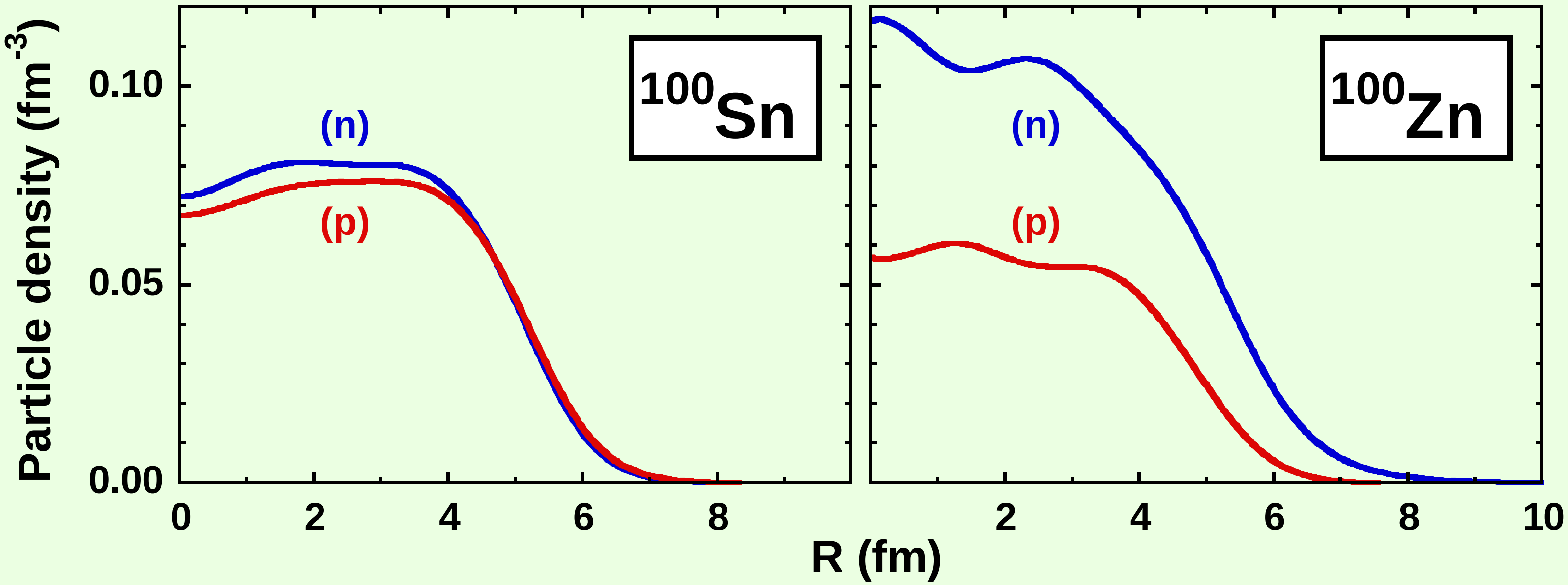
Tensor interactions in mean-field approaches

Jacek Dobaczewski

IFT UW

3rd ANL/MSU/INT/JINA RIA Theory Workshop
Argonne, 4-7 April 2006

Nuclear densities as composite fields



Modern Mean-Field Theory ≡ Energy Density Functional

$$\rho,\quad \tau,\quad \overleftrightarrow{J},\quad \vec{j},\quad \vec{T},\quad \vec{s},\quad \vec{F},$$

- Hohenberg-Kohn
- Kohn-Sham
- Negele-Vautherin
- Landau-Migdal
- Nilsson-Strutinsky

mean field ⇒ one-body densities

zero range ⇒ local densities

finite range ⇒ non-local densities

Complete local energy density

E. Perlińska, *et al.*, Phys. Rev. C69 (2004) 014316

The energy density can be written in the following form:

$$\mathcal{H}(\vec r) = \frac{\hbar^2}{2m}\tau_0(\vec r) + \sum\nolimits_{t=0,1}\left(\chi_t(\vec r) + \breve\chi_t(\vec r)\right),$$

The p-h and p-p interaction energy densities, $\chi_t(\vec r)$ and $\breve\chi_t$, for t=0 depend quadratically on the isoscalar densities, and for t=1 – on the isovector ones. Based on general rules of constructing the energy density, one obtains

Mean field

$$\begin{aligned}
\chi_0(\vec r) &= C_0^\rho\rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^\tau\rho_0\tau_0 \\
&+ C_0^{J0}J_0^2 + C_0^{J1}\vec J_0^2 + C_0^{J2}\underline{\mathsf J}_0^2 + C_0^{\nabla J}\rho_0\vec\nabla\cdot\vec J_0 \\
&+ C_0^s\vec s_0^2 + C_0^{\Delta s}\vec s_0\cdot\Delta\vec s_0 + C_0^T\vec s_0\cdot\vec T_0 \\
&+ C_0^j\vec j_0^2 + C_0^{\nabla j}\vec s_0\cdot(\vec\nabla\times\vec j_0) \\
&+ C_0^{\nabla s}(\vec\nabla\cdot\vec s_0)^2 + C_0^F\vec s_0\cdot\vec F_0, \\
\chi_1(\vec r) &= C_1^\rho\vec\rho^2 + C_1^{\Delta\rho}\vec\rho\circ\Delta\vec\rho + C_1^\tau\vec\rho\circ\vec\tau \\
&+ C_1^{J0}\vec J^2 + C_1^{J1}\vec{\vec J}^2 + C_1^{J2}\vec{\underline{\mathsf J}}^2 + C_1^{\nabla J}\vec\rho\circ\vec\nabla\cdot\vec{\vec J} \\
&+ C_1^s\vec{\vec s}^2 + C_1^{\Delta s}\vec{\vec s}\cdot\circ\Delta\vec{\vec s} + C_1^T\vec{\vec s}\cdot\circ\vec{\vec T} \\
&+ C_1^j\vec{\vec j}^2 + C_1^{\nabla j}\vec{\vec s}\cdot\circ(\vec\nabla\times\vec{\vec j}) \\
&+ C_1^{\nabla s}(\vec\nabla\cdot\vec{\vec s})^2 + C_1^F\vec{\vec s}\cdot\circ\vec{\vec F},
\end{aligned}$$

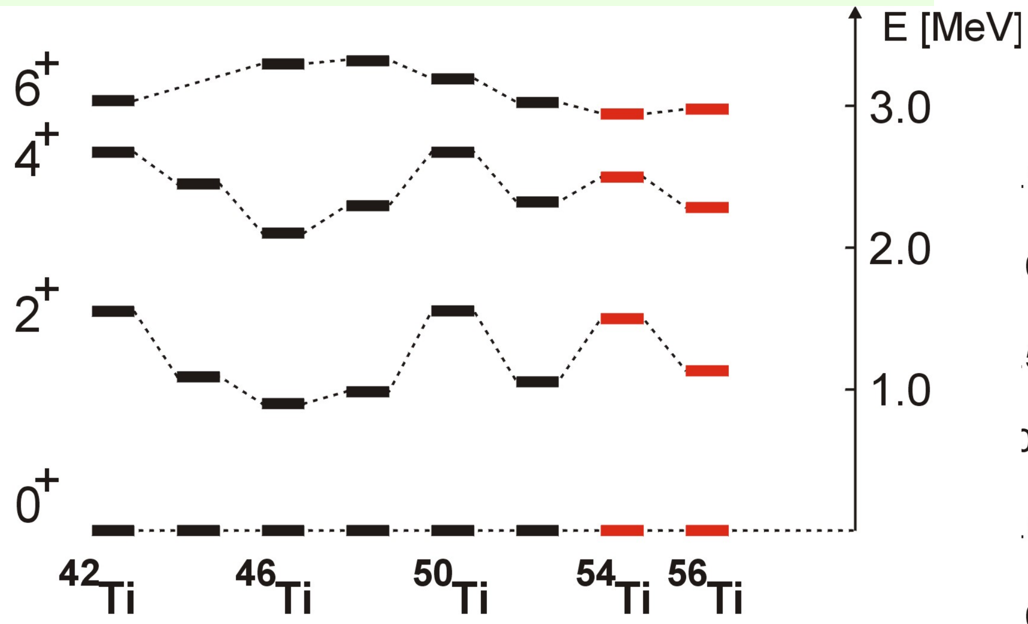
where $\times$ stands for the vector product

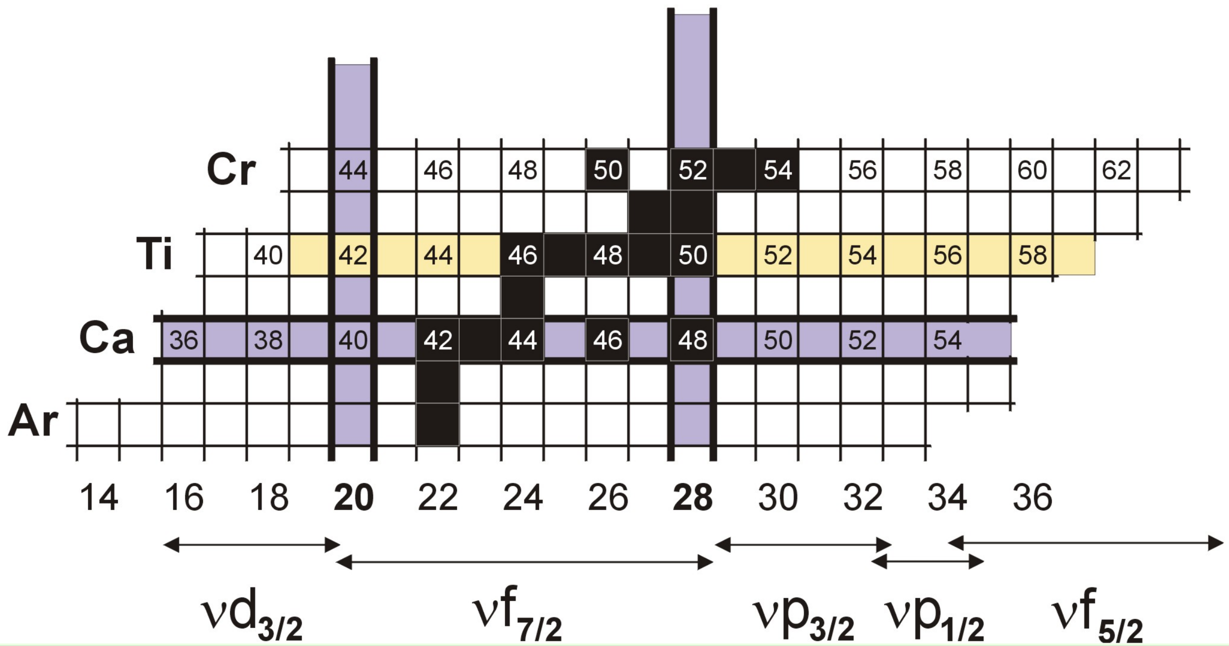
Pairing

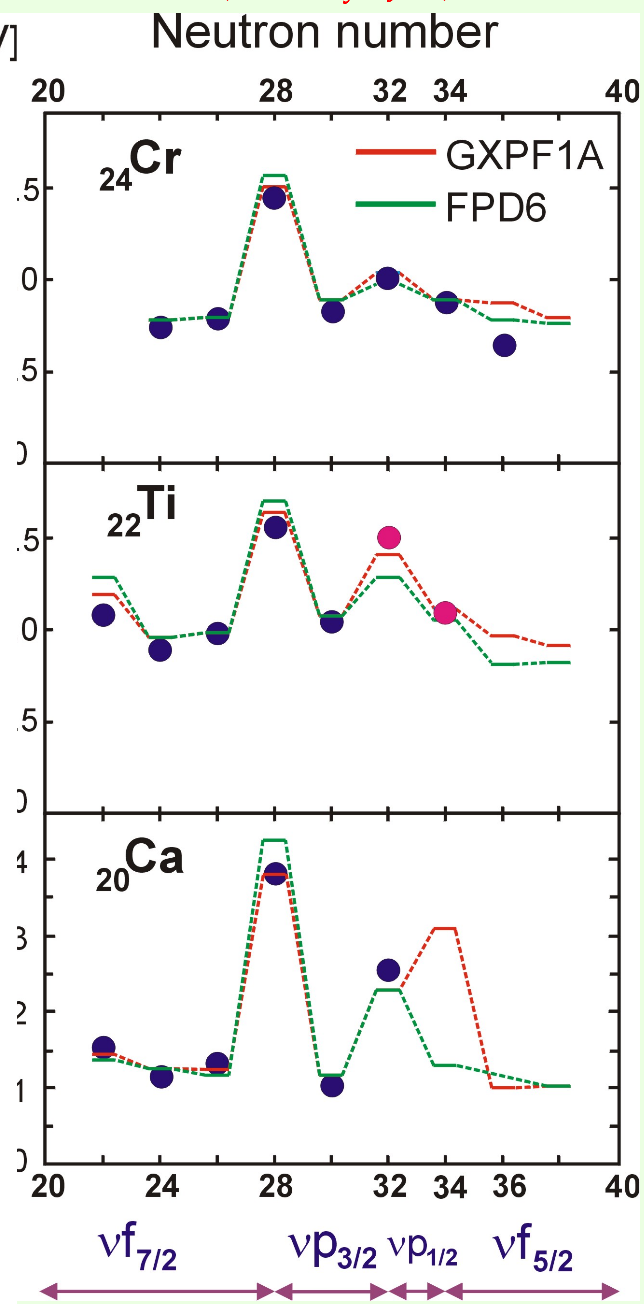
$$\begin{aligned}
\breve\chi_0(\vec r) &= \breve C_0^s|\breve{\vec s}_0|^2 + \breve C_0^{\Delta s}\Re(\breve{\vec s}_0^*\cdot\Delta\breve{\vec s}_0) \\
&+ \breve C_0^T\Re(\breve{\vec s}_0^*\cdot\breve{\vec T}_0) + \breve C_0^j|\breve{\vec j}_0|^2 \\
&+ \breve C_0^{\nabla j}\Re(\breve{\vec s}_0^*\cdot(\vec\nabla\times\breve{\vec j}_0)) \\
&+ \breve C_0^{\nabla s}|\vec\nabla\cdot\breve{\vec s}_0|^2 \\
&+ \breve C_0^F\Re(\breve{\vec s}_0^*\cdot\breve{\vec F}_0), \\
\breve\chi_1(\vec r) &= \breve C_1^\rho|\breve{\vec\rho}|^2 + \breve C_1^{\Delta\rho}\Re(\breve{\vec\rho}^*\circ\Delta\breve{\vec\rho}) \\
&+ \breve C_1^\tau\Re(\breve{\vec\rho}^*\circ\breve{\vec\tau}) \\
&+ \breve C_1^{J0}|\breve{\vec J}|^2 + \breve C_1^{J1}|\breve{\vec{\vec J}}|^2 \\
&+ \breve C_1^{J2}|\breve{\vec{\underline{\mathsf J}}}|^2 \\
&+ \breve C_1^{\nabla J}\Re(\breve{\vec\rho}^*\circ\vec\nabla\cdot\breve{\vec{\vec J}}).
\end{aligned}$$

B. Fornal, XXIX Mazurian Lakes Conference on Physics (2005)

M.Honma et al., Eur. Phys. J. 25,s01 (2005) 499



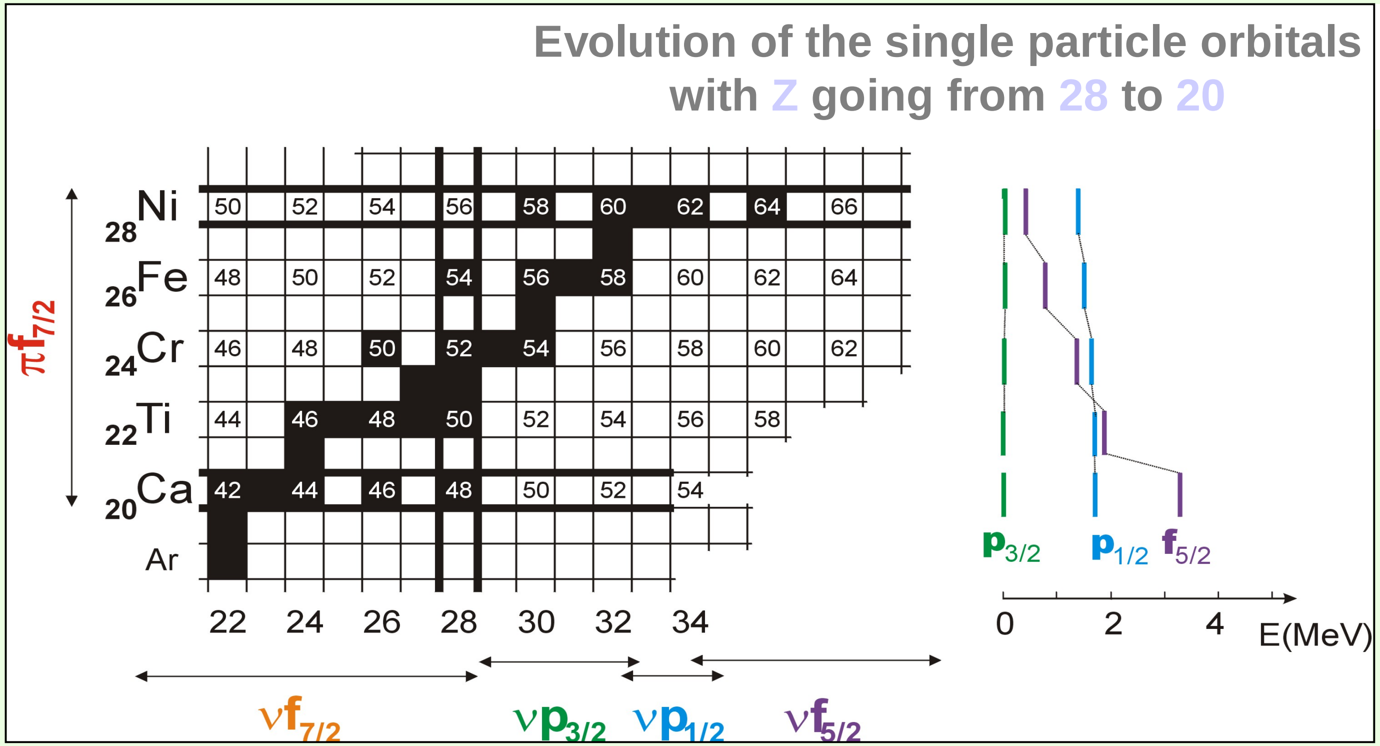




Jacek Dobaczewski

NT¤ at NuclearTheory.net

RIA Theory
Rare Isotope Accelerator Project (RIA)



Tensor Interaction

$$V_T = (\tau_1\tau_2)\ (\ [\sigma_1\sigma_2]^{(2)}\ Y^{(2)}(\Omega)\ )\ Z(r)$$

V_T couples $j_>$ and $j_<$ orbitals and favors charge exchange processes

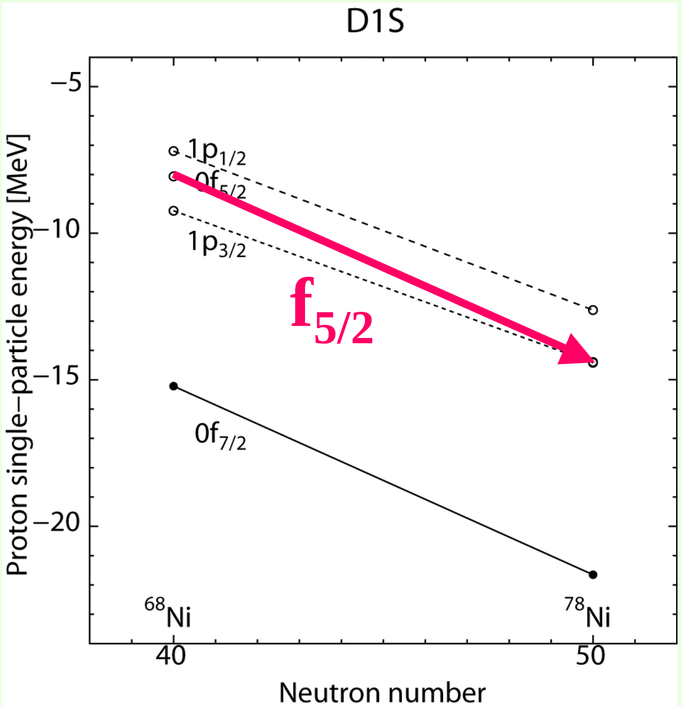
$$\pi f_{7/2} \leftrightarrow \nu f_{5/2}$$

T. Otsuka *et al.* Phys. Rev. Lett 87, 082502 (2001)

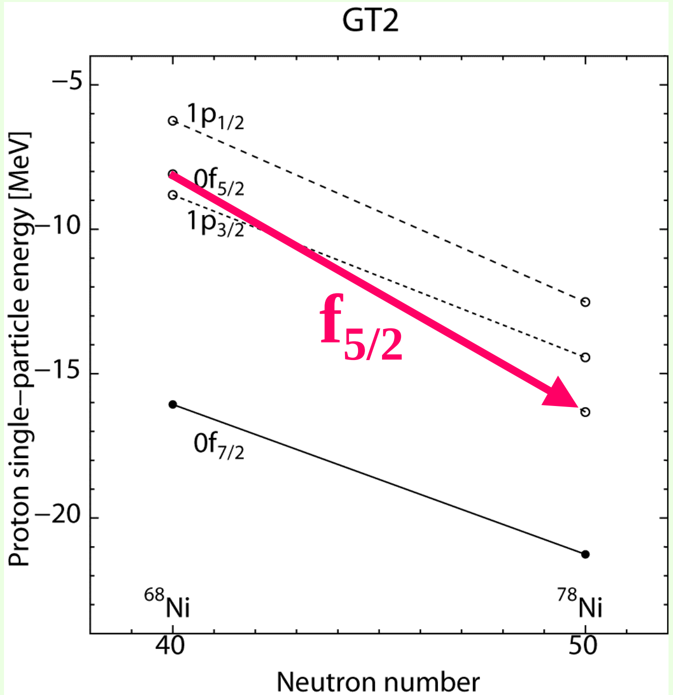
B. Fornal, XXIX Mazurian Lakes Conference on Physics (2005)

Proton effective single-particle energies of exotic Ni isotopes

Original (D1S)



GT2 (incl. tensor)



Takaharu Otsuka, Lectures in Bejing (2005)
http://jcnp.pku.edu.cn/gb/activities/summer/Otsuka/Beijing-7.ppt

Tensor-even, tensor-odd, and spin-orbit interactions

$$
\begin{aligned}
\hat{V}_{Te} &= \tfrac{1}{2}t_e[\hat{\vec{k}}' \cdot \hat{\mathsf{S}} \cdot \hat{\vec{k}}' + \hat{\vec{k}} \cdot \hat{\mathsf{S}} \cdot \hat{\vec{k}}] \\
\hat{V}_{To} &= t_o\hat{\vec{k}}' \cdot \hat{\mathsf{S}} \cdot \hat{\vec{k}} \\
\hat{V}_{SO} &= iW_0\hat{\vec{S}} \cdot [\hat{\vec{k}}' \times \hat{\vec{k}}]
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{\mathsf{S}}^{ij} &= \tfrac{3}{2}[\vec{\sigma}_1^i\vec{\sigma}_2^j + \vec{\sigma}_1^j\vec{\sigma}_2^i] - \delta^{ij}\vec{\sigma}_1 \cdot \vec{\sigma}_2 \\
\hat{\vec{S}} &= \vec{\sigma}_1 + \vec{\sigma}_2
\end{aligned}
$$

For conserved spherical and time-reversal symmetries, the corresponding energy densities read:

$$
\begin{aligned}
\mathcal{H}_T &= \tfrac{5}{8}[t_e\vec{J}_n \cdot \vec{J}_p + t_o(\vec{J}_0^2 - \vec{J}_n \cdot \vec{J}_p)] \\
\mathcal{H}_{SO} &= -\tfrac{1}{4}[3W_0\vec{J}_0 \cdot \vec{\nabla}\rho_0 + W_0\vec{J}_1 \cdot \vec{\nabla}\rho_1]
\end{aligned}
$$

Tensor and spin-orbit terms in the EDF

$$\mathcal{H}_T = \sum_{t=0,1} \left(C_t^{J0} J_t^2 + C_t^{J1} \vec{J}_t^2 + C_t^{J2} \underline{\mathsf{J}}_t^2 \right),$$

$$\mathcal{H}_{SO} = \sum_{t=0,1} C_t^{\nabla J} \rho_t \vec{\nabla} \cdot \vec{J}_t.$$

where the standard pseudoscalar, vector, and pseudotensor parts of the spin-current density:

$$J_{ab} = \frac{1}{2i} [(\vec{\nabla} - \vec{\nabla}')_a \vec{s}_b(\vec{r}, \vec{r}')]_{\vec{r}=\vec{r}'},$$

are defined as (isospin indices omitted):

$$J = \sum_{a=x,y,z} J_{aa},$$ non-axial or axial asymmetric shapes only

$$\vec{J}_a = \sum_{b,c=x,y,z} \epsilon_{abc} J_{bc},$$

$$\mathsf{J}_{ab} = \frac{1}{2} J_{ab} + \frac{1}{2} J_{ba} - \frac{1}{3} J \delta_{ab}.$$ non-spherical shapes only

Gauge invariance of the tensor terms in the EDF

$$\mathcal{H}_T = \sum_{t=0,1}\left(C_t^{J0}J_t^2 + C_t^{J1}\vec{J}_t^2 + C_t^{J2}\underline{\mathsf{J}}_t^2\right)$$

gauge-invariant combinations of the tensor densities read:

$$G^T = -\tfrac{1}{3}J^2 - \tfrac{1}{2}\vec{J}^2 - \mathsf{J}^2$$

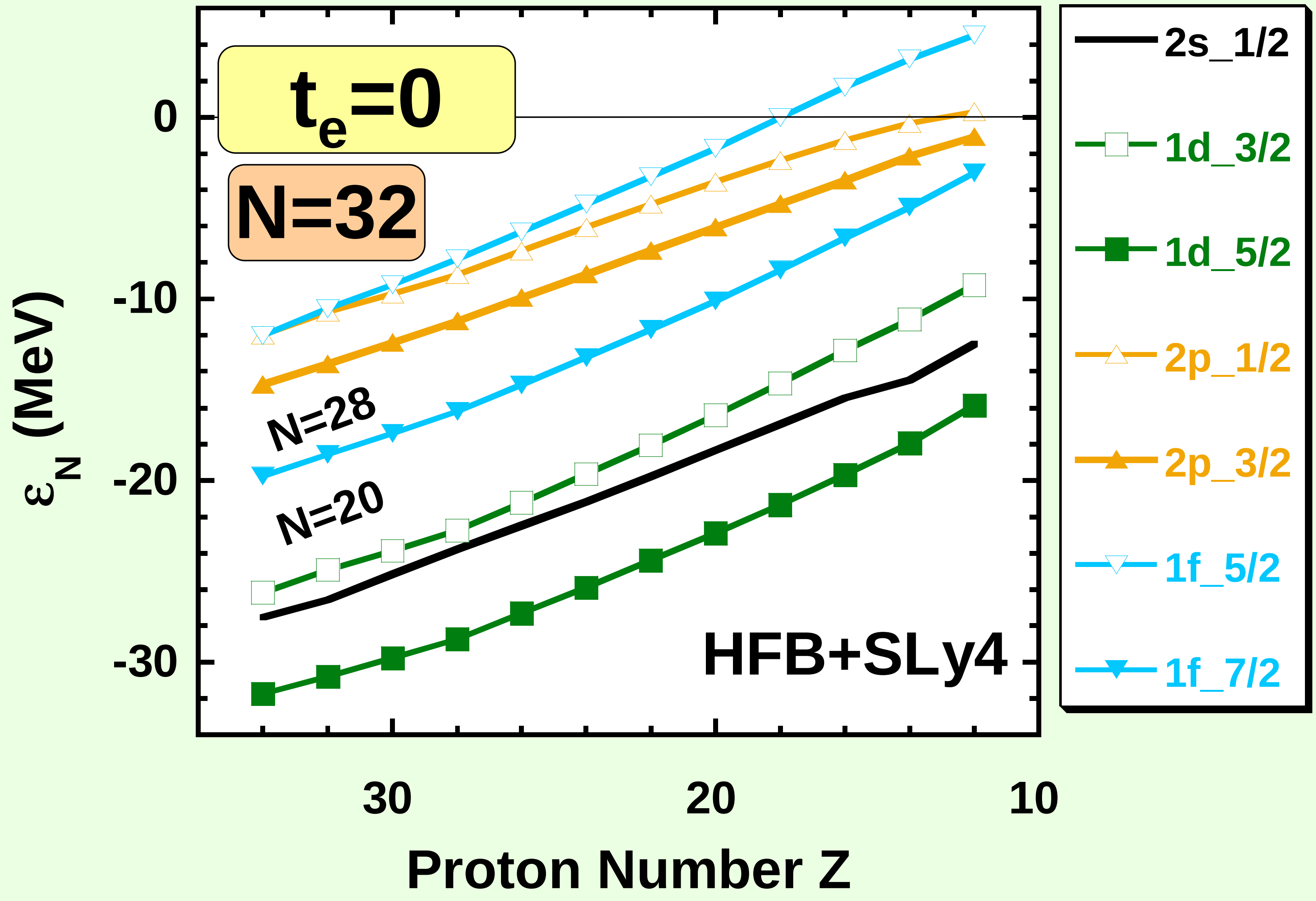
$$G^F = -\tfrac{2}{3}J^2 + \tfrac{1}{4}\vec{J}^2 - \tfrac{1}{2}\mathsf{J}^2$$

and therefore two out of three gauge-invariant tensor coupling constants are linearly independent, e.g.,

$$C^{J0} = -\tfrac{1}{3}A - \tfrac{2}{3}B$$

$$C^{J1} = -\tfrac{1}{2}A + \tfrac{1}{4}B$$

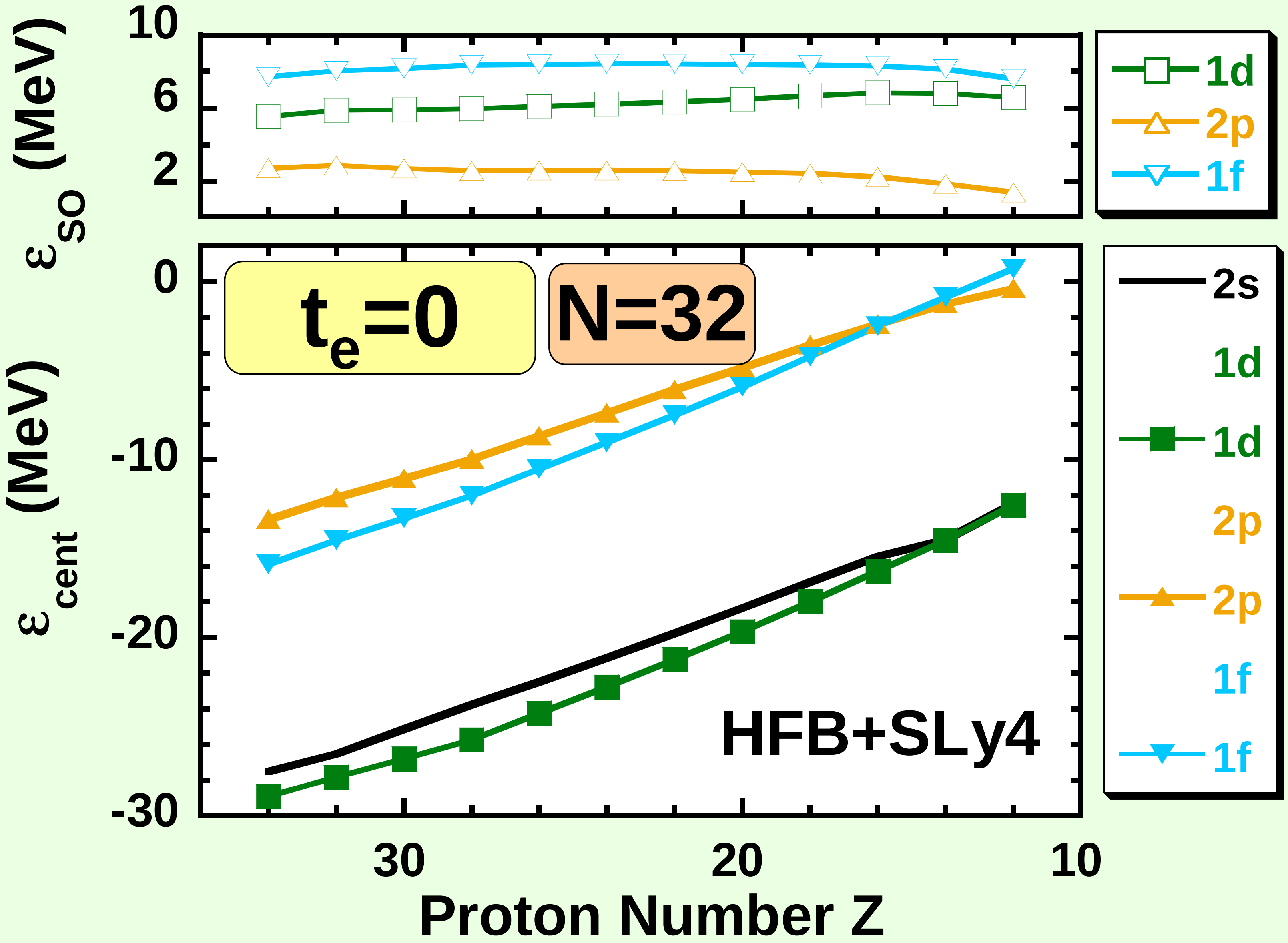
$$C^{J2} = -\,A - \tfrac{1}{2}B$$



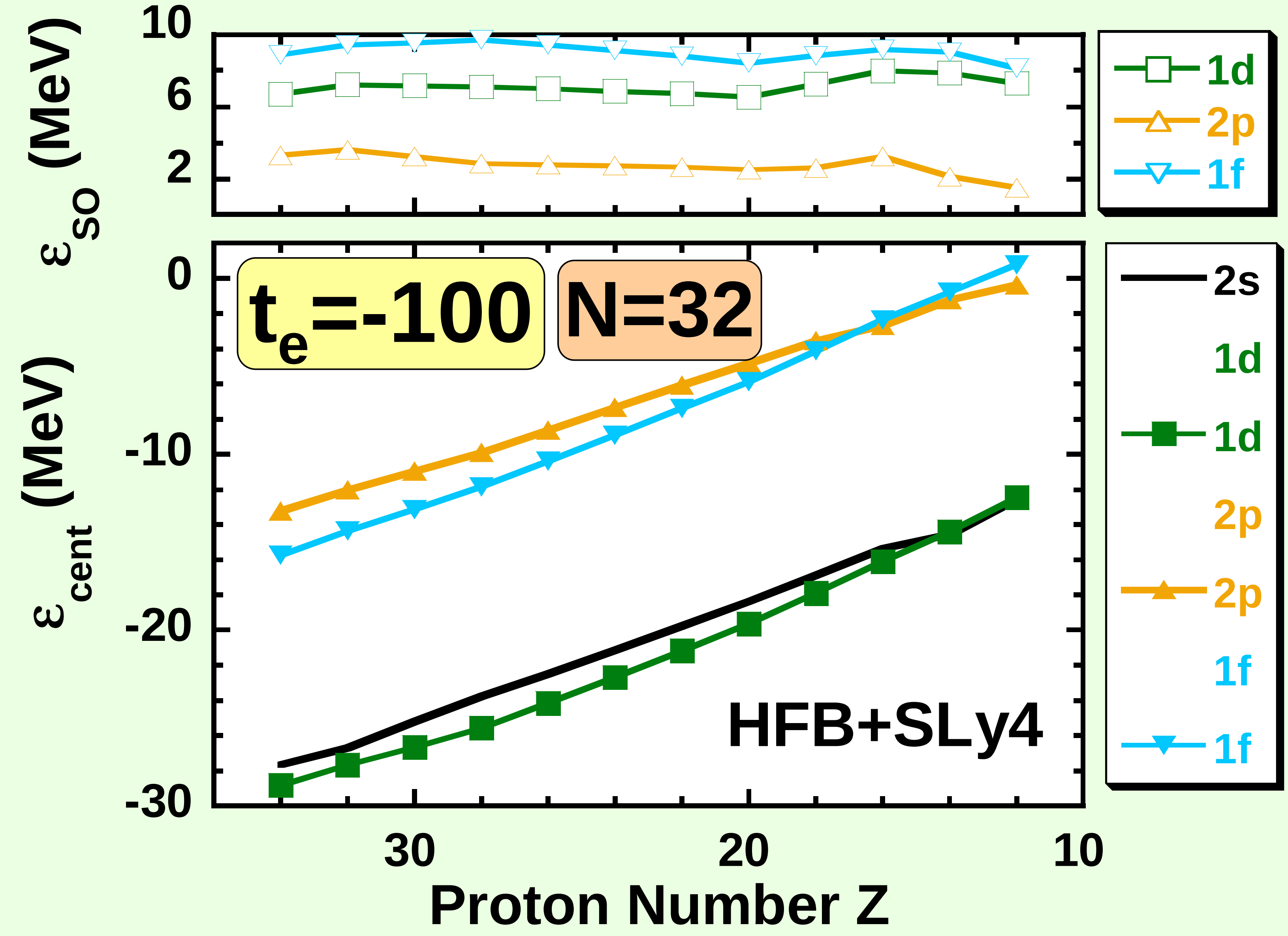
Jacek Dobaczewski

NT¤ at NuclearTheory.net

RIA Theory Rare Isotope Accelerator Project (RIA)

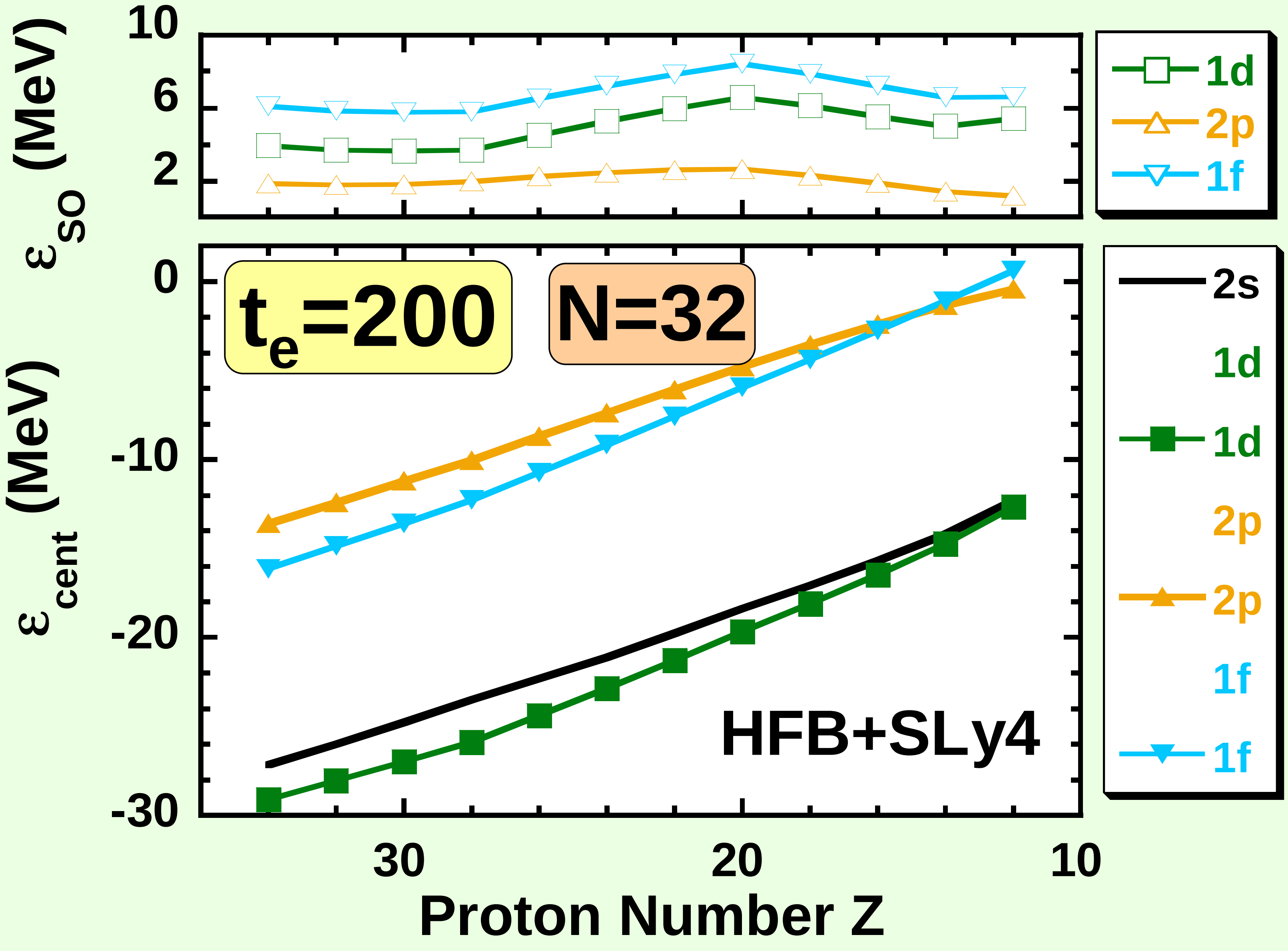


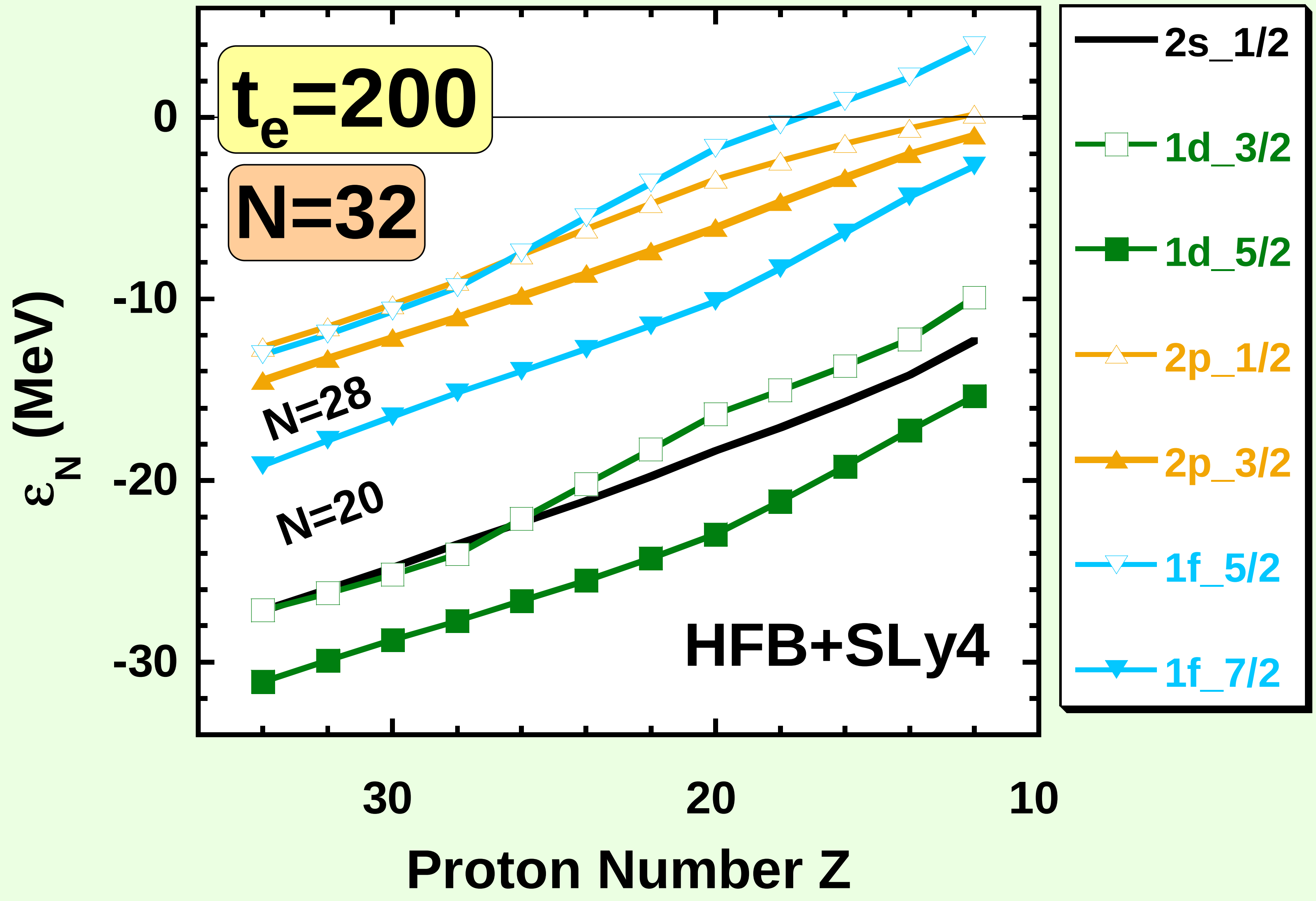
ε_{so} (MeV)
ε_{cent} (MeV)
10
6
2
0
-10
-20
-30
1d
2p
1f
2s
1d
1d
2p
2p
1f
1f
t_e=0
N=32
HFB+SLy4
30
20
10
Proton Number Z



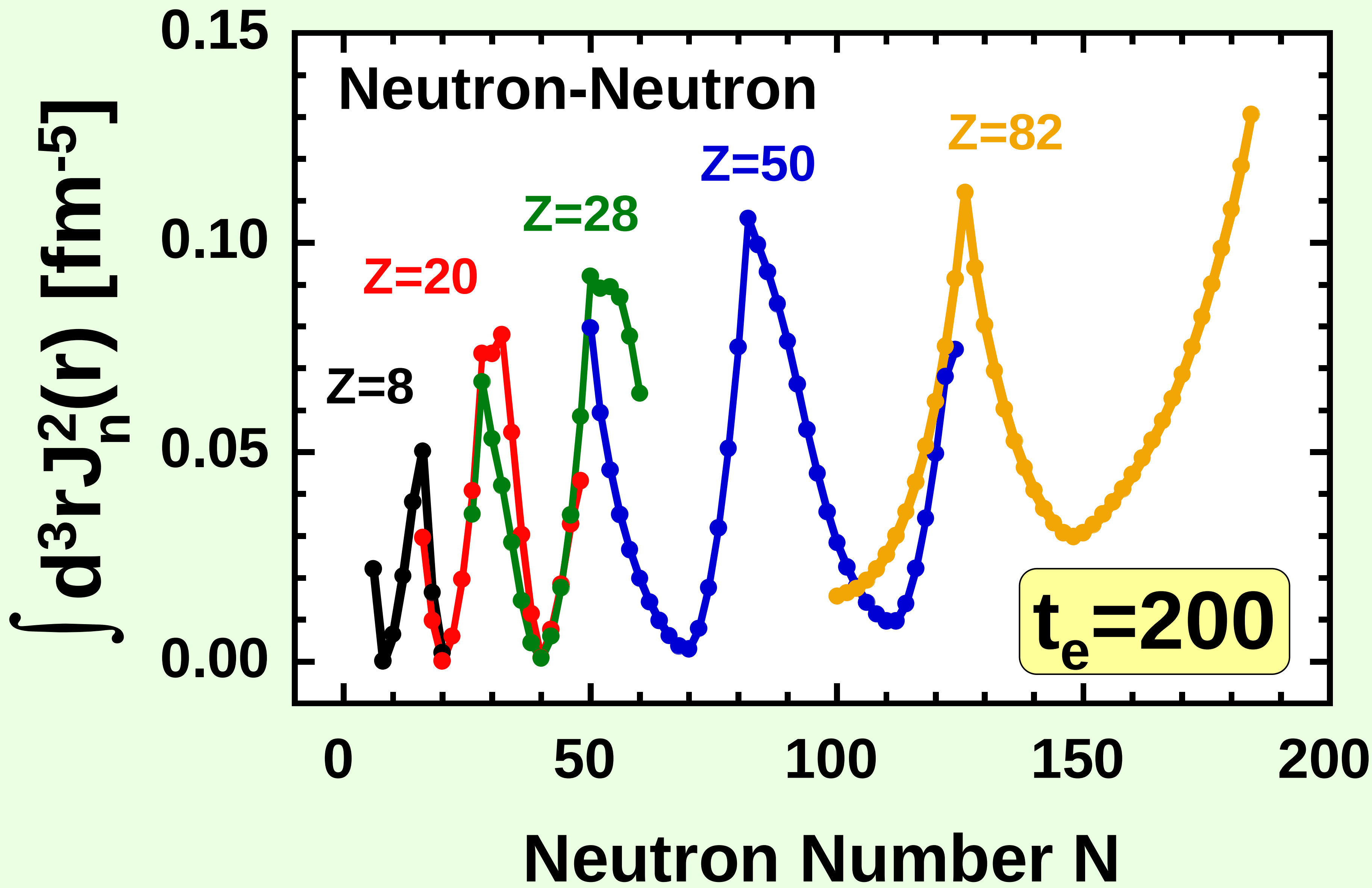
Jacek Dobaczewski

NT¤ at NuclearTheory.net


ε_{so} (MeV)
ε_{cent} (MeV)
t_e=200
N=32
HFB+SLy4
Proton Number Z
1d
2p
1f
2s
1d
1d
2p
2p
1f
1f



t_e=200
N=32
N=28
N=20
HFB+SLy4
0
-10
-20
-30
ε_N (MeV)
30
20
10
Proton Number Z
2s_1/2
1d_3/2
1d_5/2
2p_1/2
2p_3/2
1f_5/2
1f_7/2

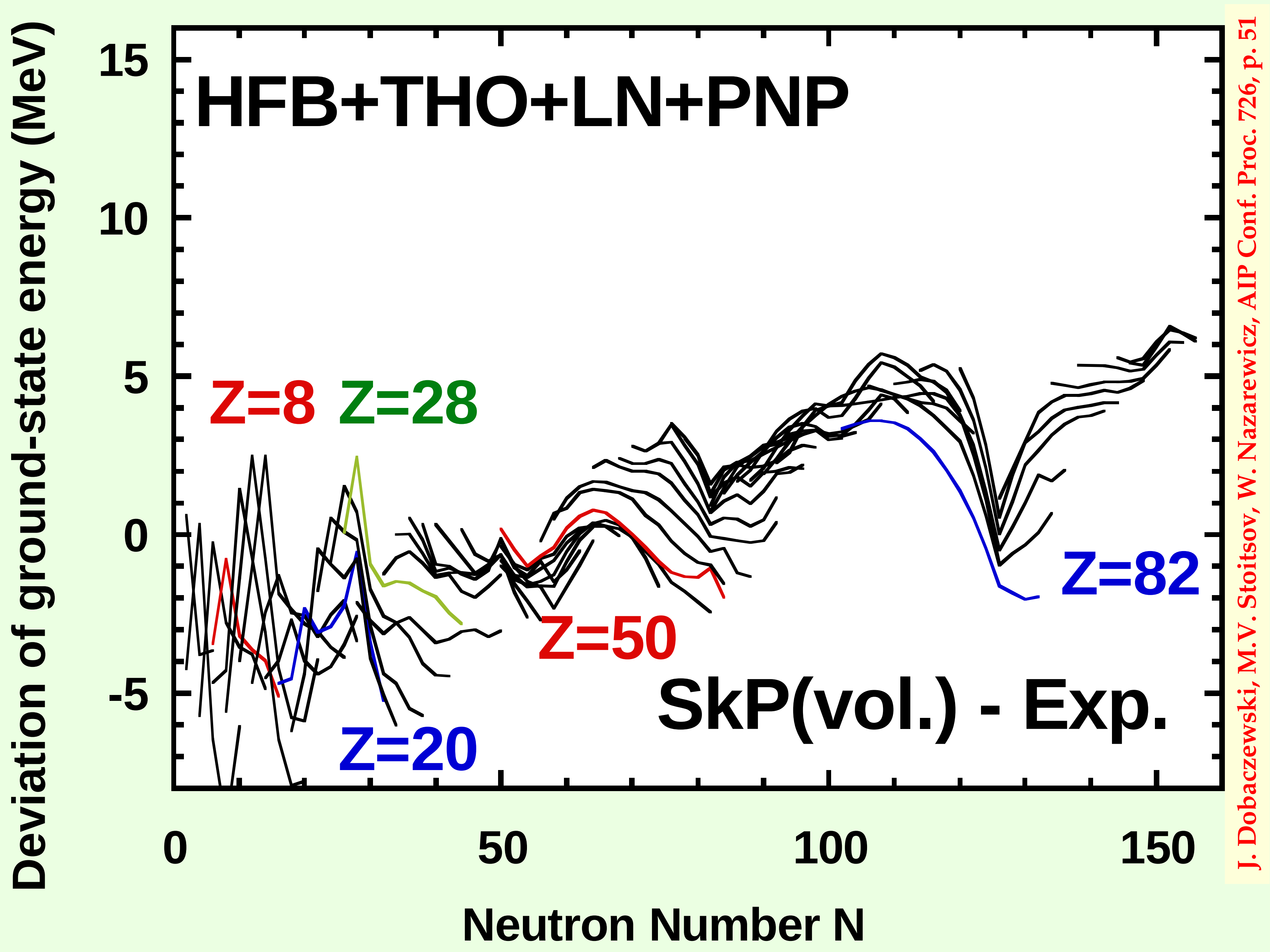


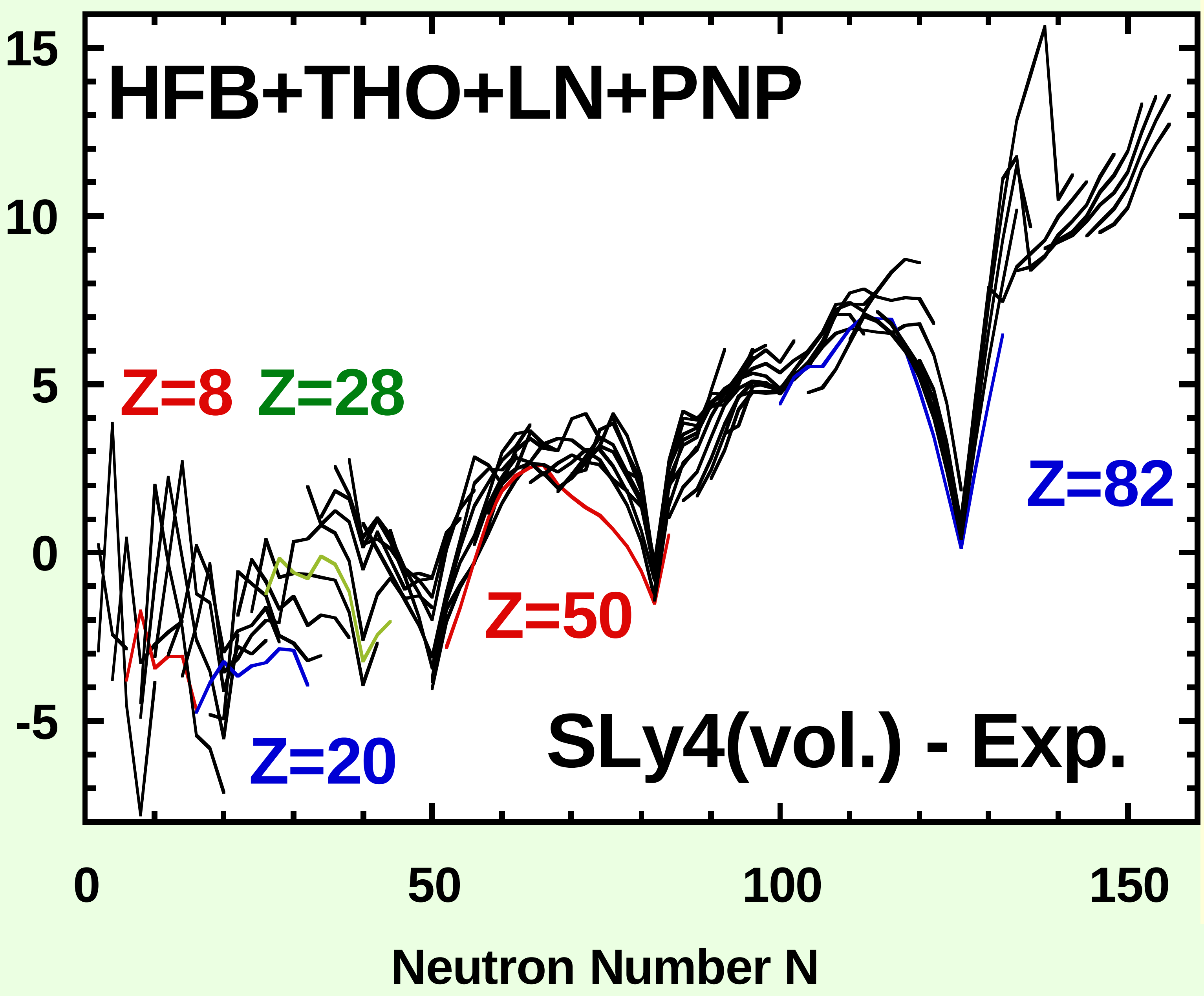
Jacek Dobaczewski

IFT UW

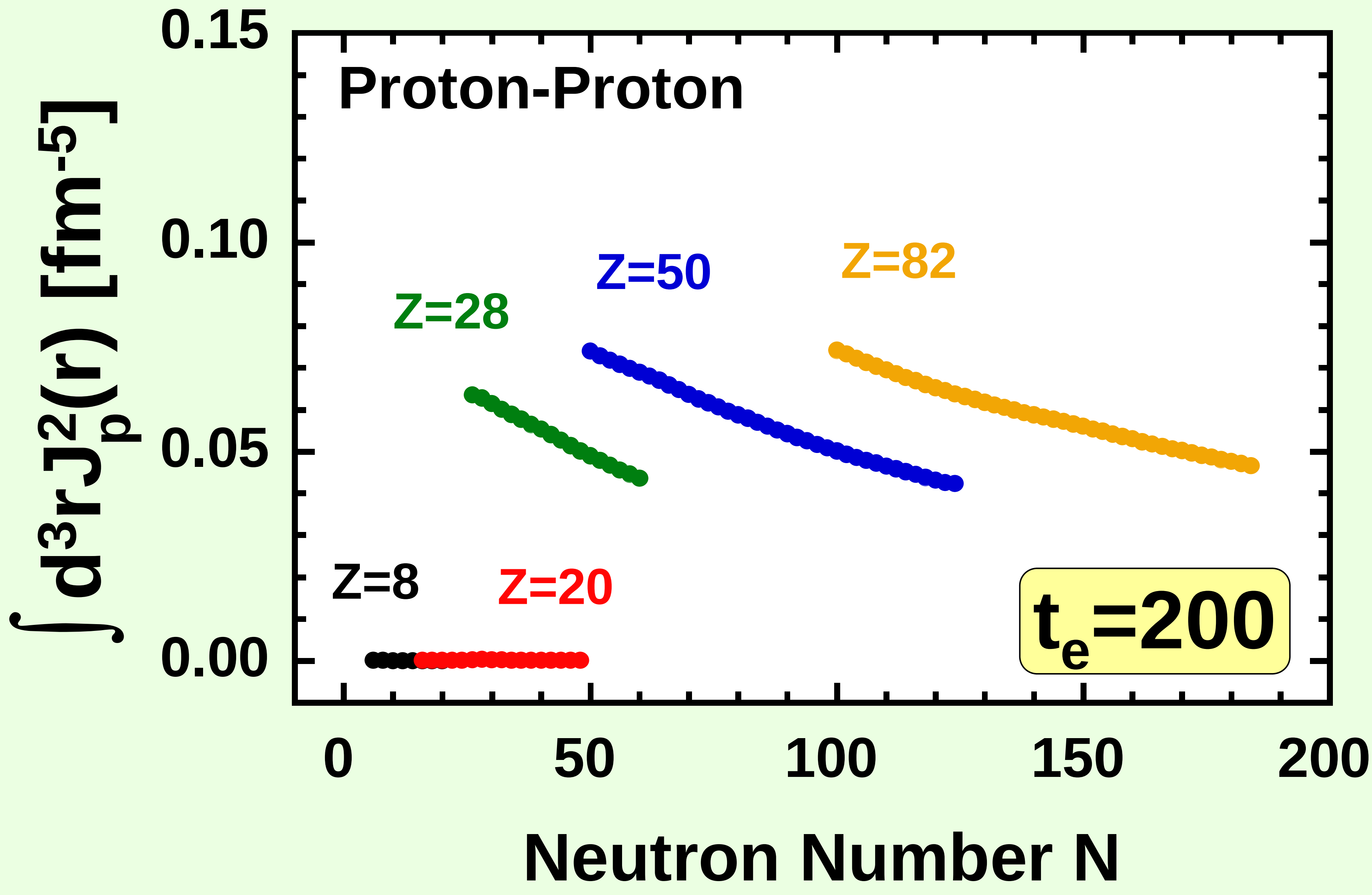
NT¤ at NuclearTheory.net

RIA Theory
Rare Isotope Accelerator Project (RIA)


HFB+THO+LN+PNP
Z=8
Z=28
Z=20
Z=50
Z=82
SkP(vol.) - Exp.
Deviation of ground-state energy (MeV)
Neutron Number N
15
10
5
0
-5
0
50
100
150
J. Dobaczewski, M.V. Stoitsov, W. Nazarewicz, AIP Conf. Proc. 726, p. 51



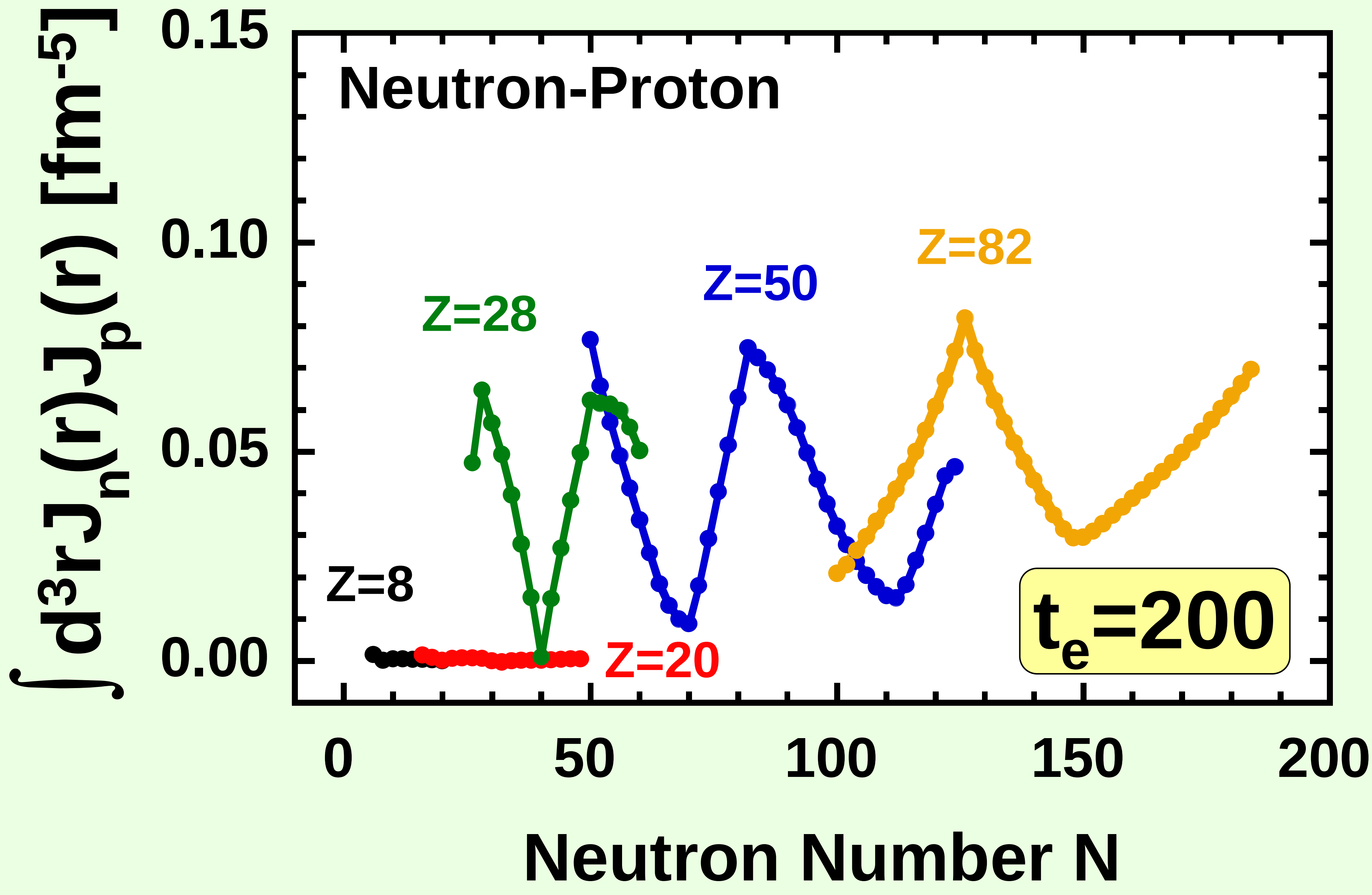
J. Dobaczewski, M.V. Stoitsov, W. Nazarewicz, AIP Conf. Proc. 726, p. 51



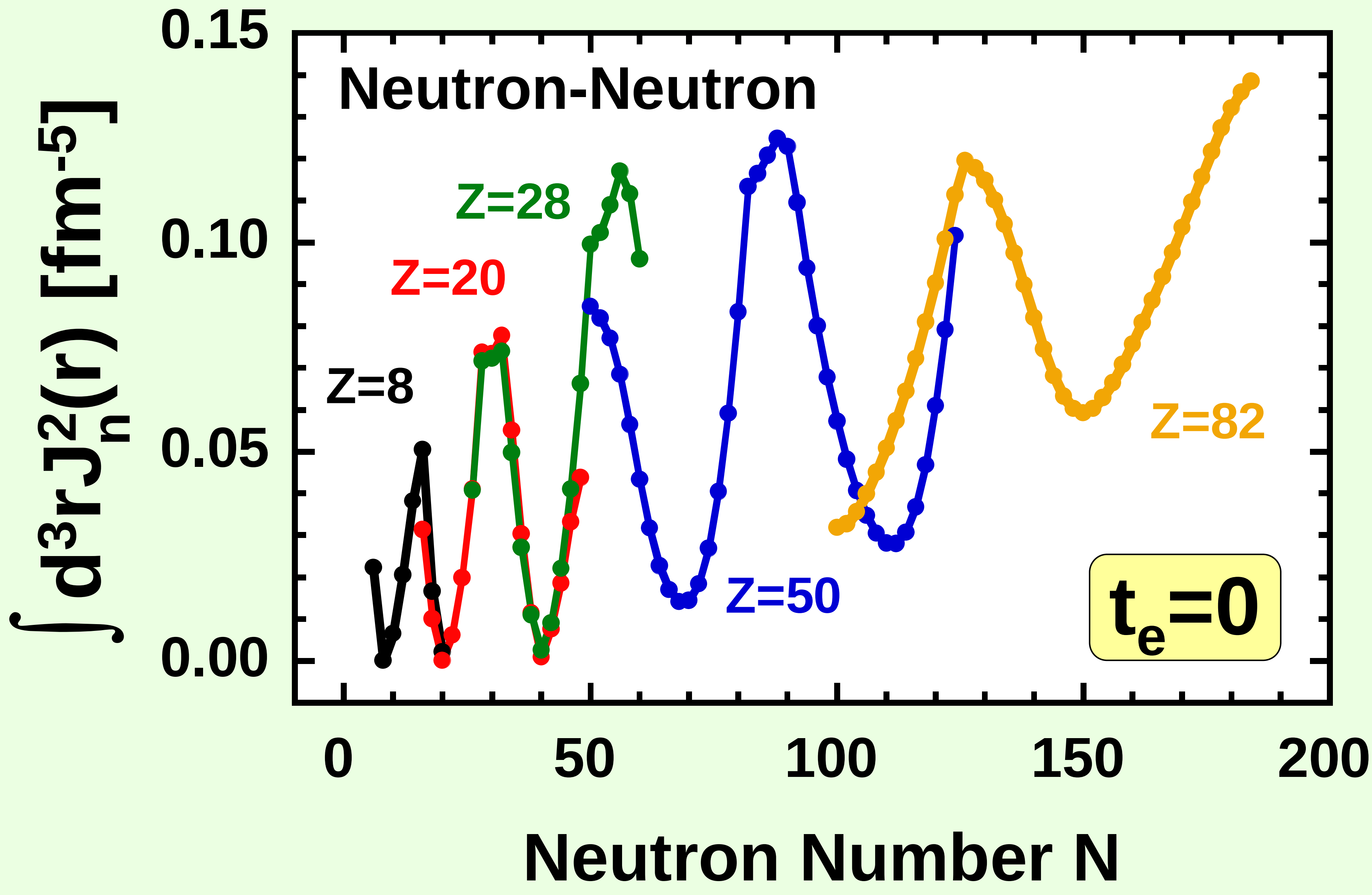
Jacek Dobaczewski

NT¤ at NuclearTheory.net

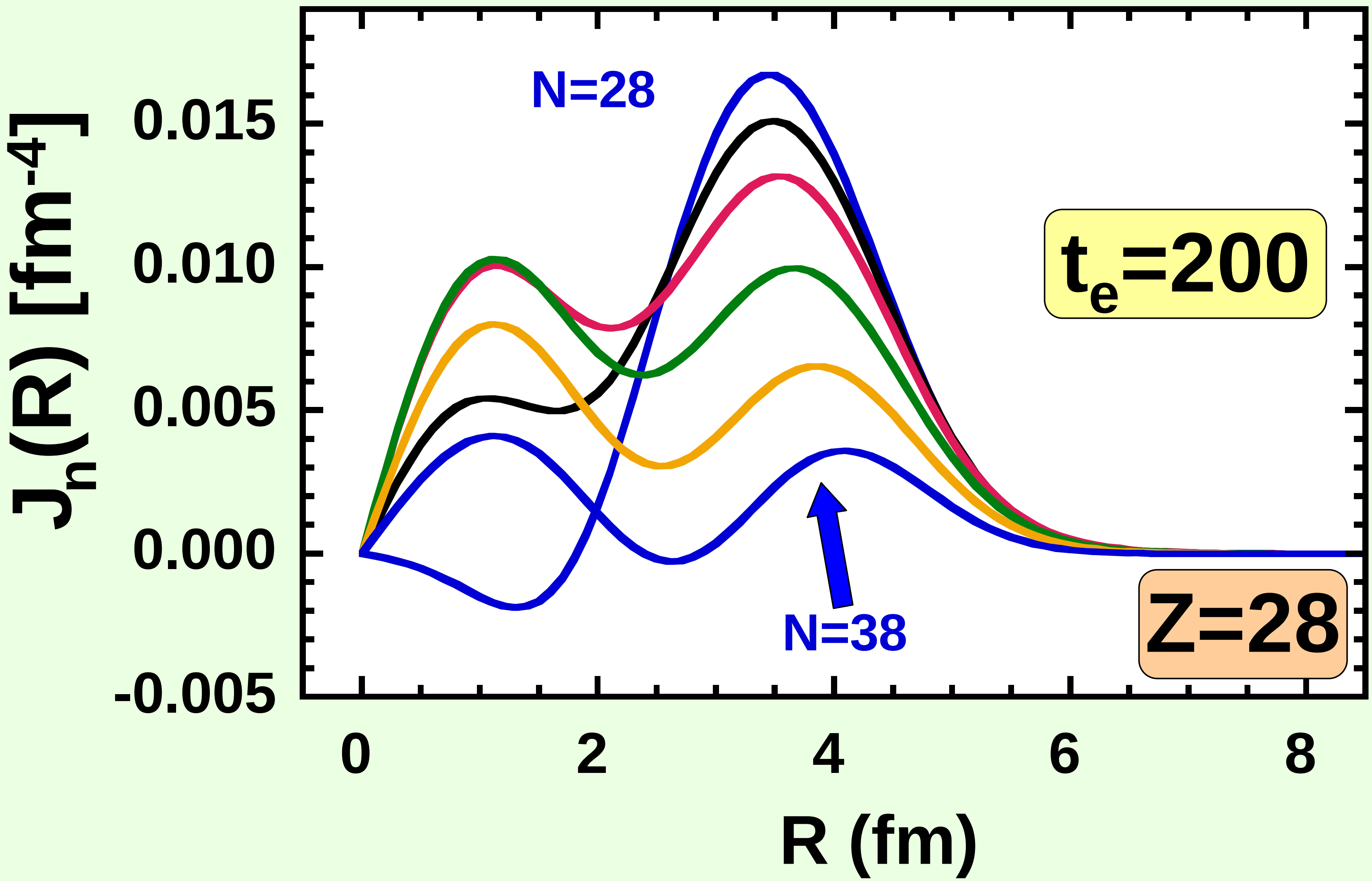
RIA Theory Rare Isotope Accelerator Project (RIA)



Neutron-Proton
$\int d^3r J_n(r) J_p(r)$ [fm^{-5}]
0.15
0.10
0.05
0.00
Z=8
Z=20
Z=28
Z=50
Z=82
t_e=200
0
50
100
150
200
Neutron Number N


Neutron-Neutron
$\int d^3r J_n^2(r)$ [fm^{-5}]
0.15
0.10
0.05
0.00
0
50
100
150
200
Neutron Number N
Z=8
Z=20
Z=28
Z=50
Z=82
t_e=0

Neutron S-O Density



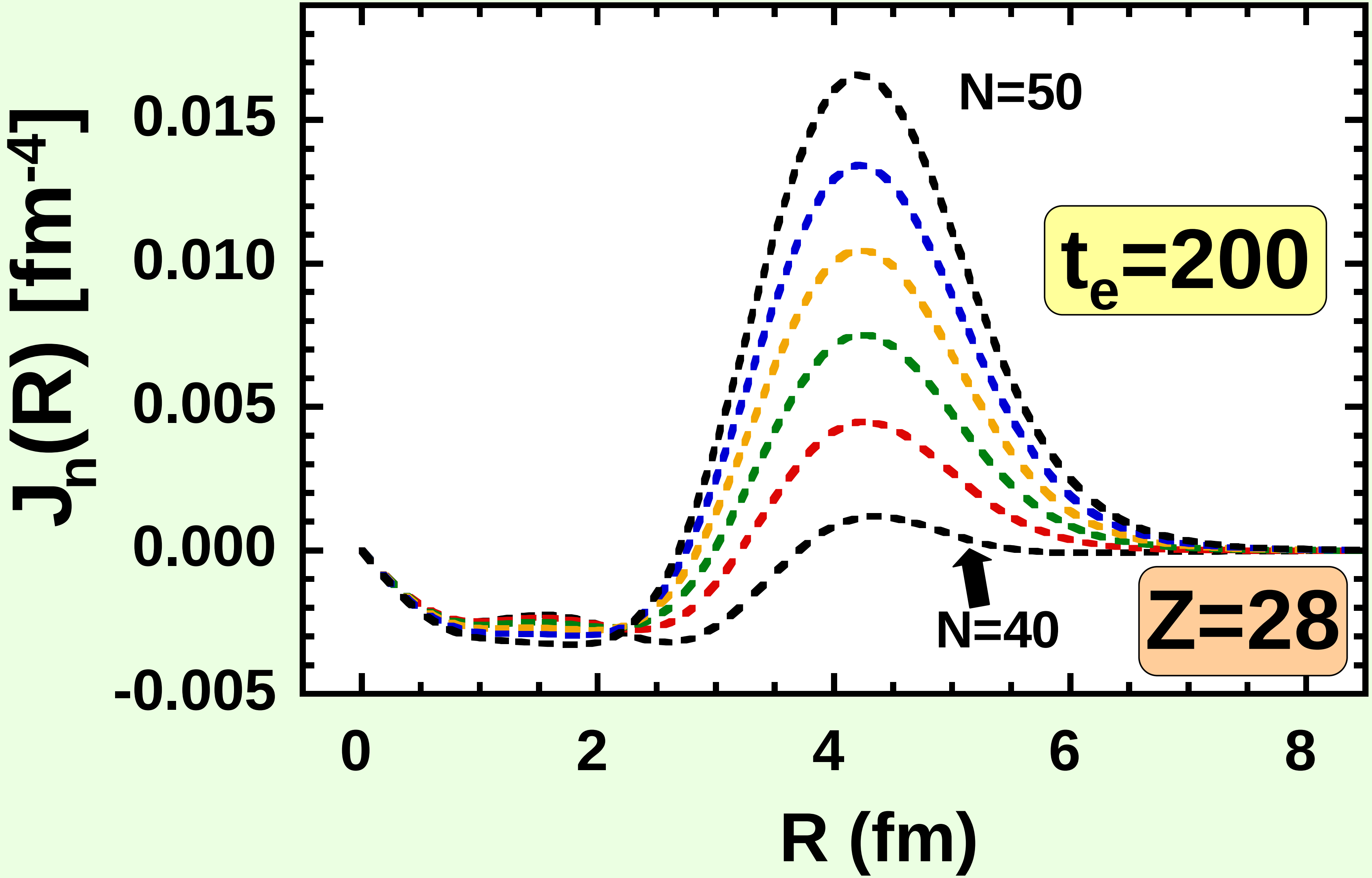
Jacek Dobaczewski

IFT UW

NT¤ at NuclearTheory.net

RIA Theory
Rare Isotope Accelerator Project (RIA)

Neutron S-O Density



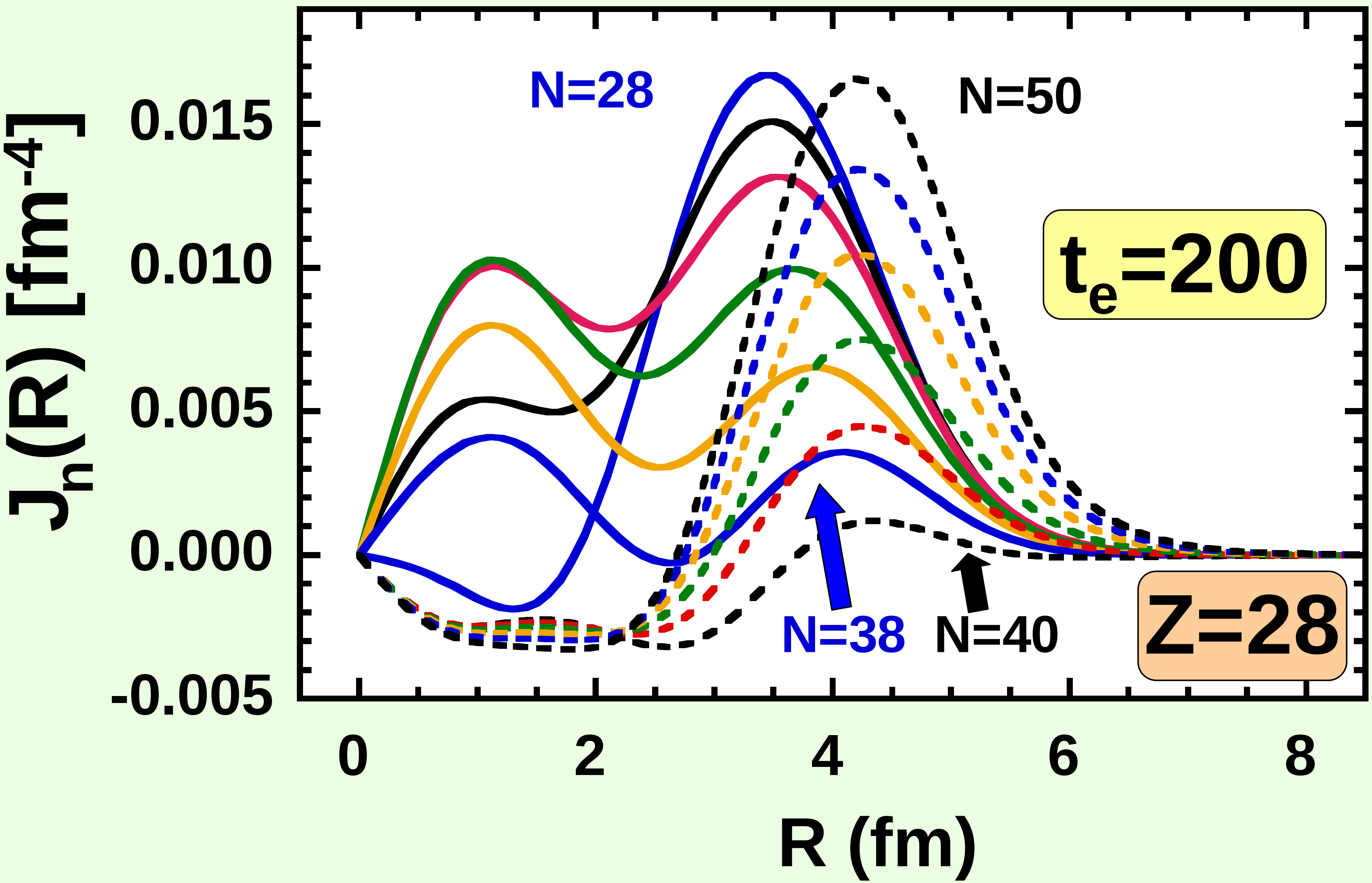
Jacek Dobaczewski

IFT UW

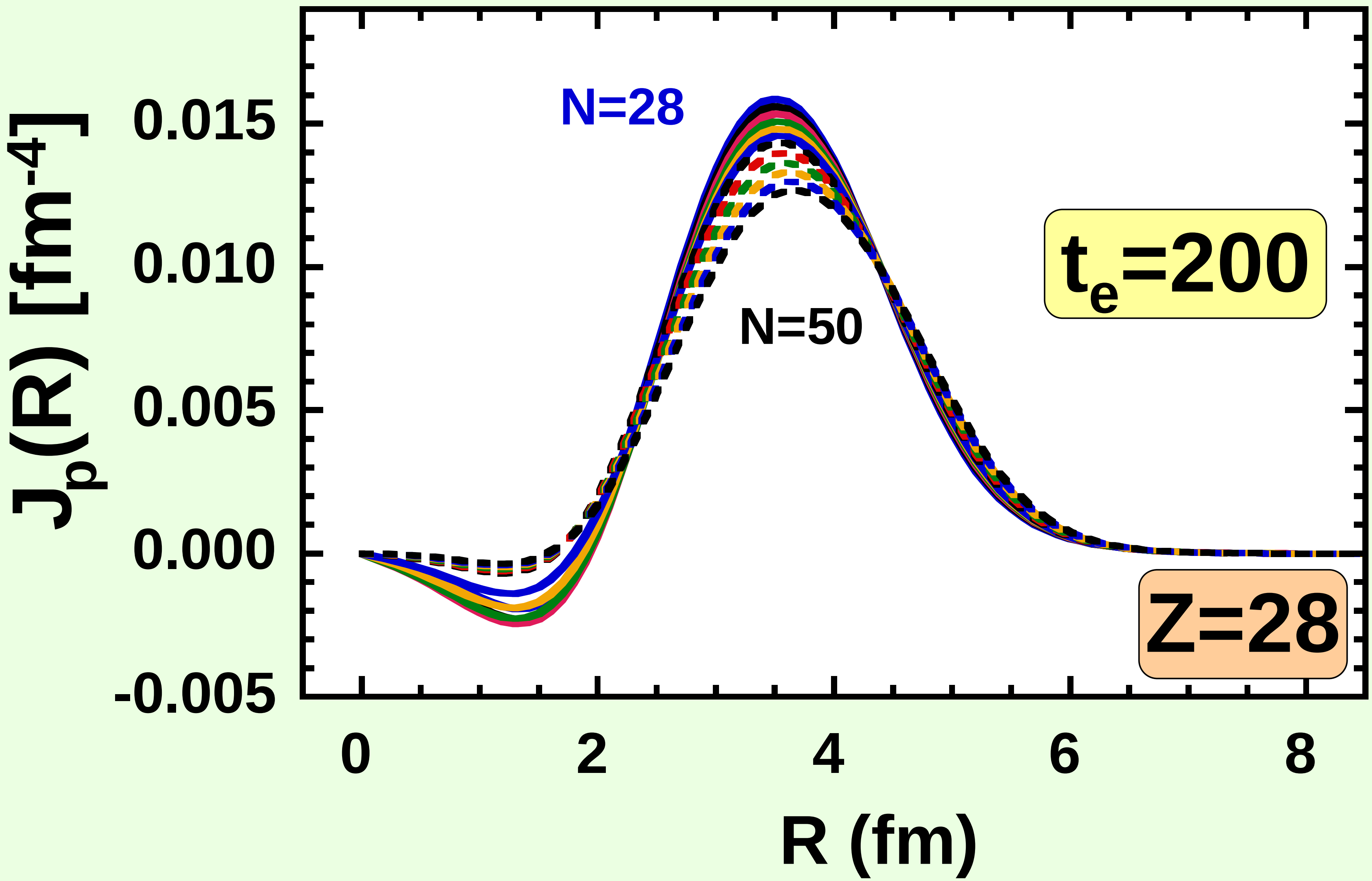
NT¤ at NuclearTheory.net

RIA Theory
Rare Isotope Accelerator Project (RIA)

Neutron S-O Density



Proton S-O Density



Conclusions

1. Tensor terms of the energy density functional constitute an unexplored channel in describing nuclear single-particle energies, nuclear masses and other global nuclear properties.
2. Tensor terms influence splitting of ALL spin-orbit partners.
3. Tensor densities depend on the imbalance in occupation of the spin-orbit partners, and therefore:
   - Are equal to zero at magic shells N=2, 8, 20.
   - Are maximum at magic shells N=28, 50, 82, 126.
4. Tensor terms are responsible for changes in the shell structure of light nuclei and should be included in systematic adjustments of nuclear effective interactions and/or energy density functionals ($t_e \approx +200$ MeV fm^5)
5. Contributions of tensor terms to nuclear masses have kinks at magic numbers of heavy nuclei and may cure systematic deficiencies of current parameterizations of the EDF.