answer we need to extrapolate to zero error spacing, infinite volume, physical pion mass.
So sources of error are

- statistical error → improve with more configurations
- fitting ⇒ there is a plateau or window in time with fluctuating results
- finite volume error → extrapolate with known or calculate L dependence
- finite lattice spacing → take continuum limit using asymptotic freedom
- chiral limit $m_q \to 0$ (or m_q physical) → use chiral perturbation theory for m_π dependence.



There is a lot of vocabulary:

a complicated story }

Fermions ⇒ different ways of handling chiral symmetry:
staggered, Wilson, Domain-wall, overlap
Improved actions ⇒ different ways of reducing discretization error:
Stout, HEX, asktad, HISQ, clover, ...
Advanced algorithms to make the simulation fast and reliable:
RHMC, DDHMC, LMA, ...
Choice of lattice: e.g. anisotropic lattice to extend time direction at cost of symmetry.

maybe look at pictures first

M26-11

7/8/2013

Let's consider the calculation for g_A.

What we want is the proton matrix element of the axial vector current:

$$\langle p' s' | A_\mu^{u-d} | p, s \rangle = \overline{u}(p' s') \left[\gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 \frac{q_\mu}{2 m_N} G_p(q^2) \right] u(p,s)$$

$\overline{u} \gamma_\mu \gamma_5 u - \overline{d} \gamma_\mu \gamma_5 d$

spinors for p,s or p's'

$\langle p' s' | p s \rangle = (2\pi)^3 2p^0 \delta(\vec{p}-\vec{p}') \delta_{ss'}$

When $q = p' - p \to 0$ then $g_A = G_A(0)$ at $p = p'$.

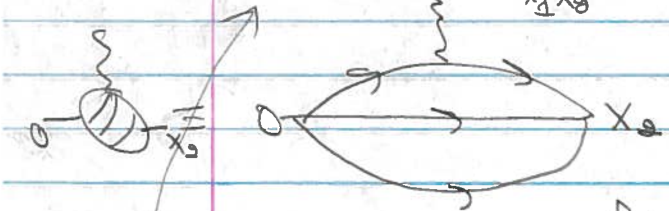
This is a 3pt function. In general (suppressing many indices):

$$G(t, \tau, \vec{p}, \vec{p}') = \sum_{\vec{x}_1 \vec{x}_2} e^{-i\vec{p}' \cdot (\vec{x}_2 - \vec{x}_1)} e^{-i \vec{p} \cdot \vec{x}_1} \langle 0 | N(\vec{x}_2, t) A_\mu^{ud}(\vec{x}_1, \tau) \overline{N}(0) | 0 \rangle$$

annihilate state at final time t

insert operator at time τ

create state with quantum numbers of proton at t=0



set $\vec{p} = \vec{p}' = 0$ here $\Rightarrow$ just sum over $\vec{x}_1, \vec{x}_2$

- use $N(\vec{x}, t) = e^{\hat{H}t} e^{-i \hat{\vec{P}} \cdot \vec{x}} N(0) e^{i \hat{\vec{P}} \cdot \vec{x}} e^{-\hat{H}t}$
- insert $I = \sum_{B,p,s} |B, p, s\rangle\langle B, p, s|$ twice

$$G(t, \tau, \vec{p}, \vec{p}') = \sum_{B B'} \sum_{s s'} e^{-E_{B'}(\vec{p}')(t-\tau)} e^{-E_B(\vec{p})\tau} \langle 0 | N(0) | B' \vec{p}' s' \rangle \times \langle B' p' s' | A_\mu^{ud}(\vec{q}) | B, p, s \rangle \langle B, p, s | \overline{N}(0) | 0 \rangle$$

what we want, for B = proton

$$\xrightarrow{0 \ll \tau \ll t} \sum_{ss'} \langle \text{proton } p'=0 \; s' | A_\mu^{ud}(0) | \text{proton } p=0 \; s \rangle e^{-E_p(t-\tau)} e^{-E_p \tau} \langle 0 | N(0) | \text{proton} \rangle \times \langle \text{proton} | \overline{N}(0) | 0 \rangle$$

with $\vec{p} = \vec{p}' = 0$

calculate 2pt function (without A_μ^{ud} insertion) and divide!

m2b-12

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See slides m2b_QCD_slides_furnstahl.pdf

- Results for f_π^R and g_π^R vs. m_π^2
  - "R" is for renormalized
  - $f_\pi^R = 89.7 \pm 1.5 \pm 1.8$ MeV at $\overbrace{m_\pi}^{\varpi} = 130$ MeV (!!!)

    ↖ below physical
  - $g_A^R = 1.24 \pm 0.04$ at $m_\pi = 130$ MeV
  - The big deal here is g_A^R jumps up to the physical value within errors.
    - consistent with ChPT extrapolation if refit
  - Are the errors under control?

    ⇒ stay tuned!!!

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Now how about the NN potential?

There are two groups with major efforts
NPLQCD → U.S. based ⇒ Wednesday
HALQCD → Japan based ⇒ today

- We will use slides by S. Aoki to test the idea that we are building the background to understand talks at a low resolution level.

- Before we look at them, however, let's think a bit.
  - If we need two nucleons, what does that say about the operators we'll need in our QCD calculation?
    ⇒ two N's and two $\bar{N}$ interpolating fields
  - But how to get phase shifts?

- Their plan:
  - construct a "wave function" that can be inverted to define a non-local potential
  - Expand the potential around local, then use it to calculate phase shifts.

} not so obvious!

Let's see!

* ⇒ look at slides [see also Aoki et al. Prog. Ther Exp Phys. **2012**, 01A105]

- Other applications:
  - baryon-baryon with strangeness
  - quark mass dependence
  - 3N forces
  - arXiv: 1305.2293 on Spin-Orbit Force from Lattice QCD