

# Clustering in Neutron-rich Nuclei

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Osaka Univ.

1. Introduction
2. Di-cluster core + neutrons
3. Di-neutron cluster
4. Multi-cluster core + neutrons
5. Summarizing discussion

RIA Theory Meeting, Argonne, 4-7 April '06

# 1. Introduction

**neutron-rich nuclei**



**neutron skin, neutron halo**

**( weakly-bound neutrons with dilute density )**



**correlations and clustering**

**due to weakly bound dilute neutrons**

**Here, two kinds of clustering generated or supported by excess neutrons**

**1.  $\alpha$  cluster structure**

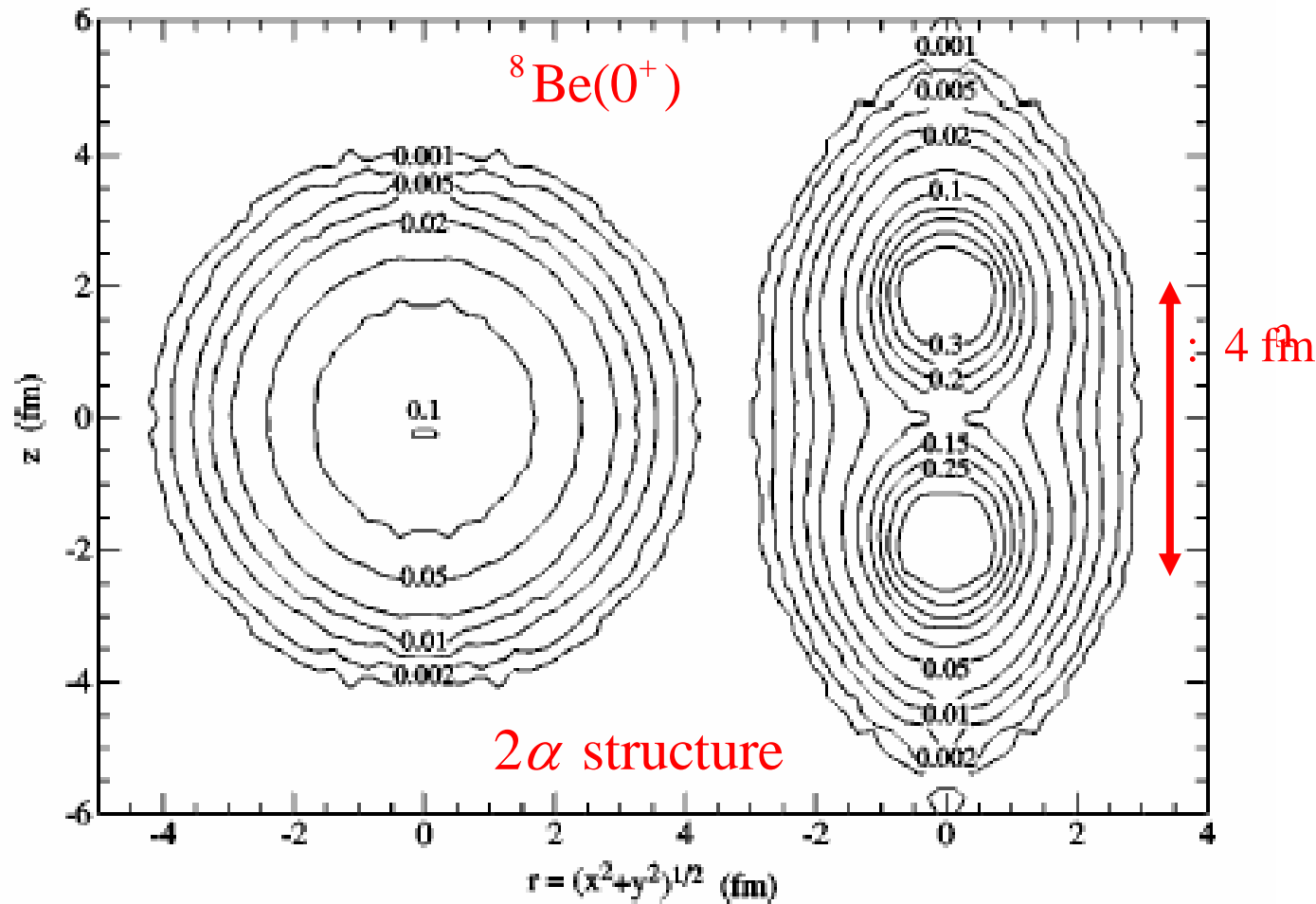
**two-cluster structure**

**multi-cluster structure**

**2. formation of di-neutron clusters**

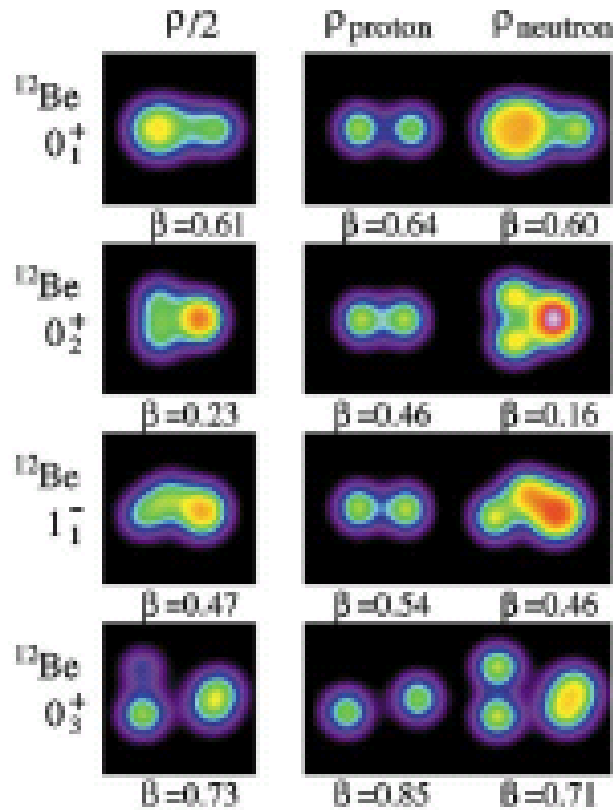
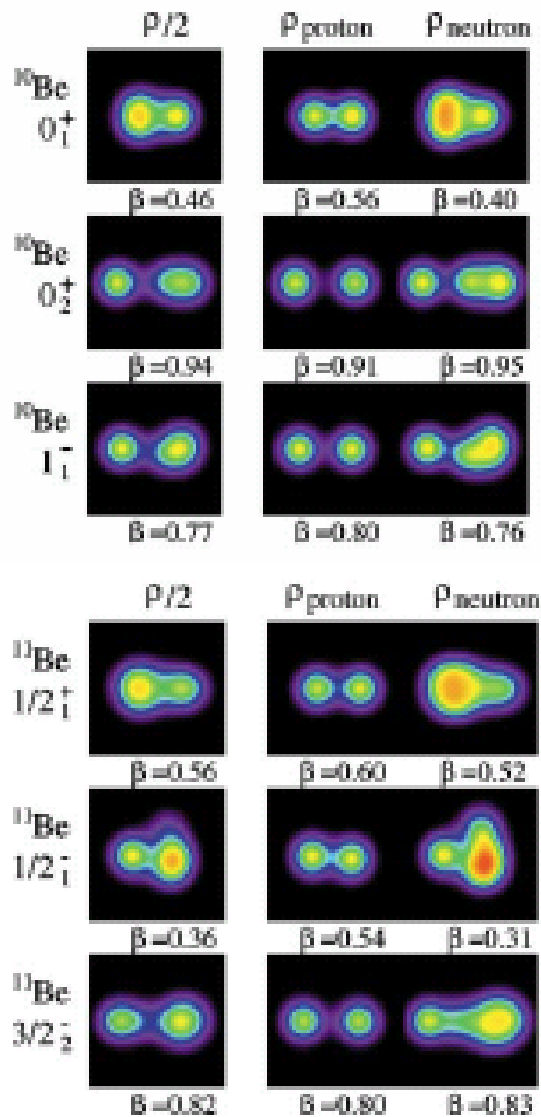
## 2. Di-cluster core + neutrons

Ab initio calculation with realistic nuclear force



Wiringer, Pieper,  
Carlson,  
Pandharipande

P.R.C. 62, 014001  
(2000)

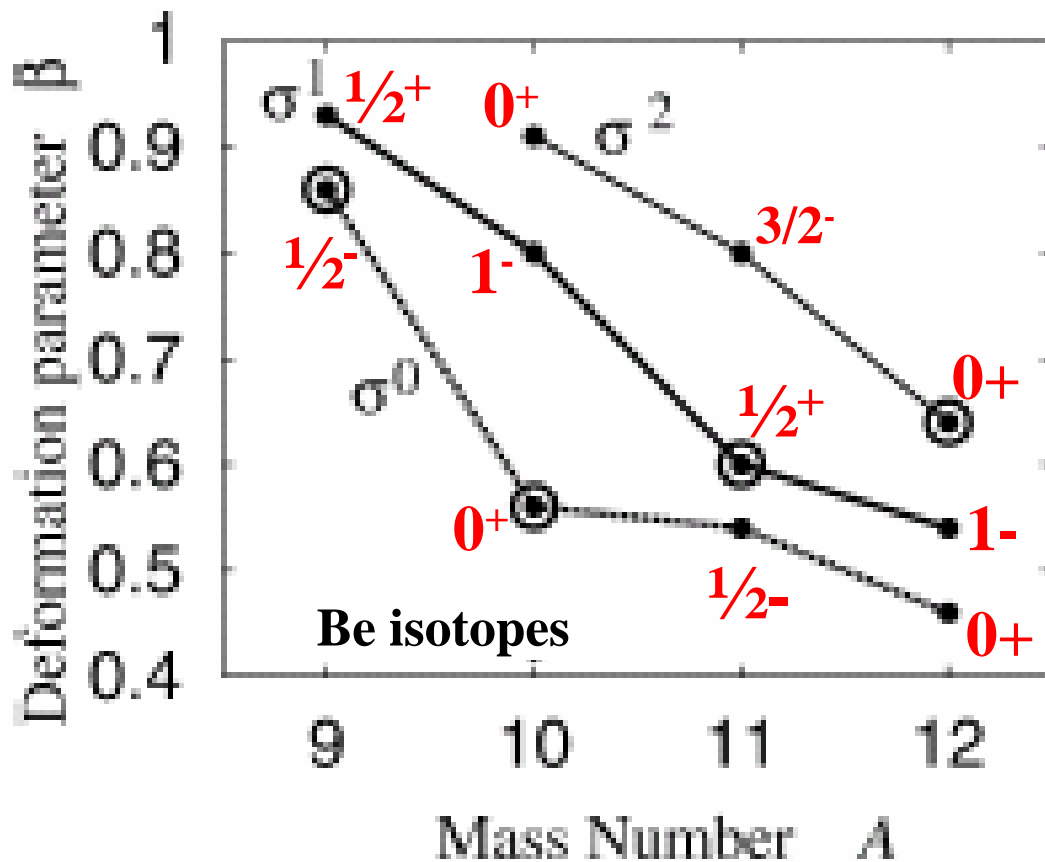


**AMD**  
(antisymmetrized  
molecular  
dynamics)

**VAP**  
calculation

**Density distribution of the intrinsic states**  
C.R.Physique 4( '03), 497

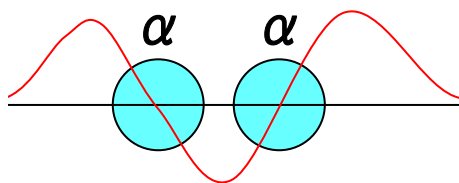
Kanada-En'yo et al.



  
 ground states

**Shell inversion  
 for  $^{11}\text{Be}$  and  $^{12}\text{Be}$**

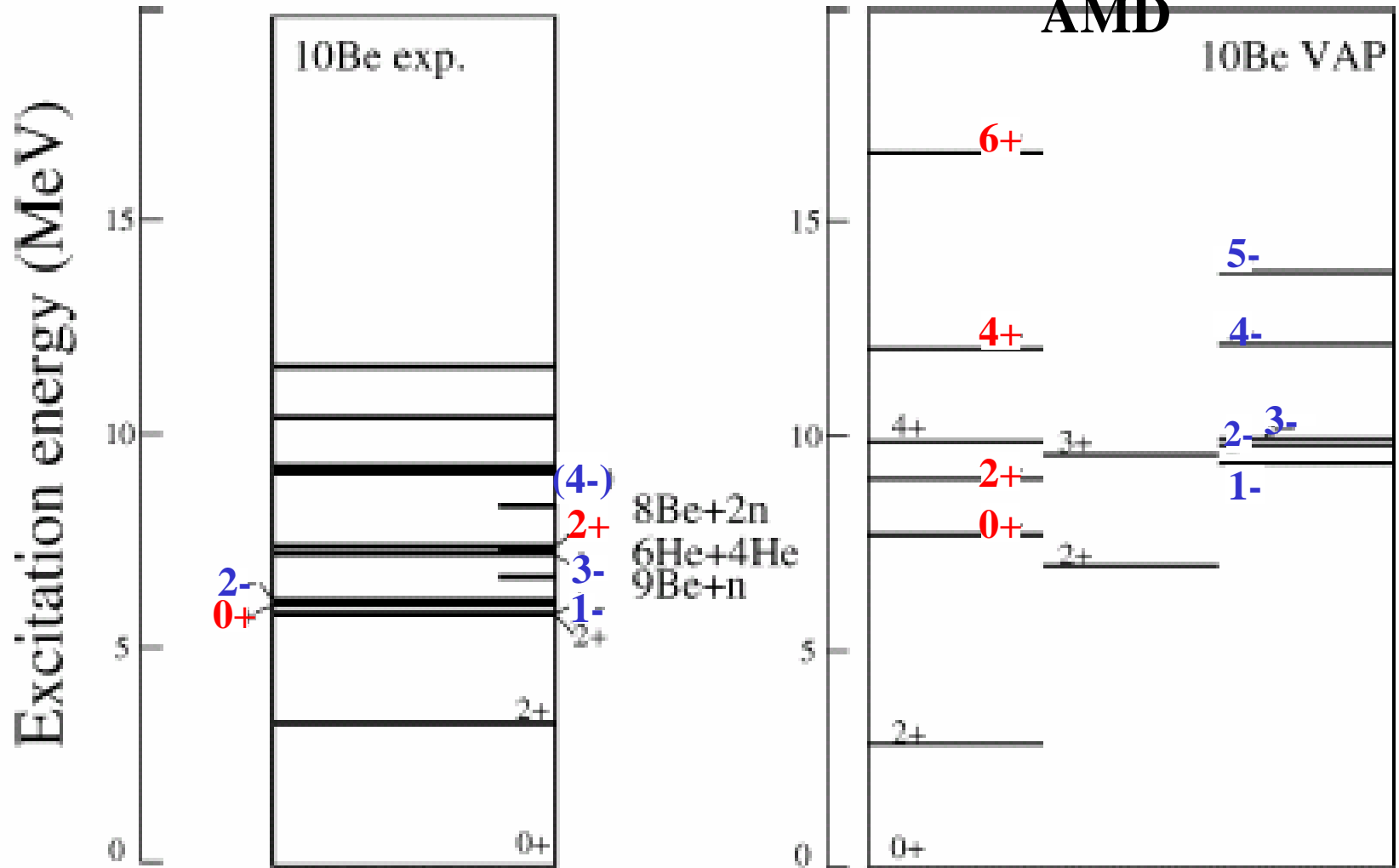
Kanada-En'yo et al.



**$\sigma$ -orbit  
 (sd-orbit)**

**$\sigma$ -orbit sustains  
 or enhances clustering**

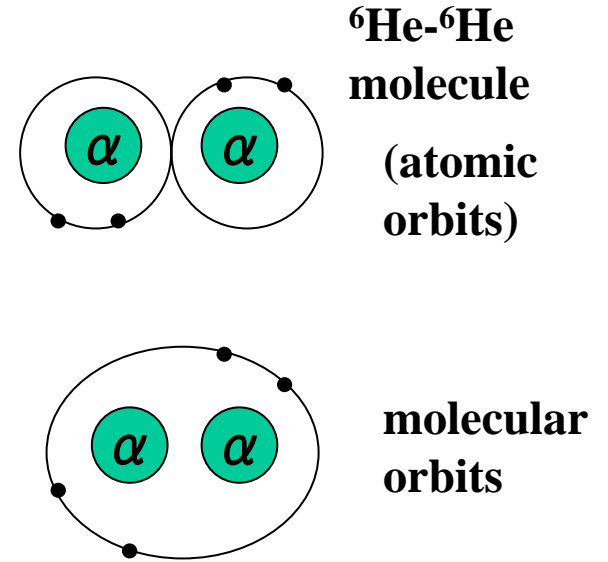
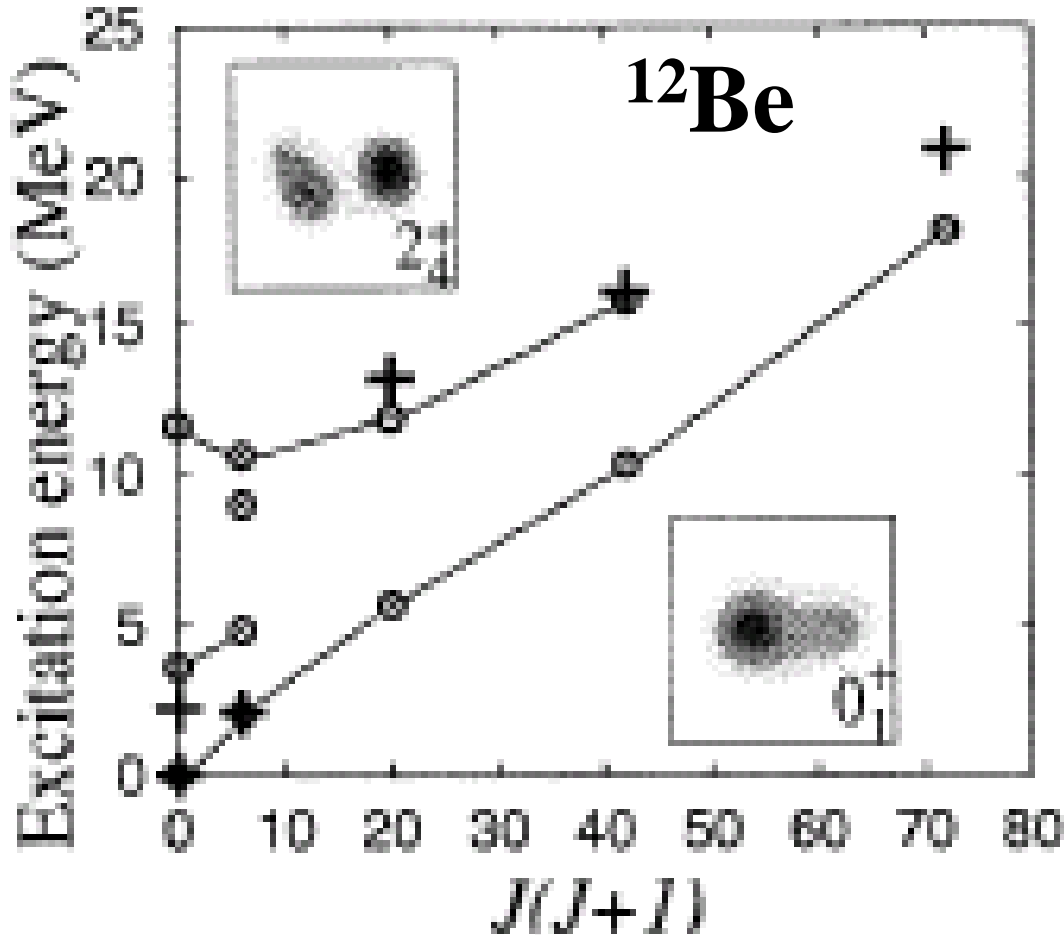
$^{10}\text{Be}$



Kanada-En'yo et al.

○ : AMD  
calculation

⊕ : experiments



Observation of shell-model-like  $0_2^+$

S. Shimoura, et al.,  
Phys.Lett. **560B**,  
31 (2003)

Observation of molecular band

M. Freer, et al., Phys. Rev. Lett. **82**, 1383 (1999)

M. Freer, et al., Phys. Rev. C **63**, 034301 (2001)

Table X.  $E2$  and  $E1$  transition strength. The theoretical results of VAP calculations with the case (g) are compared with the experimental data.<sup>58)</sup> The shell model calculations are quoted from the work with the  $(0+2)\hbar\omega$  shell model in Ref. 59).

transitions	Mult	exp	present VAP	shell model
$^{10}\text{Be}; 2_1^+ \rightarrow 0_1^+$	$E2$	$10.5 \pm 1.1$ (e fm <sup>2</sup> )	11 (e fm <sup>2</sup> )	16.26 (e fm <sup>2</sup> )
$^{10}\text{Be}; 0_2^+ \rightarrow 2_1^+$	$E2$	$3.3 \pm 2.0$ (e fm <sup>2</sup> )	0.6 (e fm <sup>2</sup> )	7.20 (e fm <sup>2</sup> )
$^{10}\text{Be}; 0_2^+ \rightarrow 1_1^-$	$E1$	$1.3 \pm 0.6 \times 10^{-2}$ (e fm)	$0.6 \times 10^{-2}$ (e fm)	
$^{10}\text{C}; 2_1^+ \rightarrow 0_1^+$	$E2$	$12.3 \pm 2.0$ (e fm <sup>2</sup> )	9 (e fm <sup>2</sup> )	15.22 (e fm <sup>2</sup> )

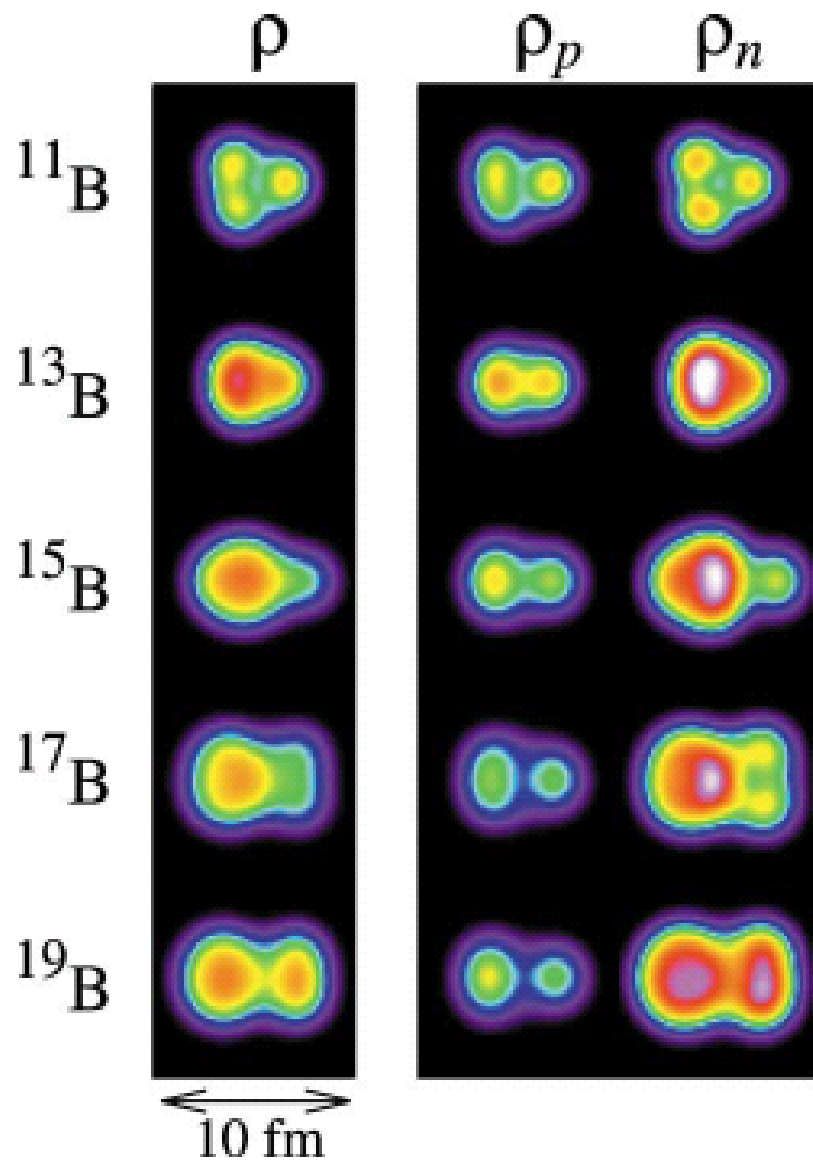
Table XI.  $B(\text{GT})$  values of  $\beta$  decays which are the square of the expectation values of Gamow-Teller operator. The experimental data are the values<sup>a)</sup> deduced from the cross sections of  $^{10}\text{B}(t, ^3\text{He})^{10}\text{Be}^*$  at  $0^\circ$  forward angle<sup>54)</sup> and the one<sup>b)</sup> from Ref. 60). The theoretical results are obtained with the case (g) and the case (h) interactions for  $^{10}\text{Be}$  and  $^{10}\text{B}$ .

initial ( $J^\pi, E_x$ ) (MeV)	exp final ( $J^\pi, E_x$ ) (MeV)	B(GT)	
$^{10}\text{B}(3^+, 0)$	$^{10}\text{Be}(2_1^+, 3.37)$	$0.08 \pm 0.03^{\text{a)}$	
$^{10}\text{B}(3^+, 0)$	$^{10}\text{Be}(2_2^+, 5.96)$	$0.95 \pm 0.13^{\text{a)}$	
$^{10}\text{B}(3^+, 0)$	$^{10}\text{Be}(2^+ \text{ or } 3^+, 9.4)$	$0.31 \pm 0.08^{\text{a)}$	
$^{10}\text{C}(0^+, 0)$	$^{10}\text{B}(1^+, 0.72)$	$3.44^{\text{b)}$	
initial	final	cal (g)	cal (h)
$^{10}\text{B}(3^+)$	$^{10}\text{Be}(2_1^+)$	0.02	0.00
	$^{10}\text{Be}(2_2^+)$	1.1	0.92
	$^{10}\text{Be}(3_1^+)$	0.40	0.38
	$^{10}\text{Be}(4_1^+)$	0.08	0.10
	$^{10}\text{Be}(2_3^+)$	0.03	0.00
$^{10}\text{Be}(0_1^+)$	$^{10}\text{B}(1^+)$	2.9	2.5

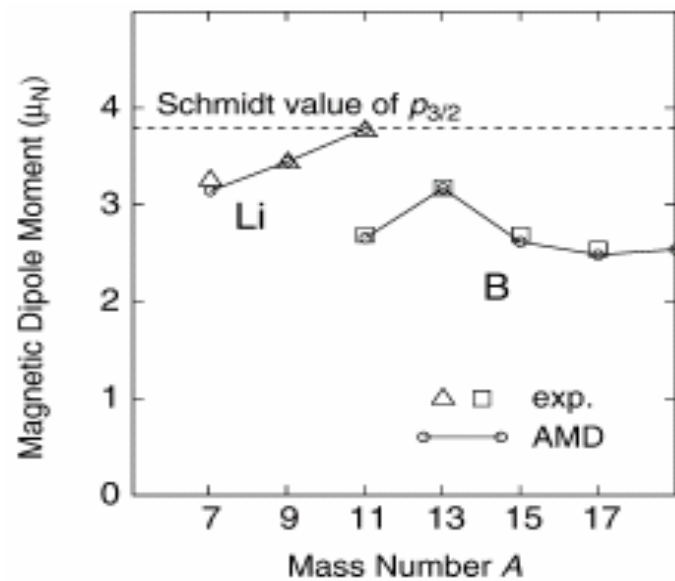
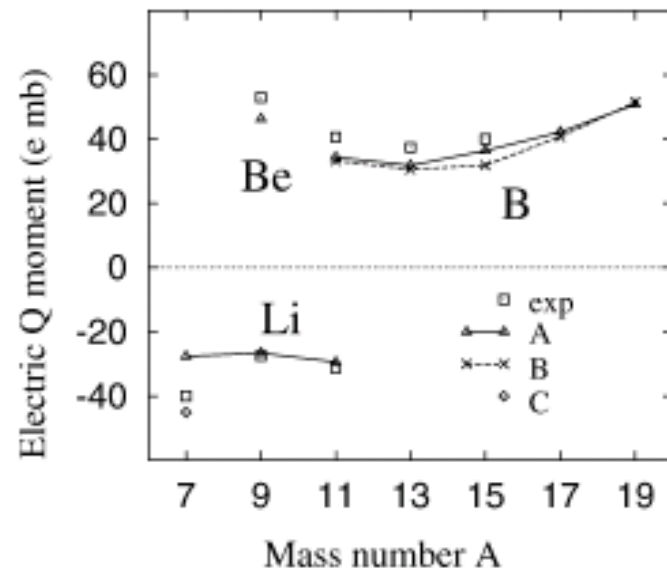
<p>B(GT) between  <math>^{12}\text{Be}(0+)</math> and  <math>^{12}\text{B}(1+)</math>  Exp: 0.59  Cal: 0.8</p>
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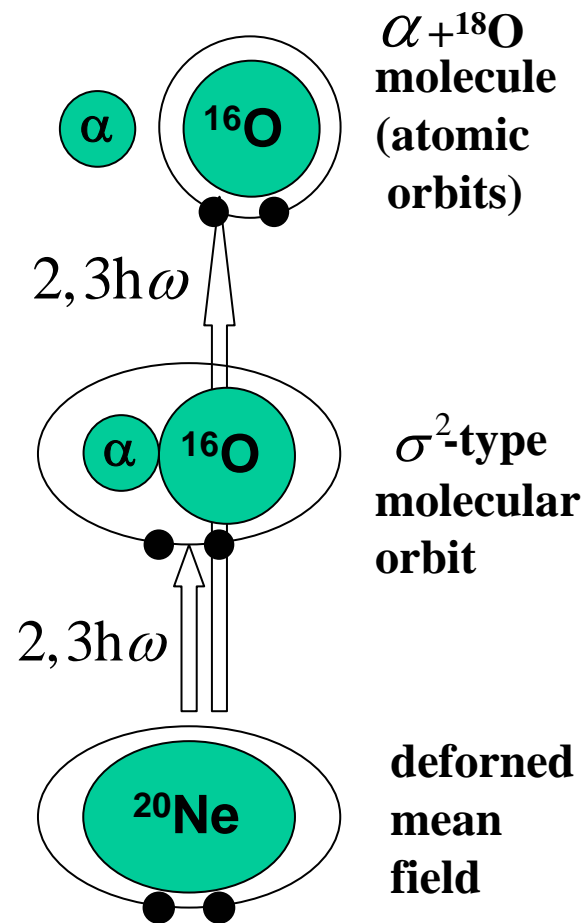
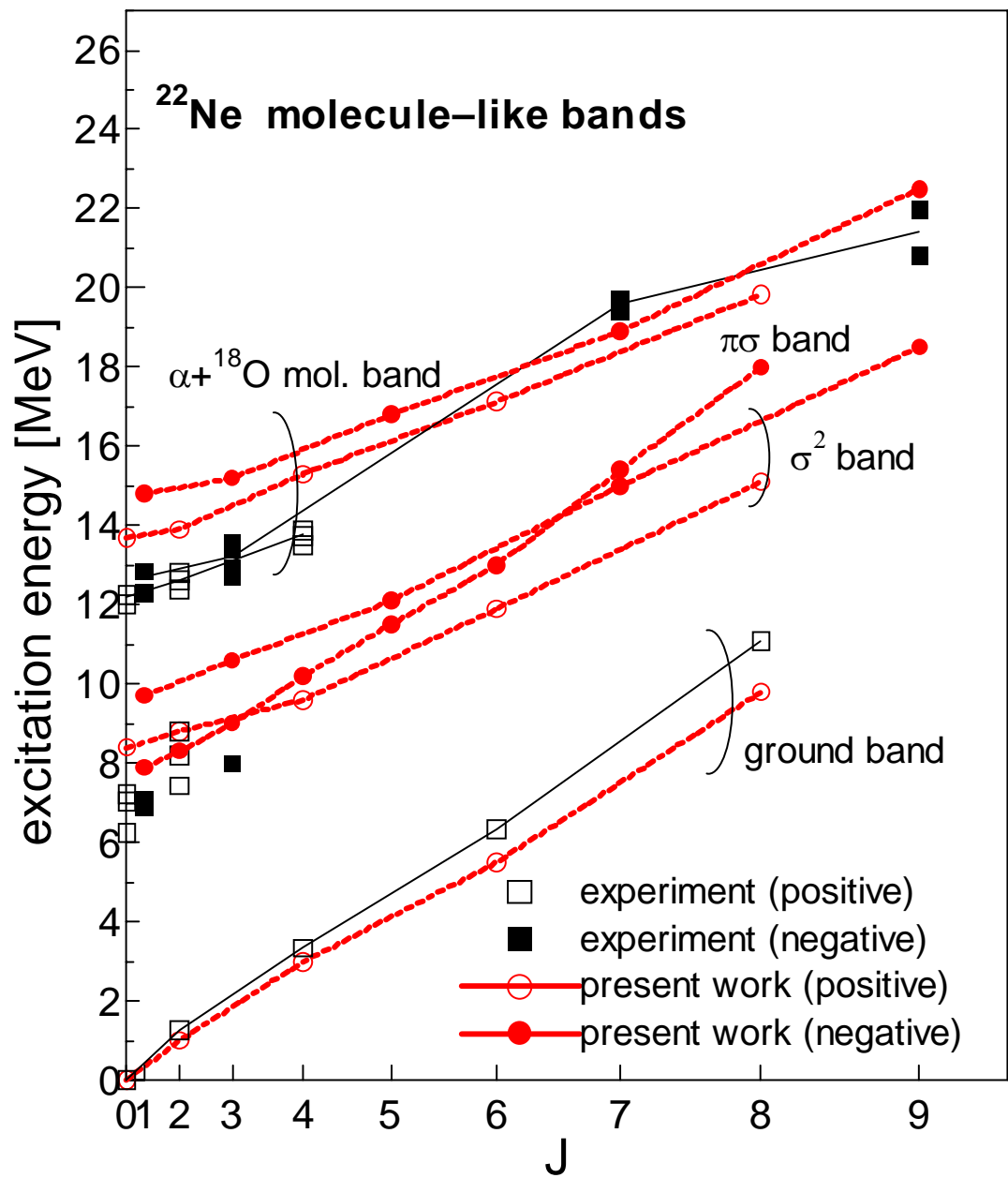
Kanada-En'yo et al.



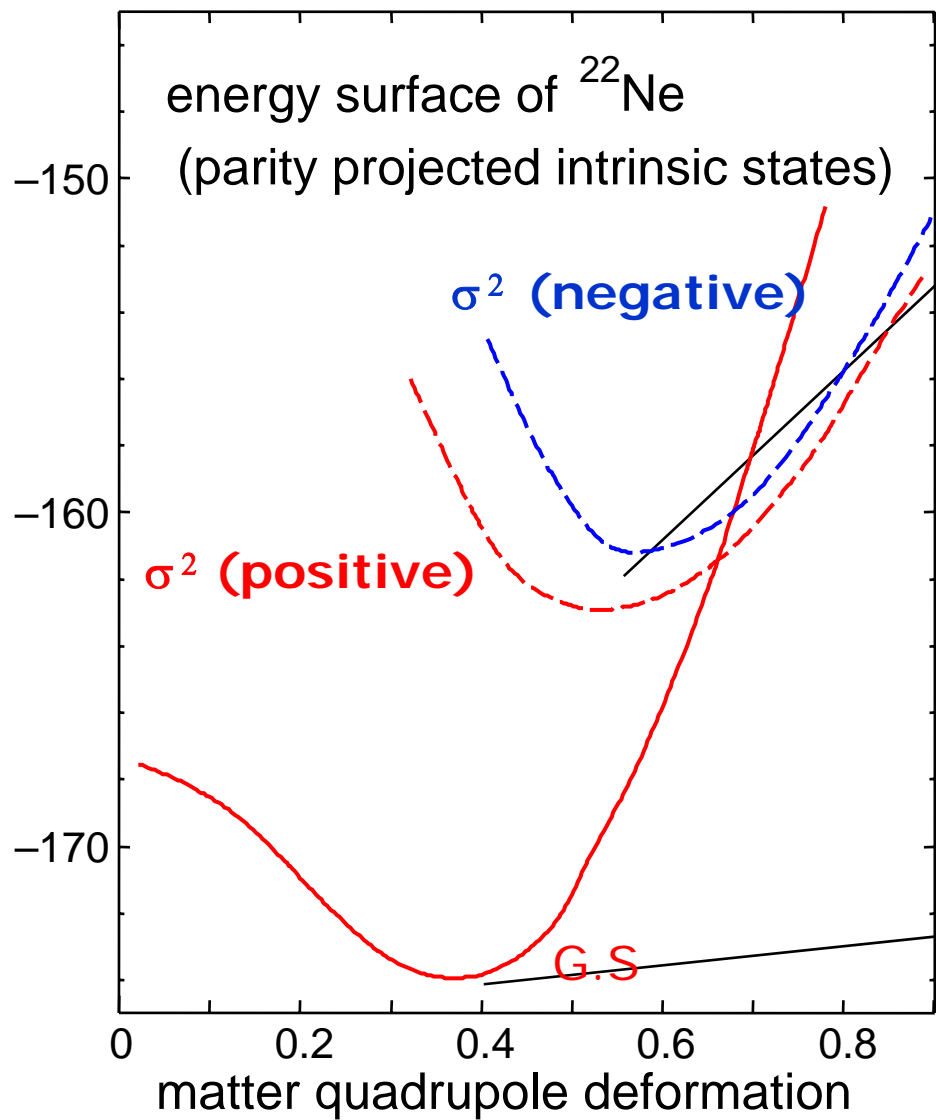


Kanada-En'yo et al.

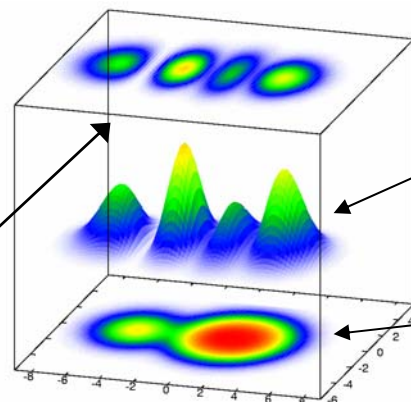




**AMD calculation  
by M. Kimura**



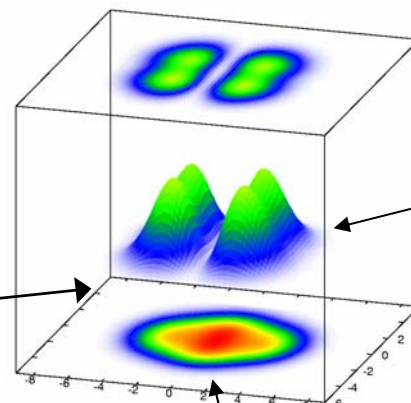
$\alpha+^{16}\text{O}+\sigma^2\text{-type}(0f_{7/2})$



staying probability  
of two excess neutrons

density distribution  
of core ( $^{20}\text{Ne}$ )

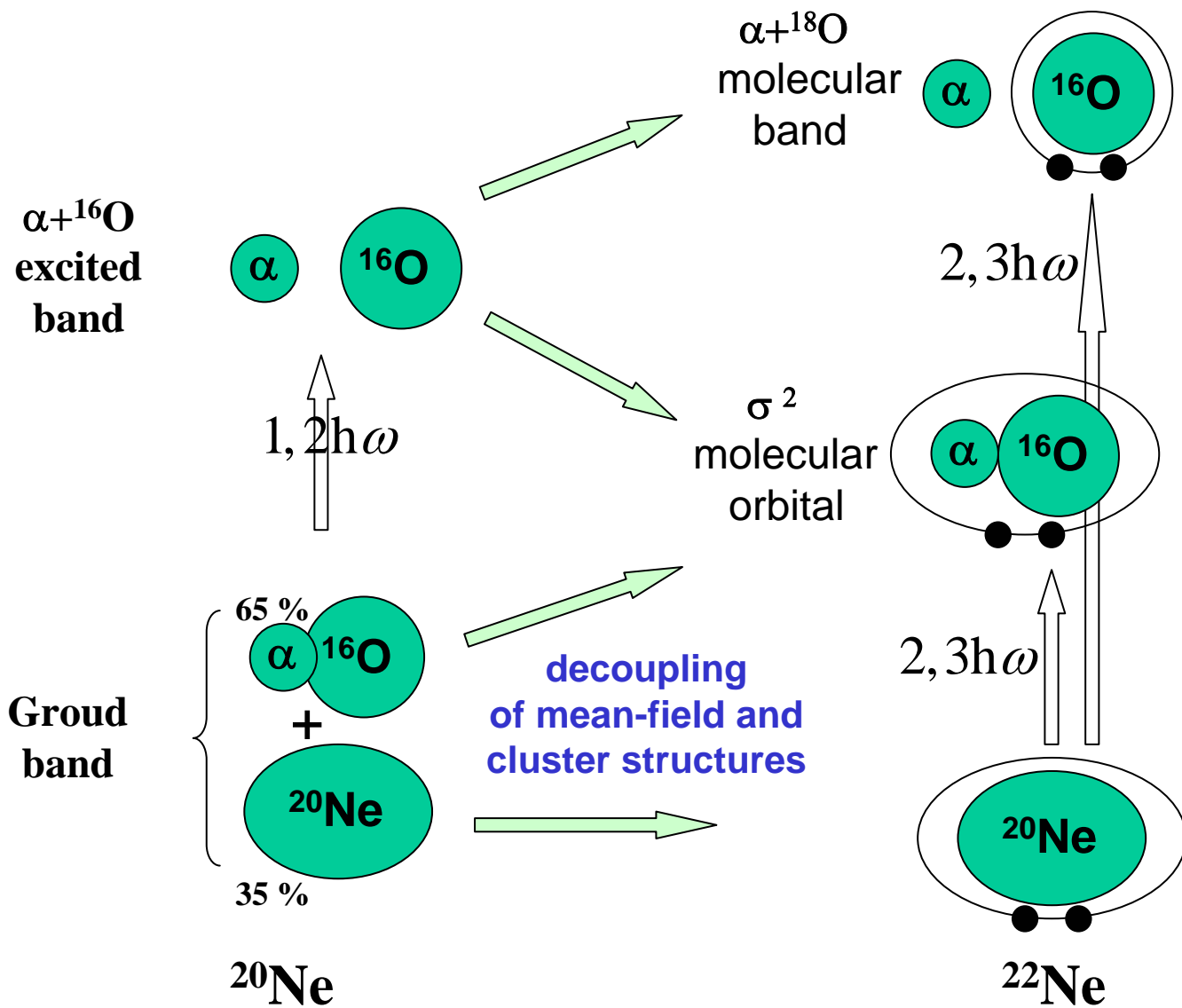
$^{20}\text{Ne}(\text{Shell}) + 2n (0d_{5/2})$



staying probability of  
two excess neutrons

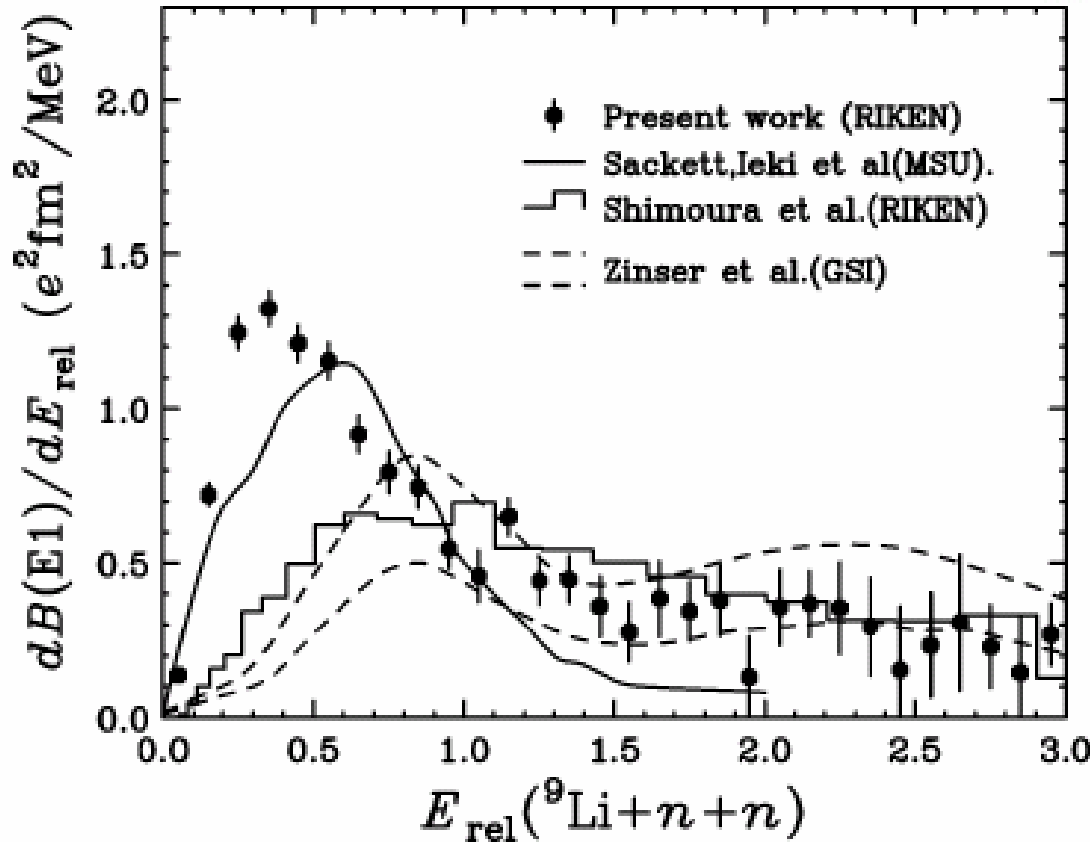
density distribution of core ( $^{20}\text{Ne}$ )





### 3. Di-neutron cluster

**<sup>11</sup>Li**



Present work:

**T.Nakamura et al.**

RIKEN Accel. Prog. Rep. 38

MSU @ 28MeV/nucleon  
PRL 70 (1993) 730.  
PRC 48(1993) 118.

RIKEN @ 43MeV/nucleon  
PLB348 (1995) 29.

GSI @280MeV/nucleon  
NPA 619 (1997) 151.

$$\frac{dB(E1)}{dE_x} \propto \left| \langle \exp(iqr) \left| \frac{Z}{A} r Y^1_m \right| \Phi_{gs} \rangle \right|^2$$

For di-neutron structure,

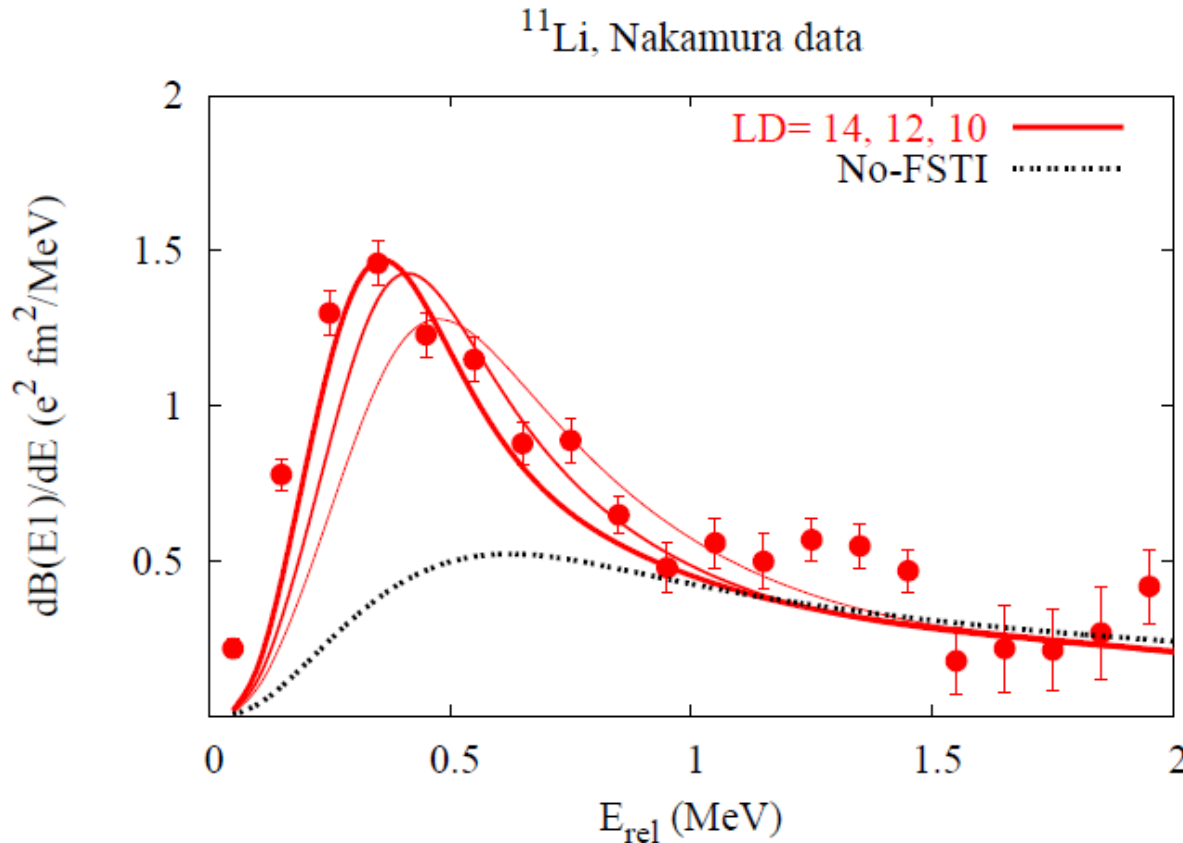
peak position is at **1.6 S<sub>2n</sub>**

S<sub>2n</sub>=0.66 MeV  $\implies$  **E<sub>peak</sub>=0.47 MeV**

# Three-body model

G.F.Bertsch and H. Esbensen, *Ann. Phys. (NY)* 209, 327 (1991).  
 H. Esbensen and G.F.Bertsch, *Nucl. Phys. A* 542, 310 (1992).  
 H. Esbensen, G.F.Bertsch, and K.Hencken, *Phys.Rev. C* 56, 3054 (1999).  
 K.Hagino and H.Sagawa, *Phys.Rev.C* 72, 044321 (2005).

$$H = \hat{h}_{nC}(1) + \hat{h}_{nC}(2) + V_{nn} + \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{A_c m}.$$



LD=14,12,10:  
 maximum angular  
 momentum of single-  
 particle states used  
 to calculating  
 $[1 + G_{1\mu}(0)V_{nn}]^{-1}$

**H. Esbensen**

# $^{11}\text{Li}$

$$B(E1) = \sum_k \frac{\Gamma}{\pi} \frac{1}{(E - E_k)^2 + \Gamma^2} B_k(E1)$$

**CAL:**

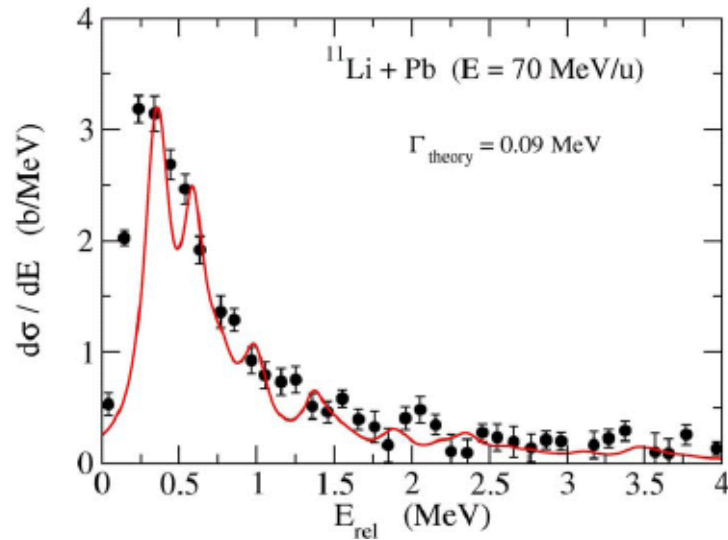
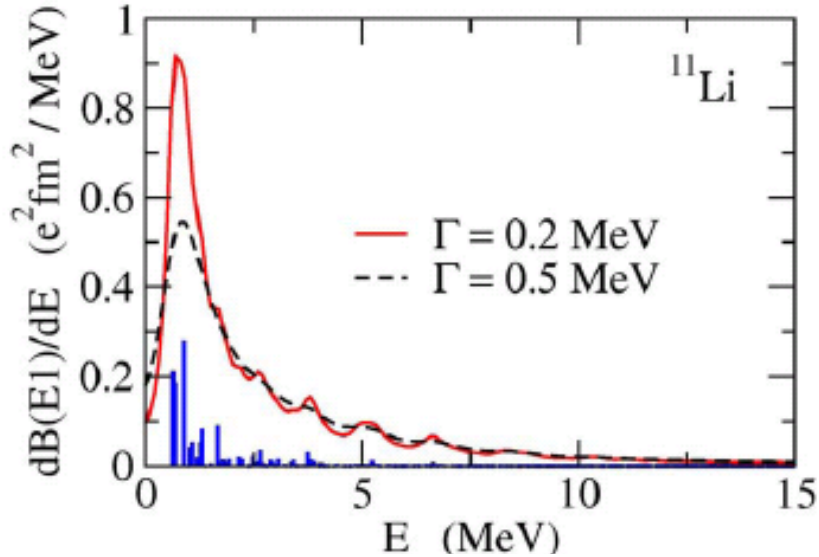
**$E_{\text{peak}} = 0.66 \text{ MeV}$**

**$B(E1) = 1.31 \text{ e2fm}^2 \text{ (} E < 3.3 \text{ MeV)}$**

**EXP:**

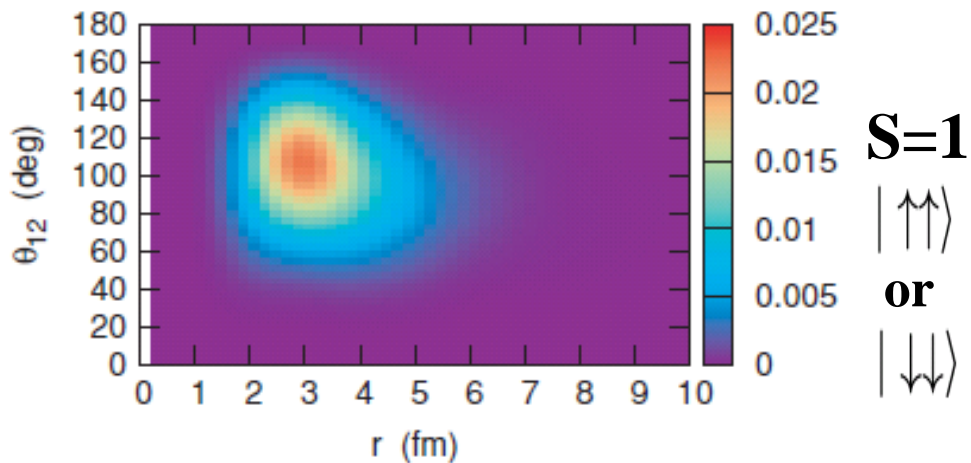
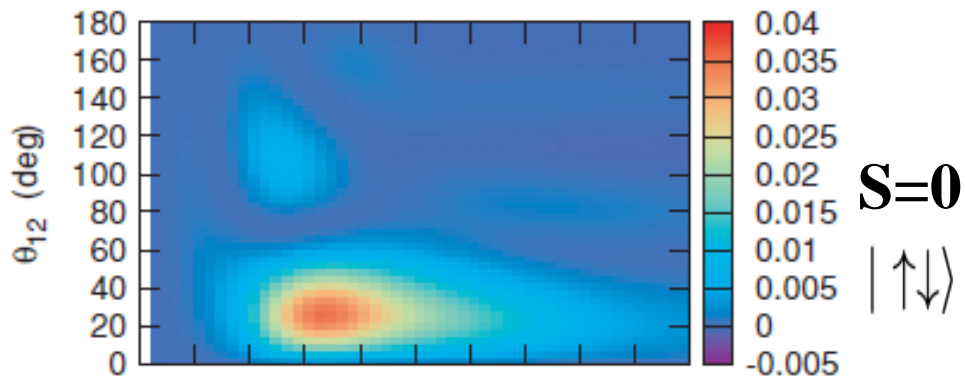
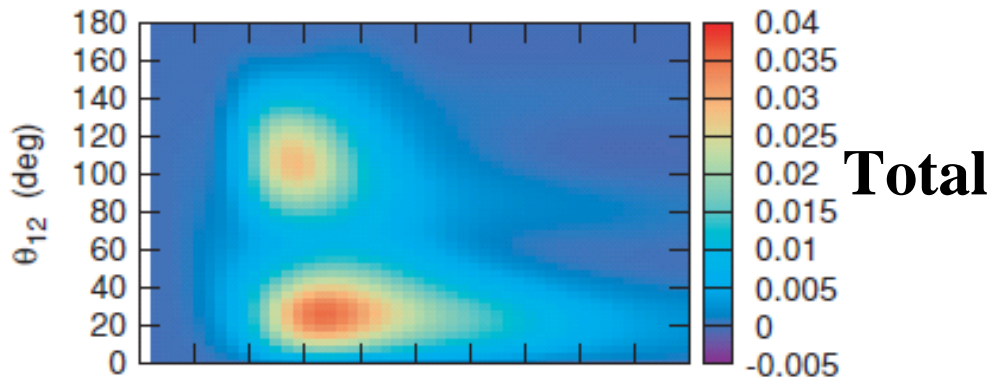
**$E_{\text{peak}} \approx 0.6 \text{ MeV}$**

**$B(E1) = 1.5 \pm 0.1 \text{ e2fm}^2 \text{ (} E < 3.3 \text{ MeV)}$**

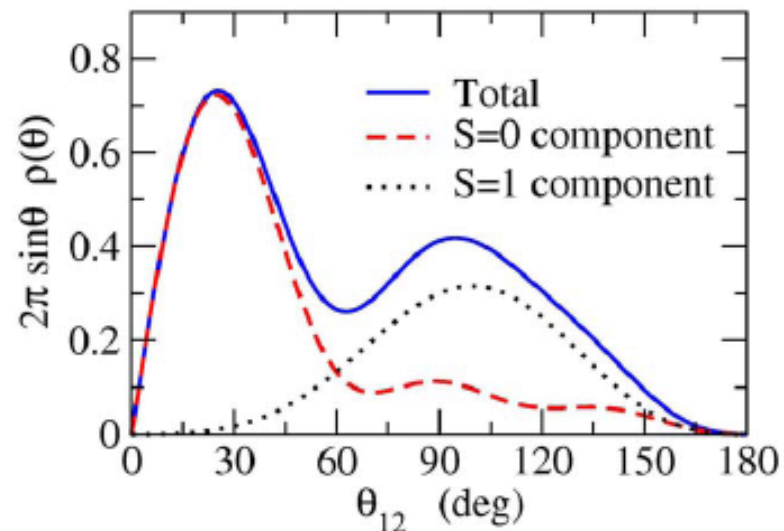
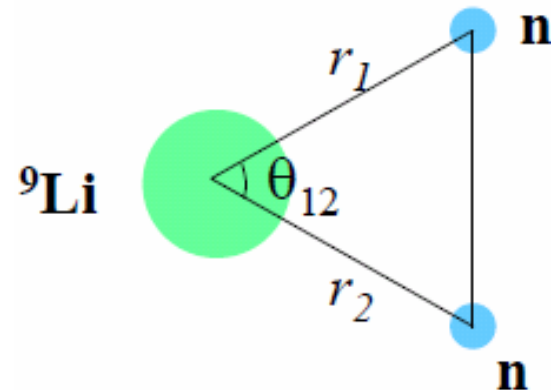


**K.Hagino and H.Sagawa**





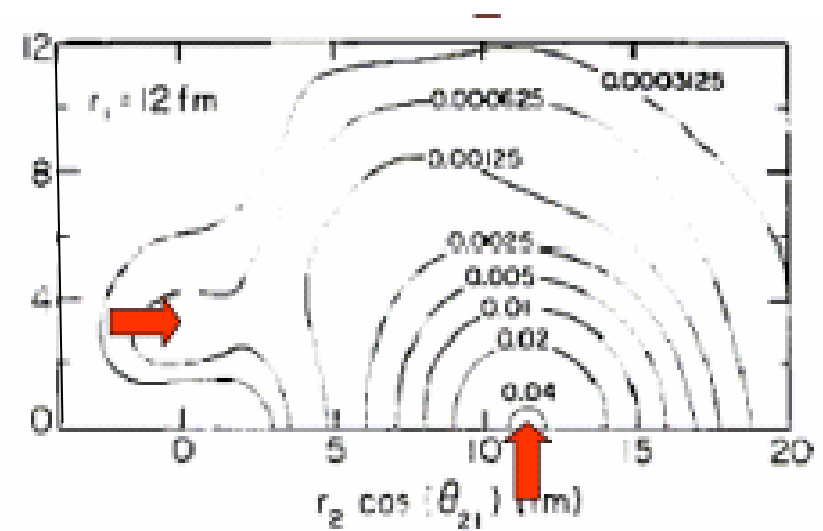
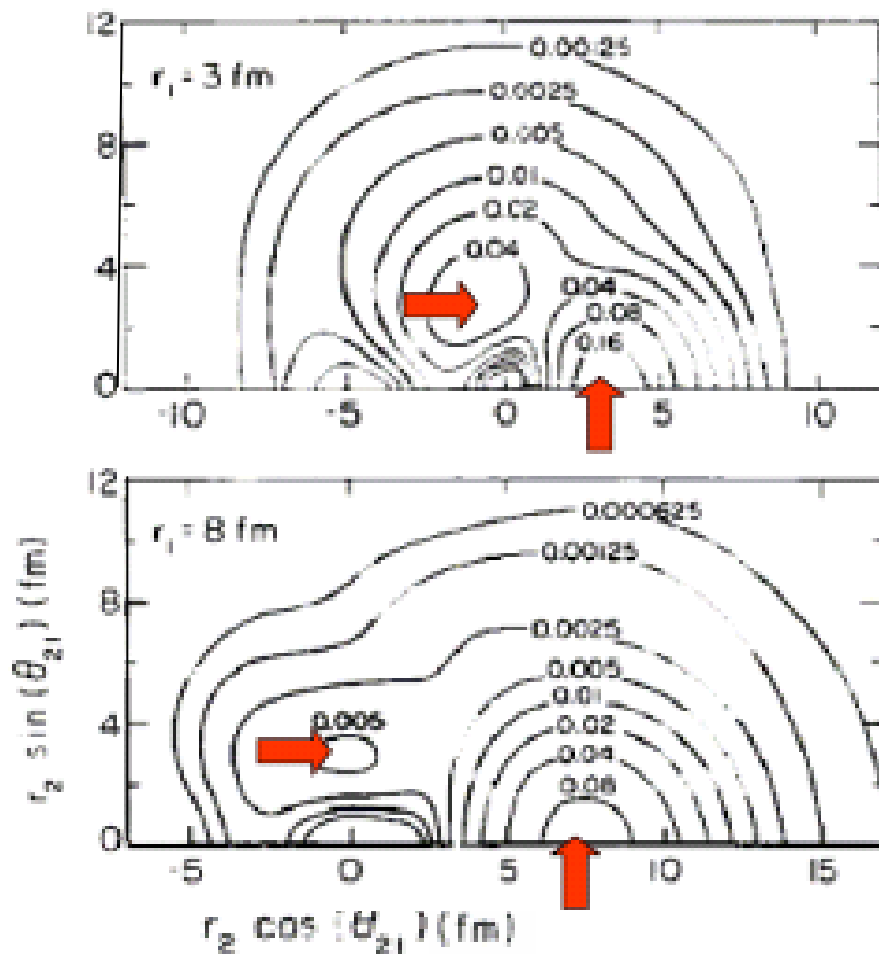
**$^{11}\text{Li}$**



**K.Hagino and H.Sagawa**

$\rho_2(r_1, r_2, \theta_{12})$  for  $^{11}\text{Li}$

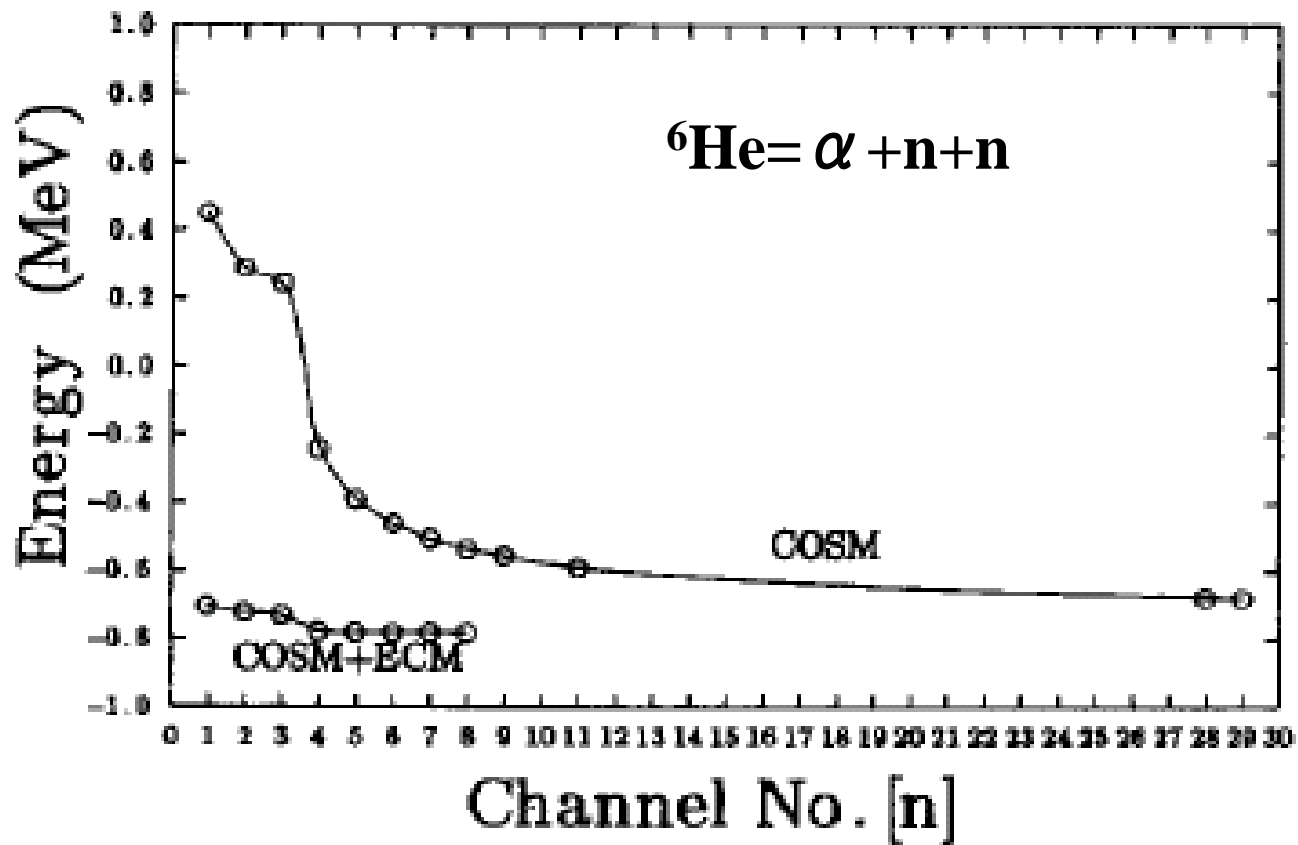
G.F.Bertsch, H.Esbensen  
Ann.Phys. 209 (1991), 327.



*“di-neutron” and “cigar-like”*  
configurations

S. Aoyama,  
S. Mukai,  
K. Kato,  
K. Ikeda

Prog.Theor.Phys.  
93 (1995), 99.



Importance of di-neutron configuration

In ECM, only  $l=L=0$  is employed.

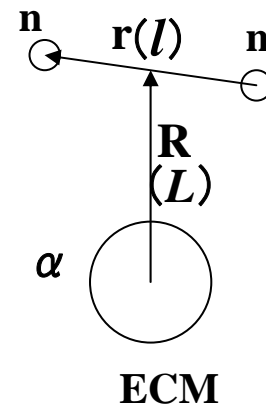
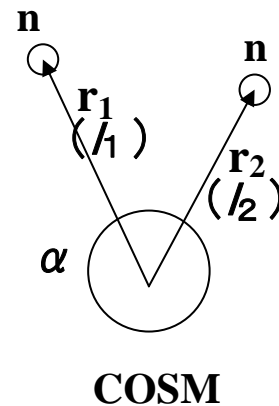


Table II. Channel configurations employed in the  $V$ -type basis functions for hybrid- $TV$  model and COSM calculations together with the ground state energies calculated in several steps. The  $l=L=0$  configuration is employed in the  $T$ -type basis functions of the hybrid- $TV$  model.

Channel No.	Configurations	Hybrid- $TV$	COSM
[1]	$(p_{3/2})^2$	-0.709 MeV	0.451 MeV
[2]	[1] + $(p_{1/2})^2$	-0.724	0.288
[3]	[2] + $(s_{1/2})^2$	-0.732	0.243
[4]	[3] + $(d_{5/2})^2$	-0.780	-0.244
[5]	[4] + $(d_{3/2})^2$	-0.782	-0.392
[6]	[5] + $(f_{7/2})^2$	-0.784	-0.463
[7]	[6] + $(f_{5/2})^2$	-0.784	-0.510
[8]	[7] + $(g_{9/2})^2$	-0.784	-0.538
[9]	[8] + $(g_{7/2})^2$		-0.559
[10]	[9] + $(h_{11/2})^2$		.
[11]	[10] + $(h_{9/2})^2$		-0.593
[12]	[11] + $(i_{13/2})^2$		.
[13]	[12] + $(i_{11/2})^2$		.
[14]	[13] + $(j_{15/2})^2$		.
[15]	[14] + $(j_{13/2})^2$		.
.	.		.
.	.		.
.	.		.
[28]	[27] + $(l=14, j=29/2)^2$		-0.681
[29]	[28] + $(l=14, j=27/2)^2$		-0.682

## Di-neutron correlation in the medium-mass region

### 2-body correlation density (spin anti-parallel)

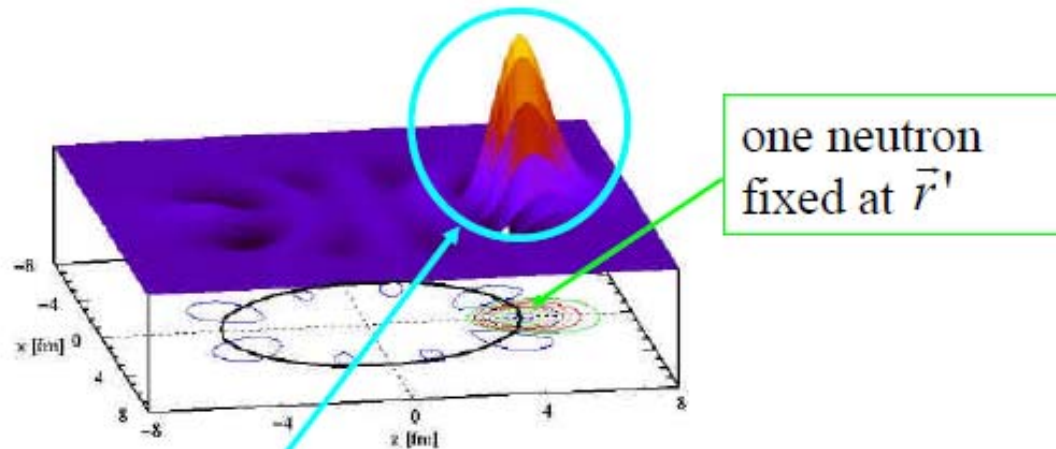
$$\rho_2^{corr}(\vec{r}'\uparrow; \vec{r}\downarrow) = \sum_{i \neq j} \delta(\vec{r} - \vec{r}_i) \delta_{\sigma_i \uparrow} \delta(\vec{r}' - \vec{r}_j) \delta_{\sigma_j \downarrow} - \rho_1(\vec{r}'\uparrow) \rho_1(\vec{r}\downarrow)$$

$$\approx |\Psi_{pair}(\vec{r}\uparrow, \vec{r}'\downarrow)|^2 \quad \text{wave function of neutron pair}$$

HFB with SLy4

Mix-type DDDI

$^{84}\text{Ni}$



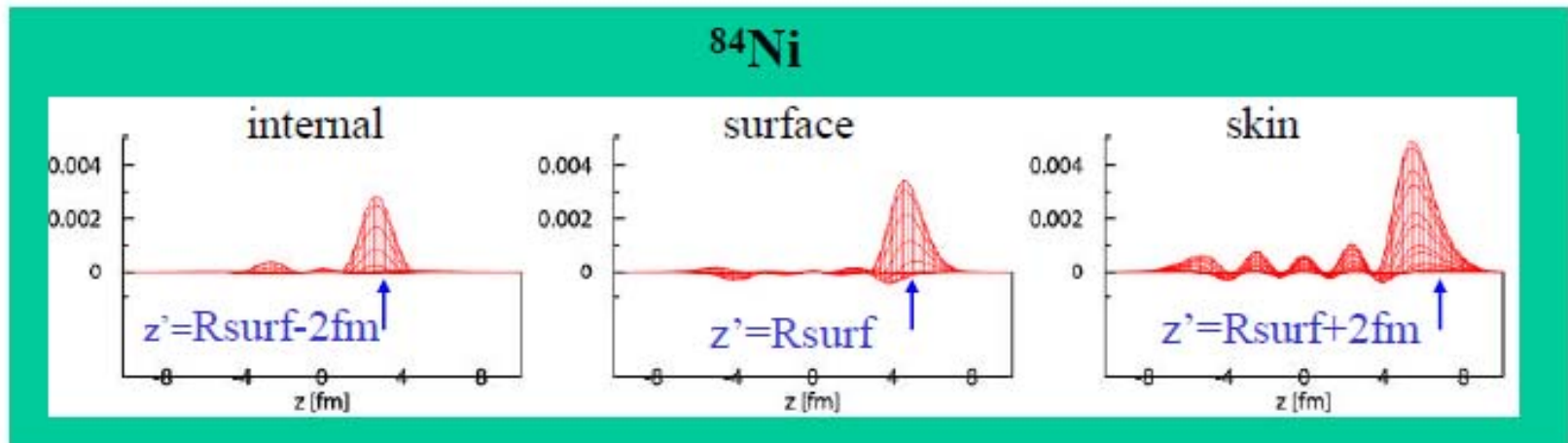
### di-neutron correlation

Strongly correlated at short relative distances  $|\mathbf{r}-\mathbf{r}'| < 2-3\text{fm}$

### Di-neutron probability

relative weight for  $|\mathbf{r}-\mathbf{r}'| < r_d$   
 $P(r_d) = 0.27 \quad (r_d = 2)$

# Di-neutron correlation is enhanced in the low-density skin region

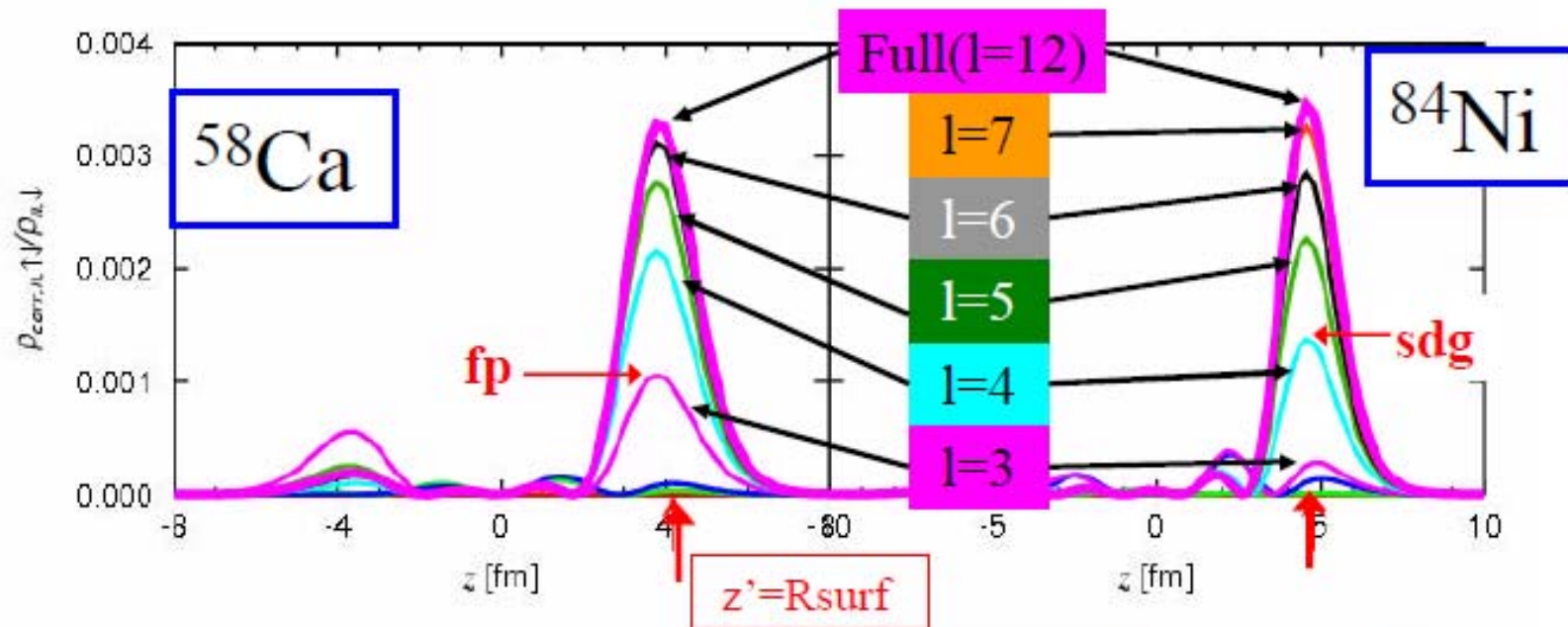


Di-neutron probability  $P(r_d)$  (relative weight within  $r_d = 2(3)\text{fm}$ )

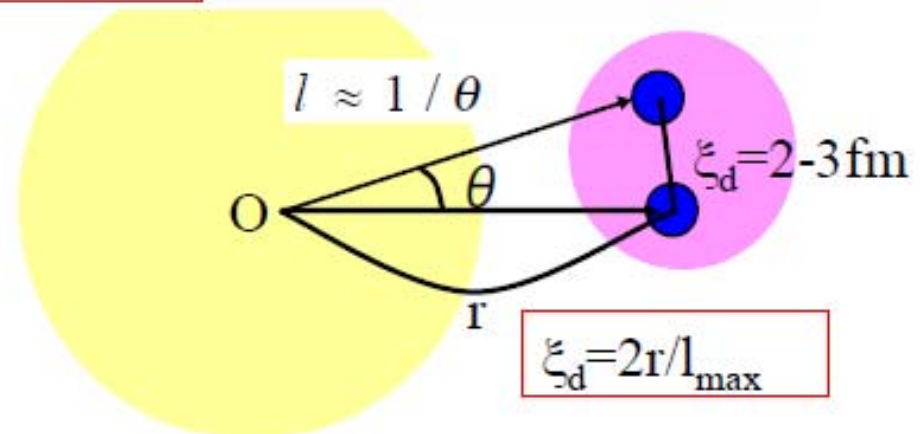
	Internal	surface	skin
$^{22}\text{O}$	0.32	0.48	0.47
$^{58}\text{Ca}$	0.39	0.53	0.59
$^{84}\text{Ni}$	0.32	0.49	0.47



# Configuration mixing: High-L orbits



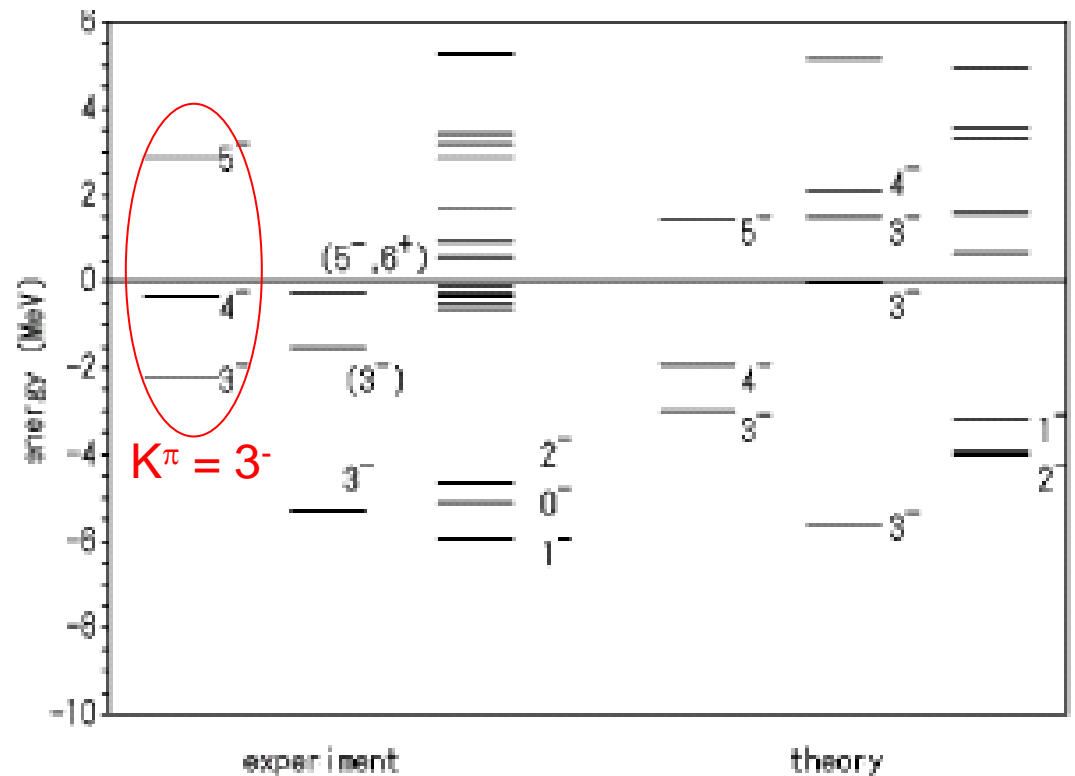
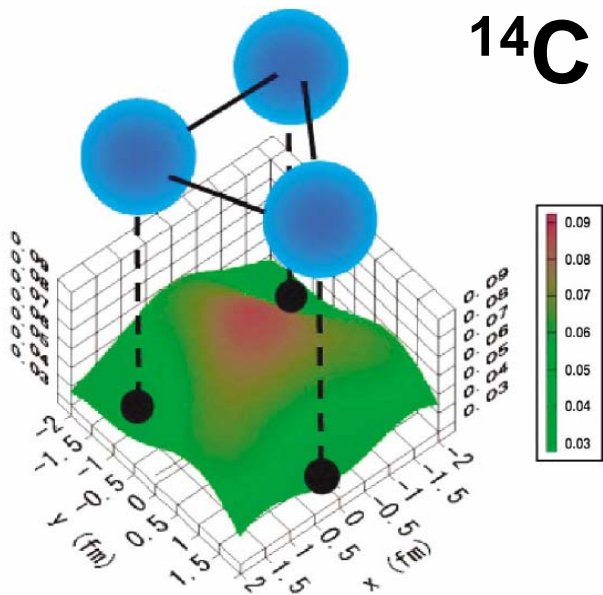
The coherent superposition of  
**high-L orbits  $l=3-8$**   
**in the continuum**  
 forms the di-neutron correlation



# 4. Multi-cluster core + neutrons

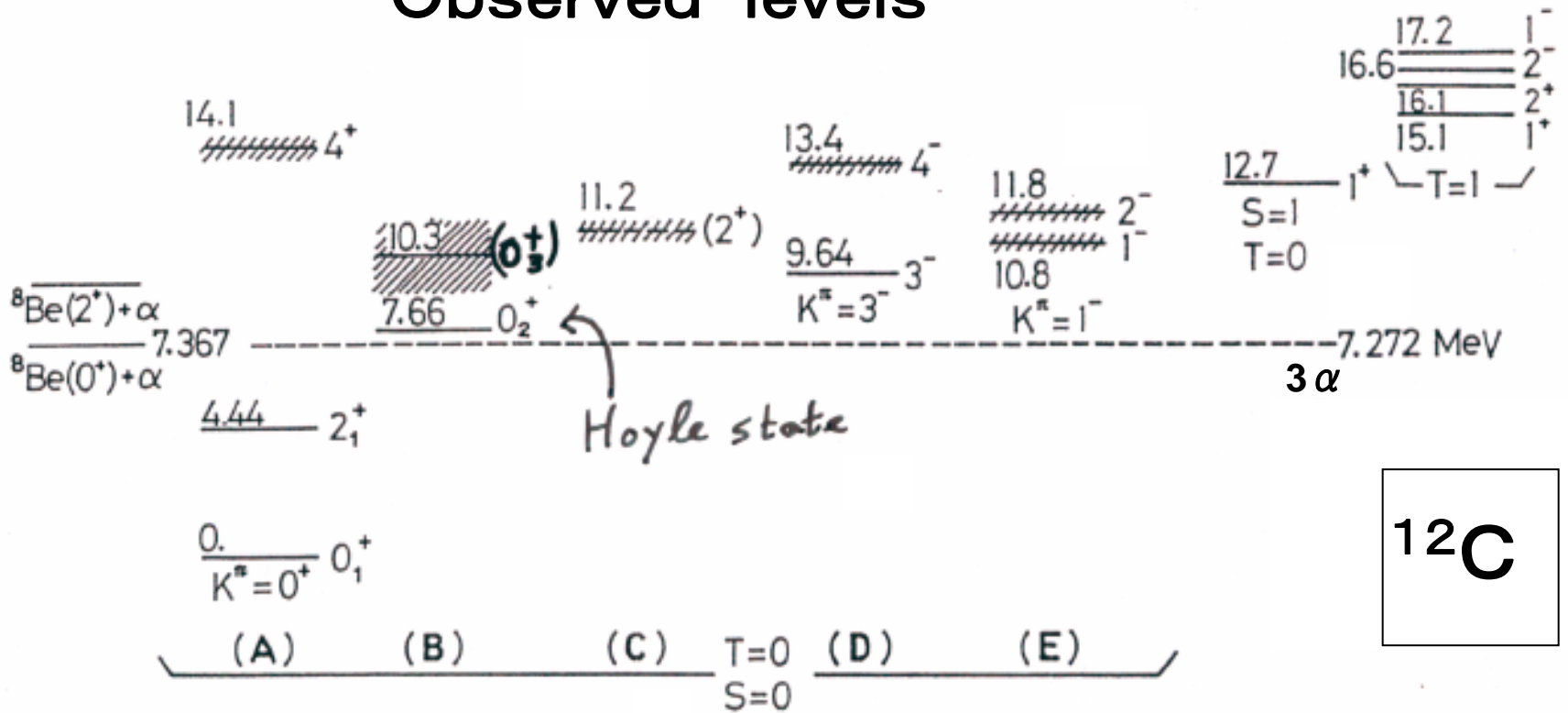
Excess neutrons stabilize  $D_{3h}$  symmetry of  $3\alpha$

N. Itagaki, T.Otsuka,  
K.Ikeda, S.Okabe  
PRL 92 142501(2004)





# Observed levels



Recent RCNP experiments  
 M.Itoh, et al.,  
 N.P.A.738,268,(2004)

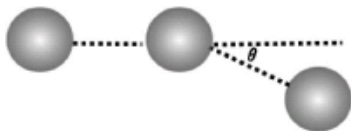
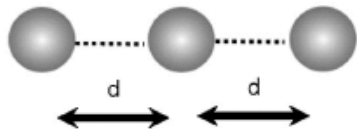
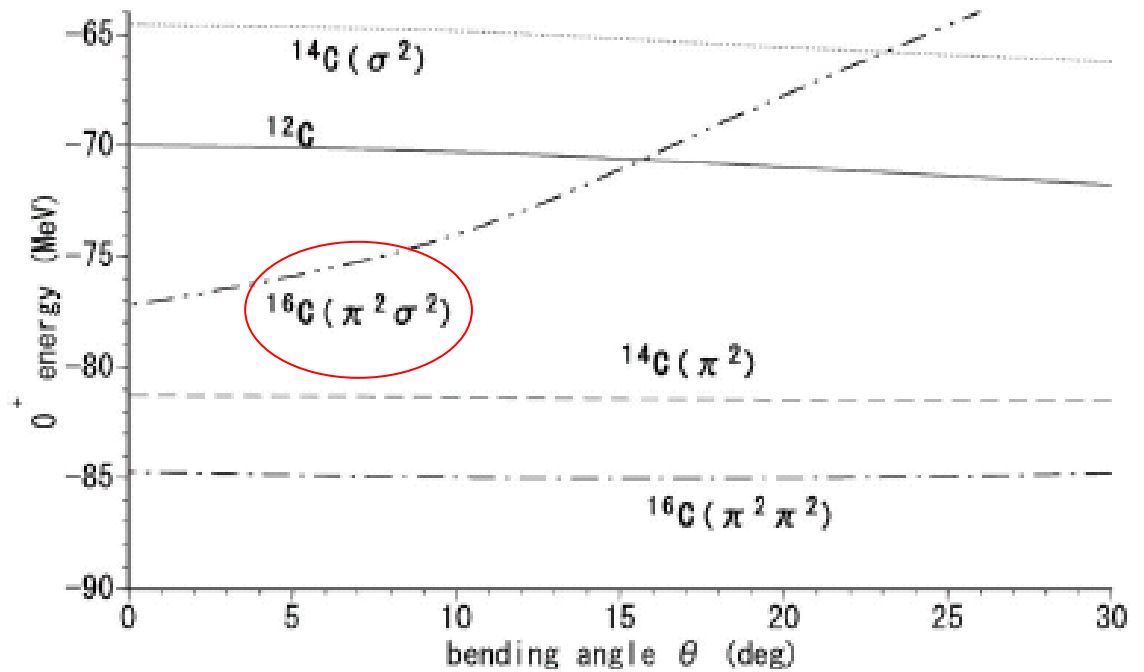
$\frac{10.3 \text{ MeV}}{(0^+)_3}$



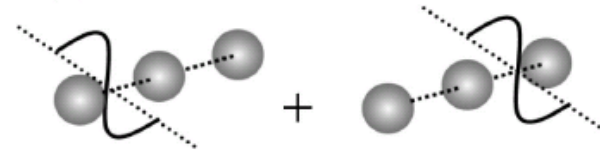
$\frac{10.0 \text{ MeV } (0^+)_3 \Gamma : 2.7 \text{ MeV}}{9.9 \text{ MeV } (2^+)_2 \Gamma : 1.0 \text{ MeV}}$

# Excess neutrons stabilize linear chain of $\alpha$

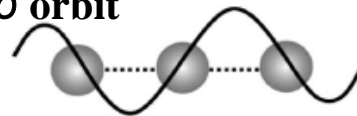
N. Itagaki, S. Okabe,  
K. Ikeda, I. Tanihata,  
P. R C 64, 014301 (2001)



(a)  $\pi$  orbit



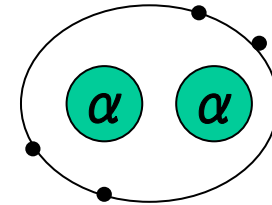
(b)  $\sigma$  orbit



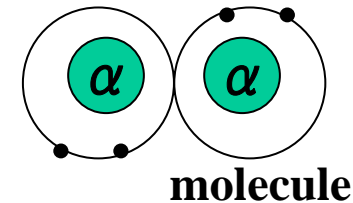
## 5. Summarizing discussion

- **Di-cluster + neutrons**

1. **Neutrons are described by molecular orbits**



2. **Neutrons are described by atomic orbits**



- **Di-neutron cluster**

1.  $^{11}\text{Li} = ^9\text{Li} + \text{di-neutron}$ ,  
probably  $^6\text{He} = \alpha + \text{di-neutron}$

2. **di-neutron condensation in neutron skin (dilute neutron matter)**  
**HFB calculation**

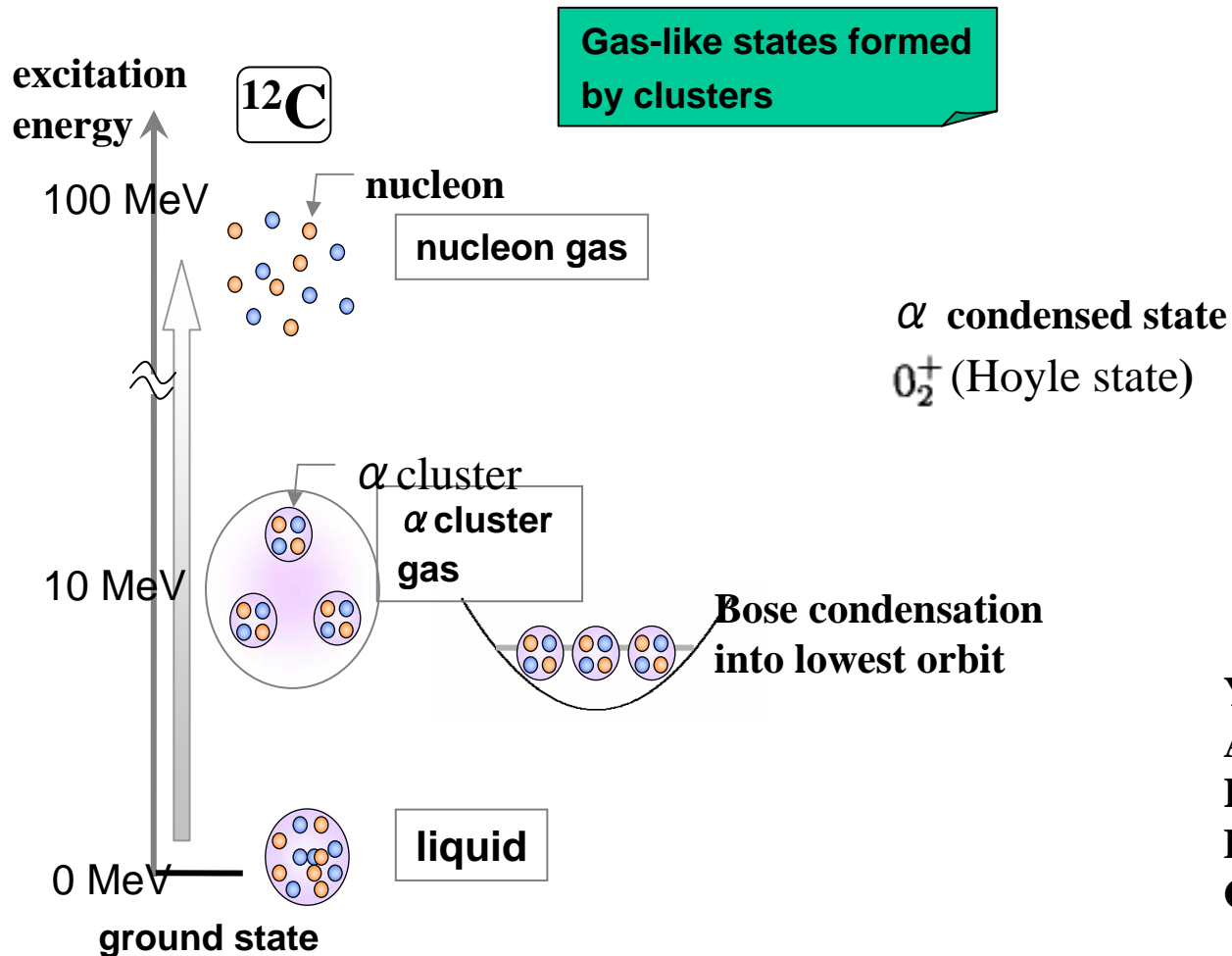
- **Multi-cluster stabilized by excess neutrons**

1. **Triangle structure of 3  $\alpha$**

2. **Linear chain structure of  $\alpha$  clusters**

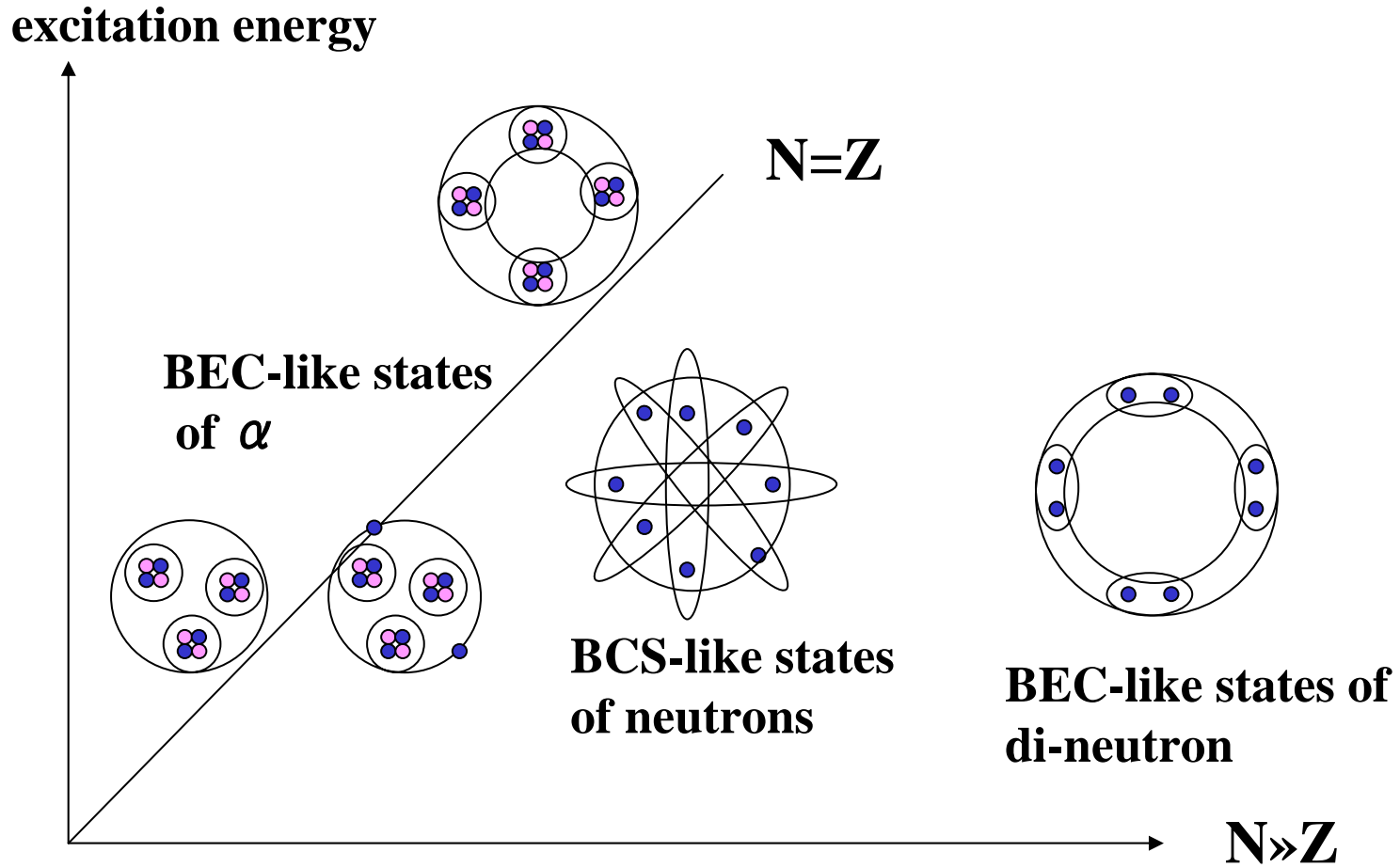
○ Clustering in dilute nuclear matter

1. Di-neutron condensation in neutron skin/halo of neutron-rich nuclei
2.  $\alpha$  cluster condensation in near-proton-dripline nuclei ?  
 $\alpha$  cluster condensation in neutron-richer nuclei ?



Y. Funaki,  
A. Tohsaki,  
H. Horiuchi,  
P. Schuck,  
G. Roepke

# Dilute nuclear states



# The studies we discussed serve to elucidate

## Richness of nuclear dynamics :

◇ New dynamics near driplines,

◇ New dynamics in excited states,

an example: no-core shell model cannot reproduce  
the second  $0+$  states and related states in  $^{12}\text{C}$

Various kinds of dynamics: mean-field dynamics,  
strong correlation dynamics,  
clustering dynamics,

# **Advantages of RIA compared with RIBF (RIKEN)**

**Variety of energies of exotic beams  
post acceleration of exotic nuclei  
in addition to in-flight exotic fragments**

**one example, fragmentation experiments of B isotopes to check their  
Li-He molecular structure**

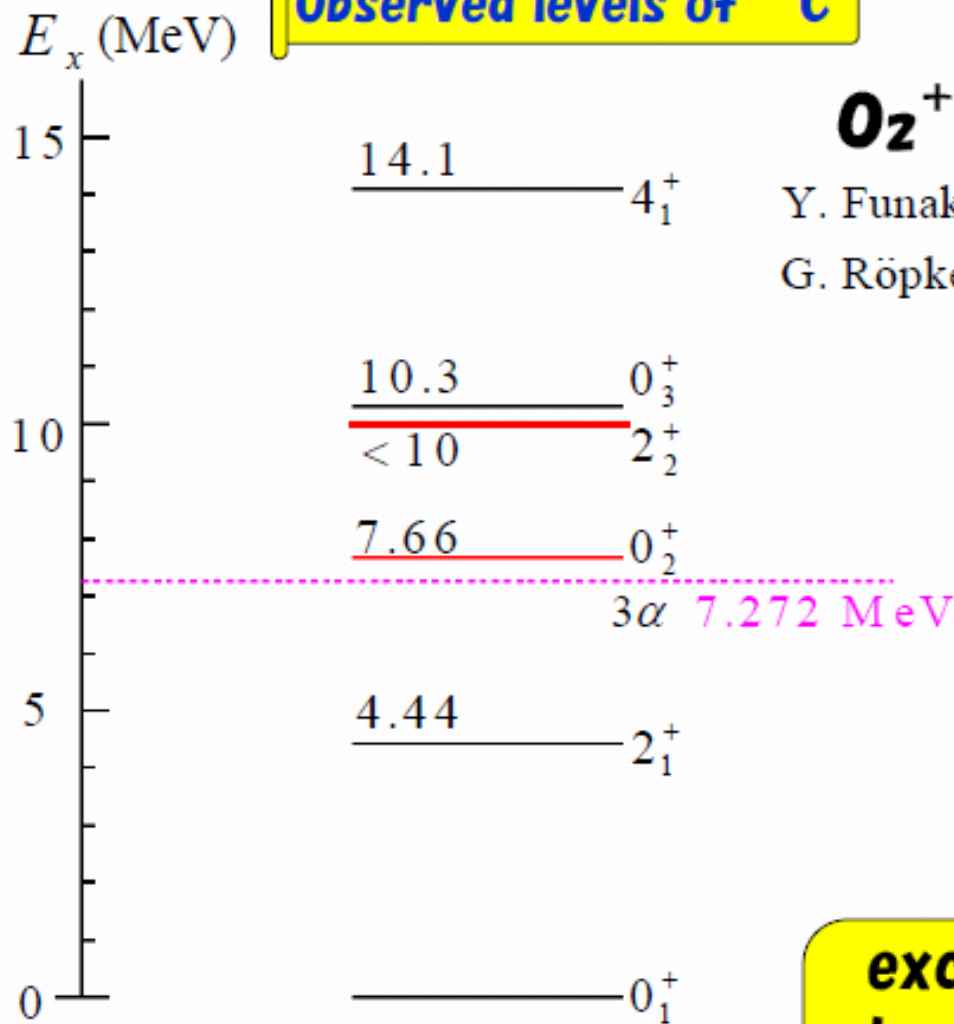
**30-50 MeV/nucleon is desirable for B beams**

**(H.Takemoto, A.Ono, & H.H. P.R.C63, 034615('01), P.T.P. 101, 101('99),  
AMD calculation of fragmentation)**

**Large reaction yields by exotic beams  
at least more than 400 MeV/nucleon ..... RIA  
at most less than 350 MeV/nucleon .....RIBF**

**necessary for more detailed studies of  
excited states of exotic nuclei**

## Observed levels of $^{12}\text{C}$



### $0_2^+$ state : $3\alpha$ condensed state

Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. C 67 (2003) 05130610

### $2_2^+$ state :

$$E = 9.9 \pm 0.3 \text{ MeV}$$

$$\Gamma = 1.0 \pm 0.3 \text{ MeV}$$

$$^{12}\text{C}(\alpha, \alpha')$$

M. Itoh et al., Nucl. Phys. A

738 (2004) 268-272

**excitation mode**

**based on  $\alpha$  condensate**

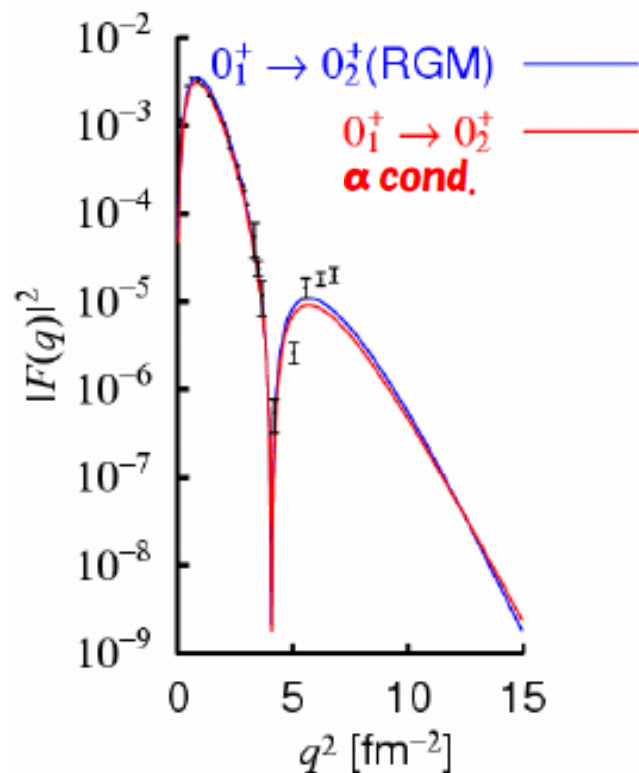
**2  $\alpha$  particles : S-orbit**

**1  $\alpha$  particle : D-orbit**



## $0_2^+$ state

	Exp.	Theor.
Excitation energy (MeV)	<b>7.65</b>	<b>7.74</b>
Width (eV)	<b><math>8.7 \pm 2.7</math></b>	<b>7.1</b>
$M(0_2^+ \rightarrow 0_1^+)$ (fm <sup>2</sup> )	<b><math>5.4 \pm 0.2</math></b>	<b>6.7</b>
$B(E2: 0_2^+ \rightarrow 2_1^+)$ (e <sup>2</sup> fm <sup>4</sup> )	<b><math>13 \pm 4</math></b>	<b>5.6</b>



## $2_2^+$ state

Exp

$E = 9.9 \pm 0.3$  MeV  
 $\Gamma = 1.0 \pm 0.3$  MeV

$\alpha$  cond. w. f.  
+ ACCC

$E = 9.38$  MeV  
 $\Gamma = 0.64$  MeV





















