Nuclear
Effective Field Theories

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Supported in part by US DOE and Sloan Foundation

Background by S. Hossenfelder

In Memoriam

Vijay Pandharipande

Outline

- Effective Field Theories ©
- Pionful EFT
- Pionless EFT
- Halo EFT
- Speculations & Conclusion

~ *Wanted* ~

Dead ♦ or ♦ Alive

QCD EXPLANATION OF NUCLEAR PHYSICS

Reward

understanding of gross features:

Why is $B/A \sim 10\,\text{MeV} = M_{QCD} \sim 1\text{GeV}$?

How large are few-nucleon forces?

Why is isospin a good symmetry?

…

Beware

coupling constants not small

Why bother?

Nuclei within the Standard Model

- the simplest complex systems

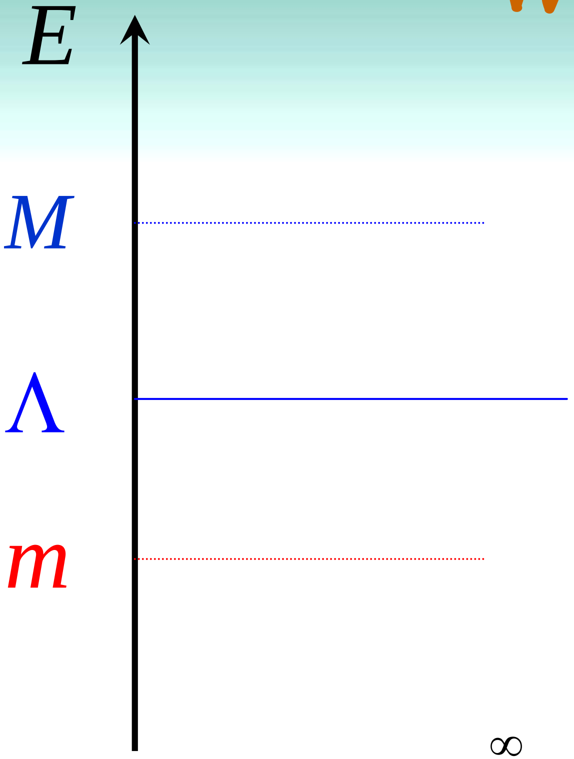
simplest: non-relativistic, shallow, ...

complex: non-perturbative, many-body, ...

- laboratories

neutron targets: nucleon properties
incubators for rare processes: beyond the SM

What is Effective?



$$Z = \int \mathcal{D}\phi_H \int \mathcal{D}\phi_L \exp\left(i \int d^4x \, \mathcal{L}_{und}(\phi_H, \phi_L) \right)$$

$$\times \int \mathcal{D}\varphi \, \delta\left(\varphi - f_\Lambda(\phi_L)\right)$$

$$= \int \mathcal{D}\varphi \, \exp\left(i \int d^4x \, \mathcal{L}_{EFT}(\varphi) \right)$$

$$\mathcal{L}_{EFT} = \sum_{d=0}^{\infty} \sum_{i(d,n)} c_i(M, \Lambda) \, O_i\left((\partial, m)^d \varphi^n \right)$$

renormalization-group invariance $\quad \dfrac{\partial Z}{\partial \Lambda} = 0$

local

underlying symmetries

$$\begin{cases} T = T^{(\infty)}(Q) \sim N(M) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_{i} c_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right) \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

normalization (under $N(M)$)

non-analytic, from loops (under $F_{\nu,i}$)

$$\nu = \nu(d, n, \mathrm{K})$$ "power counting"

↳ e.g. # loops L

For $Q \sim m$, truncate consistently with RG invariance

so as to allow systematic improvement (perturbation theory):

$$T = T^{(\nu_{\max})} + O\left(\frac{Q}{M}\right)^{\nu_{\max}+1} \qquad \Lambda \frac{\partial T^{(\nu_{\max})}}{\partial \Lambda} = O\left(\frac{Q}{\Lambda}\right)^{\nu_{\max}+1}$$

Nuclear physics scales

"His scales are His pride", Book of Job

(according to J. Friar)

$\ln Q$

perturbative QCD

~1 GeV $M_{QCD} \sim m_N,\, m_\rho,\, 4\pi f_\pi,\, \mathrm{K}$ sum ?

hadronic th with chiral symm

~100 MeV $M_{nuc} \sim f_\pi,\, 1/r_{NN},\, m_\pi,\, \mathrm{K}$ sum Q/M_{nuc}

~30 MeV $\aleph \sim 1/a_{NN}$ sum $Q/\aleph$

no small coupling → expansion in Q

Nuclear EFT

pionful EFT

$Q: \; m_\pi = M_{QCD}$

- degrees of freedom: nucleons, pions, deltas (+ roper?, …)

$$m_\Delta - m_N \sim 2m_\pi \; (m_{N'} - m_N \sim 3.5 m_\pi, \ldots)$$

- symmetries: Lorentz, ~~P~~, ~~T~~, ~~chiral~~

- expansion in: $\dfrac{Q}{M_{QCD}}: \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \ldots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$

$$\mathcal{L}_{EFT} = \sum_{\{d,p,f\}} c_{\{d,p,f\}} \left(\frac{\partial, m_\pi, m_\Delta - m_N}{M_{QCD}} \right)^d \left(\frac{\boldsymbol{\pi}}{f_\pi} \right)^p \left(\frac{N^+ N}{f_\pi^2 M_{QCD}} \right)^{f/2} f_\pi^2 M_{QCD}^2 = \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)}$$

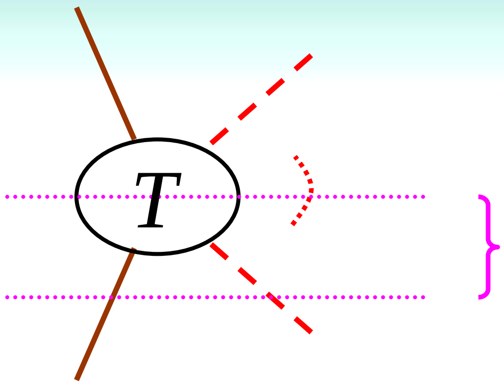
(NDA: naïve dimensional analysis) $= O(1)$ { calculated from QCD: lattice, … / fitted to data }

$$\Delta \equiv d + f/2 - 2 \geq 0$$

chiral symmetry

A= 0, 1: chiral perturbation theory

Weinberg '79
Gasser + Leutwyler '84
...
Gasser, Sainio + Svarc '87
Jenkins + Manohar '91
...



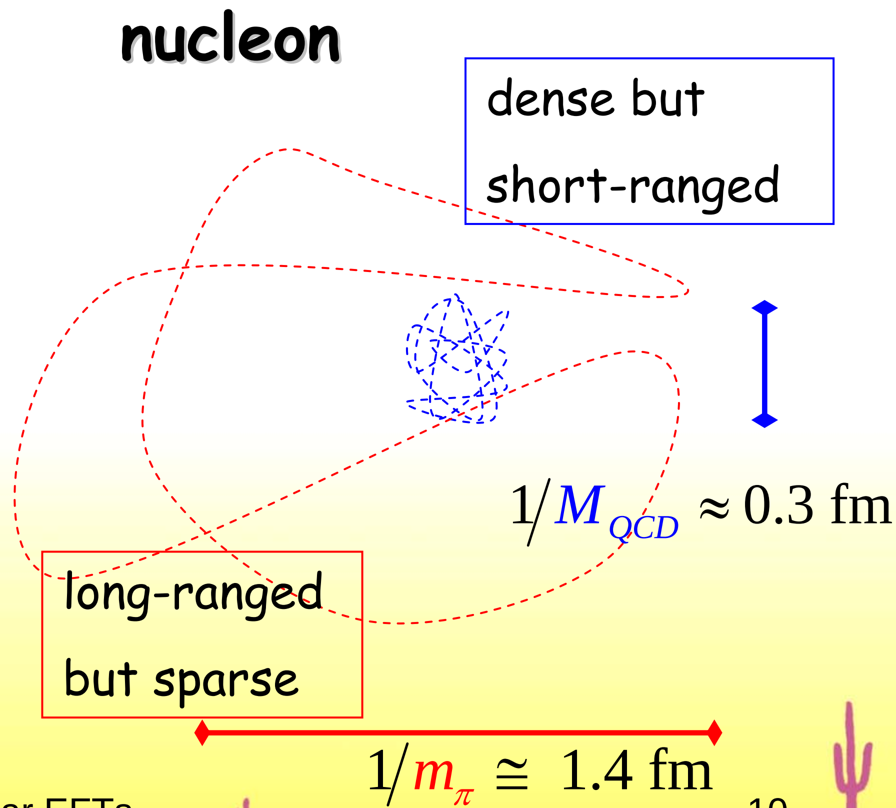
$$\frac{1}{\Delta E} : \frac{1}{Q}$$

$$: \sum_{\nu} c_{\nu} \left(\frac{Q}{M_{QCD}} \right)^{\nu} F_{\nu}\left(\frac{Q}{m_{\pi}} \right)$$

$$\nu = 2 - A + 2L + \sum_i V_i \Delta_i \geq \nu_{\min} = 2 - A$$

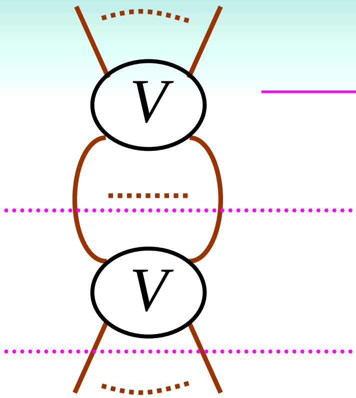
L: # loops

V_i: # vertices of type *i*



A ≥ 2: resummed chiral perturbation theory

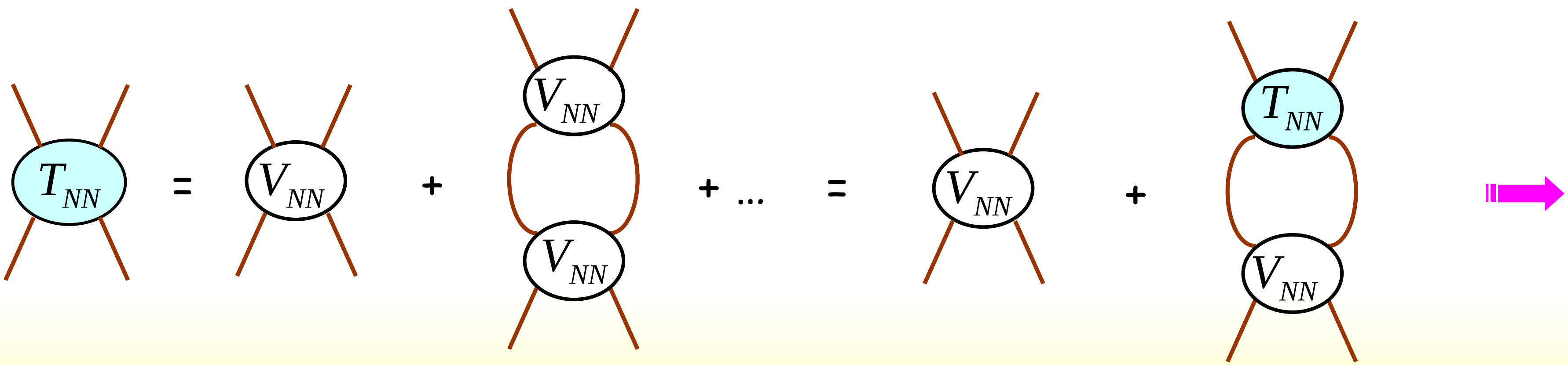
Weinberg '90, '91



A-nucleon irreducible

$\frac{1}{\Delta E} \sim \frac{m_N}{Q^2}$ infrared enhancement!

A-nucleon reducible



$$T_{NN} \sim V_{NN} + V_{NN}\frac{i m_N Q}{4\pi}V_{NN} + \ldots \sim \frac{(4\pi/m_N)}{(4\pi/m_N V_{NN}) - iQ}$$

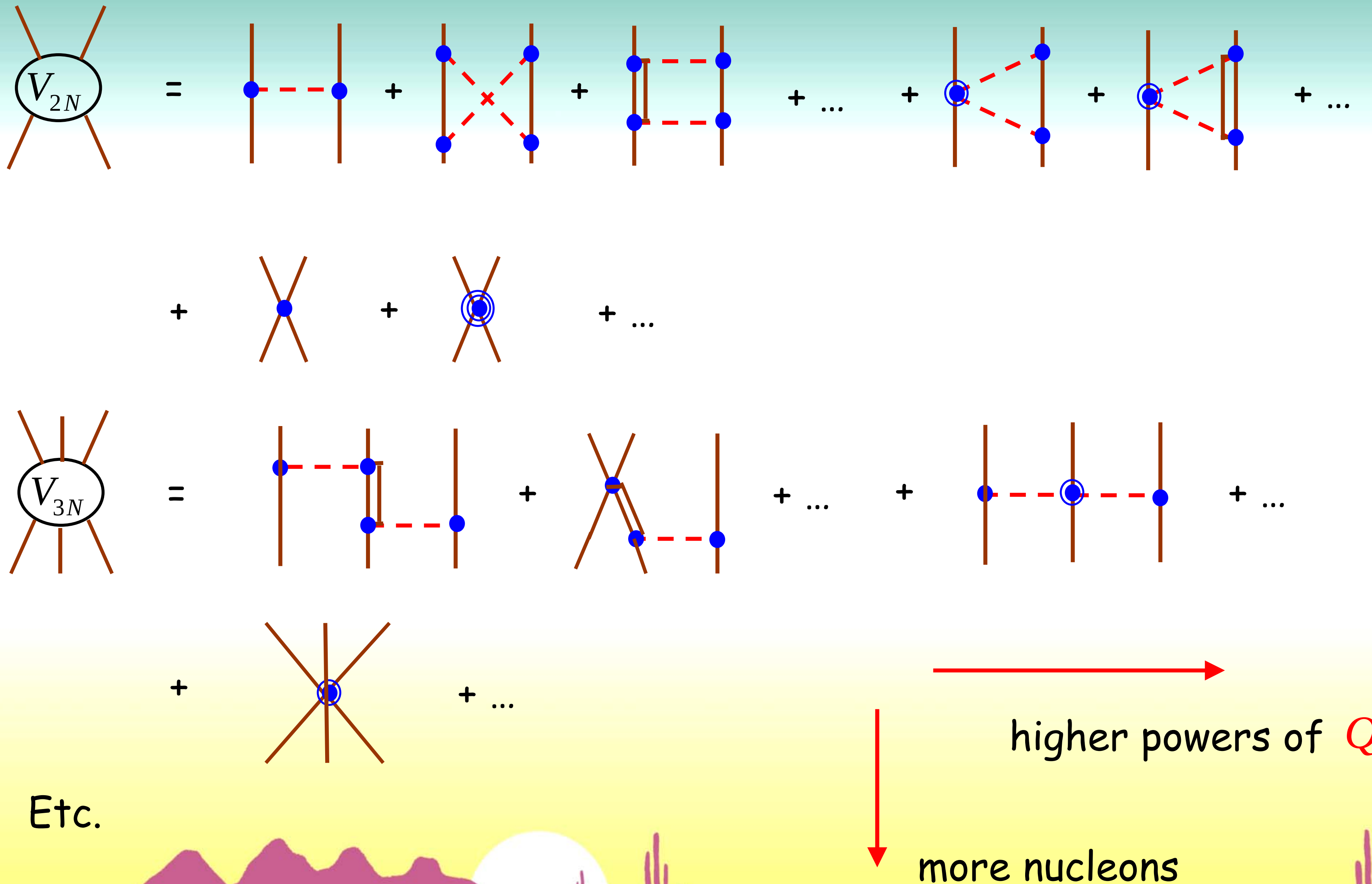
bound state at

$$Q \sim -i\frac{4\pi}{m_N V_{NN}} \equiv i\aleph$$

Ordonez + v.K. '92

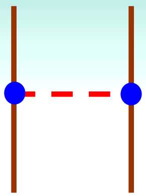
v.K. '94

...

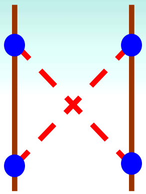


Etc.

Issue: power counting (relative sizes)



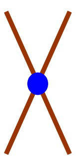
$$= O\left(\frac{1}{f_\pi^2}\right)$$



$$= O\left(\frac{1}{f_\pi^2}\frac{Q^2}{(4\pi f_\pi)^2}\right)$$

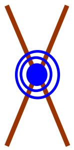
$(4\pi f_\pi) \rightarrow M_{QCD}$

$$M_0 \overset{?}{\longleftrightarrow} f_\pi$$



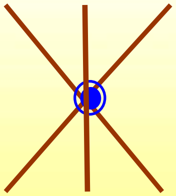
$$\equiv O\left(\frac{4\pi}{m_N M_0}\right)$$

$$M_1 \overset{?}{\longleftrightarrow} M_{QCD}$$



$$\equiv O\left(\frac{4\pi}{m_N M_0}\frac{Q^2}{M_1^2}\right)$$

$$\bar{M} \overset{?}{\longleftrightarrow} M_{QCD}$$



$$\equiv O\left(\frac{(4\pi)^2}{m_N^2 M_0^2 \bar{M}}\right)$$

etc.

Weinberg '90, '91, '92
Ordonez + v.K. '92
v.K. '94
Ordonez, Ray + v.K. '96
...

Naïve Dimensional Analysis

$$M_0 : \frac{4\pi f_\pi}{m_N} f_\pi : f_\pi$$

$$M_1 : M_{QCD}$$

$$\bar{M} : M_{QCD}$$

- LO: S-wave contacts + OPE (non-perturbative pions)
- NLO: P-wave contacts + TPE + 3N forces via delta
- ...

$$\aleph : f_\pi \rightarrow$$

$$B : \frac{f_\pi^2}{m_N} : \frac{f_\pi}{4\pi} \approx 10\,\text{MeV}$$

+ (PUNT) subLOs also iterated in Lippman-Schwinger eq.

⇒ similar to phenomenological potential models: at N2LO,

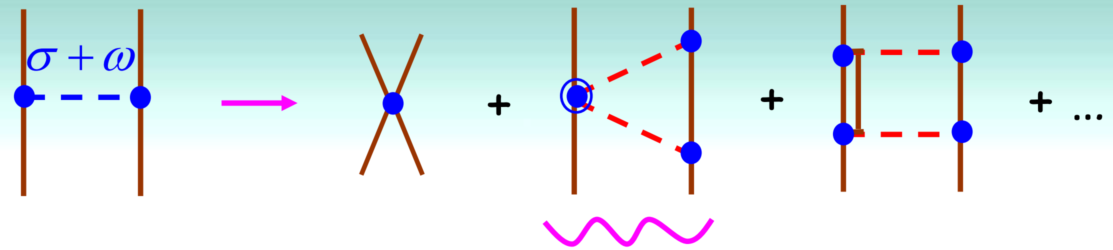
$$[V_{2N}]_{\text{short range+OPE}} : [\text{AV18}]_{\text{short range+OPE}} + \text{non-local terms}$$

$$[V_{3N}]_{\text{long range}} : \text{TM} - \text{c term} \equiv \text{TM'}$$

Note: NOT your usual potential!

Ordonez + v.K. '92
(cf. Stony Brook TPE)

e.g.,



chiral v.d. Waals force : $\frac{1}{r^6}$ for $m_\pi^2 \to 0$

Rentmeester et al. '01, '03

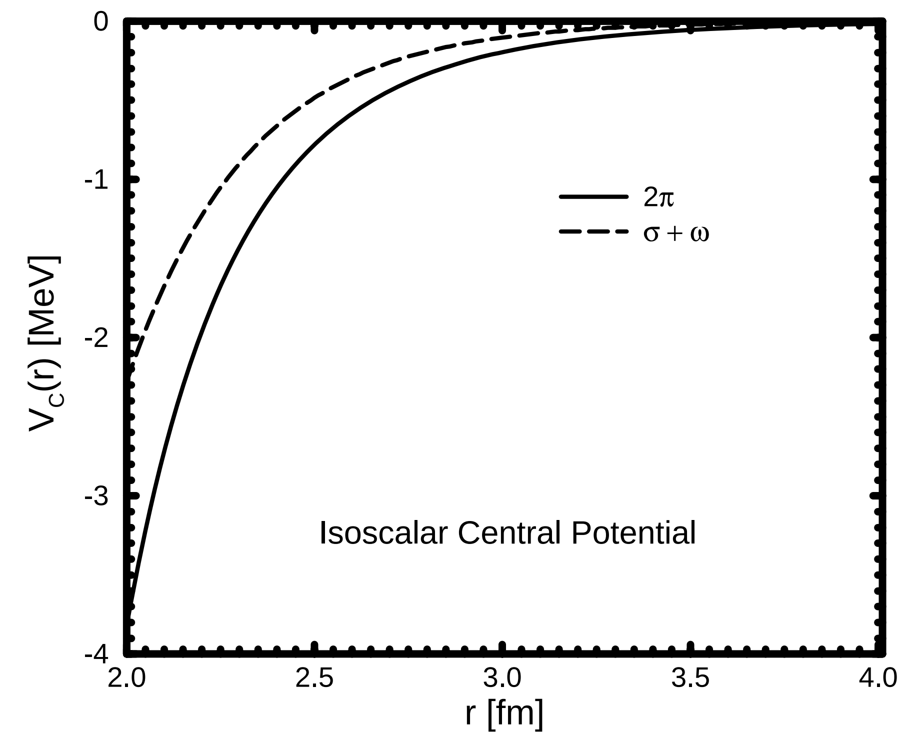
Nijmegen PSA of 1951 *pp* data

long-range pot	#bc	χ^2_{min}
OPE	31	2026.2
OPE + TPE (*lo*)	28	1984.7
OPE + TPE (*nlo*)	23	1934.5
Nijm78	19	1968.7

parameters found consistent with πN data!

at least as good!

Kaiser, Brockmann + Weise '97

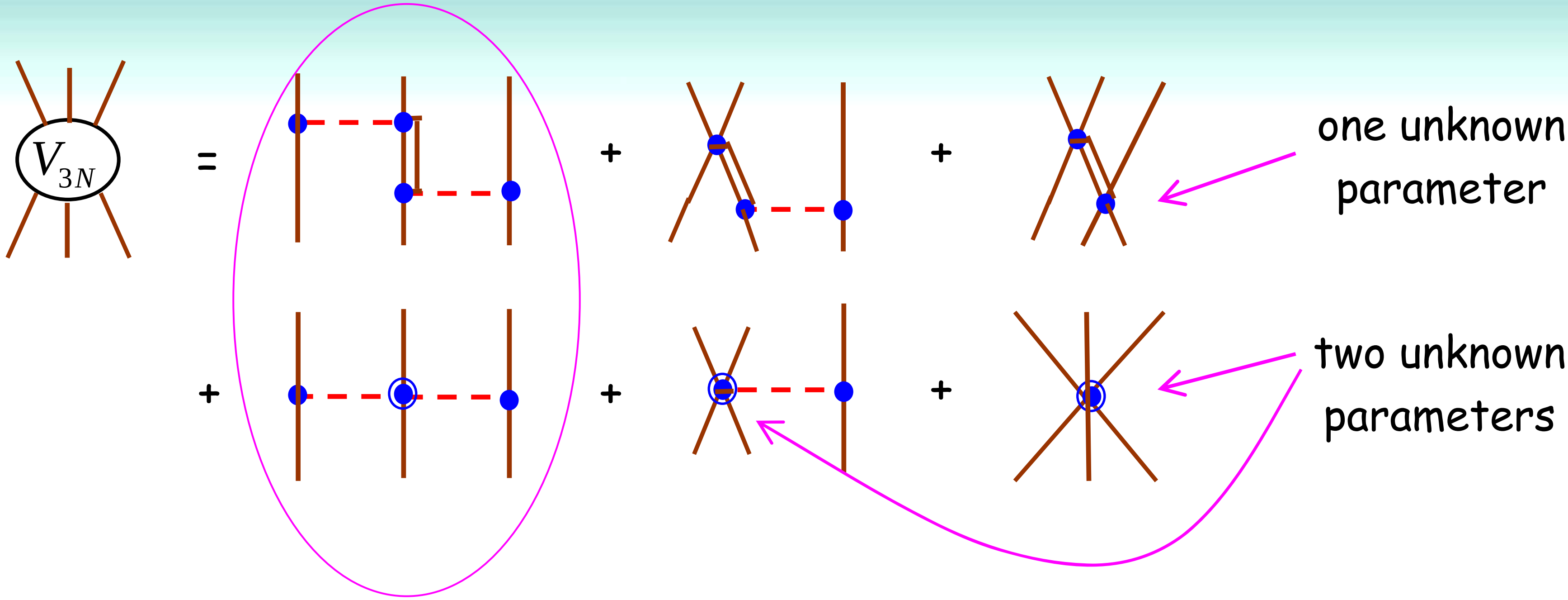


ear EFTs

models with σ, ω, ... might be misleading...

v.K. '94
Friar, Hueber + v.K. '99
Coon + Han '99
...

e.g.,



+ ...

Tucson-Melbourne pot with

$$\begin{cases} a \to a' = a - 2m_\pi^2 c \\ c \to c' = 0 \end{cases}$$

$$\left(t_{\pi N}\left(\vec{q},\vec{q}\,'\right)\right)_{\alpha\beta} = \delta_{\alpha\beta}\left[a + b\,\vec{q}\cdot\vec{q}\,' + c\left(\vec{q}^{\,2} + \vec{q}\,'^{2}\right)\right] - d\,\varepsilon_{\alpha\beta\gamma}\tau_{3\gamma}\,\vec{\sigma}\cdot\vec{q}\times\vec{q}\,' + \ldots$$

Many successes of Weinberg's counting, *e.g.*, in deltaless version

✓ At N3LO, fit to 2N phase shifts comparable to those of "realistic" phenomenological potentials

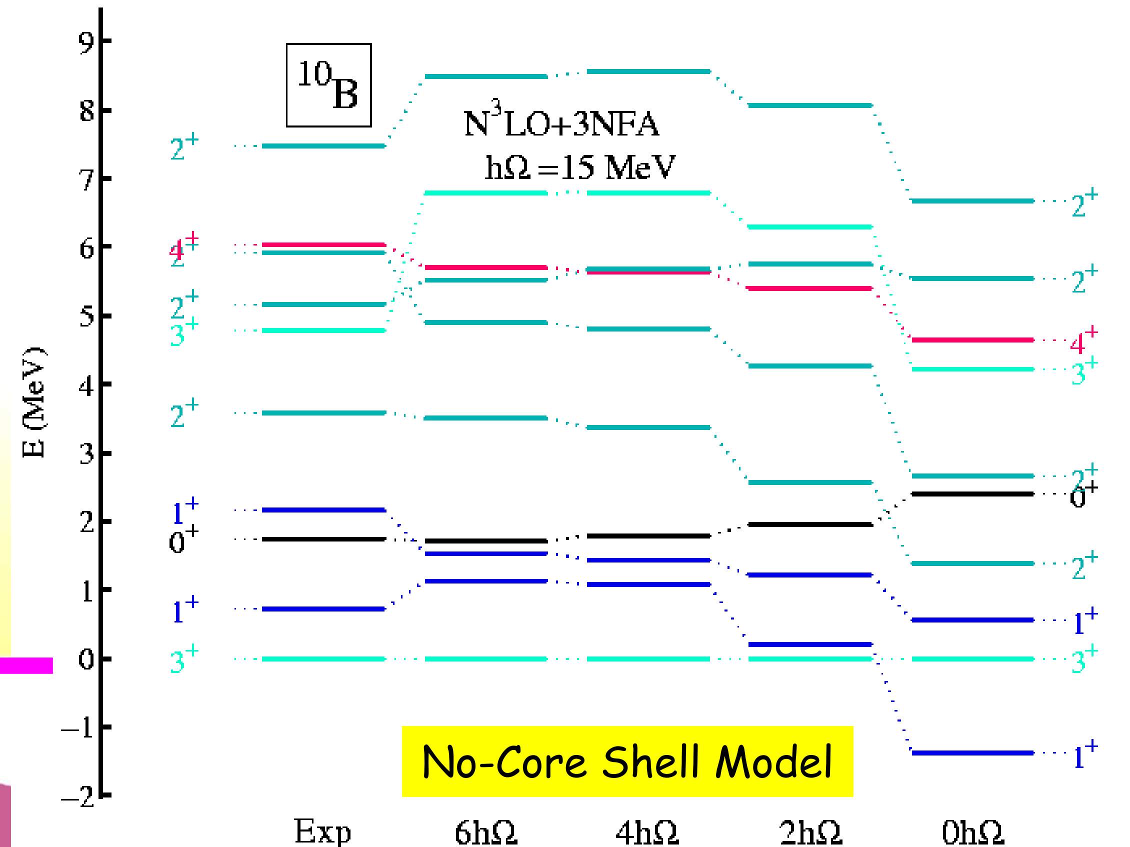
Entem + Machleidt '03…

Epelbaum, Gloeckle + Meissner '04

✓ With N3LO 2N and N2LO 3N potentials, good description of

- 3N observables and 4N binding energy — Epelbaum et al. '02
- levels of p-shell nuclei

Gueorguiev, Navratil, Ormand + Vary '05



Binding Energy (MeV)

Exp: -64.7507(3)

Thy: -64.03*

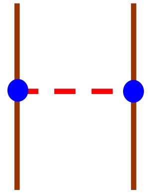
*Convergence study not completed

BUT

Is Weinberg's power counting consistent?

No!

attractive in some channels



$$: \left(\frac{g_A}{2f_\pi}\right)^2 \frac{m_\pi^3}{4\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \frac{S_{12}(\hat{r})}{(m_\pi r)^3} + \mathrm{K} \right\} e^{-m_\pi r}$$

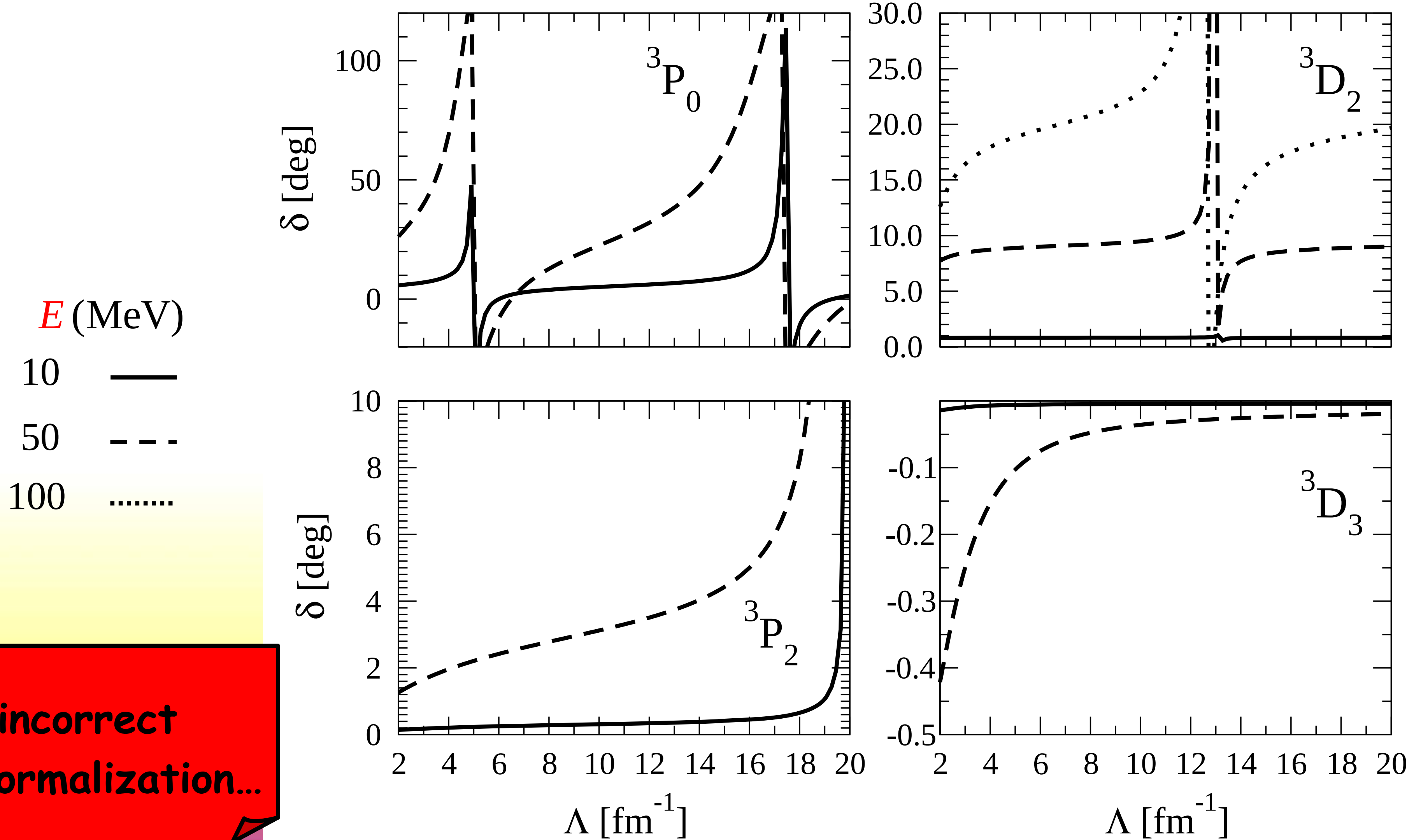
singular potential

not enough contact interactions for renormalization-group invariance even at LO

Nogga, Timmermans + v.K. '05

attractive-tensor channels in LO

cutoff dependence

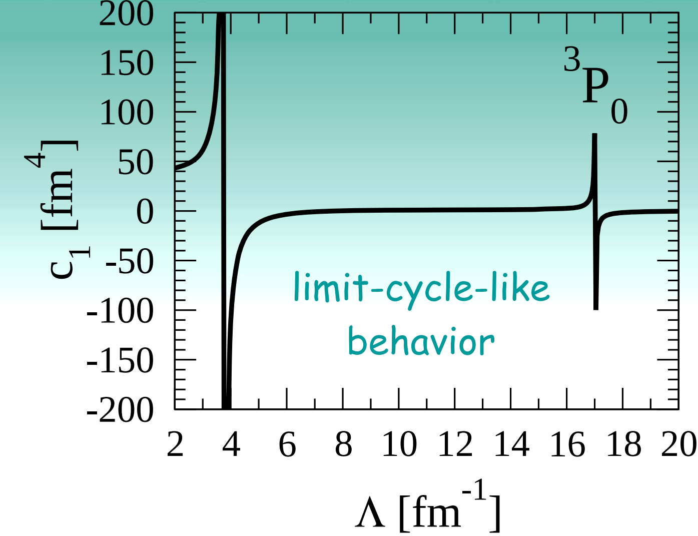


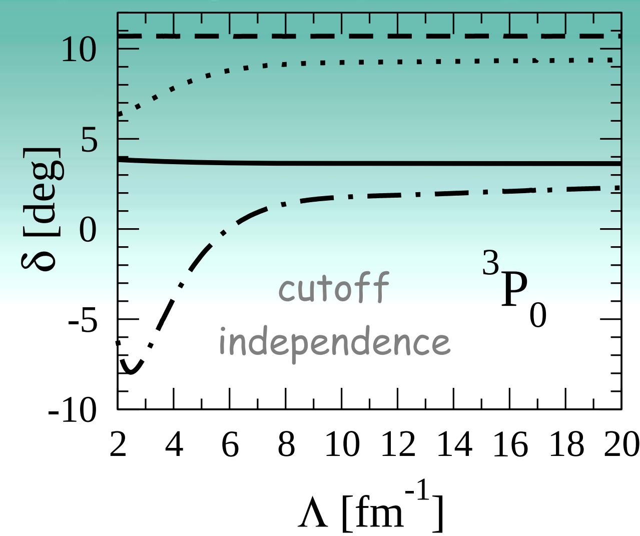
incorrect renormalization…

Promote counterterms

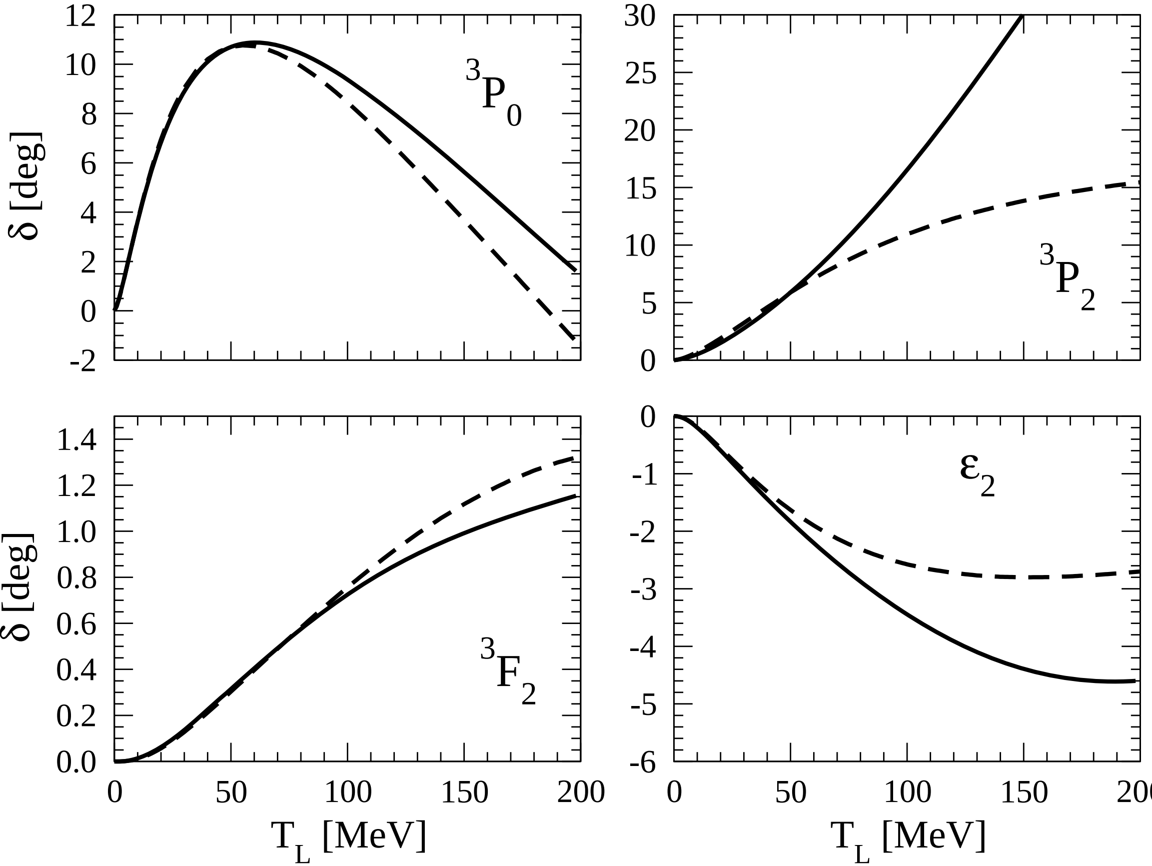
e.g.,

$$V_{l=1,\,j=0} = c_1\, pp'$$





E (MeV)
10 ——
50 – – –
100 ⋯⋯
190 — · —



LO EFT ——
($\Lambda = 20\,\mathrm{fm}^{-1}$)

Nijmegen PSA – – –

Nogga, Timmermans + v.K. '05

$\Rightarrow$ $M_l \sim (l+1) f_\pi$

centrifugal barrier

short-range interactions stronger than in Weinberg's pc for attractive tensor channels where $l <$ few $\left({}^3P_0, {}^3P_2 - {}^3F_2, {}^3D_2 ?!\right)$

c.f. Birse '05

+ subLOs in perturbation theory

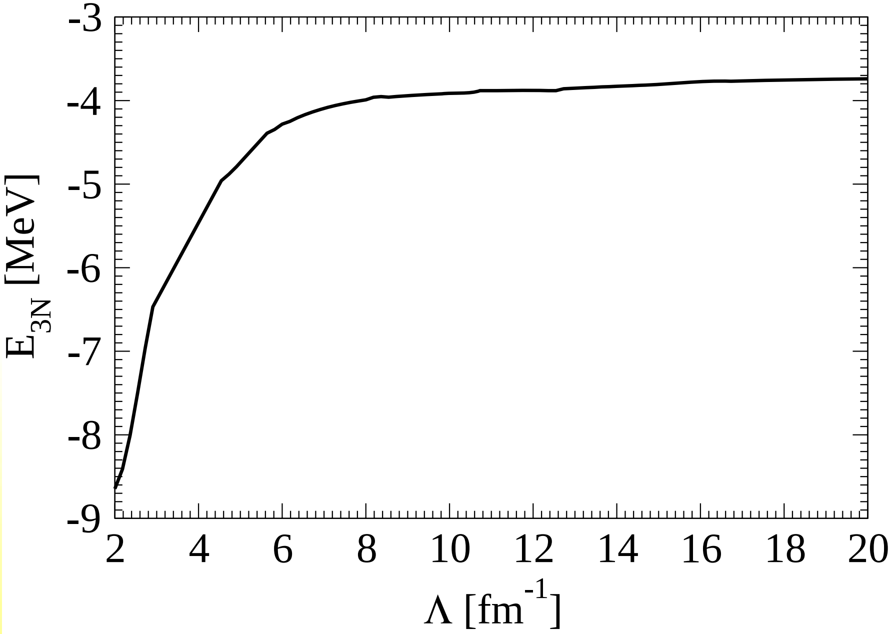
on the other hand:

correct renormalization...

triton

LO EFT

$(j \le 4)$



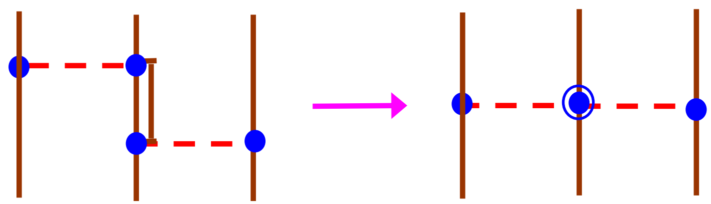
$\Rightarrow$ $M_3 \sim M_{QCD}$ indeed

Can one integrate out the delta with small error?

No!

Pandharipande, Phillips + v.K. '05

EFT folklore: in nuclei, $E \sim \frac{Q^2}{m_N} = m_\Delta - m_N$ $\Rightarrow$ can integrate out delta with small error



$+ \dots O\left(\frac{Q^2}{(m_\Delta - m_N)\, m_N}\right) = O\left(\frac{m_\pi}{m_N}\right)$

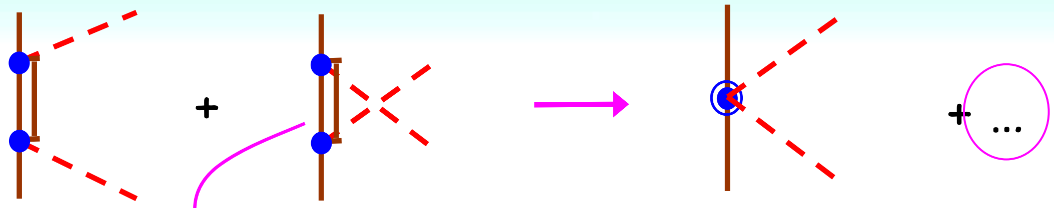
But

$$b = 4d = -\left(\frac{h_A}{2 f_\pi}\right)^2 \frac{4}{9} \frac{2}{m_\Delta - m_N} \sim \frac{1}{m_\pi}$$

$$b = 4d = -\left(\frac{h_A}{2 f_\pi}\right)^2 \frac{4}{9} \frac{2(m_\Delta - m_N)}{(m_\Delta - m_N)^2 - m_\pi^2} \sim \frac{4}{3 m_\pi}$$

30% error

πN scattering: $E : Q : m_\Delta - m_N$

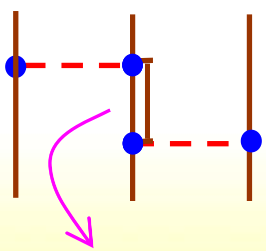


$$O\left(\frac{Q}{(m_\Delta - m_N)}\right) = O\left(\frac{1}{2}\right)$$

at threshold

$$\frac{1}{m_\Delta - m_N - Q} : \frac{1}{m_\pi} \qquad \frac{1}{m_\Delta - m_N + Q} : \frac{1}{3m_\pi}$$

while



$$\frac{1}{m_\Delta - m_N} : \frac{1}{2m_\pi}$$

relatively large error
from the πN scattering fit
leaks into 3N force

best strategy is ***not***
to integrate out delta

What needs to be done:

subLOs including deltas in perturbation theory

Nogga, Timmermans + v.K., in progress

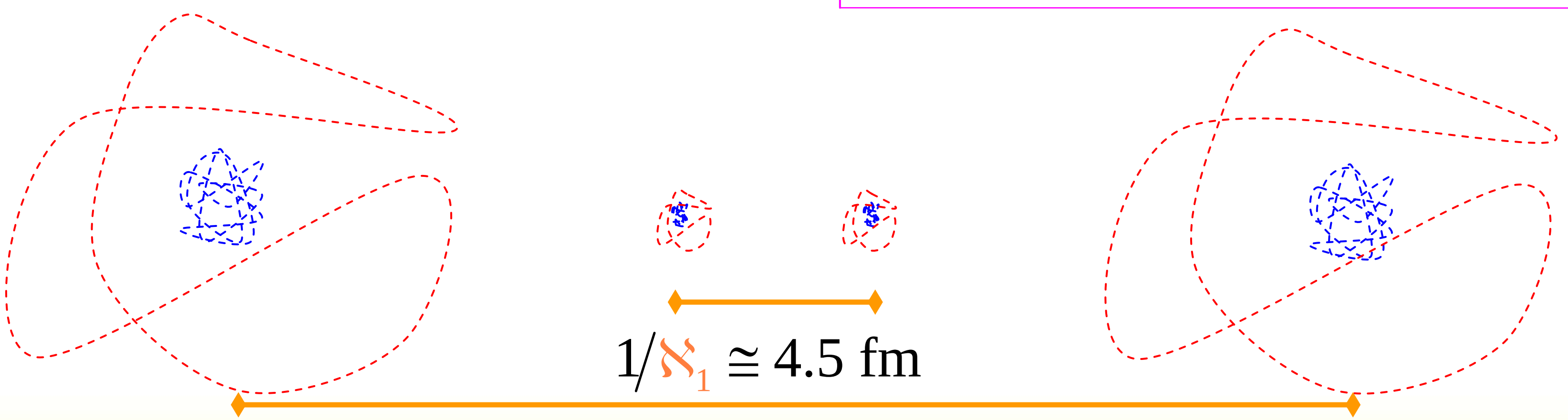
- overkill at lower energies! cf. Bethe + Peierls '35

e.g. NN s_1 channel: s_0 channel:

(real) bound state = deuteron (virtual) bound state

$\aleph_1 \sim \sqrt{m_N B_d} \cong 45 \text{ MeV} < m_\pi$

$\aleph_0 \sim \sqrt{m_N B_{d^*}} \cong 8 \text{ MeV} = m_\pi$



multipole expansion of meson cloud:

contact interactions among local nucleon fields

$$Q \sim \aleph = M_{nuc}$$

pionless EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\not{P}$, $\not{T}$
- expansion in:

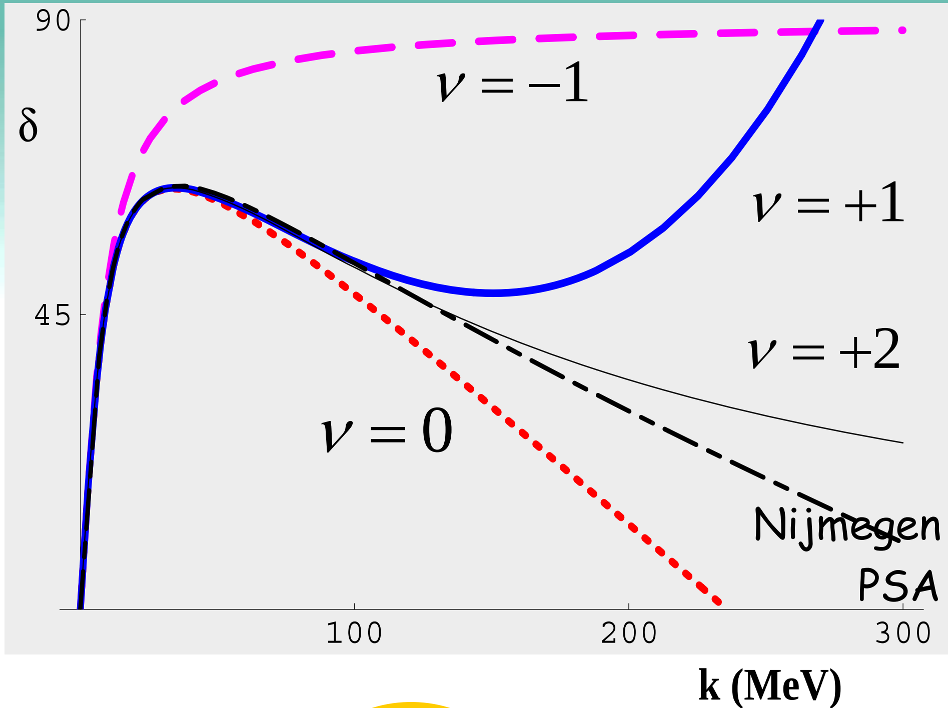
$$\frac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \ldots & \text{multipole} \end{cases}$$

$$\mathcal{L}_{EFT} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0\, N^\dagger N\, N^\dagger N$$

$$+ N^\dagger \frac{\nabla^4}{8m_N^3} N + C_2\, N^\dagger N\, N^\dagger \nabla^2 N$$

$$+ C_2'\, N^\dagger \overleftarrow{\nabla} N \cdot N^\dagger \overrightarrow{\nabla} N + \ldots$$

(omitting spin, isospin)



Chen, Rupak + Savage '99

fitted $a_1 = 5.42\ \text{fm}\ (\text{exp})$

$r_1 = 1.75\ \text{fm}\ (\text{exp})$

predicted

$B_d = 1.91\ \text{MeV}\ (\nu = 0)$

$B_d = 2.22\ \text{MeV}\ (\text{exp})$

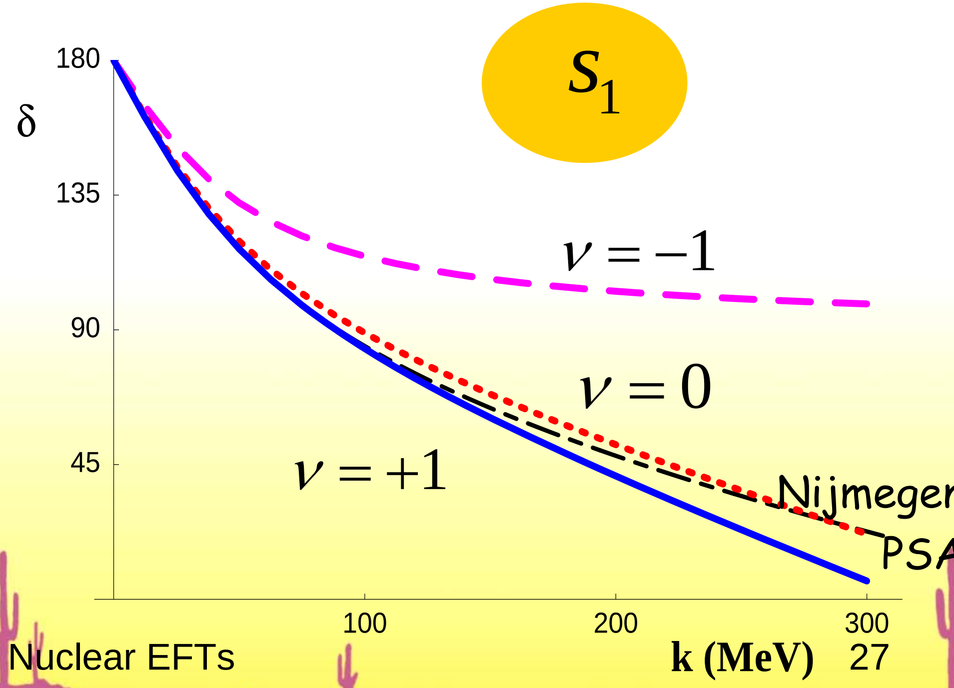
S_0

fitted $a_0 = -20.0\ \text{fm}\ (\text{exp})$

$r_0 = 2.78\ \text{fm}\ (\text{exp})$

predicted

$B_{d^*} = 0.09\ \text{MeV}\ (\nu = 0)$



$$\mathcal{L}_{EFT} = \ldots + D_0\, N^+N\, N^+N\, N^+N + \ldots$$

naïve dimensional analysis $D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{M_{nuc}^3} \quad (\nu = +1)$

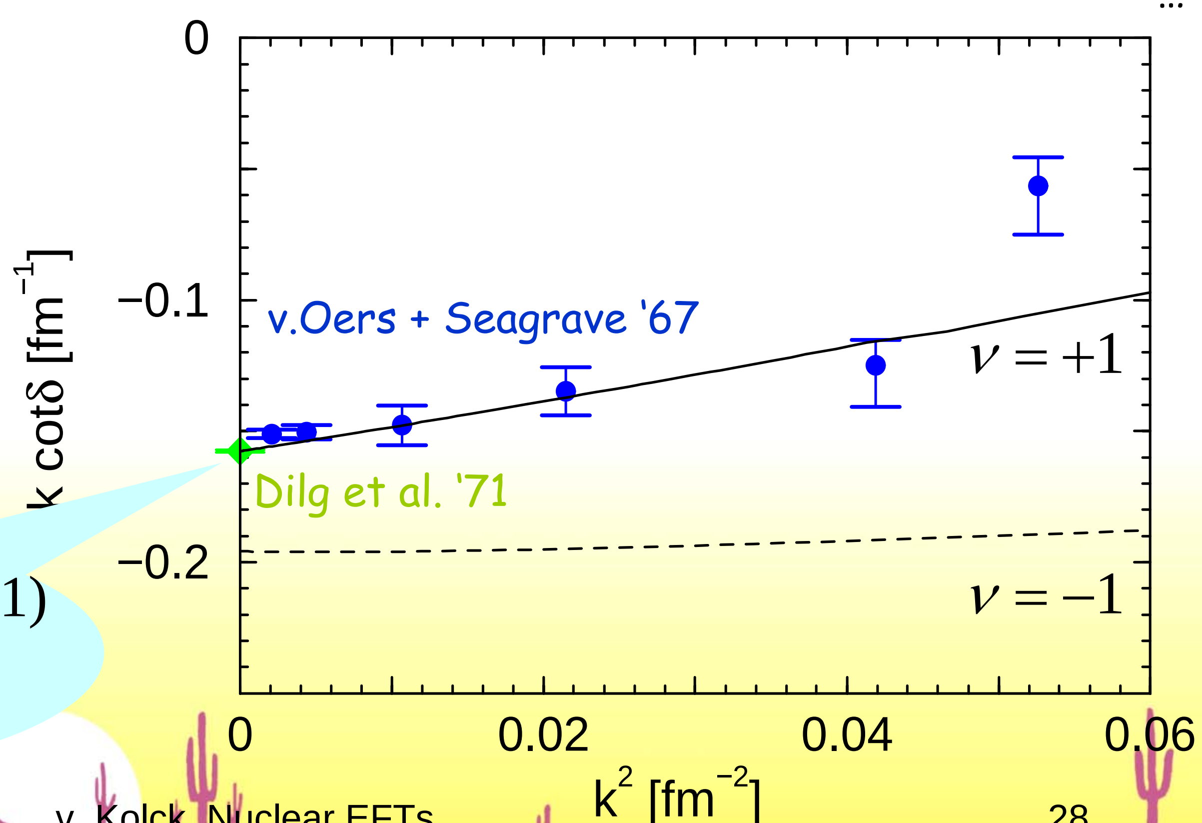
Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98
...

$S_{3/2}$ no three-body force up to $\nu = +3$

$$\lambda \le \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p \gg \aleph} \frac{1}{p^2}$$

$$\Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} 0$$



predicted

$$a_{3/2} = 6.33 \pm 0.10 \text{ fm } (\nu = +1)$$

$$a_{3/2} = 6.35 \pm 0.02 \text{ fm (exp)}$$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03
...

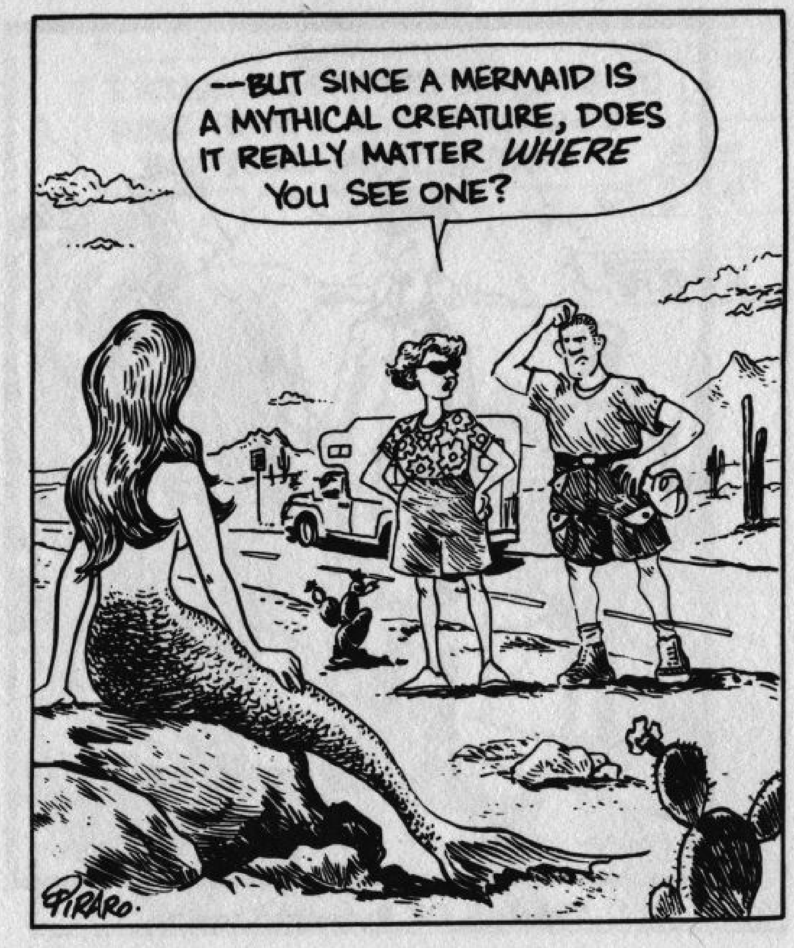
$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p?\ \aleph} A\cos\left(s_0 \ln\frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p\sim\aleph} \neq 0$$ unless

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

limit cycle!



Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03
...

$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p ? \aleph} A \cos\left(s_0 \ln\frac{p}{\Lambda} + \delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} \neq 0$$ **unless**

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

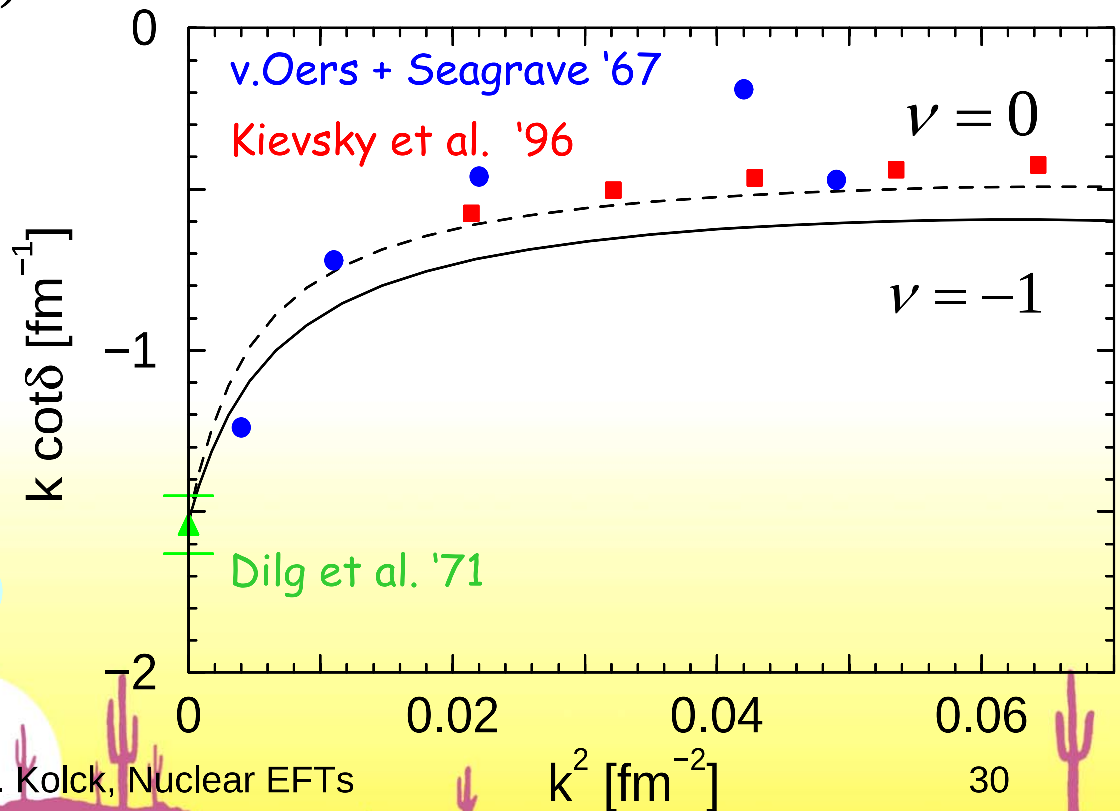
fitted

$$a_{1/2} = 0.65\,\text{fm (exp)}$$

predicted

$$B_t = 8.3\,\text{MeV} \; (\nu = 0)$$

$$B_t = 8.48\,\text{MeV (expt)}$$



+ four-body bound state can be addressed similarly

⇨ no four-body force at $\nu = -1$ Hammer, Meissner + Platter '04

~ larger nuclei?

No-Core Shell Model! Barrett, Vary + Zhang '93

...

up to now:

give'm a (preferably, EFT) potential, and they will

run the RG in a harmonic-oscillator basis of frequency $\hbar\Omega$

to a restricted space of $N = 2n + l \leq N_{max}$

alternative: Vary + v.K., in progress

Stetcu, Barrett +v.K., in progress

start with EFT in restricted space;

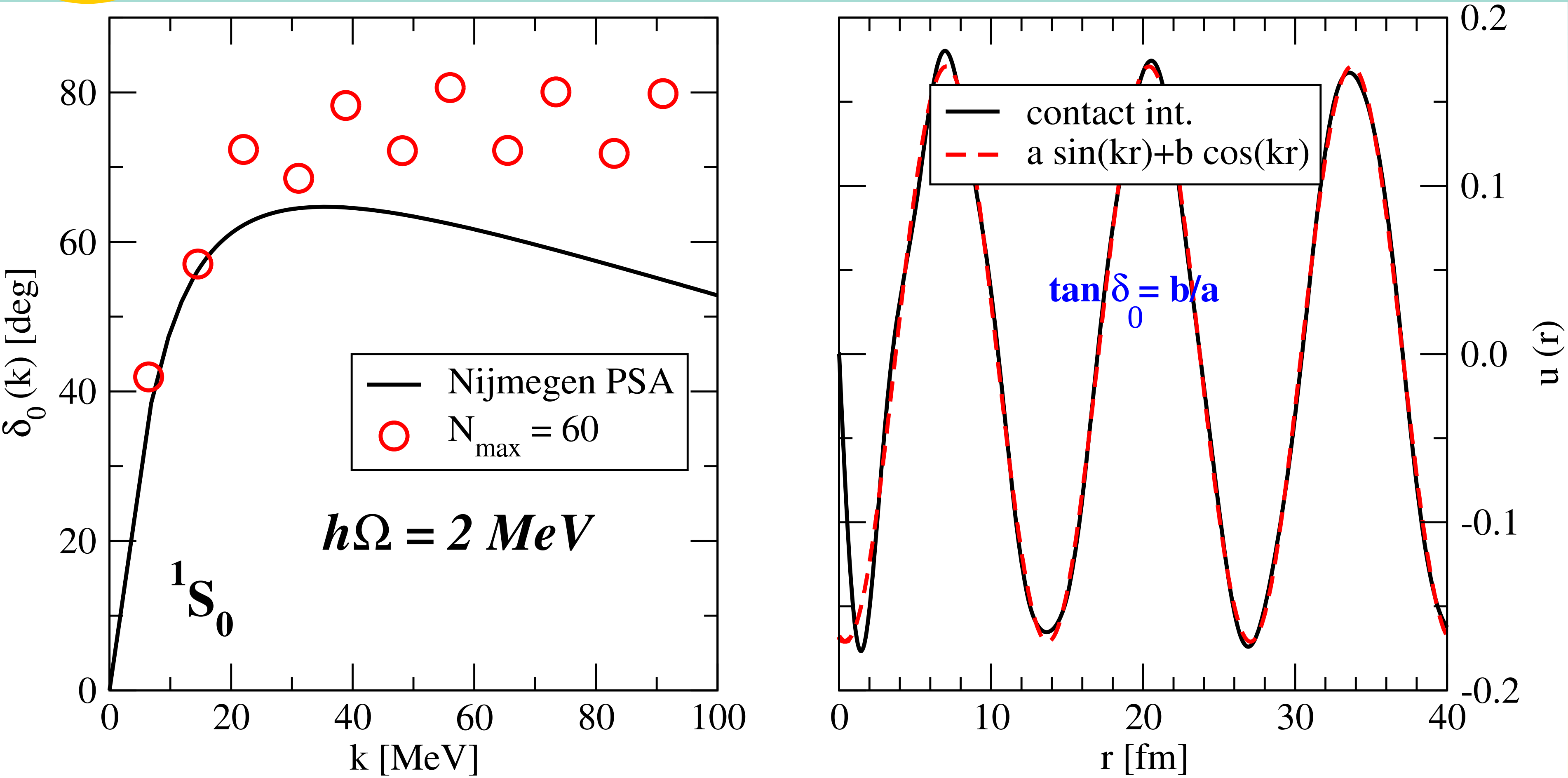
fit parameters for various $\hbar\Omega$ and N_{max} in few-nucleon systems;

and predict larger nuclei

($\hbar\Omega$ → IR, N_{max} → UV) cutoffs

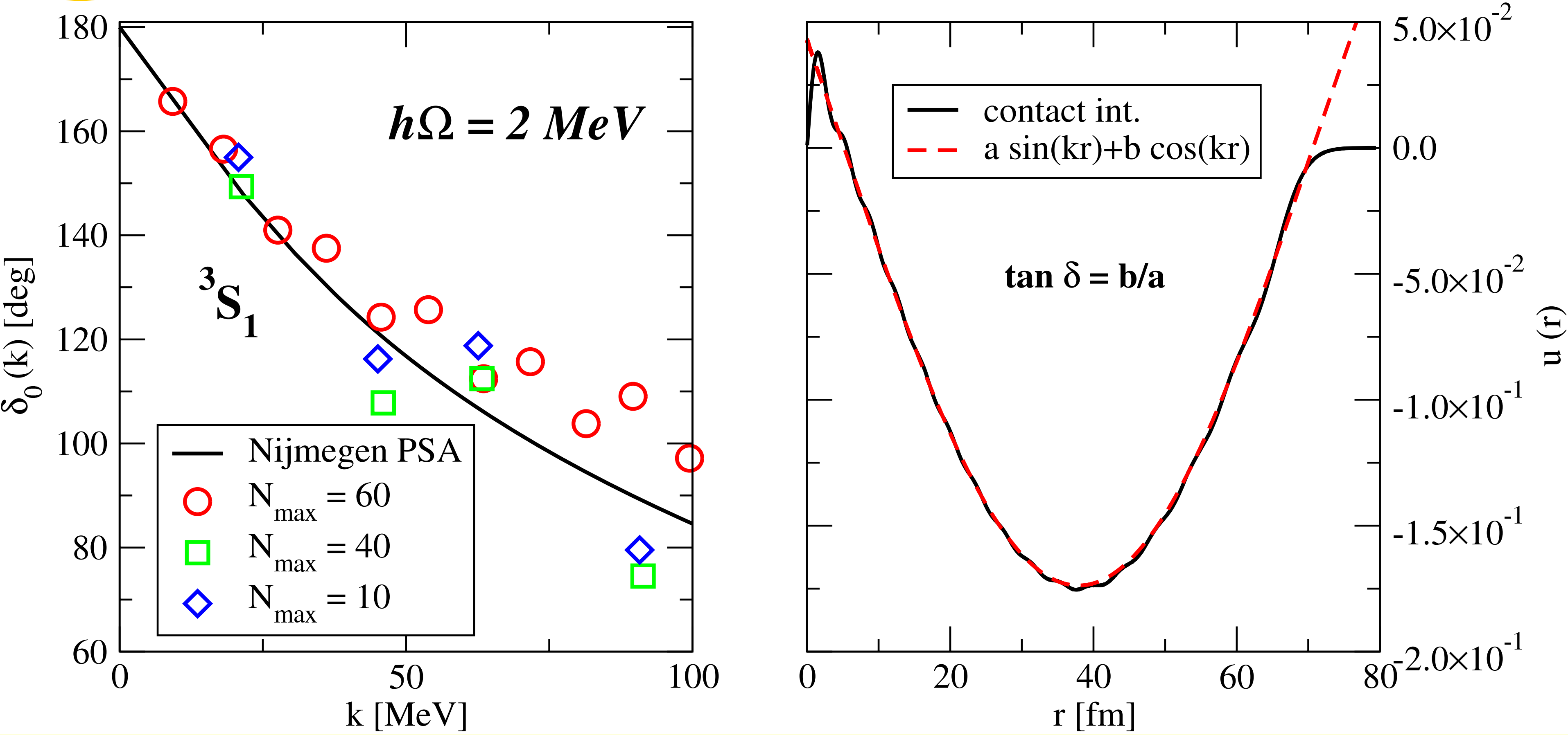
S_0

Stetcu, Barrett +v.K., in progress

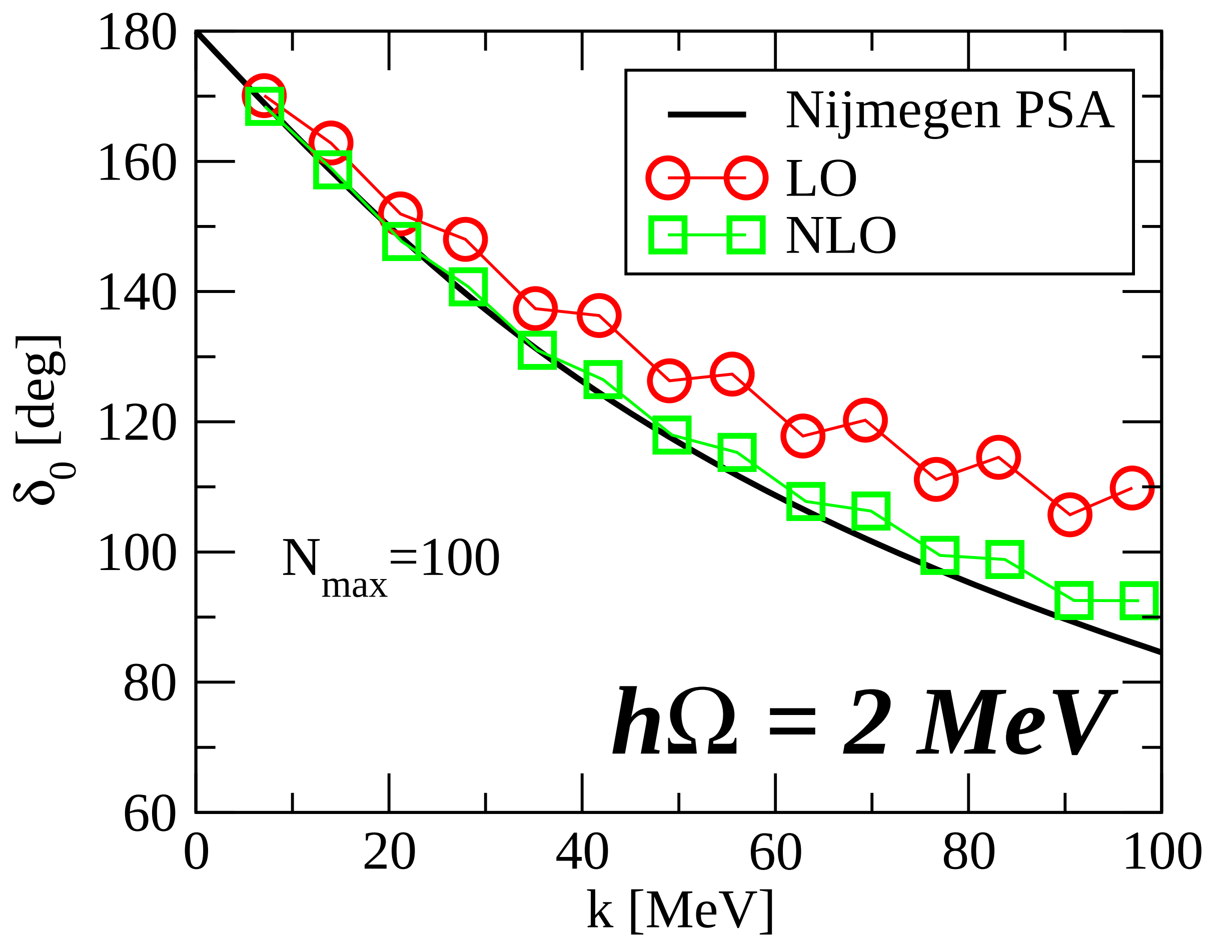


$C_0^{(0)}\left(N_{max}h\Omega\right)$ fitted to scattering length

S_1



$C_0^{(1)}\left(N_{max}h\Omega\right)$ fitted to scattering length



fitted to

scattering length

and binding energy

What needs to be done:

0') Other methods to extract phase shifts,

which work at smaller N_{max} and larger $\hbar\Omega$

1) Introduce 3N force

2) Solve three-, four- (…-) nucleon systems

and determine LO (NLO, …) parameters

3) Predict larger systems

4) Introduce pions and go back to 1)

Stetcu, Barrett +v.K., in progress

- many-body systems get complicated rapidly

+ (continuing) focus on simpler ***halo nuclei***

one or more loosely-bound nucleons (near driplines)

$$\aleph \equiv \sqrt{m_N E_N} = \sqrt{m_N E_c} \equiv M_c$$

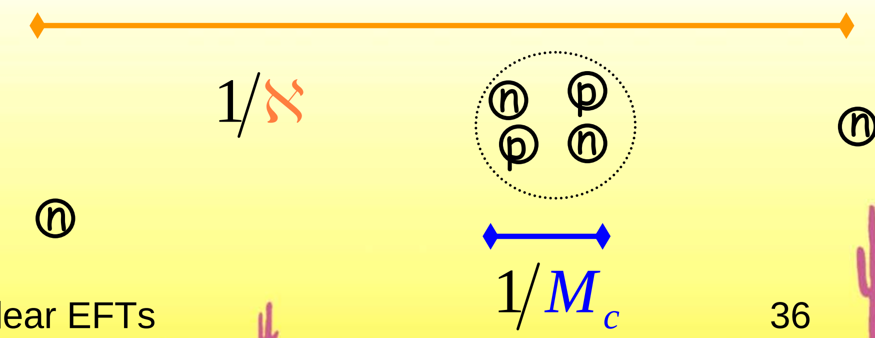
nucleon separation energy (E_N) — core excitation energy (E_c)

e.g.

$^4\mathrm{He}$ $\quad \left.\begin{array}{l} B_{\alpha^*} \cong 8\ \mathrm{MeV} \\ B_\alpha \cong 28\ \mathrm{MeV} \end{array}\right\} \quad E_\alpha = B_\alpha - B_{\alpha^*} \cong 20\ \mathrm{MeV}$

" $^5\mathrm{He}$ " $\quad p_{3/2}$ resonance at $E_n \sim 1\ \mathrm{MeV}$

$^6\mathrm{He}$ $\quad E_{2n} \sim 1\ \mathrm{MeV}$



$Q \sim \aleph = M_c$

halo EFT

- degrees of freedom: nucleons, cores
- symmetries: Lorentz, ~~P~~, ~~T~~
- expansion in:

$$\frac{Q}{M_c} = \begin{cases} Q/m_N \,,\; Q/m_c & \text{non-relativistic} \\ Q/m_\pi \,, \mathrm{L} & \text{multipole} \end{cases}$$

simplest formulation: auxiliary fields for core + nucleon states

e.g. $\;{}^4\mathrm{He} \;\; \alpha \;\; \text{scalar field}\, \varphi$

$${}^4\mathrm{He} + \mathrm{N} \quad \begin{cases} s_{1/2} \equiv 0+ \;\; \alpha \;\; \text{spin-0 field } s \\ p_{1/2} \equiv 1- \;\; \alpha \;\; \text{spin-1/2 field } T_1 \\ p_{3/2} \equiv 1+ \;\; \alpha \;\; \text{spin-3/2 field } T_3 \\ \mathrm{M} \end{cases}$$

Bertulani, Hammer + v.K. '02

Bedaque, Hammer + v.K. '03

$$\mathcal{L}_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + \varphi^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_\alpha}\right)\varphi$$

$$+ T_3^{+}\left[\sigma_3\left(i\partial_0 + \frac{\nabla^2}{2(m_\alpha + m_N)}\right) - \Delta_3\right]T_3$$

$$+ \frac{g_3}{\sqrt{2}}\left[T_3^{+}\vec{S}^{+}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

spin transition operator

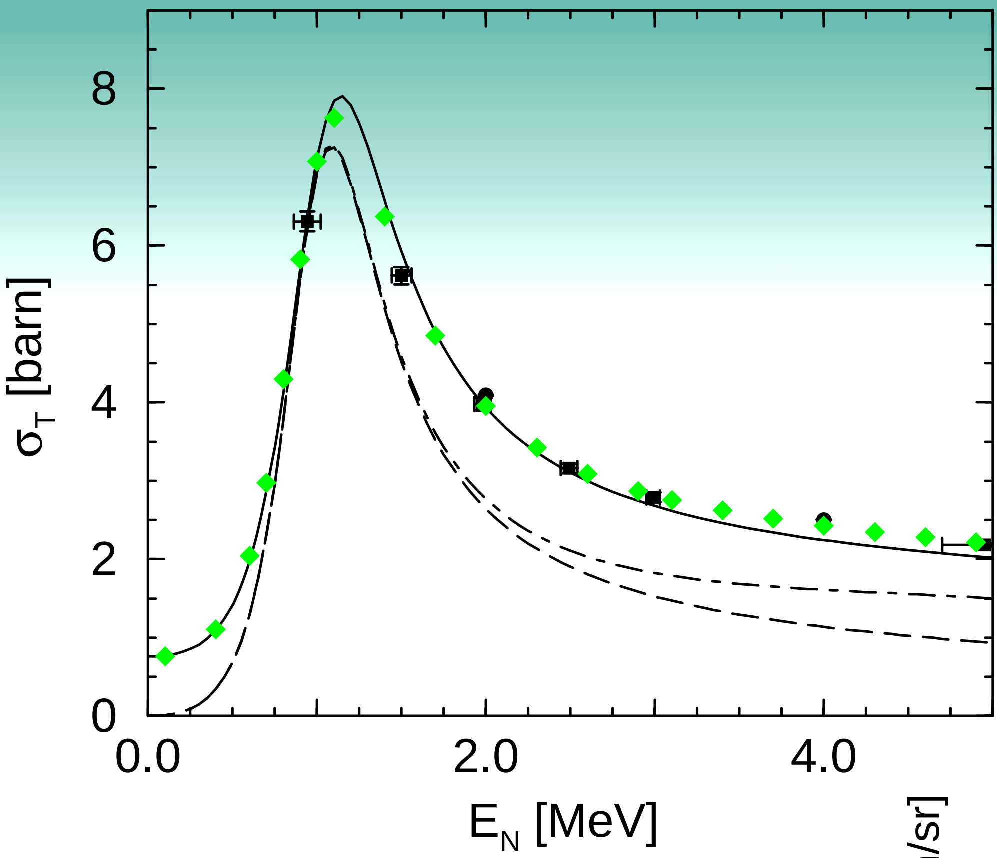
$$+ s^{+}(-\Delta_0)s + \frac{g_0}{\sqrt{2}}\left[s^{+}N\varphi + \text{H.c.}\right]$$

$$+ \ldots$$

$$+ T_1^{+}(-\Delta_1)T_1 + \frac{g_1}{\sqrt{2}}\left[T_1^{+}\vec{\sigma}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

$$+ \ldots$$

Bertulani, Hammer + v.K. '02



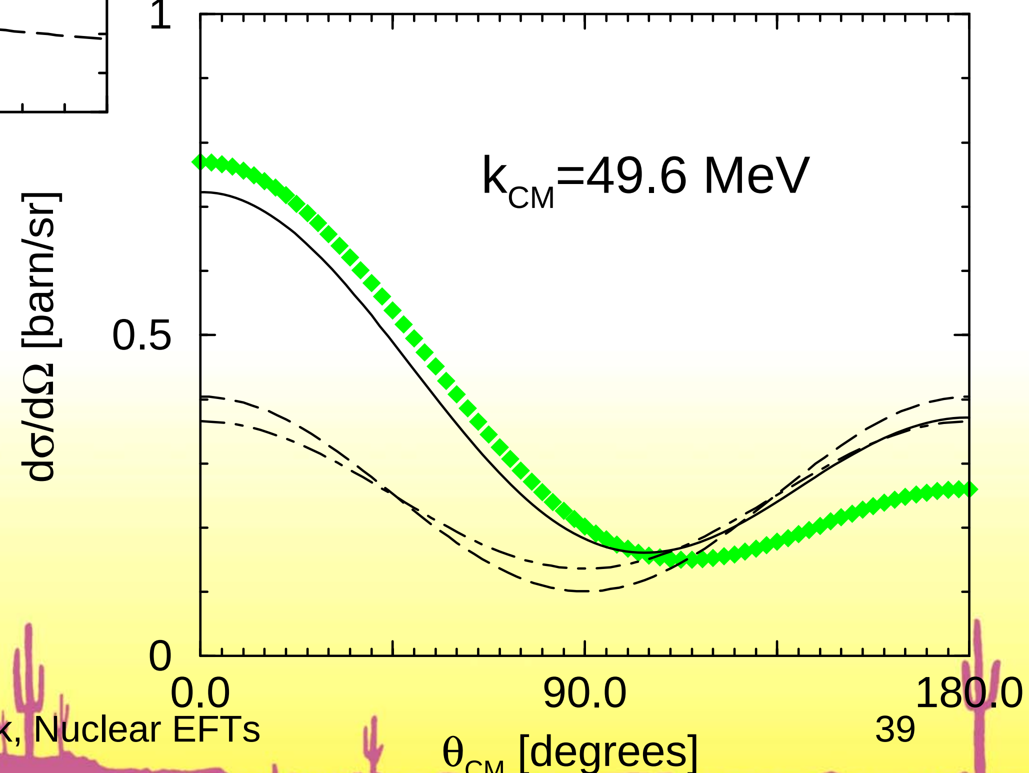
$\nu = -1$

$\nu = 0$

$\nu = -1\,\mathrm{mod}$

NNDC, BNL

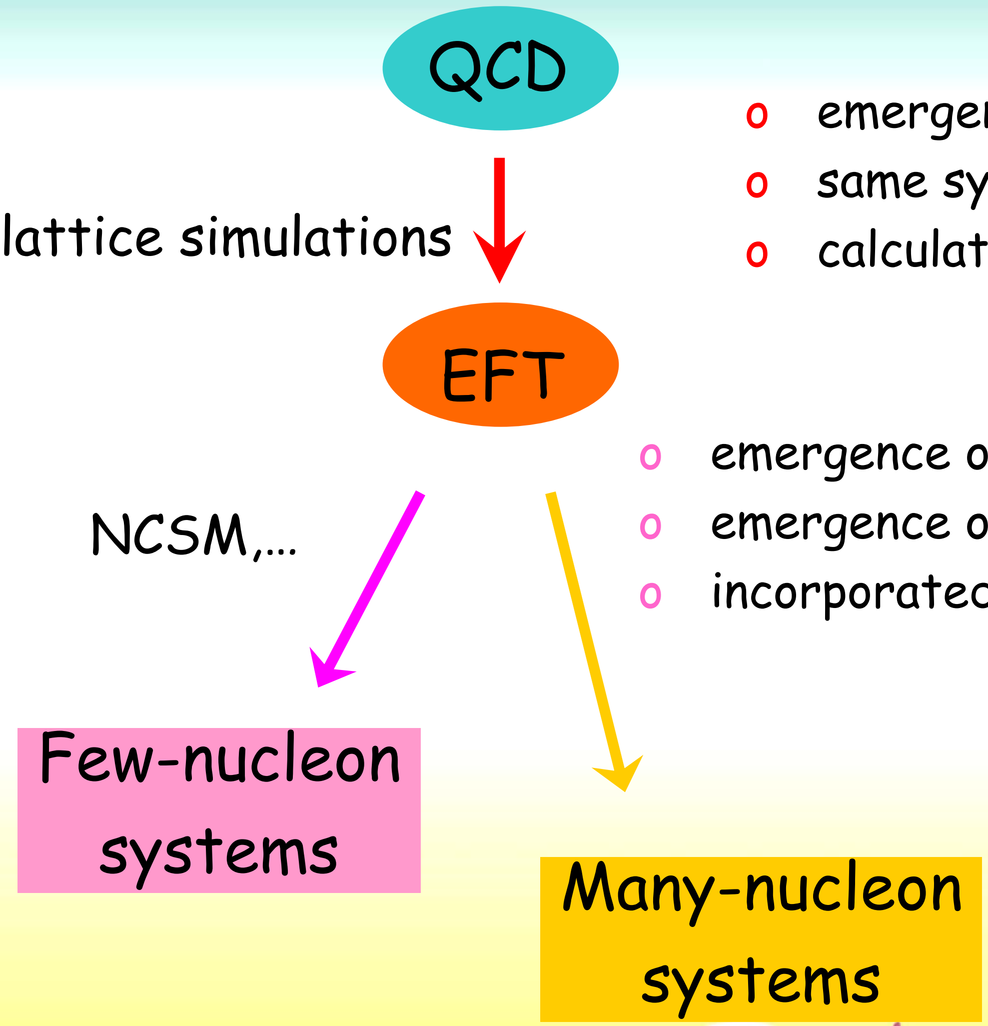
Haesner et al. '83



What needs to be done:

- three-body bound states:
  e.g. $^{6}\text{He} = \text{b.s.}\left(^{4}\text{He} + n + n\right)$ Hammer + v.K., in progress
  [c.f. $^{3}\text{H} = \text{b.s.}\left(p + n + n\right)$ Bedaque, Hammer + v.K. '99]
- Coulomb interaction:
  e.g. $p + {}^{4}\text{He} \rightarrow {}^{4}\text{He} + p$ Bertulani, Higa + v.K., in progress
  [c.f. $p + p \rightarrow p + p$ Kong + Ravndal '99]
- reactions:
  e.g. $p + {}^{7}\text{Be} \rightarrow {}^{8}\text{B} + \gamma$
  [c.f. $p + n \rightarrow d + \gamma$ Chen et al. '00]

Speculations



- emergence of new d.o.f.s
- same symmetries, but some accidental (isospin)
- calculated parameters

- emergence of new *fine-tuning* scale (halos)
- emergence of new phenomena (limit cycles, shells, ...)
- incorporated by fitting parameters

- Shell Model as EFT?
- Walecka Model as EFT?

Role of RIA-like machines?

Early to say…

Possibilities:

life on the edge:

key word: fine-tuning

Pionful EFT
- precise form of 3N (and 4N?) forces (cf. IL 269…)
- (related) role of delta?

Pionless EFT
- constrain fine-tuned parameters

Halo EFT
- constrain nucleon-core parameters
- size of few-body forces

Shell (?) EFT
- test it

+ applications { symmetry tests *(e.g., Ramsey-Musolf's talk)*
astrophysical reactions

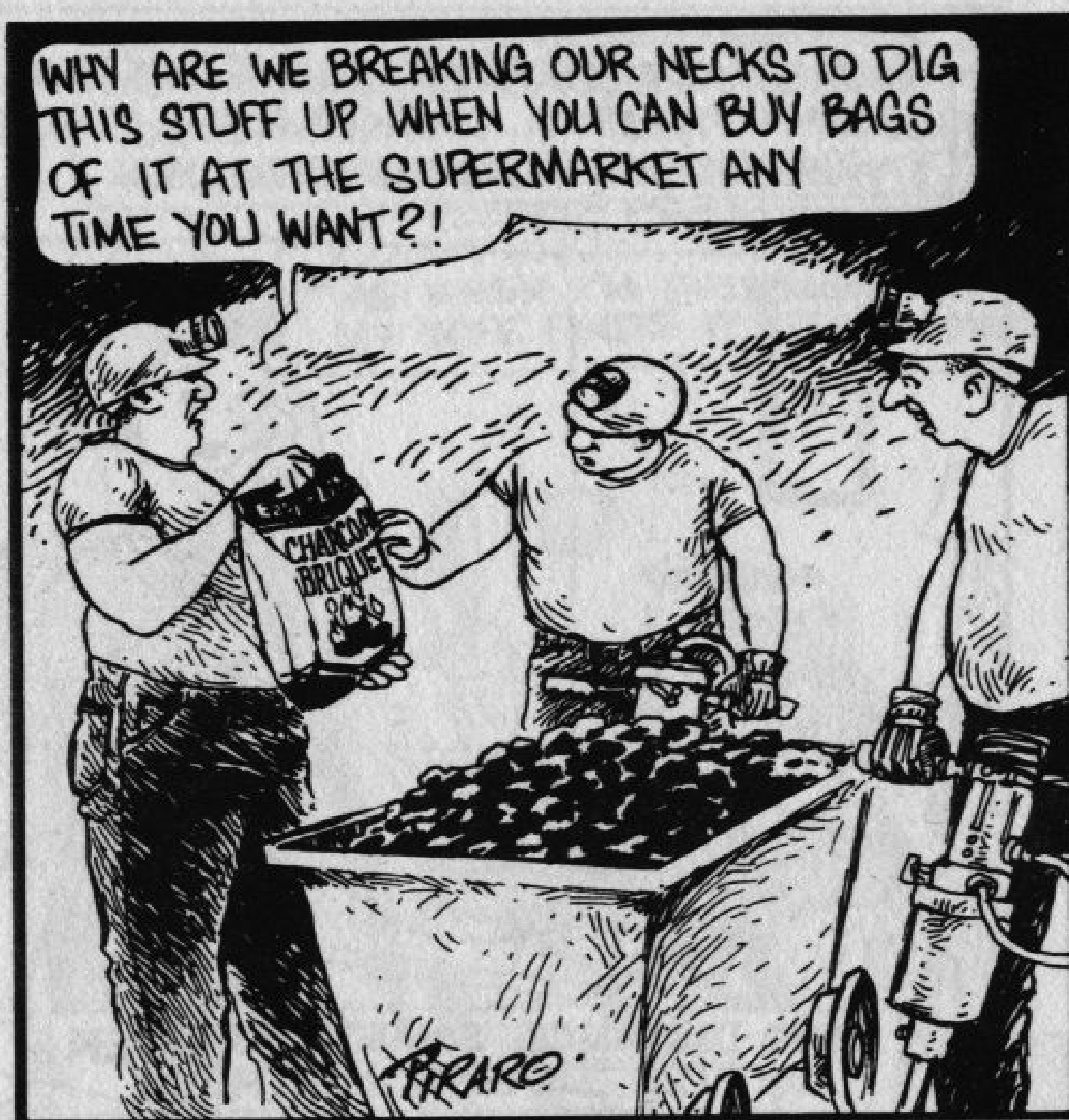
Conclusion

EFT the framework to describe nuclei within the SM

- ✓ is consistent with symmetries
- ✓ incorporates hadronic physics
- ✓ has controlled expansion

many successes so far, but still much to do

grow to larger nuclei!


WHY ARE WE BREAKING OUR NECKS TO DIG THIS STUFF UP WHEN YOU CAN BUY BAGS OF IT AT THE SUPERMARKET ANY TIME YOU WANT?!