TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions W3

[Last revised on July 17, 2013 at 08:44:15.]

Wednesday 3: More operators and nuclear matter

We have again grouped all of the two-minute and discussion questions toward the beginning. But remember to spend only about an hour working on questions and then try some of the other problems as well. When you need a break, go back and try another question!

1. Two-minute and discussion questions:

   (a) How do we know the “empirical” nuclear matter saturation energy and density?

   (b) In what ways is a nuclear momentum distribution like a QCD parton distribution?

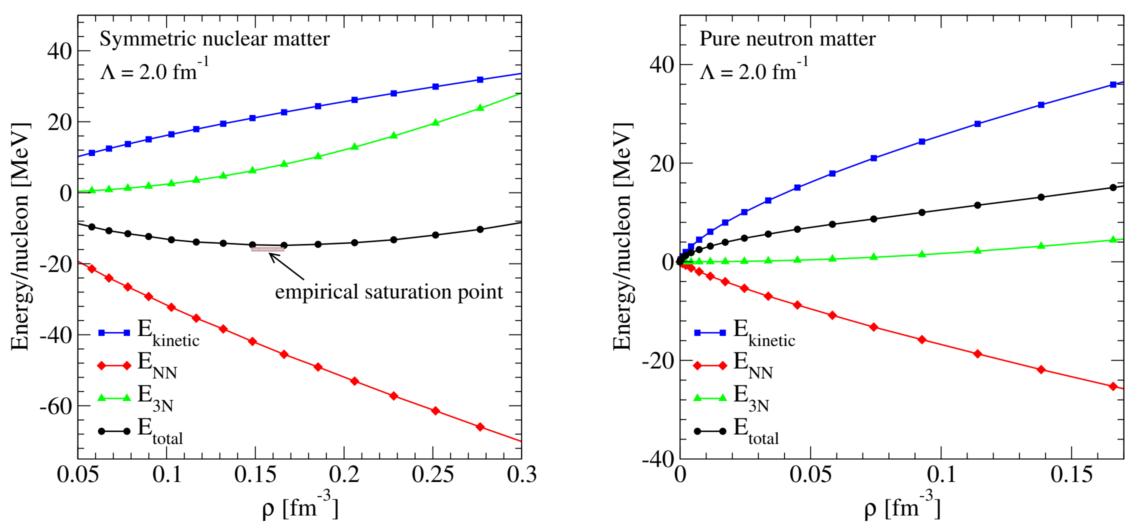
   (c) What does it mean to be *consistent* between reaction and structure theories?

   (d) Why are spectroscopic factors scheme and scale dependent but ANCs are not?

   (e) What is the impulse approximation in knock-out experiments? What are corrections to it?

   (f) How should you choose a scale (or scheme) for a nuclear calculation? (This is an open question for you to speculate on based on the entire course!)

2. Consider the figures below showing contributions to symmetric nuclear matter and pure neutron matter in MBPT calculations with a low-momentum potential.



   The line with circles is the sum of the contributions from the kinetic energy (squares), two-nucleon-only (diamonds), and terms with at least one 3N interaction (triangles).

   (a) What do you conclude about the interplay of separate contributions to achieve nuclear matter saturation? Do you think a precision calculation will be possible (e.g., at the level of a fraction of an MeV per nucleon)?

(b) How do you account for the relative size of the $E_{3\mathrm{N}}$ contributions in SNM and PNM? (That is, why is it smaller in PNM?) What is the consequence?

(c) Are the contributions from the 3NF unnaturally large? Should you judge by comparing to E_{total} or E_{NN}?

(d) If you were to evolve to lower and then to higher resolution, how would you expect these graphs to change? For example, how would the kinetic energy curves change? What about E_{total}? What about the ratio of 3NF to NN-only?

3. Exercise on deuteron asymptotic normalizations [à la R. Amado, PRC **19**, 1473 (1978)]. The asymptotic behavior of a two-body wave function $|\psi_n\rangle$ in coordinate space at long-distance distance is given by

$$\langle \mathbf{r}|\psi_n\rangle \overset{|\mathbf{r}|\to\infty}{\longrightarrow} A_n \frac{e^{-\gamma_n r}}{\sqrt{4\pi} r} ,$$

where γ_n is the binding momentum (that is, the bound-state energy is $E_n = -\hbar^2\gamma_n^2/2\mu$ with μ the reduced mass) and A_n is the asymptotic normalization constant (or ANC). One way to find an ANC is to look explicitly at the tail of the coordinate-space wave function. Here we consider how it shows up in momentum space.

(a) By starting with the Schrödinger equation for $|\psi_n\rangle$ in momentum space, show that

$$\langle \mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle \mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2} .$$

(b) Considered as a function of k^2, this function has an explicit pole at $k^2 = -\gamma_n^2$. Suppose V was the sum of Yukawa functions. Argue that $\langle \mathbf{k}|V|\psi_n\rangle$ has branch cut singularities starting at $k^2 = -(\mu_0+\gamma_n)^2$, where μ_0 is the mass of the longest-ranged Yukawa. [If you don't know how to do this, assume the answer and continue to the next part.]

(c) Now consider the explicit Fourier transform

$$\langle \mathbf{r}|\psi_n\rangle = \int \frac{d^3k}{(2\pi)^3}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \langle \mathbf{k}|\psi_n\rangle .$$

Considering the result for the previous parts and assuming a shallow bound state, show that the ANC is given by extrapolating $\langle \mathbf{k}|V|\psi_n\rangle$ in k^2 to the deuteron pole:

$$A_n = \frac{-2\mu}{\sqrt{4\pi}} \langle \mathbf{k}|V|\psi_n\rangle|_{k^2=-\gamma_n^2} .$$

(d) The consequence is that the ANC is a property of the pole but the other singularities depend on the potential. Explain why this means you can't unambiguously measure the high-momentum wave function by break-up reactions.

(e) Why is this in contrast to the situation with cold atoms in the unitary limit, where the $1/k^4$ tail at high momentum has been measured? (Hint: "high momentum" in this case means $1/a_0 \ll k \ll 1/r_0$.)