TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions Th2

[Last revised on July 11, 2013 at 11:27:15.]

Thursday 2: RG 2; 3N forces 2

This is a short exercise set (comparatively :). If you have time after going through these, please go back and work on unfinished problems from previous sets.

1. Two minute and discussion questions:

   (a) Consider an operator written in second quantization using a particular single-particle basis. How would you identify one-body, two-body, three-body, etc. parts?

   (b) If you evolve a Hamiltonian matrix with the similarity RG (SRG), which of the following do not change ($\lambda = 1/s^{1/4}$):

      i. all phase shifts $\delta_l(k)$ or only phase shifts $\delta_l(k)$ for $k \leq \lambda$;

      ii. determinant of H_s; trace of H_s; trace of H_s^2.

   (c) Explain in terms of decoupling of high and low momenta why SRG evolution generates many-body forces.

   (d) If you compare band and block diagonal SRG evolution, where is the scheme dependence and where is the scale dependence?

   (e) According to chiral effective field theory, are there five-body forces? What about 200-body forces? If you answer yes, give an example of each.

   (f) If we start with the Argonne AV18 NN potential and evolve it to a lower resolution, have we changed the potential? What about many-body forces?

   (g) What is the dominant change to the momentum-space potential when you evolve to lower resolution? How is this related to running couplings in pionless EFT?

   (h) Earlier we associate the change in sign from positive to negative of the $^1S_0$ phase shift with the large repulsive core of a local NN potential. But SRG evolution makes the core go away, yet the phase shifts are unchanged. How do we interpret why the phase shift changes sign now? If a purely local potential must have a repulsive core to change sign, what can a non-local potential have instead? [Note: this is an example of how changing resolution can change the *interpretation* of physics observables.]

   (i) To which observables of nuclei do 3N forces contribute? When we say that 3N forces contribute to the ground-state energy, why can't we measure 3N forces?

   (j) What are sources for the differences in the c_i's from the different extractions? Is the range given for the different c_i's large or as expected?

   (k) What would happen to nuclei, if 3N forces were purely attractive (assuming higher-body forces vanish)?

2. An operator O transforms under a unitary transformation U_s by

$$O_s = U(s) O U(s)^\dagger \ .$$

   (a) Using problem 2 from Wednesday's exercises as a guide, derive the flow equation for O_s (that is, the equation for dO_s/ds).

   (b) Derive the SRG flow equation for the unitary transformation U_s. [Hint: start from

$$\eta(s) \equiv \frac{dU(s)}{ds} U^\dagger(s) = [G_s, H_s]$$

   and remember that $U^\dagger(s)U(s) = 1$.]

3. According to the Particle Data Group's article on Quantum Chromodynamcs (QCD) [see `http://pdg.lbl.gov/2012/reviews/rpp2012-rev-qcd.pdf`]:
   "In the framework of perturbative QCD (pQCD), predictions for observables are expressed in terms of the renormalized coupling $\alpha_s(\mu_R)$, a function of an (unphysical) renormalization scale μ_R. When one takes μ_R close to the scale of the momentum transfer Q in a given process, then $\alpha_s(\mu_R = Q^2)$ is indicative of the effective strength of the strong interaction in that process.

   The coupling satisfies the following renormalization group equation (RGE):

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -(b_0 \alpha_s^2 + b_1 \alpha_s^3 + b_2 \alpha_s^4 + \cdots) \ ,$$

   where $b_0 = (11C_A - 4n_f T_R)/(12\pi) = (33 - 2n_f)/(12\pi)$ is referred to as the 1-loop beta-function coefficient, ..."

   (a) In our SRG formalism, what is the analog of the running coupling (or running couplings)?

   (b) What is the β function?

   (c) What plays the role of μ_R?

4. More on the leading N^{2}LO 3N forces.

   (a) Are the N^{2}LO 3N forces isospin symmetric?

   (b) Show that the c_4 part of the two-pion-exchange N^{2}LO 3N forces does not contribute in the $T = 3/2$ channel (so, e.g., to three neutrons).

   (c) We saw that the pion couples derivatively to the spin of a single nucleon by $\boldsymbol{\sigma} \cdot \mathbf{q}$. We therefore expect that, in the one-pion-exchange 3N force given by the D-term, the pion couples to the spin of the two nucleons, $\mathbf{S} = (\boldsymbol{\sigma}_m + \boldsymbol{\sigma}_n)/2$, where m, n label the two nucleons interacting at the contact interaction. Is this the case?

   (d) Does the short-range 3N force given by the E-term contribute to three neutrons? (Hint: note that this depends on the regulator.)