TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions T2

[Last revised on July 9, 2013 at 11:06:49.]

Tuesday 2: chiral EFT 2; 3N forces 1

We have again grouped all of the two-minute and discussion questions toward the beginning. But remember to spend only about an hour working on questions and then try some of the other problems as well. When you need a break, go back and try another question!

1. Two-minute and discussion questions:

   (a) How does the counting formula for ν tell us that there is a hierarchy of many-body forces ($NN > 3N > 4N > \cdots$)?

   (b) What does it mean when it is said that chiral symmetry is spontaneously broken down to its vector subgroup?

   (c) How do the c_i coefficients illustrate a unified description of πN, NN, and $3N$ interactions? Is this true for phenomenological potentials?

   (d) In what way are two-body and three-body forces in chiral EFT consistent with each other?

   (e) Compare numerical values of $\epsilon \sim e$ with the typical Q/Λ_b to justify the counting rules for isospin-symmetry-breaking corrections.

   (f) What are potential sources for the differences in the c_i determinations?

   (g) What would be alternative determinations of the D coupling compared to fitting it in 3N forces?

   (h) One of the unsolved problems in few-body physics is the A_y puzzle in nucleon-deuteron scattering. The nucleon analyzing power A_y is the difference in differential cross sections for scattering of polarized nucleons:

   $$A_y = \frac{\frac{d\sigma}{d\Omega}\big|_\uparrow - \frac{d\sigma}{d\Omega}\big|_\downarrow}{\frac{d\sigma}{d\Omega}\big|_\uparrow + \frac{d\sigma}{d\Omega}\big|_\downarrow}\,,$$

   where $\uparrow$ denotes the polarization normal to the reaction plane (spanned by the center-of-mass momentum of the incident and scattered nucleon). All existing NN plus 3N forces underpredict A_y by $\sim 30\%$ for laboratory energies $E_{\rm N} \lesssim 30\,{\rm MeV}$, whereas the predicted A_y is in very good agreement for higher energies. Which parts of nuclear forces do you expect A_y to be especially sensitive to?

2. Renormalization controversy. Renormalization in an EFT removes sensitivity to unresolved short-distance physics, order-by-order; for a cutoff EFT this sensitivity is manifested as dependence on the cutoff parameter.

(a) At each order in the EFT expansion, is it necessary that cutoff dependence be removed completely, or is it sufficient that the residual dependence is comparable to the truncation error at that order? (Note: the latter is closer to what is done in numerical analysis where there is always some residual dependence on the mesh).

(b) In the latter case, how do you really know you have successfully renormalized? (That is, what are the signatures?)

3. We use a simple EFT model, to illustrate how field redefinitions can be used to shift strength from off-shell two-body interactions to on-shell three-body interactions:

$$\mathcal{L} = \psi^\dagger \mathcal{D}\psi - g_2(\psi^\dagger\psi)^2 - \eta\left(\psi^\dagger(\psi^\dagger\psi)\mathcal{D}\psi + \psi^\dagger\mathcal{D}(\psi^\dagger\psi)\psi\right) , \tag{1}$$

where $\mathcal{D} = i\partial_t + \vec{\nabla}^2/(2m)$ is the free Schrödinger operator. The model has a two-body contact interaction with coupling g_2 and an off-shell two-body contact interaction with coupling η, which we assume to be small.

(a) Why is the part with the η coupling called an "off-shell" two-body interaction?

(b) Now consider a field transformation (a change of field variables)

$$\psi \longrightarrow \left[1 + \eta(\psi^\dagger\psi)\right]\psi\,, \quad \psi^\dagger \longrightarrow \left[1 + \eta(\psi^\dagger\psi)\right]\psi^\dagger\,. \tag{2}$$

Performing this transformation and keeping all terms of order η, show that this leads to a new Lagrangian:

$$\mathcal{L}' = \psi^\dagger\mathcal{D}\psi - g_2(\psi^\dagger\psi)^2 - 4\eta g_2(\psi^\dagger\psi)^3 + \mathcal{O}(\eta^2)\,,$$

where the off-shell two-body interaction has been traded for a three-body interaction.

(c) This shows that off-shell interactions always contribute together with many-body forces and only the sum of the two is meaningful. What do you conclude about many-body forces or off-shell interactions being observable?

4. The leading two-pion-exchange 3N interaction is given by

$$V_c = \frac{1}{2}\left(\frac{g_A}{2f_\pi}\right)^2 \sum_{i\neq j\neq k} \frac{(\boldsymbol{\sigma}_i\cdot\mathbf{q}_i)(\boldsymbol{\sigma}_j\cdot\mathbf{q}_j)}{(q_i^2+m_\pi^2)(q_j^2+m_\pi^2)}\, F_{ijk}^{\alpha\beta}\,\tau_i^\alpha\,\tau_j^\beta\,,$$

with

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left[-\frac{4c_1m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2}\,\mathbf{q}_i\cdot\mathbf{q}_j\right] + \sum_\gamma \frac{c_4}{f_\pi^2}\,\epsilon^{\alpha\beta\gamma}\,\tau_k^\gamma\,\boldsymbol{\sigma}_k\cdot(\mathbf{q}_i\times\mathbf{q}_j)\,.$$

Draw the corresponding diagram (with nucleons labelled i, j, k) and identify all parts of the expression with the vertices or line.

5. Consider the NDA formula discussed in the first lecture:

$$\mathcal{L}_{\chi\,\mathrm{eft}} = c_{lmn}\left(\frac{N^\dagger(\cdots)N}{f_\pi^2\Lambda_\chi}\right)^l \left(\frac{\pi}{f_\pi}\right)^m \left(\frac{\partial^\mu, m_\pi}{\Lambda_\chi}\right)^n f_\pi^2\Lambda_\chi^2\,.$$

(a) What should we conclude when a coefficient is unnaturally large? What if it is unnaturally small?

(b) If we combine this formula with the idea of resonance saturation by a meson with coupling g and mass m:

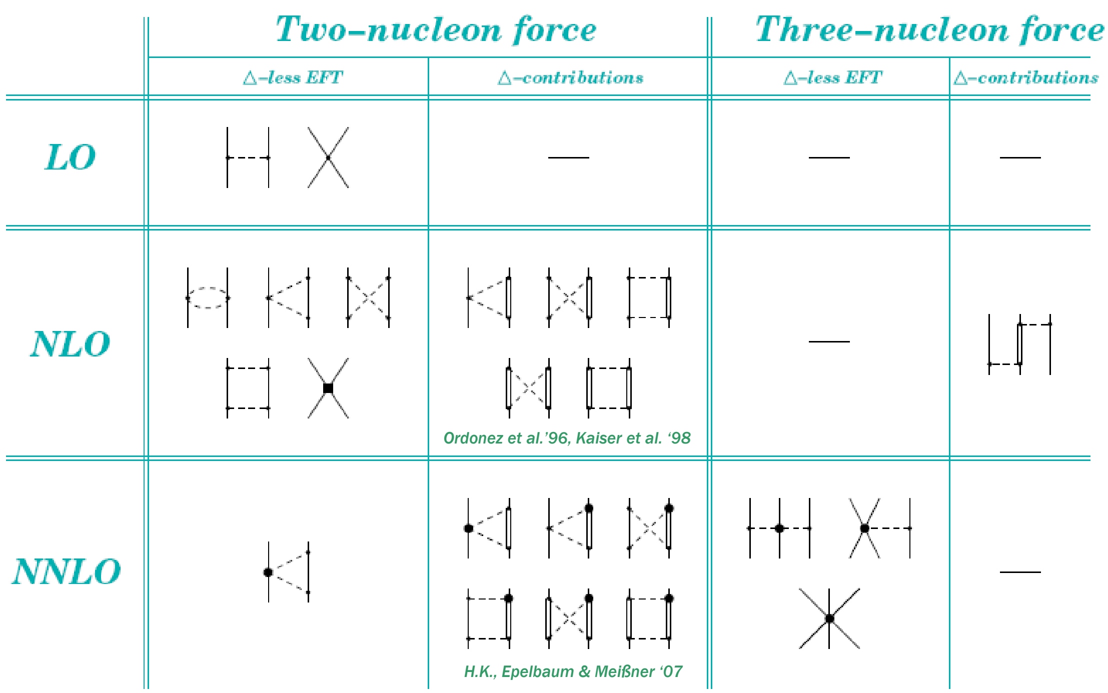
$$\frac{g^2}{\mathbf{q}^2 + m^2} \longrightarrow \frac{g^2}{m^2} - \frac{g^2}{m^2}\left(\frac{q^2}{m^2}\right) + \cdots \,,$$

what is the generic size of a meson-exchange coupling g? Find the values for the g's in a meson-exchange model by looking online; is your estimate consistent?

(c) In Eq. (4.43) of the recent review by Machleidt and Entem (arXiv:1105.2919), the $\mathrm{N^3LO}$ contact terms for the chiral EFT Lagrangian are listed as a potential. Figure out how to assign l, m, and n in the formula above to determine how to scale the D_i LECs. Then compare to the values in Table F.1. Which D_i's are unnaturally large? Speculate on why they are unnatural.

(d) What does the formula predict for the natural size of the c_i coefficients? How do they compare to their typical fit values?

6. The following chart is from H. Krebs:



(a) For each diagram in the two "Δ-contributions" columns, identify the corresponding Δ-less EFT diagrams in which the unresolved contribution will be absorbed. [Note: you may need to look up the Δ-less diagrams at $\mathrm{N^3LO}$.]

(b) Are the corresponding diagrams always at the same order? What do you conclude from that?

7. Sign of the 3N force contributions.

   (a) What was the sign of the LO three-body force in pionless EFT, when applied to the triton? (Hint: trick question.)

   (b) Do you think the two-pion-exchange (c_i), one-pion-exchange (D), and contact (E) 3N forces have definite sign (repulsive/attractive) when applied to light nuclei? If not, what could this depend on?

   (c) Check your arguments agains explicit results in the literature.

8. Show that $U = e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi}$ when substituted into

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger + m_\pi^2(U + U^\dagger)\right]$$

and expanded in powers of $\boldsymbol{\pi}$ yields the "familiar" expression (given in the first lecture) for the pion effective Lagrangian up to terms of order $\boldsymbol{\pi}^4$. Identify the constant α for this representation.