Symmetry Energy

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Outline

Symmetry Energy

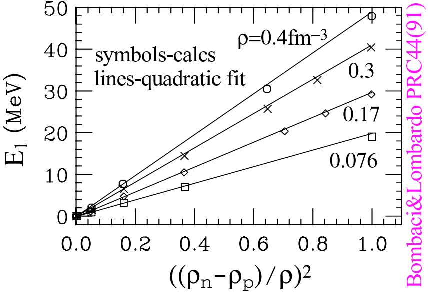
Bethe-Weizsäcker (BW) formula:

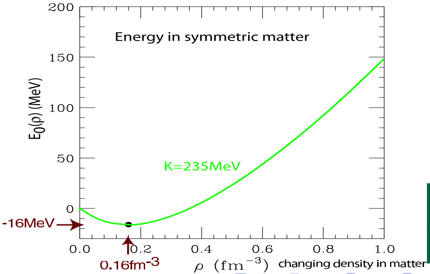
$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_A\,\frac{(N-Z)^2}{A} + \Delta$$

Symmetry energy: change in nuclear energy associated with changing neutron-proton asymmetry

In nuclear matter: $E(\rho_n, \rho_p) = E_0(\rho) + E_1(\rho_n, \rho_p)$

$$E_1 = E - E_0 \simeq S(\rho)\left(\frac{\rho_n - \rho_p}{\rho}\right)^2$$





Constraints for Symmetric Matter

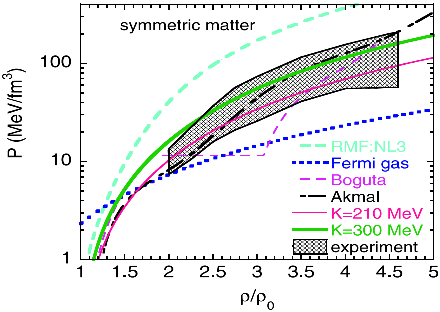
Minimum at $\rho_0 \simeq 0.16\,\mathrm{fm}^{-3}$ with $E_0(\rho_0) \simeq -16\,\mathrm{MeV}$

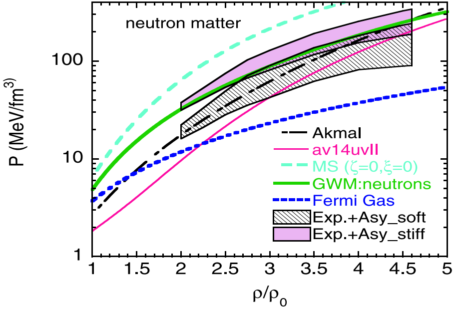
Incompressibility from giant resonances: $K \sim 235$ MeV

Youngblood, Garg, Colo *et al.* '05

At high ρ, constraints on nuclear pressure $P = \rho^2\,\partial E/\partial\rho$ from flow in semicentral reactions

PD,Lacey&Lynch Science298(02)



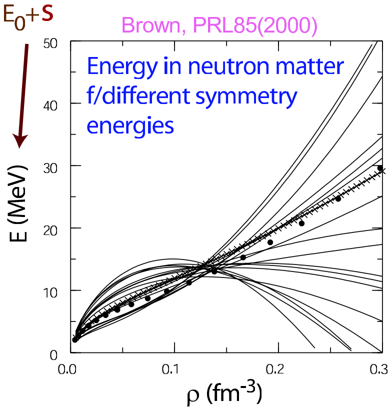


Neutron Matter: $E = E_0 + S$

Uncertain symmetry energy

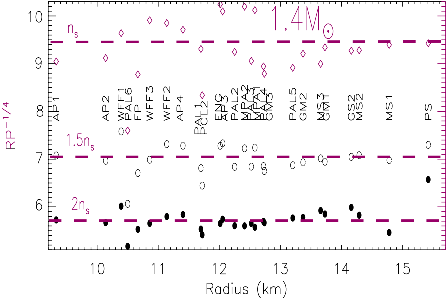
Symmetry Energy Uncertainties

Compilation of symmetry energies in literature



In neutron matter:

$E = E_0 + S \qquad P \simeq \rho^2\, dS/d\rho$



Empirical correlation

$R\, P^{-1/4} \approx$ const

Lattimer&Prakash ApJ550(01)

Symmetry Energy in Binding Formula

Standard formula: $\propto A$

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_A\,\frac{(N-Z)^2}{A} + \delta$$

Surface energy: $E_S = a_S\,A^{2/3} = \frac{a_S}{4\pi\,r_0^2}\,4\pi\,r_0^2\,A^{2/3} = \frac{a_S}{4\pi\,r_0^2}\,\mathcal{S}$

$$\frac{E_S}{\mathcal{S}} = \sigma = \frac{a_S}{4\pi\,r_0^2} \quad \text{(tension – work per area)}$$

$\rightarrow$ As nucleons at surface less bound, increasing surface requires work.

Symmetry energy reduces the binding, so, as n-p asymmetry increases, the work to create surface should drop (you cannot subtract same thing twice from volume!)

$$\sigma = \frac{\partial\,E_S}{\partial\,\mathcal{S}} \quad \searrow \qquad \text{(in the general definition of tension)}$$

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From Tension to Surface n-p Excess

σ as intensive should depend on an intensive quantity characterizing neutron-proton (n-p) asymmetry $\rightarrow \mu_A$

$$\mu_A = \frac{\partial E}{\partial (N - Z)} \quad = \frac{1}{2} (\mu_n - \mu_p)$$

Since tension should drop no matter whether more neutrons or protons $\rightarrow$ quadratic in chemical potential

$$\sigma = \sigma_0 - \gamma \, \mu_A^2$$

Surface energy E_S must then also depend on μ_A...

Partial-derivative consistency for E [$\Phi = \mu_A(N - Z) - E$; $\partial\Phi/\partial\mu_A = N - Z$] then requires: Surface must contain n-p excess!

$$(N_S - Z_S) \propto \mu_A$$

Surface energy must be quadratic in the excess and/or μ_A.
?How can surface hold particles?!

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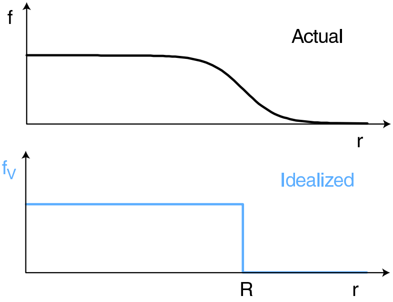
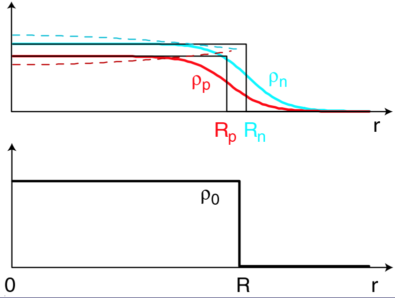
Volume-Surface Separation á la Gibbs

Gibbs definition for surface quantities - difference between actual and idealized where volume contribution only: $F_S = F - F_V$



result depends on surface position R: $A_S = A - A_V = 0$



2-component system: surfaces for neutrons and protons may be displaced.

Net surface position set demanding: $A_S = 0$.

However, $N_S - Z_S \neq 0$!

Symmetry Energy Modification

With derivative consistency resolved, $\sigma = \sigma_0 - \gamma\, \mu_A^2$ yields for surface energy

$$\begin{aligned} E_S &= \sigma_0\, \mathcal{S} + \gamma\, \mu_A^2\, \mathcal{S} = E_S^0 + \frac{1}{4\gamma}\, \frac{(N_S - Z_S)^2}{\mathcal{S}} \\ &= E_S^0 + a_A^S\, \frac{(N_S - Z_S)^2}{A^{2/3}} \qquad \text{(surface capacitor)} \end{aligned}$$

Volume similarly: $E_V = E_V^0 + a_A^V\, \frac{(N_V - Z_V)^2}{A}$ (volume capacitor)

Net Energy & Asymmetry: $E = E_S + E_V, \quad N - Z = N_S - Z_S + N_V - Z_V$

Minimization of E with respect to the asymmetry partition: analogous to coupled capacitors, $q_X = N_X - Z_X$, $E_X = E_X^0 + q_X^2/2C_X$, with the result

$$E = E^0 + \frac{q^2}{2C} = E^0 + \frac{(N-Z)^2}{\frac{A}{a_A^V} + \frac{A^{2/3}}{a_A^S}}$$

Modified Binding Formula

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \frac{a_A^V}{1 + A^{-1/3}\, a_A^V / a_A^S} \, \frac{(N-Z)^2}{A}$$

$a_A(A)$

Regular formula for $a_A^V/a_A^S = 0$ - i.e. surface not accepting the asymmetry excess ($a_A^S = \infty$) - or for $A \to \infty$.
Modified formula: weakening of the symmetry term for low A.

Whether one can replace $a_A(A)$ by a_A^V for large A depends on the ratio a_A^V/a_A^S.

The ratio may be determined from surface asymmetry excess, as surface-to-volume asymmetry ratio:

$$\frac{N_S - Z_S}{N_V - Z_V} = \frac{C_S}{C_V} = \frac{A^{2/3}/a_A^S}{A/a_A^V} = A^{-1/3}\, a_A^V / a_A^S$$

Asymmetry Skins

Measuring n-p skin sizes difficult: 2 different probes needed.
E.g. electrons + protons, $\pi^+ + \pi^-$, protons + neutrons

Issues: 1. Data in terms of difference of n and p rms radii.
Conversion straightforward, if diffuseness similar for n and p.

2. For heavy nuclei, Coulomb competes with symmetry energy, pushing protons out to surface and polarizing interior.
$\Rightarrow$ minimize sum of 3 energies w/respect to asymmetry:

$$E = E_V + E_S + E_C \qquad E_C = \frac{e^2}{4\pi\epsilon_0}\frac{1}{R}\left(\frac{3}{5}Z_V^2 + Z_V Z_S + \frac{1}{2}Z_S^2\right)$$

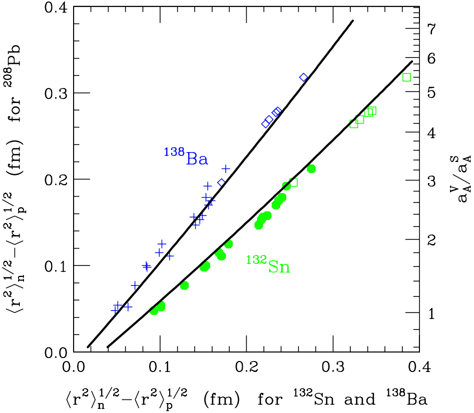
From the modified minimization, analytic difference of rms radii:

$$\frac{\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2}}{\langle r^2\rangle^{1/2}} = \frac{A}{6NZ}\frac{N-Z}{1+A^{1/3}a_A^S/a_A^V} - \frac{a_C}{168a_A^V}\frac{A^{5/3}}{N}\frac{\frac{10}{3}+A^{1/3}a_A^S/a_A^V}{1+A^{1/3}a_A^S/a_A^V}$$

symmetry energy only Coulomb correction

Testing Macroscopic Theory

Comparison of the analytic formula (lines) with a multitude of nonrelativistic and relativistic mean-field calculations by Typel and Brown PRC64(01)027302 (symbols)



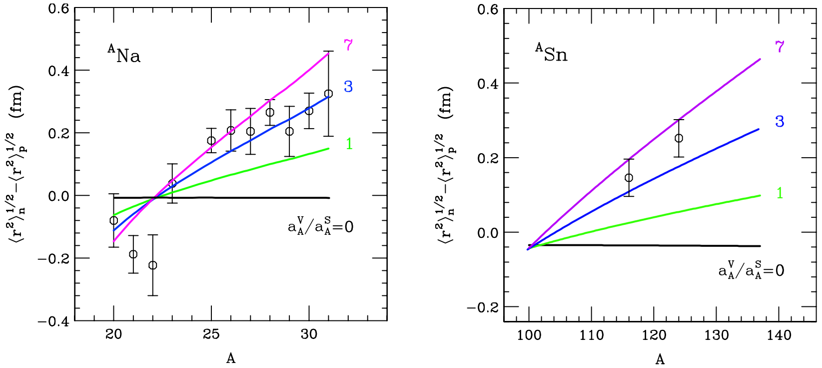
Accuracy, in reproducing microscopic theory, of $\sim$0.01 fm ?!

Other tests: Thomas-Fermi

$\Rightarrow$ next data

Comparison to Skin Data

Systematic of n-p skin sizes for different Na isotopes by Suzuki et al., PRL75(95)3241 + other data



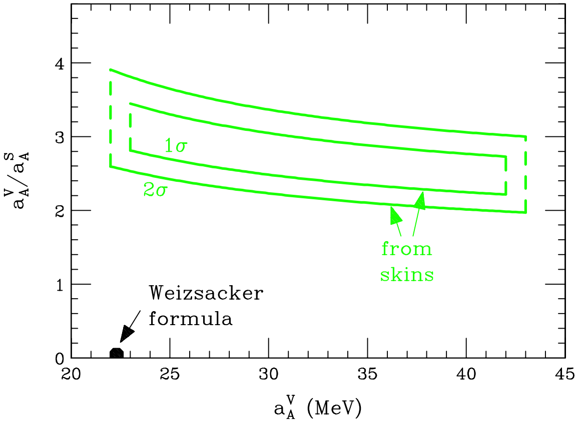
difference between the rms n and p radii vs A

$a_A^V/a_A^S \sim 3$

Global Fit to Skin Data

1-σ & 2-σ limits on a_A^V/a_A^S as a function of a_A^V

dependence on a_A^V due to Coulomb



As $A^{-1/3}\, a_A^V/a_A^S$ never small, symmetry term not expandable; Bethe-Weizsäcker not acceptable at the macroscopic level.

Charge Invariance

Conclusions on symmetry term details, following mass-formula fits, are interrelated with conclusions on other terms: isospin-dependent Coulomb, Wigner & pairing + isospin-independent, due to $(N-Z)/A$ - A correlations along line of stability (PD NPA727(03)233)!

Best would be to study the symmetry term in isolation from the rest of mass formula! Absurd?!

Charge invariance comes to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

$$E_A = a_A(A)\,\frac{(N-Z)^2}{A} = 4\,a_A(A)\,\frac{T_z^2}{A}$$

$$\rightarrow\ E_A = 4\,a_A(A)\,\frac{T^2}{A} = 4\,a_A(A)\,\frac{T(T+1)}{A}$$

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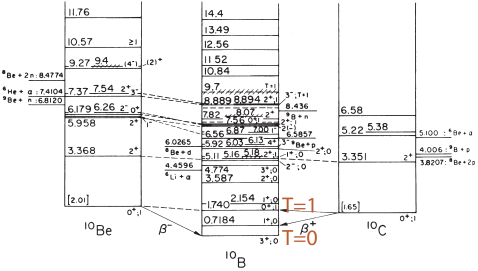
A-Dependent Symmetry Energy from IAS Data

$$\rightarrow \; E_A = 4\, a_A(A)\, \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T. Pairing term contributes depending on evenness of T.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



Study of changes in the symmetry term possible nucleus by nucleus

IAS Data Analysis

In the same nucleus, when pairing drops out:

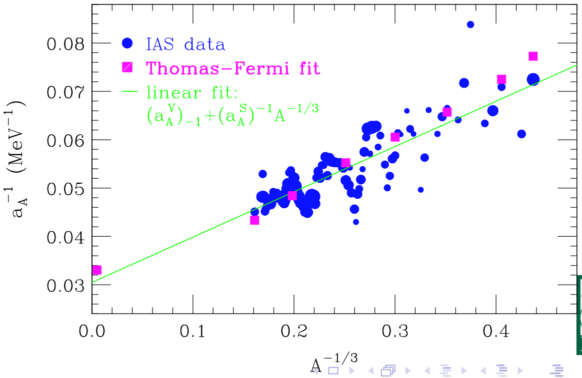
$$E_2(T_2) - E_1(T_1) = \frac{4\,a_A}{A}\{T_2(T_2+1) - T_1(T_1+1)\}$$

$$a_A^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E} \qquad \stackrel{?}{=} (a_A^V)^{-1} + (a_A^S)^{-1}\,A^{-1/3}$$

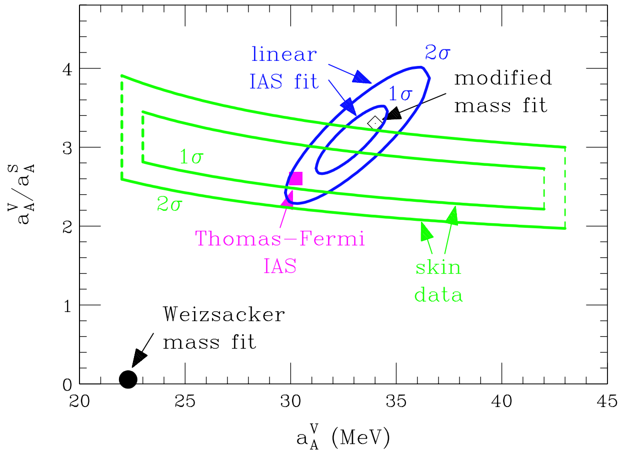
extracted inverse symmetry coefficient

available IAS with largest energy differences used

Antony *et al.* ADNDT66(97)1



Fit Combination



Conclusions: $30.0\,\text{MeV} \lesssim a_A^V \lesssim 32.5\,\text{MeV}$, $2.6 \lesssim a_A^V / a_A^S \lesssim 3.0$

next: Symmetry-coeff ratio constraints low-ρ dependence of E_A.

Microscopic Background

In TF approx with $E = E_0 + \int d^3r \, \rho \, S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2$, where S - symmetry energy ($S(\rho_0) = a_A^V$), Gibbs prescription for semiinfinite matter yields:

$$\frac{a_A^V}{a_A^S} = \frac{3}{r_0} \int dr \, \frac{\rho(r)}{\rho_0} \left[\frac{S(\rho_0)}{S(\rho(r))} - 1 \right]$$

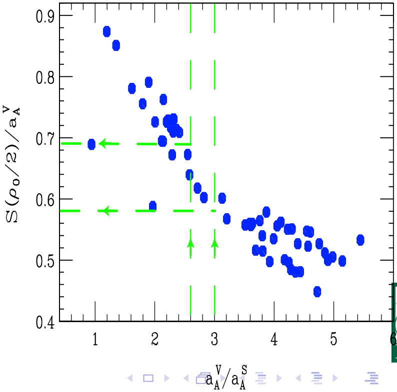
$\Rightarrow$ a_A^V / a_A^S probes shape of $S(\rho)$!

For $S(\rho) \equiv a_A^V$, $a_A^V / a_A^S = 0$!

Surface capacitance emerges, because S drops with ρ.

From $2.6 \lesssim a_A^V / a_A^s \lesssim 3.0$ for mean-field structure calcs (Furnstahl, NPA706(02)85 - symbols), we deduce symmetry energy reduction at $\rho_0/2$:

$$0.58 \lesssim S(\rho_0/2)/a_A^V \lesssim 0.69$$



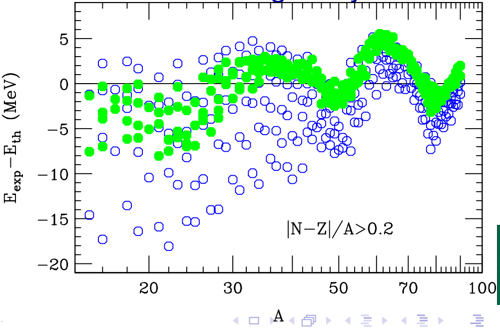
Further Consequences

In $S(\rho) \simeq a_A^V(\rho/\rho_0)^\gamma$: $\gamma = (0.54 - 0.77)$.

Neutron Stars: Pressure estimate from $S(\rho)$ + Lattimer-Prakash scaling, $R\,P^{1/4} \simeq$ const, yields $11.5\,\text{km} \lesssim R \lesssim 13.5\,\text{km}$ for an $1.4\,M_\odot$ star.

Density dependence too weak for the direct Urca cooling.

Mass Formula Performance: Fit residuals for light asymmetric nuclei, when either following the Bethe -Weizsäcker formula (open symbols) or the modified formula with $a_A^V/a_A^S = 2.8$ imposed (closed), i.e. the same parameter No.



Conclusions

- Macroscopic consistency puts surface symmetry energy into binding formula, with volume and surface symmetry energies combining as energies of coupled capacitors.
- Extension implies surface asymmetry skins and weakening of the symmetry term for light nuclei.
- Skins restrict ratio of symmetry coefficients; charge invariance allows to study symmetry term in one nucleus.
- Skin/IAS fits: $30.0\,\text{MeV} \lesssim a_A^V \lesssim 32.5\,\text{MeV}$ and $2.6 \lesssim a_A^V/a_A^S \lesssim 3.0$.
- Surface symmetry energy emerges due to weakening of symmetry energy with density. a_A^V/a_A^S ratio places S within $(0.58 - 0.69)a_A^V$ at $\rho_0/2$. Consequences for neutron stars.
- Description of giant dipole resonances improves with inclusion of surface symmetry energy. Resonances more of a GT type for light nuclei and of an SJ type for heavy.
- Next: Shell-correction for IAS

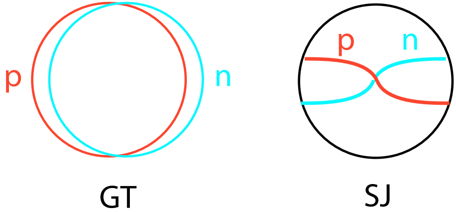
Questions for RIA

- Skin-size vs asymmetry (high-T low-A data)
- High-T IAS
- Dependence on asymmetry for central-collision observables:
  - collective flow
  - yields ($S(\rho > \rho_0)$)
  - stopping
  - asymmetry transport
  - low-velocity correlations

Asymmetry Oscillations

Movement of neutrons vs protons - giant resonances visible in excitation cross sections

2 classical models of the simplest giant dipole resonance (GDR)



Goldhaber-Teller (GT): n & p distributions oscillate against each other as rigid entities:

$$E_{GDR} = \hbar\Omega \propto \sqrt{A^{2/3}/A} = A^{-1/6}$$

Steinwedel-Jensen (SJ): Standing wave of n-p in the interior with vanishing flux at the surface

$$E_{GDR} = \hbar c_a/\lambda \propto A^{-1/3}$$

GDR Evolution with Mass

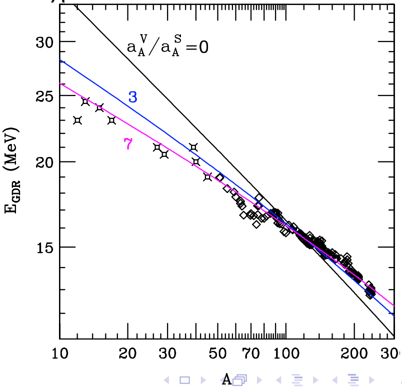
GT model: $a_A^V \to \infty$ SJ model: $a_A^S \to \infty$

Realistic model: SJ but asymmetry flux may flow in and out of the surface. . . The boundary condition produces:

$$qR\, j_1(qR) = \frac{3\, a_A^S\, A^{1/3}}{a_A^V}\, j_1'(qR)$$

j_1 - spherical Bessel function, typical for waves when spherical symmetry; q - wavenumber, $E_{GDR} = \hbar\, c_A\, q$

As $a_A^S\, A^{1/3}/a_A^V$ changes, the condition changes between that of open and close pipe and the resonance evolves between GT and SJ

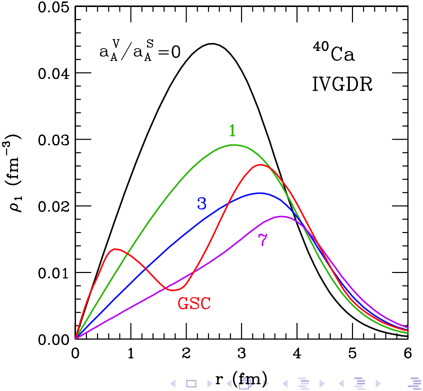


Transition Densities

Local Amplitude $\equiv$ Transition Density

$$\rho_1(r) = \frac{D_V}{\rho_0} j_1(qr) \left[\rho(r) - \frac{a_A^V}{3\, a_A^S\, A^{1/3}}\, r\, \frac{d\rho}{dr} \right]$$

Compared to microscopic calculations (Khamerdzhiev *et al.*, NPA624(97)328) GSC, including 2p-2h excitations and ground-state correlations



Liquid Droplet Model

Liquid droplet model (Myers & Swiatecki '69)

$$E = \left(-a_1 + J\,\bar{\delta}^2 - \frac{1}{2}K\,\bar{\epsilon}^2 + \frac{1}{2}M\,\bar{\delta}^4\right)A + \left(a_2 + Q\,\tau^2 + a_3\,A^{-1/3}\right)A^{2/3} + c_1\,\frac{Z^2}{A^{1/3}}\left(1 + \frac{1}{2}\,\tau\,A^{-1/3}\right) - c_2\,Z^2\,A^{1/3} - c_3\,\frac{Z^2}{A} - c_4\,\frac{Z^{4/3}}{A^{1/3}}$$

where

$$\bar{\epsilon} = \frac{1}{K}\left(-2a_2\,A^{-1/3} + L\,\bar{\delta}^2 + c_1\,\frac{Z^2}{A^{4/3}}\right), \qquad \tau = \frac{3}{2}\,\frac{J}{Q}\,(\bar{\delta} + \bar{\delta}_s)$$

$$\bar{\delta} = \frac{I + \frac{3}{8}\,\frac{c_1}{Q}\,\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\,\frac{J}{Q}\,A^{-1/3}}, \qquad \bar{\delta}_s = -\frac{c_1}{12J}\,\frac{Z}{A^{1/3}}, \qquad I = \frac{N-Z}{N+Z}$$

$Q = H/(1 - \frac{2}{3}\,P/J)$. Expansion in asymmetry yields results consistent with current, but approach more complex...

Liquid Drop Model

The current formula:

$$E = -a_V\,A + a_S\,A^{2/3} + a_C\,\frac{Z^2}{A^{1/3}} + a_A^V\,\frac{(N-Z)^2}{A}\,\frac{1}{1+\frac{a_A^V}{a_A^S}\,A^{-1/3}}$$

Liquid drop model [LDM] (Myers & Swiatecki '66)

$$\begin{aligned} E \;&=\; -a_V\left(1-\kappa_V\,I^2\right)A + a_S\left(1-\kappa_S\,I^2\right)A^{2/3} \\ &\quad + a_C\,\frac{Z^2}{A^{1/3}} - a_4\,\frac{Z^2}{A} \end{aligned}$$

with $I=(N-Z)/A$. LDM corresponds to the expansion in the current formula:

$$\frac{1}{\frac{A}{a_A^V}+\frac{A^{2/3}}{a_A^S}} \simeq \frac{a_A^V}{A}\left(1-\frac{a_A^V}{a_A^S}\,A^{-1/3}\right)$$

But that expansion only accurate for $A \gtrsim 1000$, i.e. never!