TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions Th3

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Thursday 3: Nuclear forces impact on (exotic) nuclei

We have again grouped all of the two-minute and discussion questions toward the beginning. But remember to spend only about an hour working on questions and then try some of the other problems as well. When you need a break, go back and try another question!

1. Two-minute and discussion questions:

   (a) Why did we conclude that we can calculate neutron matter better (at nuclear densities and below) than the level of uncertainty in nuclear forces?

   (b) Why are neutrons in the core of neutron stars expected to form a $^3\mathrm{P}_2$ superfluid? Why not $^3\mathrm{P}_0$ or $^3\mathrm{P}_1$?

   (c) If P-wave superfluidity would be turned off (due to some many-body effects), do you expect the neutrons in the core of neutron stars to be superfluid in some other channel? If yes, in which one?

   (d) For the slide on $\mathrm{N}^3\mathrm{LO}$ 3N interactions in neutron matter (p. 6 of the lecture today), why are there different bands for the two-pion-exchange–contact 3N interactions?

   (e) Why did we conclude that the long-range two-pion-exchange parts of 3N forces dominate for the contribution to valence neutrons compared to the shorter-range 3N forces? Why do you expect this physically?

   (f) Which parts of 3N forces are mainly responsible for the $^{48}\mathrm{Ca}$ shell closure? (Hint: Recall that the $N = 28$ shell closure after $N = 20$ is due to the $f_{7/2}$ shell being below the other pf-shell orbits.)

2. RG analysis of infrared poles.

   (a) Recall the running coupling in pionless EFT $C_0(\Lambda) = \frac{4\pi}{m}\frac{1}{\frac{1}{a}-\frac{2}{\pi}\Lambda}$. For positive scattering length, $C_0(\Lambda)$ has an infrared ($\Lambda \to 0$) pole at $\Lambda \sim \frac{1}{a}$. What is the reason for this pole?

   (b) More generally, infrared poles in running couplings signal the formation of bound states in the theory. Consider an effective theory with a cutoff Λ_F around the Fermi surface at $k_\mathrm{F} \pm \Lambda_\mathrm{F}$ and study the infrared behavior as $\Lambda_\mathrm{F} \to 0$. Integrating out particle-particle/hole-hole scattering beyond $k_\mathrm{F} \pm \Lambda_\mathrm{F}$ gives an in-medium T-matrix for two particles back-to-back on the Fermi surface

$$T_\mathrm{med} = \left[\frac{1}{C_0(\Lambda)} - \left(\int_{k_\mathrm{F}+\Lambda_\mathrm{F}}^{\Lambda} \frac{p^2 dp}{2\pi^2} \frac{m}{k_\mathrm{F}^2 - p^2} - \int_0^{k_\mathrm{F}-\Lambda_\mathrm{F}} \frac{p^2 dp}{2\pi^2} \frac{m}{k_\mathrm{F}^2 - p^2} \right) \right]^{-1} , \qquad (1)$$

where the first integral term is due to integrating out particle-particle and the second due to hole-hole scattering. Why do they have opposite sign? Why is the E in the T-matrix replaced by k_{F}^2/m? Why did we drop the $i\epsilon$?

(c) Show that the poles of the in-medium T-matrix are given by

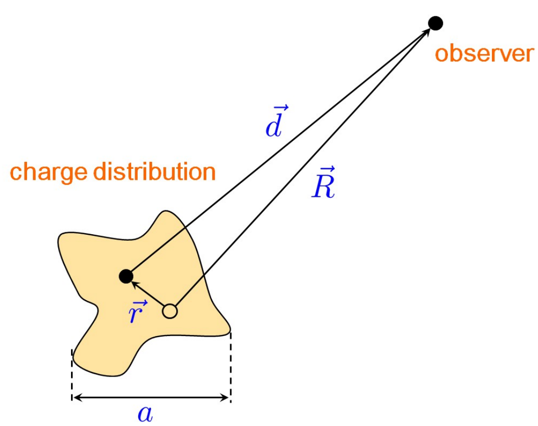
$$\frac{\pi}{2k_{\mathrm{F}}a} - 2 + \frac{1}{2}\ln\frac{4k_{\mathrm{F}}^2 - \Lambda_{\mathrm{F}}^2}{\Lambda_{\mathrm{F}}^2} = 0\,. \tag{2}$$

(d) Consider weak coupling $k_{\mathrm{F}}a \ll 1$. Show that there is an infrared ($\Lambda_{\mathrm{F}} \to 0$) pole only for negative scattering length a, so for an arbitrary attractive interaction. Show that the pole is at an energy $E \sim \frac{k_{\mathrm{F}}}{m}\Lambda_{\mathrm{F}}$ with

$$E \sim \frac{2}{e^2}\frac{k_{\mathrm{F}}^2}{m}\,e^{\frac{\pi}{2k_{\mathrm{F}}a}}\,. \tag{3}$$

This pole is due to the formation of Cooper pairs and signals the instability towards superfluidity or superconductivity.

3. YAEFTA (Yet Another Effective Field Theory Analogy). Almost every introduction to effective (field) theory mentions the analogy to a multipole expansion. Let's review and extend that analogy. Here we use the cartoon and notation from E. Epelbaum's lectures.



Recall that if we have a localized charge distribution $\rho(\mathbf{r})$ within a volume characterized by distance a, we can use the expression for the Coulomb potential to find the electrostatic potential

$$V(\mathbf{R}) \propto \int d^3r\,\frac{\rho}{|\mathbf{R}-\mathbf{r}|}\,.$$

If we expand $1/|\mathbf{R}-\mathbf{r}|$ for $r \ll R$, we get the multipole expansion

$$\int d^3r\,\frac{\rho}{|\mathbf{R}-\mathbf{r}|} = \frac{q}{R} + \frac{1}{R^3}\sum_i R_i P_i + \frac{1}{6R^5}\sum_{ij}(3R_iR_j - \delta_{ij}R^2)Q_{ij} + \cdots$$

where the total charge q, dipole moment P_i, and quadrupole moment Q_{ij} are defined as

$$q = \int d^3r\,\rho(\mathbf{r})\,, \qquad P_i = \int d^3r\,\rho(\mathbf{r})\,r_i\,, \qquad Q_{ij} = \int d^3r\,\rho(\mathbf{r})(3r_ir_j - \delta_{ij}r^2)\,.$$

(a) This is a "top down" approach to the effective theory, which starts from the underlying theory. Can we do the analogous top down construction of chiral effective field theory from QCD? What about pionless EFT from chiral EFT?

(b) Two-minute questions:

   i. We can change degrees of freedom in an effective by eliminating modes (e.g., high momentum states) or by switching to more collective dofs. What happens here? (Identify the underlying degrees of freedom and the degrees of freedom in the effective theory.)

   ii. What are the low-energy constants (LECs) for the multipole expansion?

   iii. How would you fit the LECs to experimental data?

   iv. What is the expansion parameter? How do we do power counting?

   v. EFTs have breakdown scales beyond which the EFT expansion fails to converge. What is the breakdown scale of the multipole expansion?

   vi. Given a finite set of multipole moments, can you invert the expressions for them to find a unique charge distribution $\rho(\mathbf{r})$? If not, how many different distributions could there be?

   vii. If you knew $\rho(\mathbf{r})$ somehow (maybe by a microscopic calculation), why might there still be advantages to using the multipole expansion?

(c) Provide naive dimensional analysis estimates of the multipole moments. Do this by identifying the relevant dimensionful quantity (or quantities).

(d) Now consider more than one localized charge distributions interacting.

   i. For two charge distributions, what is the difference between having the charges fixed and allowing them to rearrange within their localized volumes (polarize)?

   ii. For three charge distributions, what is the consequence for the low-energy theory of polarization?

(e) Rather than a top-down construction (given the Coulomb interaction), can you construct the multipole expansion by the EFT "bottom-up" approach of considering the most general terms consistent with the long-distance physics and the symmetries? What is the long-distance physics here? What are the symmetries?

(f) Now imagine a different world replacing $1/|\mathbf{R}-\mathbf{r}| \to f(|\mathbf{R}-\mathbf{r}|)$ by f, which has a range of order L. For example, suppose f was a gaussian (or a Yukawa form). Describe the features of the analogous low-energy theory (multipole expansion).