Evaluation of Modern ^{3}He$(\alpha,\gamma)^7$Be Data

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In both the Sun and the early universe, the ^{3}He$(\alpha,\gamma)^7$Be reaction plays a key role. The rate of this reaction is the least certain nuclear input needed to calculate both the primordial ^{7}Li abundance in big bang nucleosynthesis (BBN) and the solar neutrino flux. Taking advantage of several recent highly precise experiments, we analyse modern ^{3}He$(\alpha,\gamma)^7$Be data using a robust and minimally model dependent approach capable of handling discrepant data sets dominated by systematic rather than statistical errors. We find $S_{34}(0) = 0.580 \pm 0.043(0.054)$ keV b at the 68.3(95.4)% confidence level.

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I. INTRODUCTION

As the nearest star, the Sun is the best studied. Models predict the neutrino fluxes produced by the radioactive decay of ^{7}Be and ^{8}B in the solar core [1]. Over the last decade, various neutrino observatories have measured the flux and flavour composition of the ^{8}B neutrinos from the Sun [2, 3], allowing constraints to be placed on neutrino mixing angles and mass-squared differences. New experiments sensitive to the ^{7}Be flux [4] are ongoing. Both the ^{7}Be and ^{8}B neutrino flux predictions are nearly directly proportional to the astrophysical S factor for the ^{3}He$(\alpha,\gamma)^7$Be reaction, S_{34}, at a relative energy of ~ 20 keV; in fact, $\phi_\nu(^7\mathrm{Be}) \propto S_{34}(0)^{0.86}$ and $\phi_\nu(^8\mathrm{B}) \propto S_{34}(0)^{0.81}$ [5, 6]. This sensitivity allowed S_{34} to be constrained using the measured solar neutrino fluxes and laboratory measurements of S_{17}, the astrophysical S factor for the ^{7}Be$(p,\gamma)^8$B reaction [7]. The ^{3}He$(\alpha,\gamma)^7$Be reaction is important not only in determining the solar neutrino fluxes, but also in other astrophysical environs.

Primordial nucleosynthesis has been a robust prediction of hot big bang cosmology for over 40 years [8–12]. The theory explains the large universal abundance of ^{4}He as well as the origin of trace quantities of the light isotopes D, ^{3}He, and ^{7}Li. It is a theory with three free parameters, the cosmic baryon density, the neutron mean lifetime, and the number of active light neutrino species. Via fits to the standard electroweak theory, measurements at the Large Electron Positron Collider have determined the number of active light neutrino species to be $N_\nu = 2.984 \pm 0.008$ [13], justifying the choice $N_\nu = 3$. Many experiments have been performed to determine the mean lifetime of the neutron; the current Particle Data Group recommendation is $\tau_n = 885.7 \pm 0.8$ sec [13]. The cosmic baryon density has been determined by analysing anisotropies in the cosmic microwave background radiation; the latest WMAP satellite results are $\Omega_b h^2 = 0.02273 \pm 0.00062$ [14], where Ω_b is the universal mean density of baryons expressed in units of the critical density required to close the universe and the Hubble constant is $100h$ km sec^{-1} Mpc^{-1}. Using the Friedmann-Robertson-Walker cosmological model, the standard model of particle physics, and nuclear reaction rates one can therefore predict the light element abundances very precisely and compare directly with primordial abundance estimates based on observations of the oldest systems.

When observations of the Li abundances in the oldest stars in the Milky Way were compared with the predictions of BBN, a 2-3σ discrepancy was found [15]. Several possibilities exist for resolving this discrepancy, including particle physics beyond the standard model, improvements to the stellar models used to interpret astronomical observations, better approximations of the matter distribution in relativistic cosmology [16, 17], modifications of gravitational theory (e.g., [18]), and improved nuclear reaction rates. In this paper, we examine the last possibility. Lithium is made as beryllium in the early universe via the ^{3}He$(\alpha,\gamma)^7$Be reaction. Of course the rates of reactions that destroy ^{7}Be must be known in addition to those that create it. Recent studies of the ^{7}Be$(d,p)2\alpha$ reaction cross section suggest that this reaction is not the source of the discrepancy [19, 20]. The primordial ^{7}Li abundance prediction is nearly directly proportional to the ^{3}He$(\alpha,\gamma)^7$Be cross section at a relative energy of ~ 300 keV; the primordial abundance

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ratio ^{7}Li/H $\propto S_{34}^{0.96}$ [15]. On account of the obvious importance of this reaction for both solar neutrinos and BBN, it has been subjected to extensive theoretical as well as experimental study.

Since the first potential model calculations [21, 22], many theoretical studies of the ^{3}He$(\alpha,\gamma)^7$Be reaction have been performed [23–31]. Examining the potential models, a hard sphere potential yields $S_{34}(0) = 0.47$ keV b and $S'(0)/S(0) = -0.60$ MeV^{-1} [22], while a more physical potential yields $S_{34}(0) = 0.46$ keV b and $S'(0)/S(0) = -0.79$ MeV^{-1} [31]. Though the zero-energy S factor values are in good agreement, the shapes as measured by the logarithmic derivatives are quite different. Similarly, in the single channel resonating group method (RGM) calculations of Ref. [25], $S(0)$ varies widely with the nucleon-nucleon potential. The most commonly used fitting function for experimental data is the microscopic cluster model calculation of Kajino [23, 25, 27]. Ref. [25] reports $S_{34}(0) = 0.50 \pm 0.03$ keV b and $S'(0)/S(0) = -0.548 \pm 0.033$ MeV^{-1}. The estimated theoretical uncertainty of $\pm 6\%$ [25] on both of these quantities must be considered when fitting this predicted shape to experimental data. The commonly cited potential model and microscopic cluster model calculations are shown in Figure 1. Another RGM calculation that includes the ^{6}Li+p configuration in addition to the ^{3}He+α configuration [30] finds that the energy dependence of the astrophysical S factor is not uniquely determined and yields a range of $S'(0)/S(0)$ from -0.70 to -0.50 MeV^{-1}. Analyses that renormalize theoretically calculated S factors must include an additional systematic error in order to account for uncertainties in the true shape of the S factor when extrapolating to zero energy. It would be desirable to use the data themselves to determine the shape of the S factor.

In this paper, we evaluate the modern prompt capture γ ray and induced ^{7}Be activity measurements of the ^{3}He$(\alpha,\gamma)^7$Be cross section in a nearly model-independent way. We use well known physics to constrain the energy dependence of the low energy cross section and disuss the treatment of systematic errors and their propagation into the final uncertainties, providing reliable values for S_{34} at energies relevant to solar neutrino production and BBN with realistic uncertainties derived from Markov Chain Monte Carlo calculations.

II. MODERN DATA

In the past 50 years, there have been many experimental efforts to measure the ^{3}He$(\alpha,\gamma)^7$Be cross section, including those of Holmgren and Johnston [32], Parker and Kavanagh [33], Nagatani *et al.* [34], Kräwinkel *et al.* [35], Robertson *et al.* [36], Volk *et al.* [37], Alexander *et al.* [38], Osborne *et al* [39], and Hilgemeier *et al.* [40]. With the exception of Ref. [32], these measurements were all considered in the Adelberger *et al.* review [6], which recommended $S_{34}(0) = 0.53 \pm 0.05$ keV b. This review noted a 2.5σ discrepancy between the weighted averages of the measurements based on prompt γ ray detection and those based on induced ^{7}Be activity. At the time of this evaluation, we have the benefit of several subsequent, independent ^{7}Be activity measurements as well as two prompt γ ray detection studies. These new measurements employed improved detectors, better background suppression, and underground accelerators. Moreover, they were published almost two decades after the last of the measurements considered in Ref. [6]. We opt therefore to restrict this analysis to modern experimental data, namely the recent measurements of Bemmerer *et al.* [41], Brown *et al.* [42], Confortola *et al.* [43], Gyürky *et al.* [44], and Nara Singh *et al.* [45]. As the data of Ref. [41] are included in the data of Ref. [44], we will refer henceforth only to the latter. The data considered here are shown in Table I.

III. EVALUATION

Several papers have discussed the energy dependence of nonresonant low energy radiative capture cross sections [21, 31, 46]. When reactions are dominated by external capture, the astrophysical S factor exhibits a subthreshold pole of the form

$$S(E) \propto \frac{1}{E+Q}, \tag{1}$$

where E is the relative energy and Q is the energy released by the reaction. This pole affects the convergence of the usual Maclaurin series expansions and the shape of cross sections at low energy, resulting in an upturn as $E \to 0$. If one wants better convergence, this subthreshold pole must be taken into account, expanding the quantity $(E+Q)S(E)$.

As shown in Ref. [47], a completely model independent approach such as a Maclaurin series expansion does not work well when there are large discrepancies among data sets. In such cases one requires a physical constraint on the shape of the S factor. Since there have been many studies of the ^{3}He$(\alpha,\gamma)^7$Be reaction, we can constrain the form of the fitting function beyond just a Maclaurin series using known physics. For instance, this radiative capture is dominated by $E1$ transitions at low energies [22]. Therefore, given the spin of nuclei involved, only $\ell = 0$ and $\ell = 2$ incoming partial waves contribute significantly to the external capture.

TABLE I: The modern data used in this evaluation. Shown are the S factors for capture into the ground (S_0) and first excited state (S_1) individually, or the sum of the two contributions (S_{tot}). Relative systematic errors for prompt measurements (δ_p), activity measurements (δ_a), and those that are common to the two methods (δ_c) are shown separately.

	Brown *et al.* [42]	
$\delta_p = 0.0387$[a]	$\delta_a = 0.030$	$\delta_c = 0.027$
	Prompt	
$E = 0.3274$	$S_0^{(p)} = 0.349 \pm 0.012$	$S_1^{(p)} = 0.143 \pm 0.007$
$E = 0.4260$	$S_0^{(p)} = 0.311 \pm 0.004$	$S_1^{(p)} = 0.126 \pm 0.002$
$E = 0.5180$	$S_0^{(p)} = 0.302 \pm 0.005$	$S_1^{(p)} = 0.119 \pm 0.002$
$E = 0.5815$	$S_0^{(p)} = 0.280 \pm 0.007$	$S_1^{(p)} = 0.118 \pm 0.002$
$E = 0.7024$	$S_0^{(p)} = 0.268 \pm 0.005$	$S_1^{(p)} = 0.114 \pm 0.002$
$E = 0.7968$	$S_0^{(p)} = 0.260 \pm 0.002$	$S_1^{(p)} = 0.111 \pm 0.001$
$E = 1.2337$	$S_0^{(p)} = 0.227 \pm 0.002$	$S_1^{(p)} = 0.100 \pm 0.001$
$E = 1.2347$	$S_0^{(p)} = 0.236 \pm 0.002$	$S_1^{(p)} = 0.104 \pm 0.001$
	Activity	
$E = 0.3274$	$S_{tot}^{(a)} = 0.495 \pm 0.015$	
$E = 0.4260$	$S_{tot}^{(a)} = 0.458 \pm 0.010$	
$E = 0.5180$	$S_{tot}^{(a)} = 0.440 \pm 0.010$	
$E = 0.5815$	$S_{tot}^{(a)} = 0.400 \pm 0.011$	
$E = 0.7024$	$S_{tot}^{(a)} = 0.375 \pm 0.010$	
$E = 0.7968$	$S_{tot}^{(a)} = 0.363 \pm 0.007$	
$E = 1.2337$	$S_{tot}^{(a)} = 0.330 \pm 0.006$	
$E = 1.2347$	$S_{tot}^{(a)} = 0.324 \pm 0.006$	
	Confortola *et al.* [43]	
$\delta_p = 0.038$	$\delta_a = 0.032$	$\delta_c = 0.023$
	Prompt	
$E = 0.0933$	$S_0^{(p)} = 0.3819 \pm 0.0170$	$S_1^{(p)} = 0.1451 \pm 0.0058$
$E = 0.1061$	$S_0^{(p)} = 0.3661 \pm 0.0132$	$S_1^{(p)} = 0.1519 \pm 0.0046$
$E = 0.1701$	$S_0^{(p)} = 0.3599 \pm 0.0076$	$S_1^{(p)} = 0.1501 \pm 0.0026$
	Activity	
$E = 0.0929$	$S_{tot}^{(a)} = 0.534 \pm 0.016$	
$E = 0.1057$	$S_{tot}^{(a)} = 0.493 \pm 0.015$	
$E = 0.1695$	$S_{tot}^{(a)} = 0.507 \pm 0.010$	
	Gyürky *et al.* [44]	
	$\delta_a = 0.031$	
	Activity	
$E = 0.1056$	$S_{tot}^{(a)} = 0.516(1 \pm 0.052)$	
$E = 0.1265$	$S_{tot}^{(a)} = 0.514(1 \pm 0.020)$	
$E = 0.1477$	$S_{tot}^{(a)} = 0.499(1 \pm 0.017)$	
$E = 0.1689$	$S_{tot}^{(a)} = 0.482(1 \pm 0.020)$	
	Nara Singh *et al.* [45]	
	$\delta_a = 0.0363$[b]	
	Activity	
$E = 0.4200$	$S_{tot}^{(a)} = 0.420 \pm 0.014$	
$E = 0.5060$	$S_{tot}^{(a)} = 0.379 \pm 0.015$	
$E = 0.6145$	$S_{tot}^{(a)} = 0.362 \pm 0.010$	
$E = 0.9500$	$S_{tot}^{(a)} = 0.316 \pm 0.006$	

[a] This combines the 3.5% systematic uncertainty with the 1% varying systematic and 1.3% energy uncertainties.
[b] The effective systematic error is derived from the energy dependent systematic errors with $\delta_{eff}^{-2} = \sum_i \delta_i^{-2}/N$.

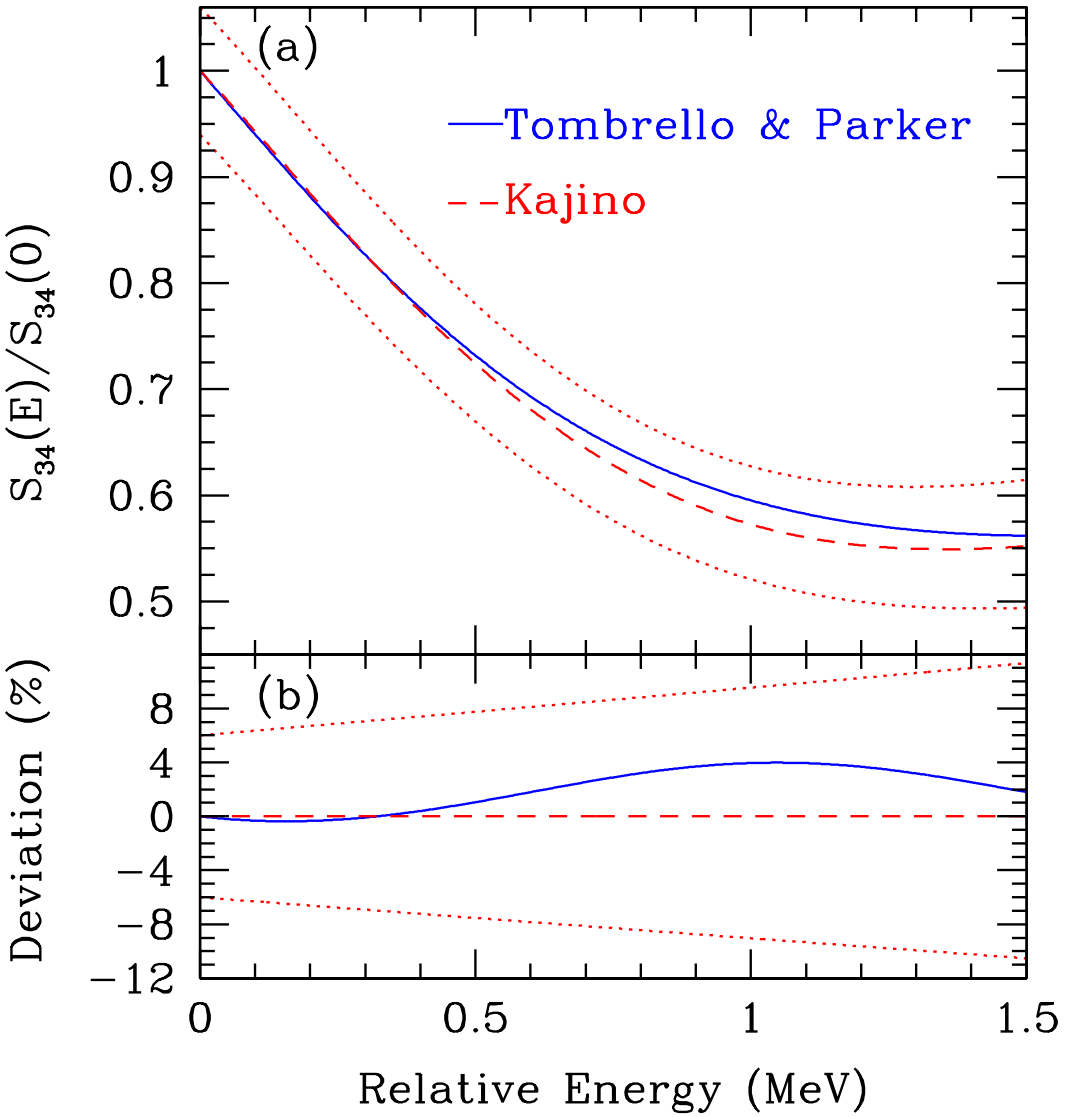

FIG. 1: (Color online) (a) Theoretical S_{34} calculations of Tombrello and Parker [22] (solid) and of Kajino [27] (dashed) are plotted relative to their values at $E = 0$. (b) The deviation relative to the Kajino calculation. In both plots, the dotted lines delineate the uncertainty due to the ± 6% theoretical uncertainty in the zero-energy S factor and logarithmic derivative estimated in Ref. [25].

Following Ref. [48], we expand the remaining terms accounting for Coulomb and pole effects and find

$$S(E) = \frac{Q}{E+Q}\left[s_0(1+aE+\cdots)^2 + s_2(1+4\pi^2 E/E_G)(1+16\pi^2 E/E_G)(1+cE+\cdots)^2\right]. \quad (2)$$

Here, E_G is the Gamow energy given numerically by $E_G = 0.97913 Z_1^2 Z_2^2 A$ MeV where $Z_1 e$ and $Z_2 e$ are the charges of the reactants and A their reduced mass in atomic mass units. For S_{34}, $E_G = 26.9437$ MeV. s_0 and s_2 are the amplitudes of the s and d wave $E1$ capture components respectively, whilst the terms with the coefficients a and c are

higher order contributions. At leading order ($\propto E^0$) inside the square brackets, only s_0 contributes, with $s_2, a, c \equiv 0$. At next-to-leading order ($\propto E^2$), s_2 and a are also finite. This ordering scheme does not preclude c from being finite, but it does allow us systematic control over which terms to include given the data.

The ^{3}He$(\alpha,\gamma)^7$Be reaction can proceed via capture into either of the spin-orbit partners, the ^{7}Be ground state ($Q = 1.5861$ MeV) and the first excited state at 429 keV ($Q = 1.1570$ MeV). When determining best fits and zero-energy extrapolations, one must bear this in mind and use experiments sensitive to not only the total capture cross section, but also the partial capture cross sections into the ground and first excited states. We are therefore left with 3 parameters for the S factor fits for each capture (i.e. ground or excited state).

In general, modern experiments are dominated by systematic errors. As such, standard statistical treatments used to combine data from different, possibly discrepant experiments are not valid. The traditional method of scaling the error in the mean by $\sqrt{\chi^2/\nu}$, where ν is the number of degrees of freedom, has been shown to poorly estimate the error when systematics dominate the error budget [15, 49, 50]. Such treatments lead to an underestimation of the true uncertainties. We require a more reliable prescription for this analysis.

If one assumes that the dominant systematic error is the overall normalization error, several techniques can be used to estimate the true uncertainty. It was shown in Ref. [15] that a simple robust measure of systematic uncertainty is determined by the weighted dispersion of the fit. In another approach, d'Agostini [49] suggests floating the normalizations of individual experiments weighted by their uncertainties as a way of compensating for the shortcomings of the standard statistical approach. This method works well, and agrees with the error estimate of Ref. [15]. In the modern era, the size of systematic errors is often greater than or equal to the size of statistical errors. Hence systematic errors must be treated properly in order to obtain a reliable central value and uncertainties .

We use a χ^2 minimization procedure to determine the parameters of our calculated S_{34} that best fit the data. We break up the χ^2 into two components, χ^2_{data} and χ^2_{norm}, such that $\chi^2 = \chi^2_{data} + \chi^2_{norm}$. Thus for a single data set n,

$$\chi^2_{data}(n) = \sum_i \left(\frac{S(E_i) - \alpha_n S_i}{\alpha_n \sigma_i} \right)^2 \text{ and} \tag{3}$$

$$\chi^2_{norm}(n) = \left(\frac{\alpha_n - 1}{\delta_n} \right)^2 . \tag{4}$$

Here, α_n and δ_n are the floating renormalization factor and the normalization uncertainty of the dataset n, $S(E_i)$ is the theoretically calculated S factor at E_i, S_i is the measured S factor at E_i, and σ_i is the standard deviation of the measured S factor at E_i.

In the case where two experiments have correlated normalizations, χ^2_{norm} is modified and becomes

$$\chi^2_{norm}(n1, n2) = \frac{\left[\delta^2_{n2}(\alpha_{n1} - 1)^2 - 2\delta^2_c(\alpha_{n1} - 1)(\alpha_{n2} - 1) + \delta^2_{n1}(\alpha_{n2} - 1)^2\right]}{[\delta^2_{n1}\delta^2_{n2} - \delta^4_c]} \tag{5}$$

where δ_{n1} and δ_{n2} are the total systematic errors of the data sets $n1$ and $n2$ and δ_c is the systematic error common to both data sets. This equation is the result of inverting the covariance matrix.

We use a Markov Chain Monte Carlo (MCMC) approach to evaluate the data, following the two-step procedure laid out by Ando *et al.* [51]. First, we find the best fit by varying the parameters randomly, with ever decreasing step sizes until the results converge to a predetermined number of significant figures. Then, with step sizes determined by $\Delta\chi^2 = 1$ variations, we start a random walk away from the best fit point using the Metropolis algorithm [52, 53]. Once the sample variance of each of the parameters converges within the specified resolution, we stop the MCMC. We tested convergence with chains of length up to 10^8, finding convergence after about 10^6 steps. We use the 10^8 step chain for our final results.

IV. RESULTS

To test the validity of the adopted three parameter S_{34} model, we found the best fits for both two and four parameter models as well, fixing $a = 0$ for the two parameter fit and letting both a and c vary for the four parameter fit. We find best fits with $\chi^2_{tot} = 85, 63, 61$ or $\chi^2_{tot}/d.o.f. = 2.74, 2.17, 2.26$ for the two, three, and four parameter fits, respectively, where the number of degrees of freedom $d.o.f. = 31$, 29, 27. One can see readily that given the modern data considered here, we can not adequately determine more than 3 parameters. Also, the two parameter fit is clearly unrepresentative of the data, whilst the quality of the four parameter fit is no better than that of the three parameter fit. Hence including a fourth parameter is unwarranted. The uniqueness of the solution was tested by starting the minimization procedure from different parameter values, finding the same minimum.

Parameter	Mode	Mean ± Std. Dev.	Norm. Error
α_p(Brown)	0.95	0.95 ± 0.02	0.0387
α_a(Brown)	0.95	0.95 ± 0.02	0.030
α_p(Confortola)	0.99	0.99 ± 0.02	0.038
α_a(Confortola)	1.01	1.01 ± 0.02	0.032
α_a(Gyürky)	1.02	1.02 ± 0.02	0.031
α_a(Nara Singh)	1.04	1.04 ± 0.02	0.0363
s_0(gs)	0.406	0.406 ± 0.009	
s_2(gs)	0.007	0.007 ± 0.001	
a(gs)	-0.207	-0.203 ± 0.038	
s_0(ex)	0.163	0.163 ± 0.004	
s_2(ex)	0.004	0.004 ± 0.001	
a(ex)	-0.134	-0.127 ± 0.055	
χ^2_{norm}	4.83	6.48 ± 2.02	$N_{norm} = 6$
χ^2_{data}	58.38	69.06 ± 4.70	$N_{data} = 41$
χ^2_{tot}	63.21	75.54 ± 4.95	$N_{par} = 6$

TABLE II: Results of the Markov Chain Monte Carlo parameter estimation.

After the best fit is found by minimizing χ^2 and the MCMC has converged, we find the limits of central confidence intervals by determining parameter values that obey the relation $P(\nu/2, \Delta\chi^2/2) = CL$ [54]. Here $P(a,x) = \gamma(a,x)/\Gamma(a) = 1 - \Gamma(a,x)/\Gamma(a)$ is the regularized incomplete gamma function [55], $\nu = N_{par} + N_{norm}$ is the total number of varied parameters and normalizations, $\Delta\chi^2 = \chi^2 - \chi^2_{min}$, and CL is the desired confidence level.

The results of this analysis are presented in Table II and Figure 2. We find a mode (most likely value) and central 68.3% CL interval of $S_{34}(0) = 0.580 \pm 0.043$ keV b ($\Delta S_{34}(0)/S_{34}(0) = 7.4\%$). We find a mean and standard deviation of $S_{34}(0) = 0.580 \pm 0.013$ keV b ($\Delta S_{34}(0)/S_{34}(0) = 2.2\%$). The small size of the interval given by the mean ± the standard deviation compared to the central 68.3% CL results from marginalizing over all parameter distributions. The standard deviation here corresponds to defining errors with $\Delta\chi^2 = 1$, known to underestimate uncertainties when there is more than one fitted parameter [13, 54]. The central 95.4% confidence interval is $S_{34}(0) = 0.580 \pm 0.054$ keV b. The size of this relative uncertainty is slightly smaller than that given in the evaluation of Adelberger *et al.* [6], which was a 1σ error. Hence the modern data considered here permit a considerably more precise recommendation for $S_{34}(0)$, even when allowing the shape of the S factor to be determined by the data rather than a theoretical model and when fitting the branching ratio as well as the total S factor.

To test the robustness of our procedure, we compare the uncertainty determined here to the discrepancy and normalization errors defined in Reference [15]. Using the definitions

$$\delta^2_{disc} = \frac{\sum_n \chi^2_n}{\sum_n f^2_n} \text{ and} \tag{6}$$

$$\delta^2_{norm} = \frac{\sum_n \frac{N_n}{\chi^2_n}\delta^2_n}{\sum_n \frac{N_n}{\chi^2_n}}, \tag{7}$$

where $\chi^2_n = \chi^2_{data}(n)$ with renormalization $\alpha_n = 1$, $f^2_n = \sum_{i_n}(S^{(i_n)}_{tot}/\sigma_{i_n})^2$, N_n and δ_n are the number of data points in and total normalization error of set n, we find a discrepancy error of 5.3% and a normalization error of 3.5%, yielding a total systematic uncertainty of 6.4%. When added in quadrature to the 2.2% statistical error, we find a total uncertainty of 6.8%, in very good agreement with the present 7.4% MCMC results. The difference between the two arises because the former takes into account neither discrepancies in the branching ratio S_1/S_0 nor correlated normalization errors, as the MCMC method does. The MCMC results for the branching ratio are shown in Figure 3. The s- and d-wave contributions to the ground and excited state transitions determined in the MCMC analysis are shown in Figure 4.

We have calculated the product of the Avogadro constant and the thermally averaged rate of the ^{3}He(α,γ)^{7}Be

Parameter	Low	Adopted	High
a_0	15.531721	15.609867	15.679639
n_1	-0.100208	-0.020478	0.037757
n_2	0.235187	0.211995	0.196645
d_1	0.114322	0.255059	0.353050
d_2	0.373802	0.338573	0.316019

TABLE III: Shown are the fit parameters for the product of the Avogadro constant and the recommended thermal rate $N_A\langle\sigma v\rangle$ and its central 68.3% confidence level limits, with $a_2 = -12.82707707$, $a_6 = -2/3$, and $a_0 = \ln[1.03762 \times 10^7 S_{34}(0)\mathrm{keV}^{-1}\mathrm{b}^{-1}]$ fixed.

reaction per particle pair and fit it using the form

$$N_A\langle\sigma v\rangle = \exp\left[a_0 + a_2/T_9^{1/3} + a_6 \ln T_9\right] \frac{\left(1 + n_1 T_9^{2/3} + n_2 T_9^{4/3}\right)}{\left(1 + d_1 T_9^{2/3} + d_2 T_9^{4/3}\right)} \mathrm{cm}^3\ \mathrm{mol}^{-1}\ \mathrm{s}^{-1}. \quad (8)$$

The fit parameters are shown in Table III. This functional form provides a better fit to the reaction rate than the fitting functions of Caughlan and Fowler [56] and REACLIB [57, 58] and agrees with the numerically calculated rate within 0.5%.

Figure 5 compares the energy dependence of S_{34} determined here from a nearly model-independent analysis of modern data with that calculated by Kajino *et al.* [27]. It is apparent that in the energy range of principal interest below 500 keV, the shapes of the two curves differ substantially. Extrapolation using the RGM model of Kajino from the energy range important for BBN to that of interest in the Sun would differ by some 5% from extrapolation based on the S factor determined here. This is quite consistent with the estimates of the theoretical error given in Ref. [25].

V. CONCLUSIONS

We have performed a minimally model dependent analysis of modern ^{3}He(α,γ)^{7}Be data. This analysis properly takes into account the systematic errors of potentially discrepant data and yields reliable central values and uncertainties. At the 68.3% confidence level, we find $S_{34}(0) = 0.580 \pm 0.043$ keV b and $S'(0)/S(0) = -0.92 \pm 0.18$ MeV^{-1}. We have used this value of $S_{34}(0)$ to compute the thermally averaged rate of the ^{3}He(α,γ)^{7}Be reaction per particle pair, and have fit this rate with an analytical form accurate within 0.5%.

This reaction rate is about 9% higher than the recommendation of [6], but is quite consistent with it. It is also perfectly consistent with the recommendations of References [59, 60]. Moreover, access to precise modern data has allowed us to minimize the uncertainties associated with extrapolation and arrive at a more precise and better founded value that does not depend on nuclear structure models.

Our recommended reaction rate implies that the predicted ^{7}Be and ^{8}B solar neutrino fluxes should both be increased by 8% compared with predictions based on the $S_{34}(0)$ value from Ref. [6]. Similarly, the present analysis implies that the value of S_{34} in the energy range relevant to big bang nucleosynthesis is 17% larger than the value adopted in Ref. [15], resulting in a 17% increase in the predicted primordial ^{7}Li abundance. The uncertainties found here reduce the total error in the ^{7}Li abundance prediction by a factor of ~ 2. The discrepancy between the predicted and observed primordial ^{7}Li abundances increases from $\sim 3\sigma$ to $\sim 5\sigma$ [61], implying a rather serious disagreement that must be resolved.

In order to improve our knowledge of the rate of the ^{3}He(α,γ)^{7}Be reaction, future experiments should collect data at at least six different energies. No modern data exist in the Gamow window for big bang nucleosynthesis, so this would be of primary interest. Information on capture into the ground and first excited states would also be useful, arguing in favour of prompt γ ray measurements rather than those based on induced ^{7}Be activity. Finally, precise measurements from 1 - 3 MeV would allow better constraints on the shape of the S factor at the low energies of astrophysical interest.

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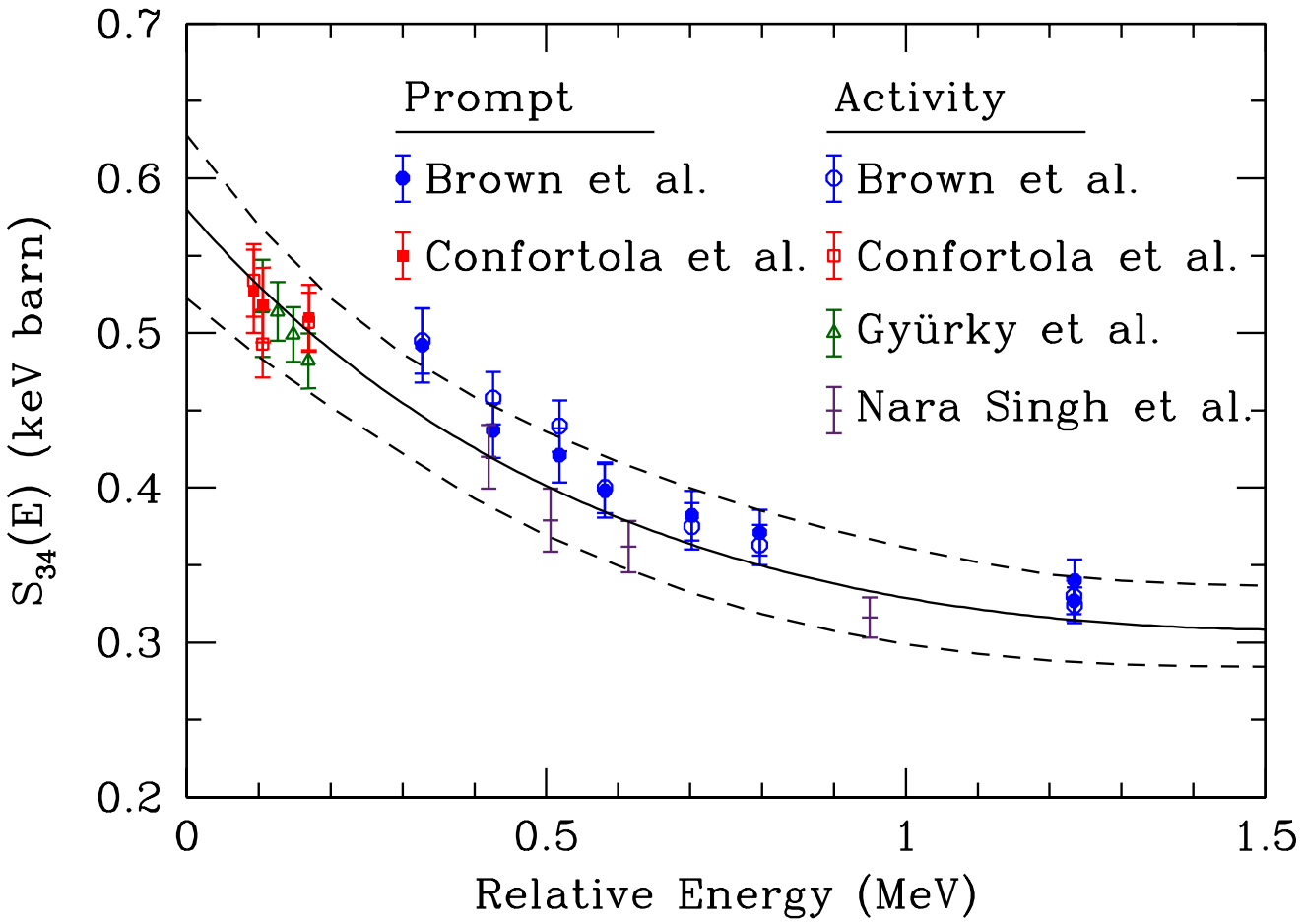

FIG. 2: (Color online) The best fit S factor for the ^{3}He(α,γ)^{7}Be reaction (solid curve). Also shown are the central 68.3% confidence level limits (dashed curves). The experimental data and their total uncertainties are plotted (Activity-open points, Prompt-solid points).

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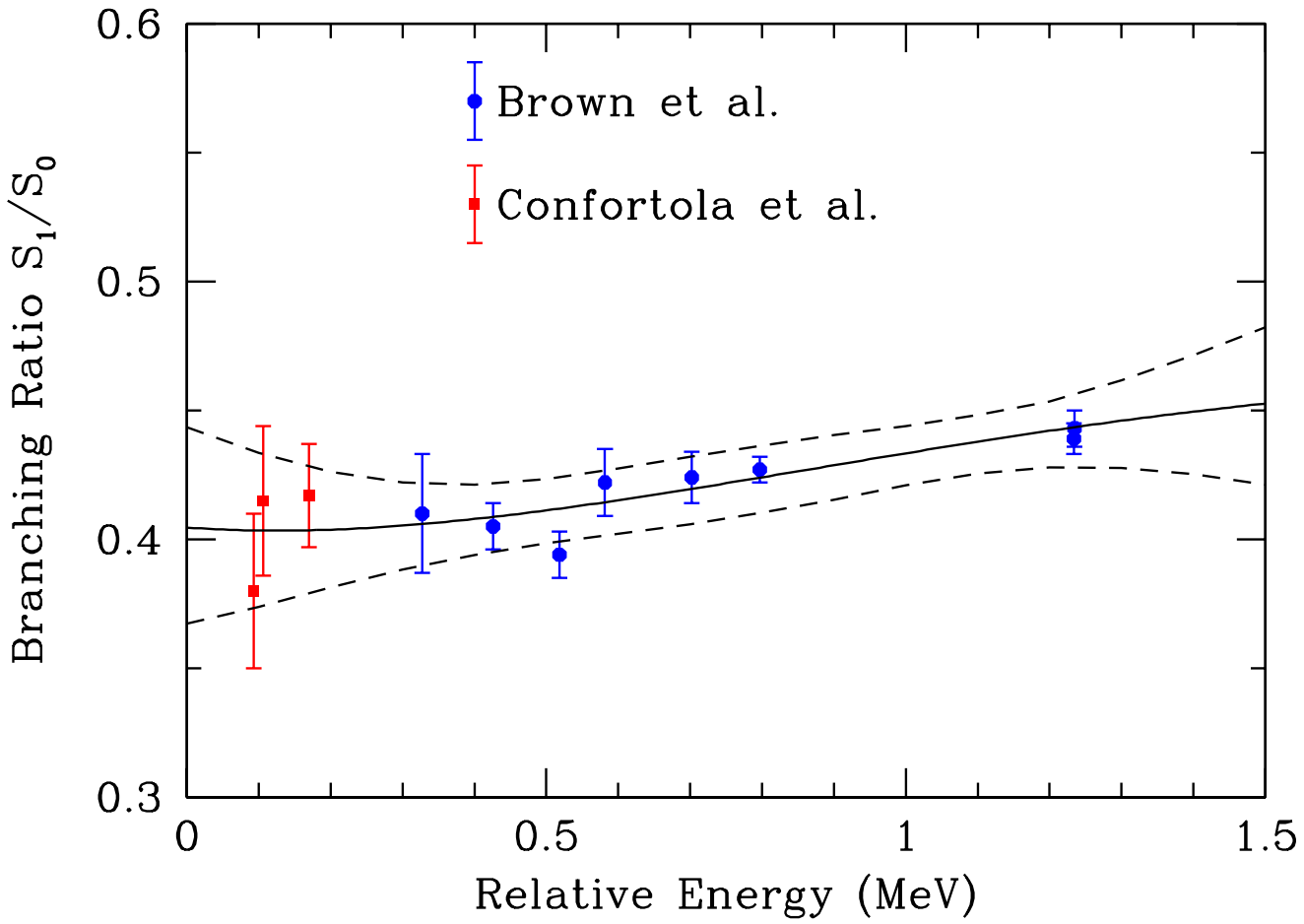

FIG. 3: (Color online) The best fit branching ratio S_1/S_0 for the ^{3}He(α,γ)^{7}Be reaction (solid curve). Also shown are the central 68.3% confidence level limits (dashed curves). The experimental data and their total uncertainties are plotted.

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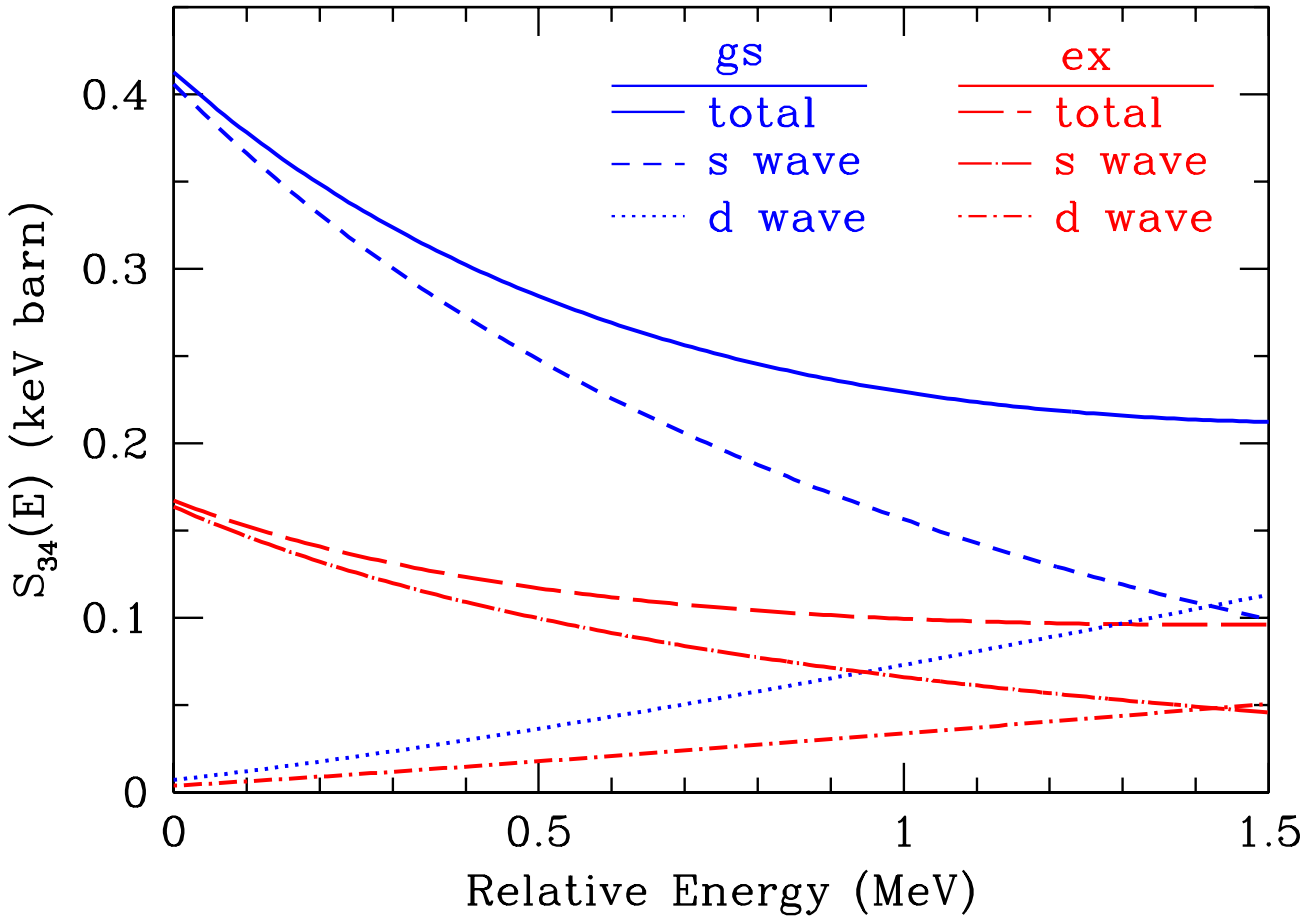

FIG. 4: (Color online) This figure shows the fitted contributions of various partial waves to the S factor. The s-wave (short dashed), d-wave (dotted) and total (solid) S factors for capture into the ground state and the s-wave (long dash-dotted), d-wave (short dash-dotted) and total (long dashed) S factors for capture into the first excited state are all shown.

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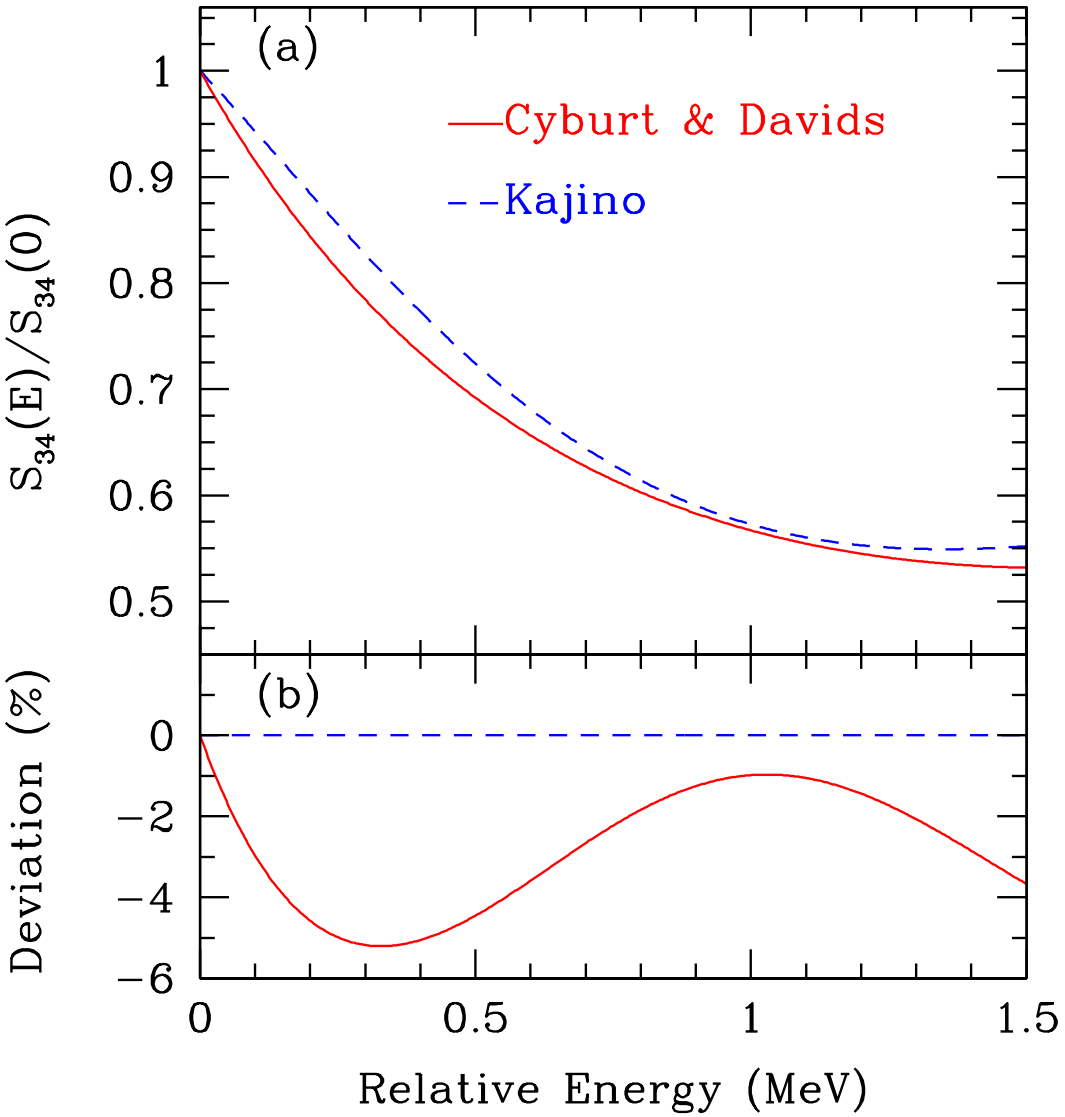

FIG. 5: (Color online) (a) The S_{34} calculation of Kajino [27] (dashed) and of this evaluation (solid) are plotted relative to their values at $E = 0$. (b) The deviation relative to the Kajino calculation.

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