TALENT/INT Course on Nuclear Forces
Exercises and Discussion Questions F2

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Friday 2: Many-body overview; electroweak interactions

In preparation for next week's lectures, please review the lectures from the first two weeks and look through the review by Bogner, Furnstahl, and Schwenk.

1. Two-minute and discussion questions.

   (a) The Weinberg eigenvalue analysis at finite density indicates that the S-waves, which are non-perturbative because of the bound-state or near-bound-state in those channels, become perturbative. How can this happen? Where does the deuteron go?

   (b) Why is it advantageous to convert numerical calculations to matrix form?

   (c) Why do we usually use Gaussian quadrature (rather than some other quadrature like Simpson's rule) to do numerical integrals, such as over angles?

   (d) If you evolve a Hamiltonian in the $A = 2$ space (that is, the two-body space) by the SRG, you identify the two-body potential. Now you evolve the Hamiltonian in the $A = 3$ space. How can you isolate the three-body part of the Hamiltonian?

   (e) How does using a reference state to define your second-quantized operators change the identification of what is 0-body, 1-body, 2-body, and 3-body physics in your Hamiltonian?

   (f) Why are NCSM matrices so sparse? Which part is more sparse: two-body or three-body matrix elements (and justify your answer)?

   (g) Why do importance truncated no-core shell model (IT-NCSM) calculations make it possible to calculate larger nuclei? What type of Hamiltonian can be used to do these calculations? [Hint: you need to use perturbation theory.]

   (h) Why are spectroscopic factors (SFs), which are a type of wave function overlap between A and $A + 1$ body nuclei, scale and scheme dependent? Are coupled cluster calculations of SFs still useful if SFs are not directly measurable?

   (i) Is calculating the quark mass dependence of nuclear observables such as the triple-alpha rate just of intellectual interest or are there real-world applications?

   (j) In the first lecture the question "Why do we need so many different methods?" (meaning theoretical many-body methods using microscopic interactions as input) was addressed. What is your answer to this question?

   (k) Do you expect two-body currents to contribute to the magnetic moment of the deuteron?

2. Some very basic two-minute nuclear physics questions (which *everyone* should know) about the figure in the first lecture showing the table of the nuclides.

   (a) Why is the slope of the black region less than a 45 degree angle once it is past $Z = N = 20$ or so?

   (b) How do the binding energies of the stable nuclei in black compare to each other? (E.g., do they vary over a wide range? Do they have a regular pattern?)

   (c) What happens to the binding energy as you move perpendicular to the black line?

   (d) What is the difference between being unstable and unbound? What are the driplines?

   (e) [This one may not be so obvious!] Why is the location of the proton dripline so much better known than the neutron dripline?

   (f) Last week there was a discussion of pairing. Where is pairing important in the table of nuclides?

3. Review of the spectra of light nuclei: Pick one of the p-shell nuclei from the GFMC or NCSM calculations shown in the lecture.

   (a) Draw the simple approximation to the ground-state configuration by filling the lowest orbits in the shell model.

   (b) Describe which excitations give rise to the different quantum numbers in the spectra.

4. The quadrupole moment of the deuteron is predicted by the EGM potentials to be $Q_d = 0.273$–0.275, 0.271–0.275, and 0.264–0.268 fm^2 at NLO, N^{2}LO, and N^{3}LO, respectively. Experimentally, $Q_d = 0.2859(3)$ fm^2. To what do you attribute the difference between theory vs. experiment?

5. How does the leading long-range axial-vector two-body current look diagrammatically in chiral EFT with explicit Δ's? At which order does this enter?

6. The lattice spacings for lattice EFT calculations were given in a slide to be $1\,\text{fm} \le a \le 5\,\text{fm}$.

   (a) What are the ultraviolet cutoffs Λ in momentum corresponding to these spacings? (Be precise; factors of two are relevant here.)

   (b) How does this range compare to typical chiral EFT potentials used for other many-body methods? Are they hard or soft interactions?

7. Review: Using lattice EFT one can calculate the correlation function for A nucleons:

$$Z_A(t) = \langle \Psi_A | e^{-\tau H} | \Psi_A \rangle \ ,$$

   where Ψ_A is a Slater determinant for A free nucleons and τ is the Euclidean time. [See the U. Meißner slides.]

(a) Show that if you calculate the time derivative of the log correlator you can find the ground-state energy E_A^0 in the large time limit:

$$\frac{d}{d\tau} \ln Z_A(\tau) = -E_A(\tau) \stackrel{\tau\to\infty}{\longrightarrow} -E_A^0 \; .$$

[Hint: Use a complete set of eigenstates of H.]

(b) Suppose that we want the ground-state expectation value of a normal-order operator $\mathcal{O}$. We can calculate the correlator with an insertion of $\mathcal{O}$:

$$Z_A^{\mathcal{O}}(t) = \langle \Psi_A | e^{-\tau H/2} \mathcal{O} e^{-\tau H/2} | \Psi_A \rangle \; .$$

Show how we can use $Z_A^{\mathcal{O}}(t)$ and $Z_A(t)$ is the large τ limit to find the desired expectation value.

8. Including contact interactions in the lattice EFT Hamiltonian through auxiliary fields [schematic]. If we have fermion fields (e.g., for the nucleons) appearing only as quadratics, like $N^\dagger N$, then the path integral is a Gaussian that can be done analytically. So the difficulty is dealing with Fermion terms that aren't quadratic, such as a contact interaction $(N^\dagger N)^2$. The "trick" is to introduce an additional path integral over a new field s.

(a) Show that if $\rho \sim N^\dagger N$, then

$$\exp(\rho^2/2) \propto \int_{-\infty}^{\infty} ds \, e^{-s^2/2 - s\rho} \; .$$

Show this without actually evaluating the integral by completing the square (note that you can shift the integration variable s by whatever you want).

(b) Explain how this solves the problem. In the end, what fields are you doing path integrals over numerically?

9. In the Th2a lecture slides, a schematic version of the SRG flow equation was used to claim that three-body forces are always induced, even if the initial interaction was two-body only:

$$\frac{dV_s}{ds} = \Big[\Big[\sum \underbrace{a^\dagger a}_{G_s}, \sum \underbrace{a^\dagger a^\dagger a a}_{\text{2-body}}\Big], \sum \underbrace{a^\dagger a^\dagger a a}_{\text{2-body}}\Big] = \cdots + \sum \underbrace{a^\dagger a^\dagger a^\dagger a a a}_{\text{3-body!}} + \cdots$$

Verify explicitly by carrying out commutators on an initial Hamiltonian with a one-body G_s (like $T_{\rm rel}$) and a two-body potential that a three-body interaction is generated after one iteration.