Proton-neutron asymmetry in exotic nuclei

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CENTER FOR THEORETICAL PHYSICS
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Collective properties of exotic nuclei

Extensive new set of nuclei

- Proton-neutron imbalance
- Changes in shell structure?

Theoretical effort: Anticipate new collective phenomena

- Signatures by which phenomena can be recognized
- Estimates of where phenomena may occur

Most low-energy collective phenomena essentially *isoscalar*

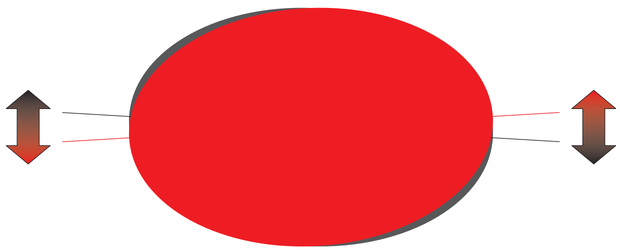
Similar proton and neutron distributions in the ground state

Deformation arises from strong proton-neutron quadrupole interaction, which couples proton and neutron deformations

Proton-neutron asymmetry in collective excitations

Scissors mode

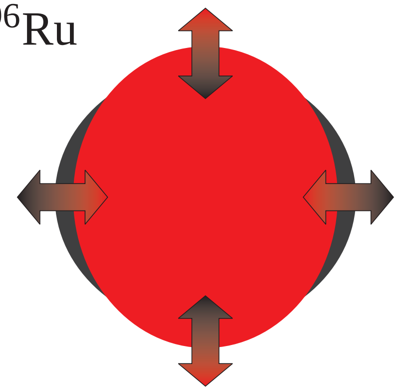
e.g., ^{156}Gd



N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
F. Iachello, Nucl. Phys. A **358**, 89c (1981).
D. Bohle *et al.*, Phys. Lett. B **137**, 27 (1984).

Mixed symmetry states

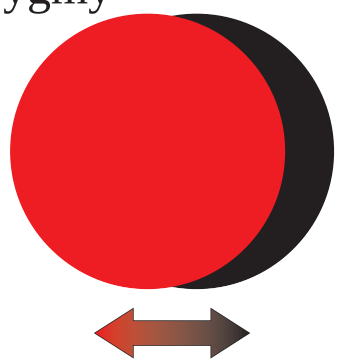
e.g., ^{94}Mo, ^{96}Ru



F. Iachello, Phys. Rev. Lett. **53**, 1427 (1984).
N. Pietralla *et al.*, Phys. Rev. Lett. **84**, 3775 (2000).

Dipole resonances

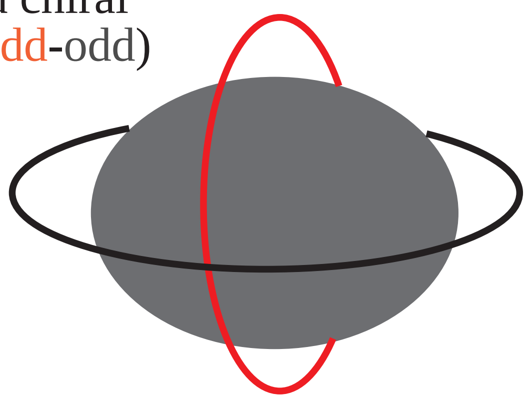
Giant and pygmy resonances



M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).
A. Zilges *et al.*, Phys. Lett. B **542**, 43 (2002).

Asymmetry in coupling

Shears and chiral rotation (odd-odd)



S. Frauendorf, Rev. Mod. Phys. **73**, 463 (2001).

Proton-neutron asymmetry in the ground state?

Very neutron-rich nuclei

Well-separated proton and neutron valence spaces

$\Rightarrow$ Reduced proton-neutron coupling strengths?

$\Rightarrow$ Larger role for proton-neutron asymmetry in ground state?

Nuclear structure

Ground state properties, excitation modes, transition radiations ($M1$)

Mechanisms for triaxiality

– Higher-order interactions in one-fluid Hamiltonian ($d^\dagger d^\dagger d^\dagger \tilde{d}\tilde{d}\tilde{d}$ or $\cos^2 3\gamma$)

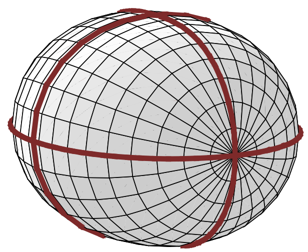
P. Van Isacker and J. Chen, Phys. Rev. C **24**, 684 (1981).

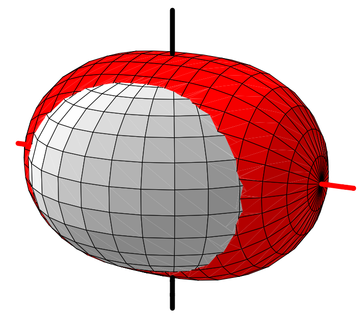
– Higher-multipolarity pairs (hexadecapole)

K. Heyde *et al.*, Nucl. Phys. A **398**, 235 (1983).

– Unaligned proton and neutron symmetry axes

A. E. L. Dieperink and R. Bijker, Phys. Lett. B **116**, 77 (1982).
J. N. Ginocchio and A. Leviatan, Ann. Phys. (N.Y.) **216**, 152 (1992).





The interacting boson model (IBM-1)

Truncation to s-wave ($J=0$) and d-wave ($J=2$) nucleon pairs

$$s_0 \;\; d_{+2} \;\; d_{+1} \;\; d_0 \;\; d_{-1} \;\; d_{-2}$$

States: Linear combinations of $(s_0^\dagger)^n (d_{+2}^\dagger)^{n'} \cdots |0\rangle$

Operators (H, $\hat{L}$, $\hat{T}$, ...): Polynomials in $b^\dagger b$ *e.g*, $\hat{L} = \sqrt{10}[d^\dagger \times \tilde{d}]^{(1)}$

Algebraic model: Constructed from elements of Lie algebra

$$\mathrm{U}(6): \;\; s_0^\dagger s_0 \;\; s_0^\dagger d_{+2} \;\; \ldots \;\; d_{-2}^\dagger d_{-2}$$

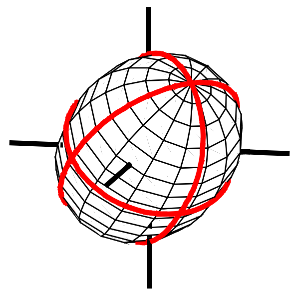
Dynamical symmetry

$$\mathrm{U}(6) \supset \begin{pmatrix} \left.\begin{matrix} \mathrm{U}(5) \\ \mathrm{SO}(6) \end{matrix}\right\} \supset \;\mathrm{SO}(5) \\ \mathrm{SU}(3) \\ \overline{\mathrm{SU}(3)} \end{pmatrix} \supset \underbrace{\mathrm{SO}(3) \supset \mathrm{SO}(2)}_{\text{Angular momentum}}$$

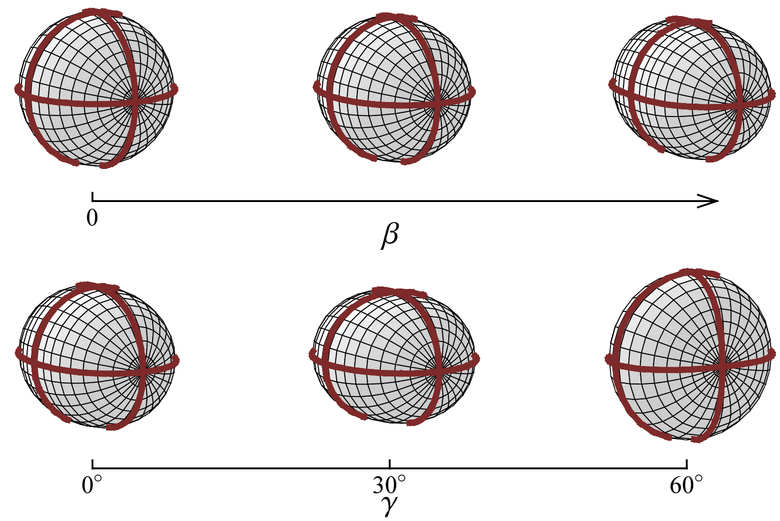
– H constructed from Casimir (invariant) operators of subalgebra chain
– Eigenstates have good quantum numbers
– Problem exactly soluble (energies, eigenstates, transition MEs)
– Defines distinct form of ground state configuration (“phase”)

Classical limit of the IBM-1

Quadrupole-deformed liquid drop



$\beta, \gamma, \theta_1, \theta_2, \theta_3$



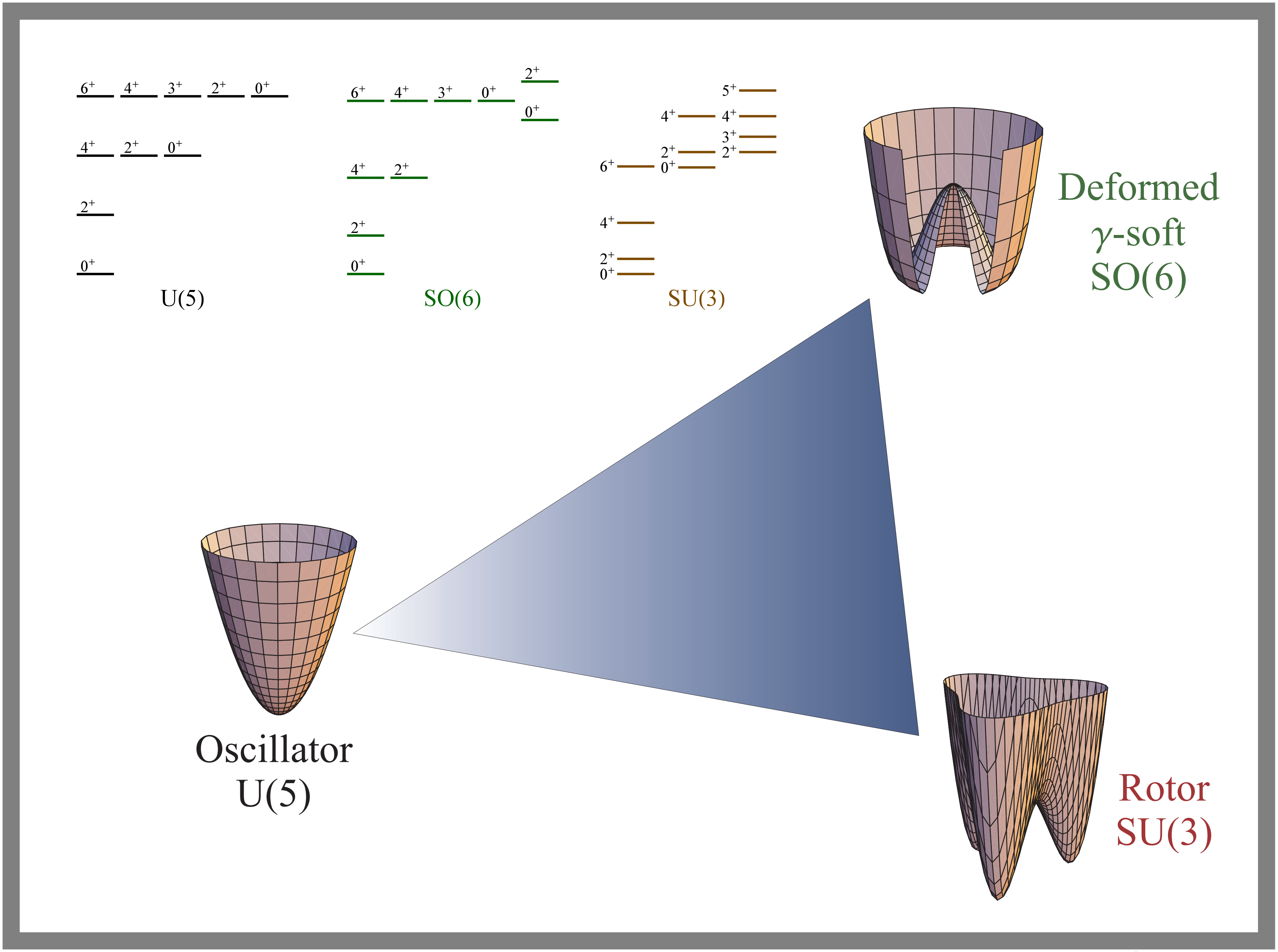
Coherent states $|\beta,\gamma\rangle$

$$|\beta,\gamma\rangle = \left[s_0^\dagger + \beta\cos\gamma d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_{+2}^\dagger + d_{-2}^\dagger)\right]^N |0\rangle$$

Classical energy surface

$$\mathscr{E}(\beta,\gamma) = \langle\beta,\gamma|H|\beta,\gamma\rangle$$

Minimization of $\mathscr{E}$ $\Rightarrow$ ground state energy, equilibium coordinate values



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Phase diagram of the IBM-1

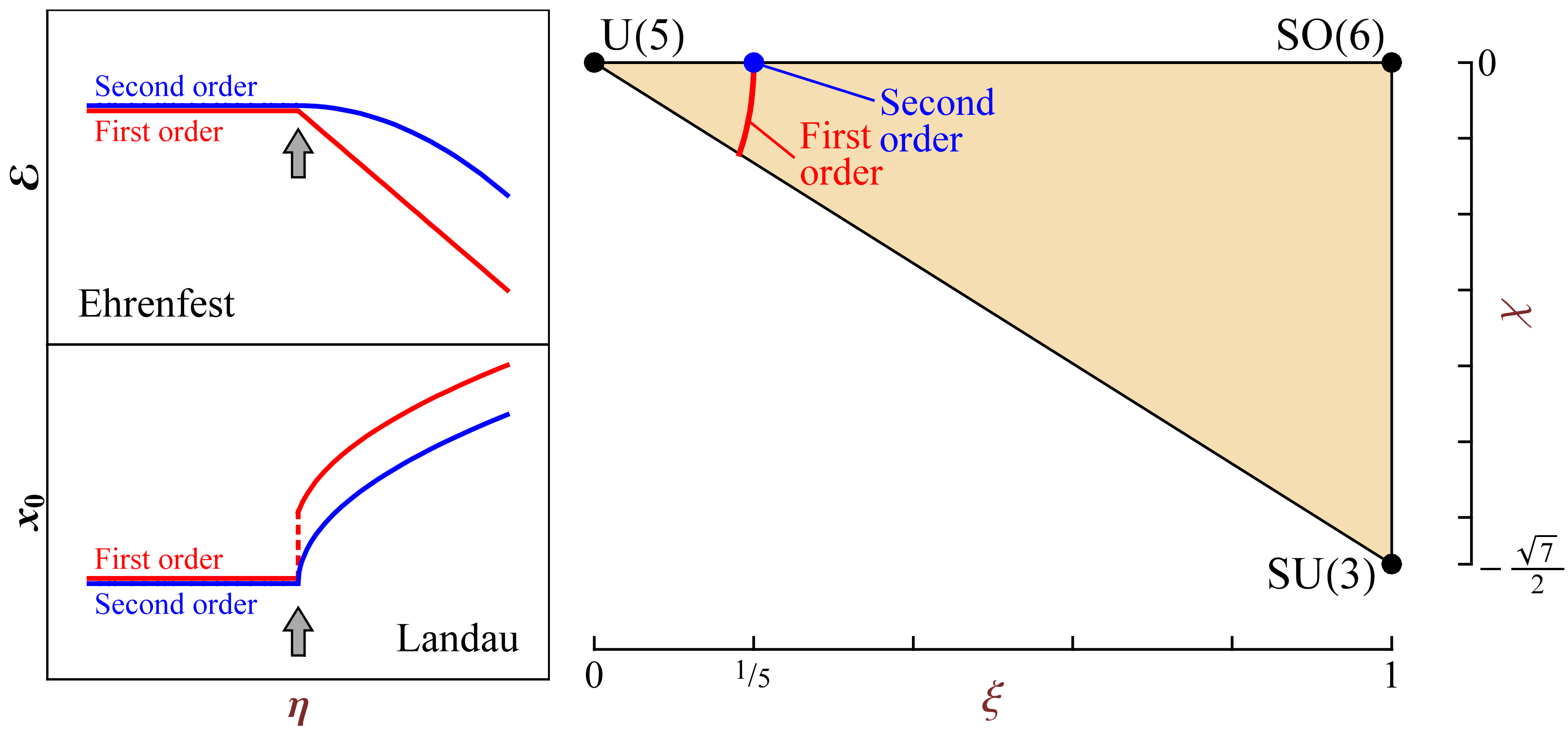
$$H=(1-\xi)\frac{1}{N}\hat{n}_d-\xi\frac{1}{N^2}\hat{Q}^\chi\cdot\hat{Q}^\chi$$

ξ: (spherical) $\leftrightarrow$ (deformed)

$\hat{n}_d=d^\dagger\cdot\tilde{d}$ $\quad\hat{Q}^\chi=(s^\dagger\times\tilde{d}+d^\dagger\times\tilde{s})^{(2)}+\chi(d^\dagger\times\tilde{d})^{(2)}$

χ: (prolate) $\leftrightarrow$ (γ-soft) $\leftrightarrow$ (oblate)

A. E. L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980).
D. H. Feng, R. Gilmore, and S. R. Deans, Phys. Rev. C **23**, 1254 (1981).



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The proton-neutron interacting boson model (IBM-2)

Proton and neutron pairs as separate boson species

$$\underbrace{s^\dagger_{\pi,0}\ d^\dagger_{\pi,-2}\ d^\dagger_{\pi,-1}\ d^\dagger_{\pi,0}\ d^\dagger_{\pi,+1}\ d^\dagger_{\pi,+2}}_{\text{Proton}} \qquad \underbrace{s^\dagger_{\nu,0}\ d^\dagger_{\nu,-2}\ d^\dagger_{\nu,-1}\ d^\dagger_{\nu,0}\ d^\dagger_{\nu,+1}\ d^\dagger_{\nu,+2}}_{\text{Neutron}}$$

$$H = \varepsilon_\pi \hat{n}_{d\pi} + \varepsilon_\nu \hat{n}_{d\nu} + \kappa_{\pi\pi} Q_\pi \cdot Q_\pi + \kappa_{\pi\nu} Q_\pi \cdot Q_\nu + \kappa_{\nu\nu} Q_\nu \cdot Q_\nu + \cdots$$

Symmetric dynamical symmetries (isoscalar)

$U_{\pi\nu}(5)$	$SO_{\pi\nu}(6)$	$SU_{\pi\nu}(3)$	$\overline{SU_{\pi\nu}(3)}$
Spherical	γ-soft	Prolate	Oblate

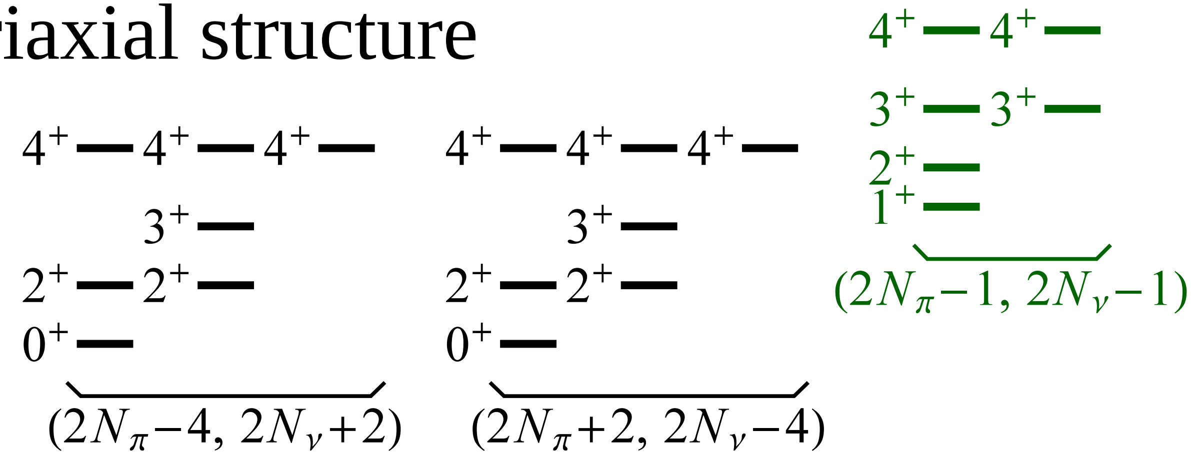
P. Van Isacker, K. Heyde, J. Jolie, and A. Sevrin, Ann. Phys. (NY) **171**, 253 (1986).

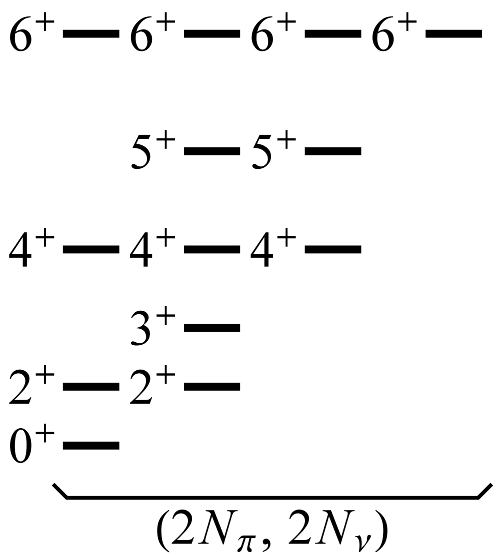
Asymmetric dynamical symmetries (isovector)

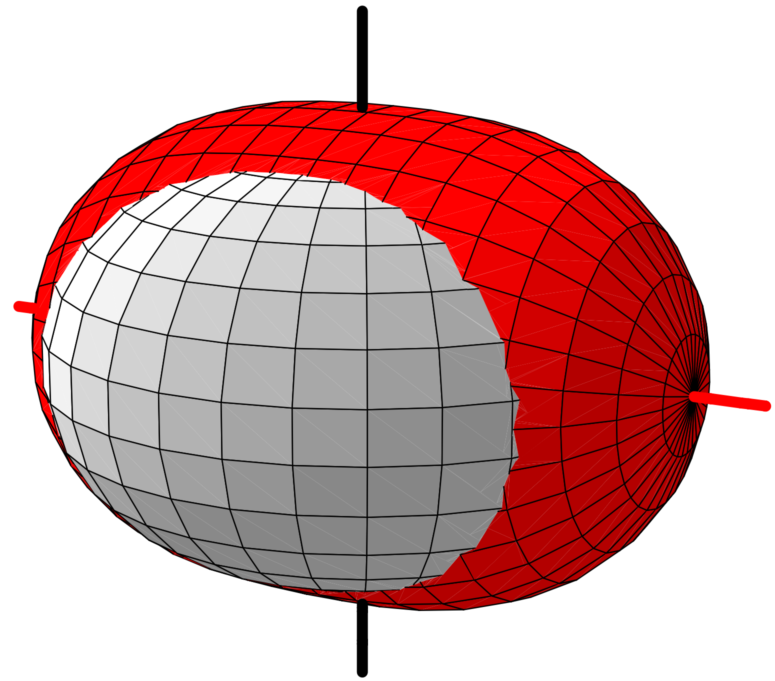
$$U_\pi(6) \otimes U_\nu(6) \supset \left\{ \begin{array}{lcl} SU_\pi(3) \otimes \overline{SU_\nu(3)} & \supset & \boxed{SU^*_{\pi\nu}(3)} \\ \overline{SU_\pi(3)} \otimes SU_\nu(3) & \supset & \boxed{\overline{SU^*_{\pi\nu}(3)}} \end{array} \right\} \supset SO_{\pi\nu}(3) \supset SO_{\pi\nu}(2)$$

A. E. L. Dieperink and R. Bijker, Phys. Lett. B **116**, 77 (1982).
A. Sevrin, K. Heyde, and J. Jolie, Phys. Rev. C **36**, 2621 (1987).
N. R. Walet and P. J. Brussaard, Nucl. Phys. A **474**, 61 (1987).

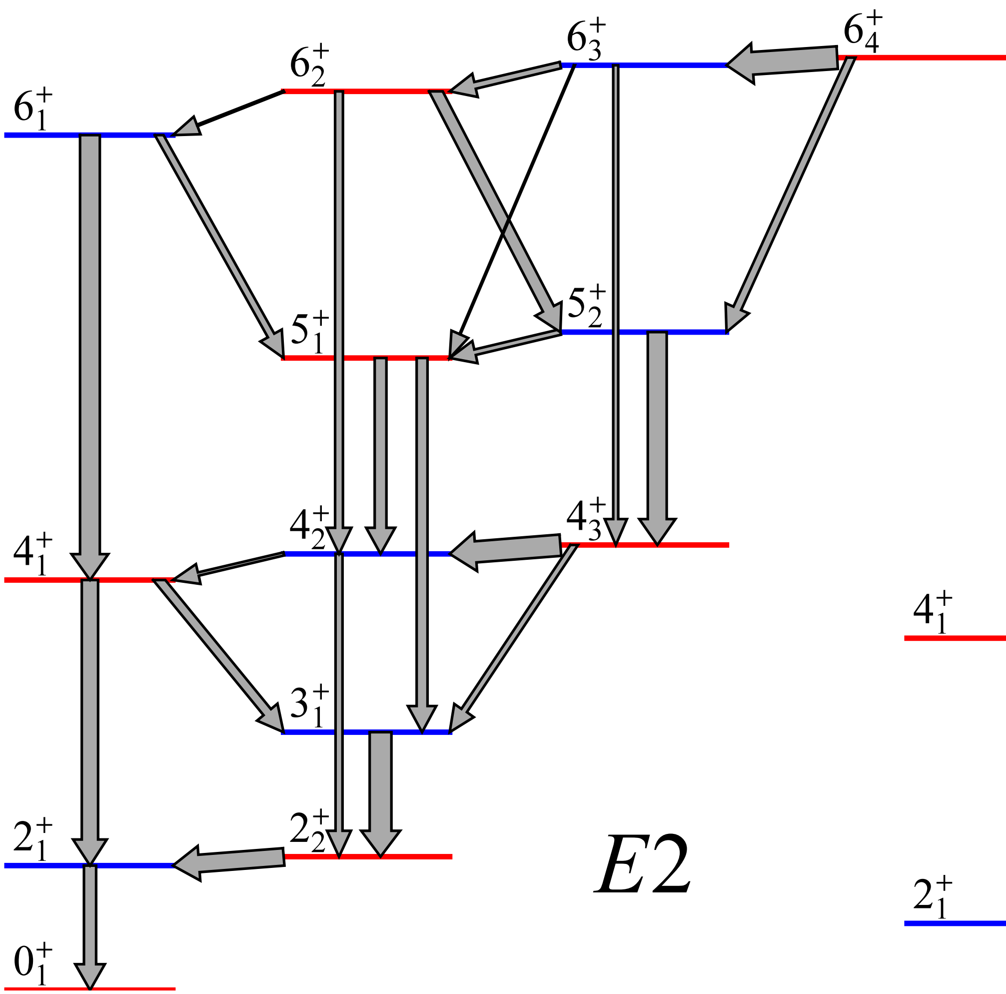
The $SU^*_{\pi\nu}(3)$ dynamical symmetry: Proton-neutron triaxial structure

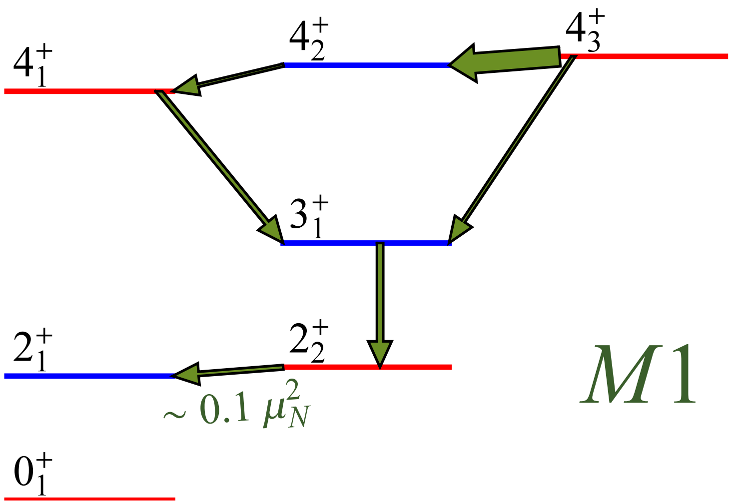






The SU$^*_{\pi\nu}$(3) dynamical symmetry: Electromagnetic transitions





Essential parameters and coordinates for the IBM-2

Collective coordinates ("order parameters")

$$\left\{\begin{matrix}\beta_\pi & \gamma_\pi & \theta_{1\pi} & \theta_{2\pi} & \theta_{3\pi}\\ \beta_\nu & \gamma_\nu & \theta_{1\nu} & \theta_{2\nu} & \theta_{3\nu}\end{matrix}\right\} \Rightarrow \left\{\begin{matrix}\beta_\pi & \gamma_\pi\\ \beta_\nu & \gamma_\nu\end{matrix}\ \vartheta_1\ \vartheta_2\ \vartheta_3\right\} \Rightarrow \left\{\begin{matrix}\beta_\pi & \gamma_\pi\\ \beta_\nu & \gamma_\nu\end{matrix}\right\}$$

Coherent state energy surface $\mathscr{E}(\beta_\pi, \gamma_\pi, \beta_\nu, \gamma_\nu, \vartheta_1, \vartheta_2, \vartheta_3)$

Four order parameters: β_π, γ_π, β_ν, and γ_ν

Hamiltonian parameters ("control parameters")

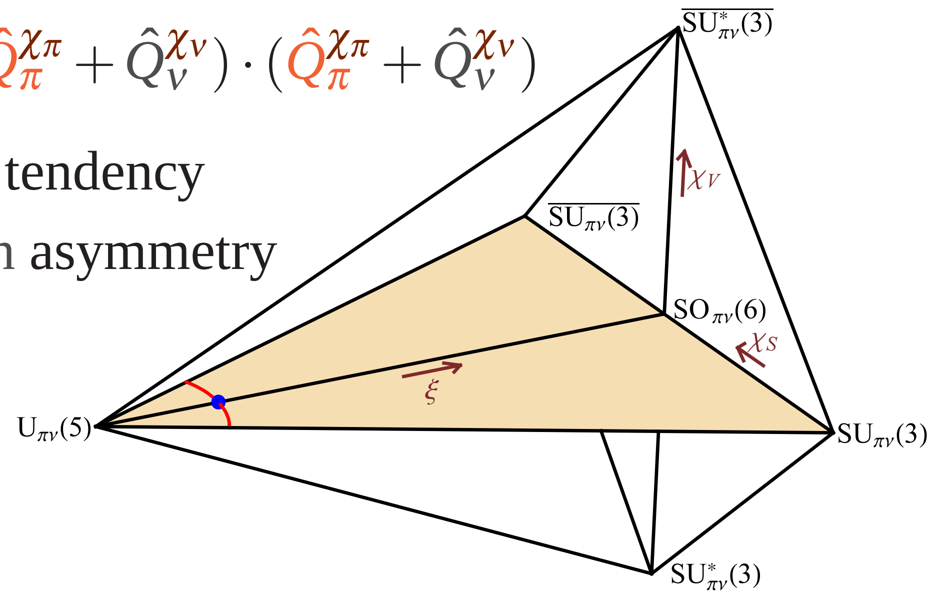
$$H = (1-\xi)\frac{1}{N}(\hat{n}_{d\pi} + \hat{n}_{d\nu}) - \xi\frac{1}{N^2}(\hat{Q}_\pi^{\chi_\pi} + \hat{Q}_\nu^{\chi_\nu})\cdot(\hat{Q}_\pi^{\chi_\pi} + \hat{Q}_\nu^{\chi_\nu})$$

$\chi_S = \frac{1}{2}(\chi_\pi + \chi_\nu)$ prolate-oblate tendency

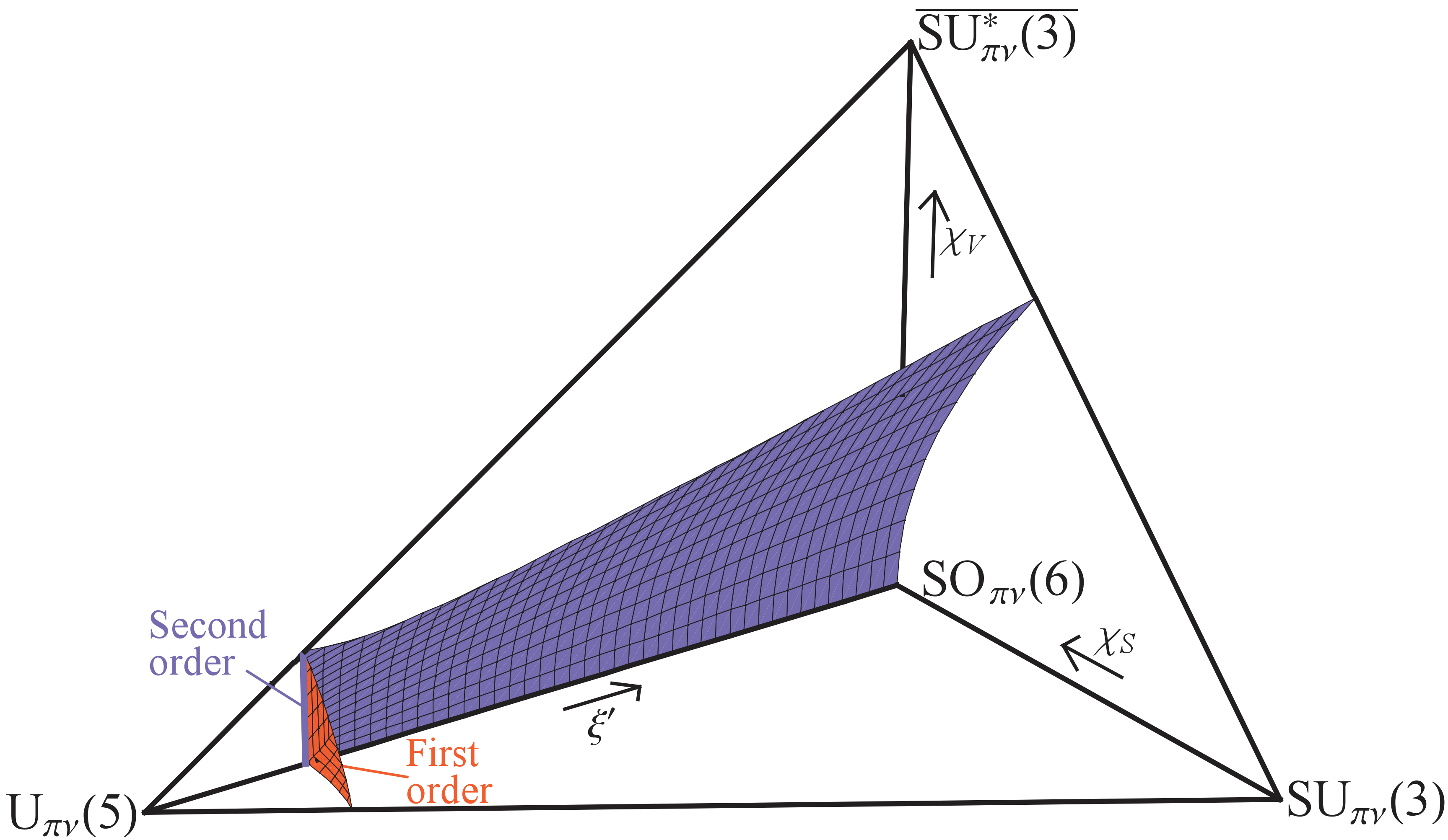
$\chi_V = \frac{1}{2}(\chi_\pi - \chi_\nu)$ proton-neutron asymmetry

Three control parameters:

ξ, χ_S, and χ_V



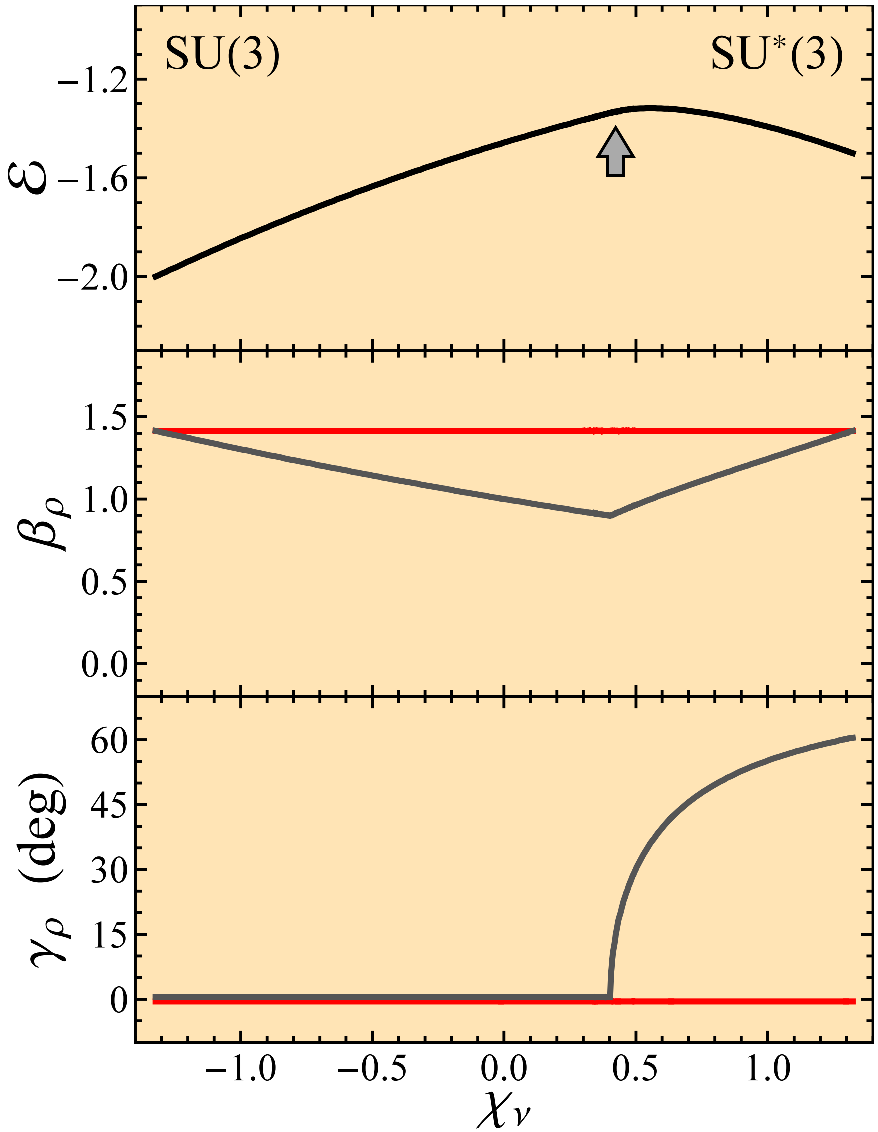
Phase diagram of the IBM-2

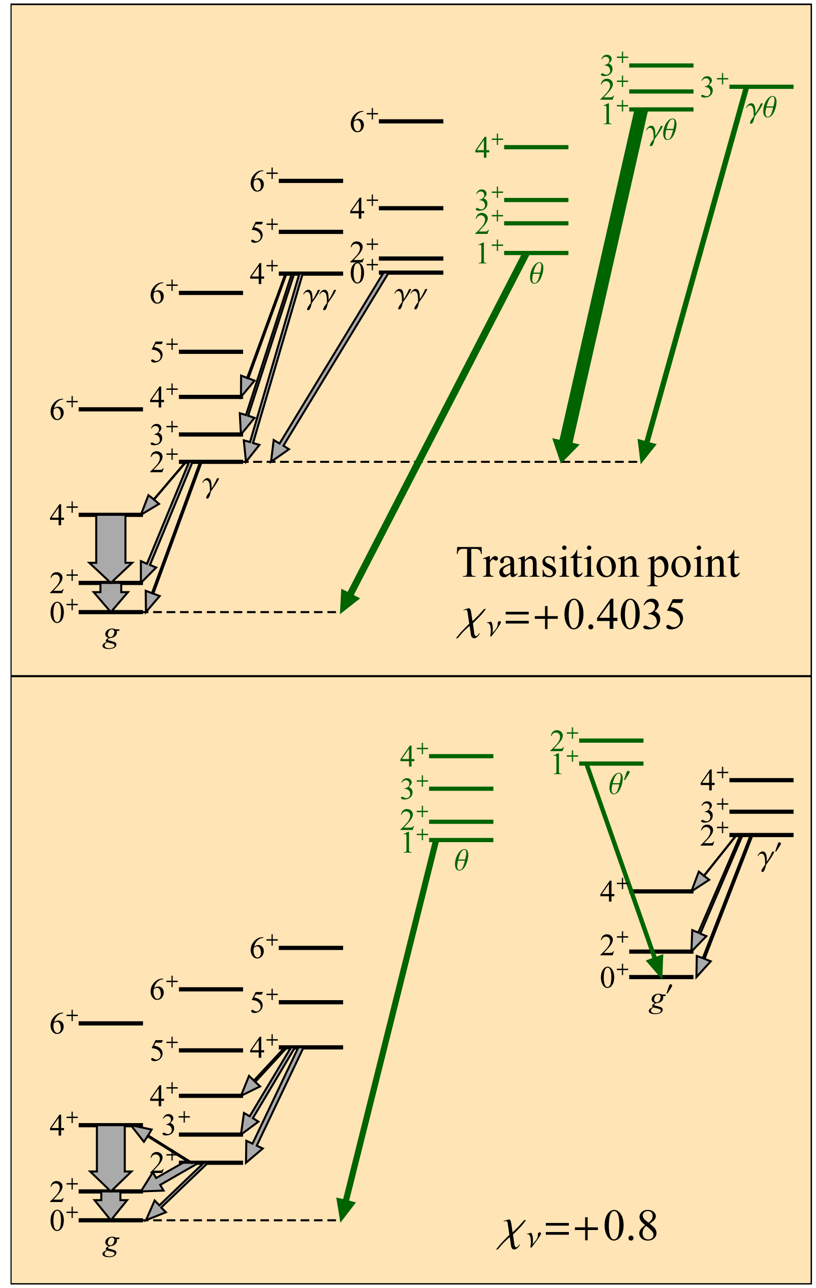


M. A. Caprio and F. Iachello, Phys. Rev. Lett. **93**, 242502 (2004).
M. A. Caprio and F. Iachello, Ann. Phys. (N.Y.) **318**, 454 (2005).

$N_\pi/N_\nu = 1$

$SU_{\pi\nu}(3)$-$SU^*_{\pi\nu}(3)$ transition





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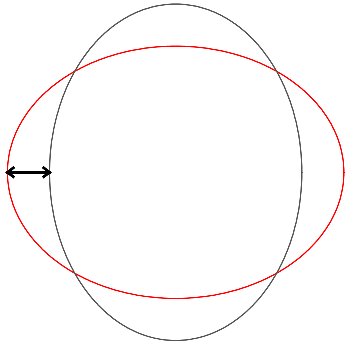
Proton-neutron symmetry energy (Majorana operator)

Difference between proton and neutron deformation tensors

$$\hat{M} \equiv -2 \sum_{k=1,3} (d_\pi^\dagger \times d_\nu^\dagger)^{(k)} \cdot (\tilde{d}_\pi \times \tilde{d}_\nu)^{(k)} + (s_\pi^\dagger \times d_\nu^\dagger - s_\nu^\dagger \times d_\pi^\dagger)^{(2)} \cdot (\tilde{s}_\pi \times \tilde{d}_\nu - \tilde{s}_\nu \times \tilde{d}_\pi)^{(2)}$$

$$\approx |\alpha_\pi - \alpha_\nu|^2$$



Major ingredient in realistic Hamiltonian

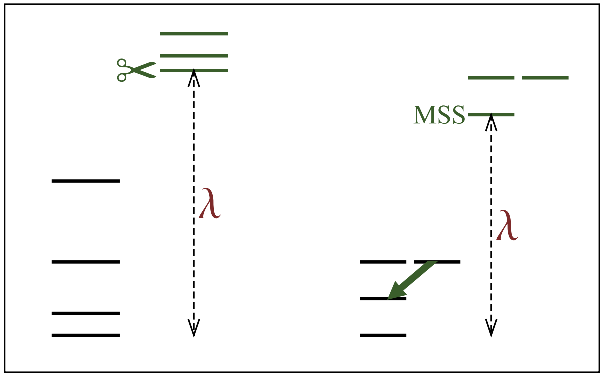
$$H = \underbrace{\varepsilon_\pi \hat{n}_{d\pi} + \varepsilon_\nu \hat{n}_{d\nu}}_{\text{Pair energy}} + \underbrace{\kappa_{\pi\pi} Q_\pi \cdot Q_\pi + \kappa_{\pi\nu} Q_\pi \cdot Q_\nu + \kappa_{\nu\nu} Q_\nu \cdot Q_\nu}_{\text{Quadrupole}} + \underbrace{\lambda \hat{M}}_{\text{Symmetry}}$$

Strength λ approximately known

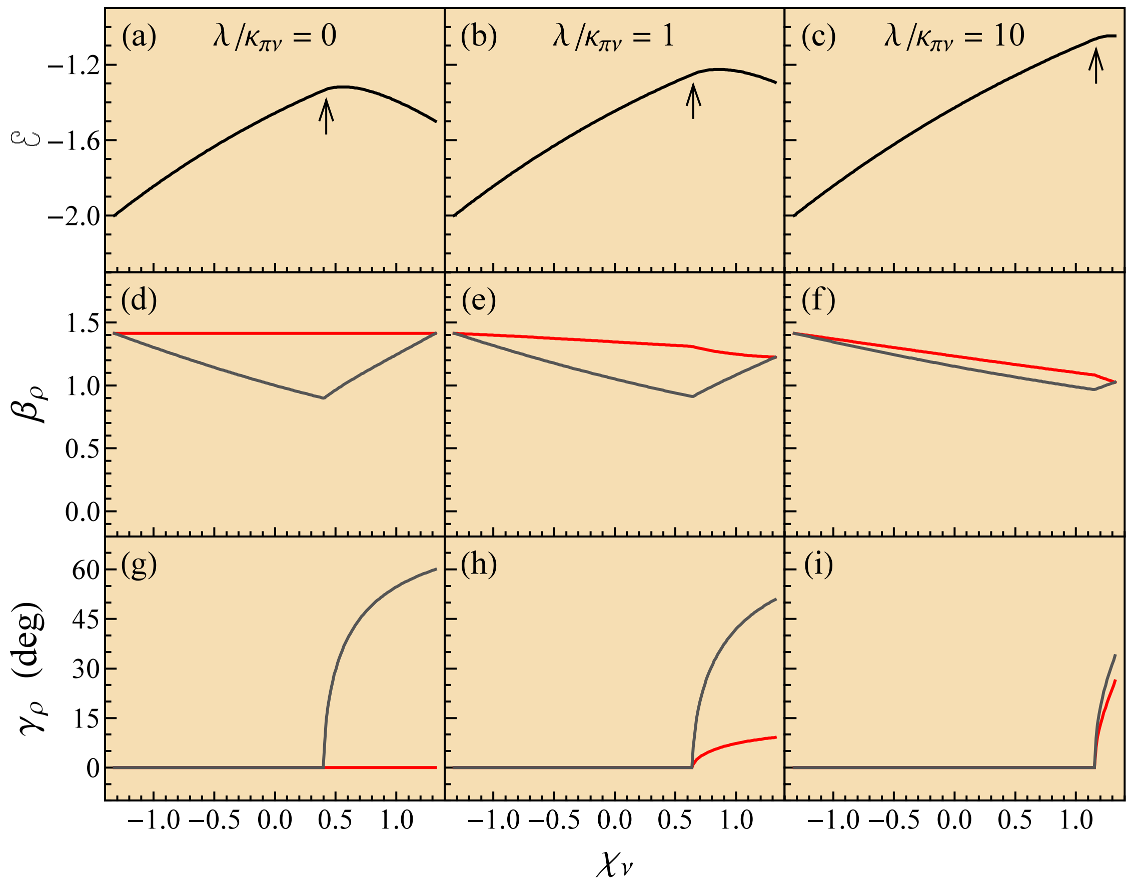
– From scissors and mixed-symmetry energies

– From $M1$ mixing ratios

$$\frac{\lambda}{\kappa_{\pi\nu}} \approx 5$$



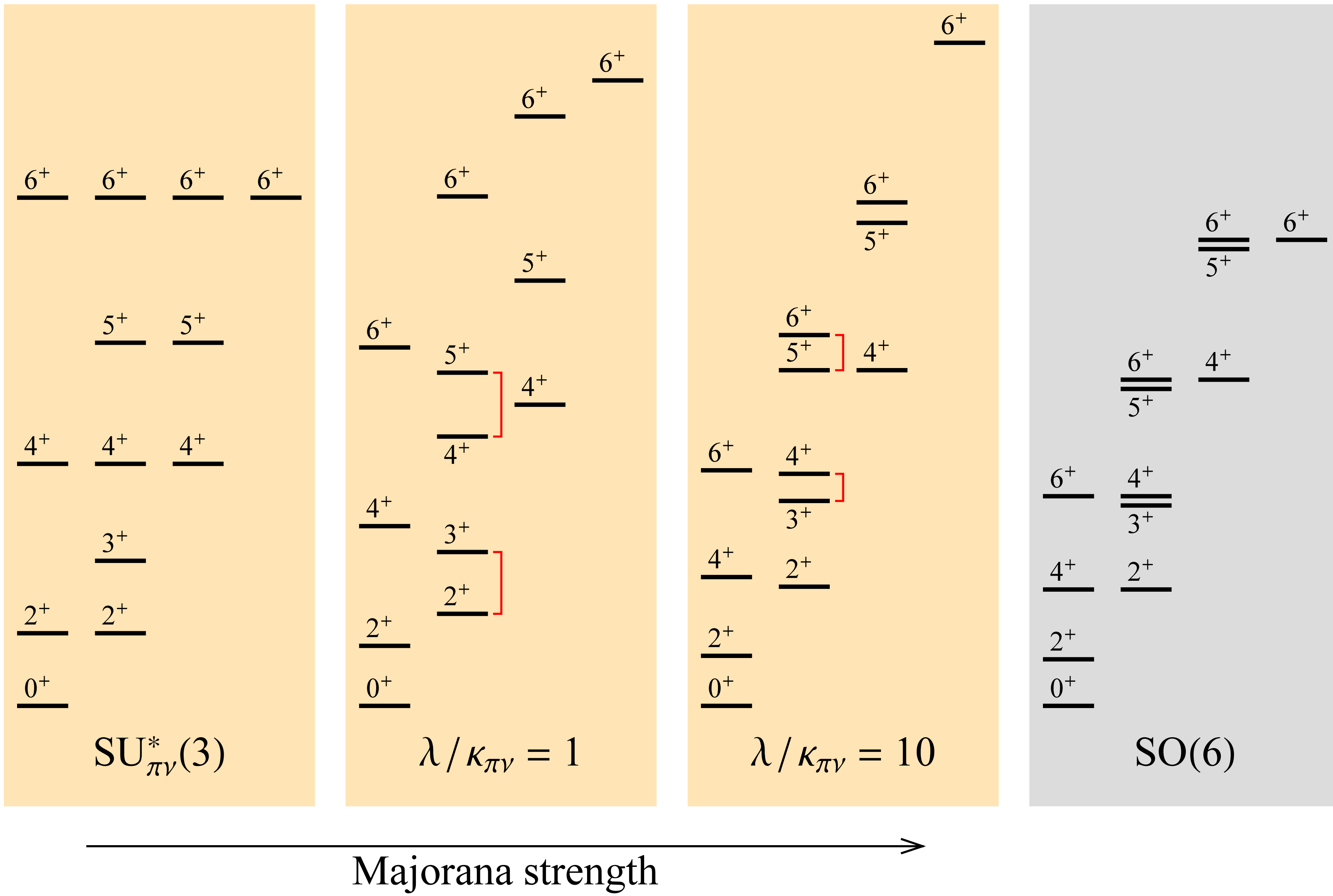
Effect of Majorana operator on phase transition



- Phase transition to triaxiality delayed
- Proton and neutron equilibrium coordinates values brought together
- *But also* energy minimum at triaxial deformation shallower

$\mathrm{SU}^*_{\pi\nu}(3)$ triaxial ⇒ one-fluid triaxial ⇒ one-fluid γ-soft

Effect of Majorana operator on $SU^*_{\pi\nu}(3)$ structure



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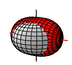
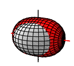
Proton-neutron triaxiality

Main signatures

- Low-lying $K=2$ band

  but rotational $L(L+1)$ energy sequence
- Unusual $B(E2)$ strength pattern

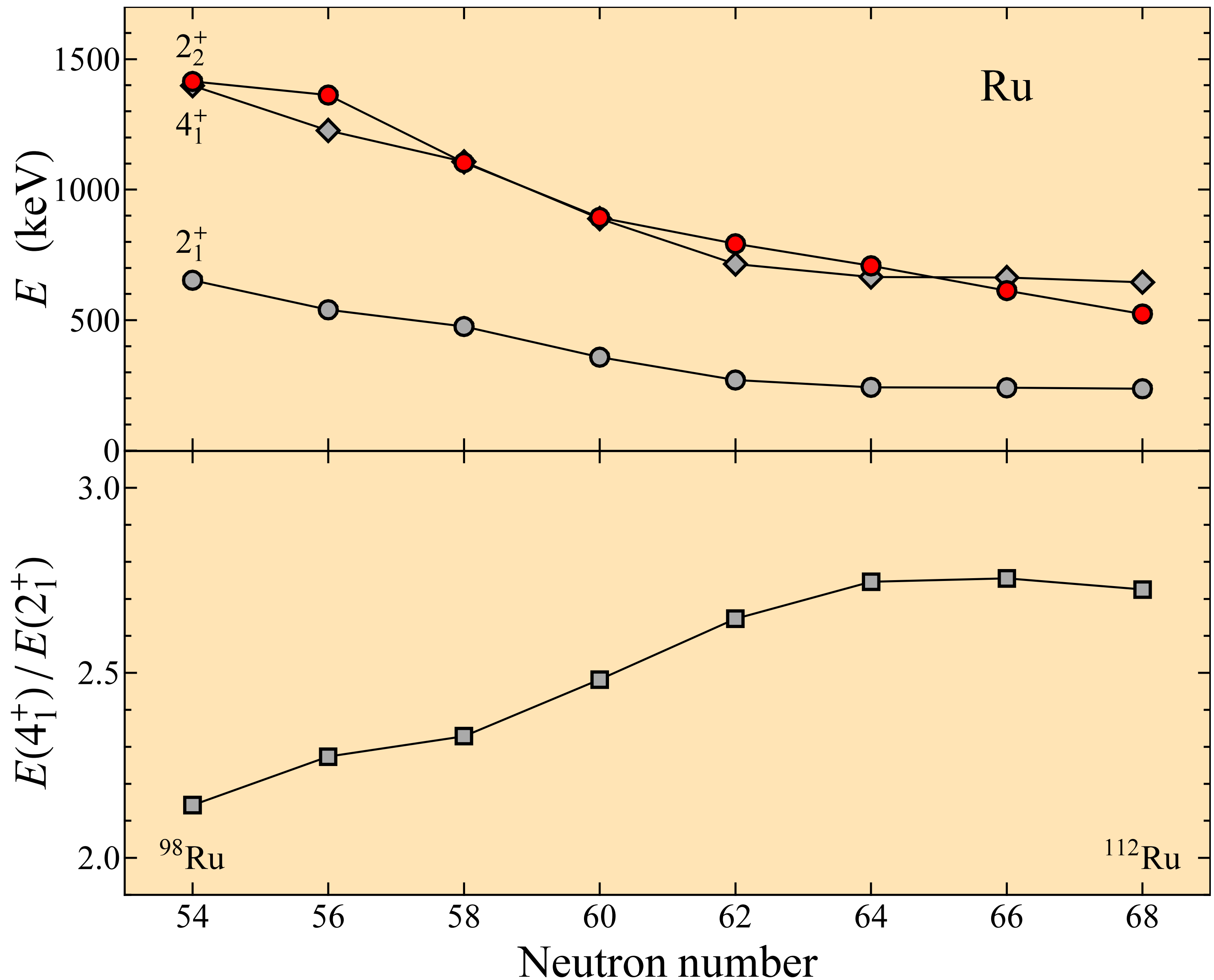
  similar to classic rigid triaxial rotor (Davydov)
- Anharmonically low $K=4$ band
- Strong $M1$ admixtures 
- Orthogonal scissors mode 

But attenuated by Majorana operator

$\mathrm{SU}^*_{\pi\nu}(3)$ triaxial

$\Rightarrow$ one-fluid triaxial

$\Rightarrow$ one-fluid γ-soft



Ru
2_2^+
4_1^+
2_1^+
1500
1000
500
0
E (keV)
3.0
2.5
2.0
$E(4_1^+)\,/\,E(2_1^+)$
^{98}Ru
^{112}Ru
54
56
58
60
62
64
66
68
Neutron number

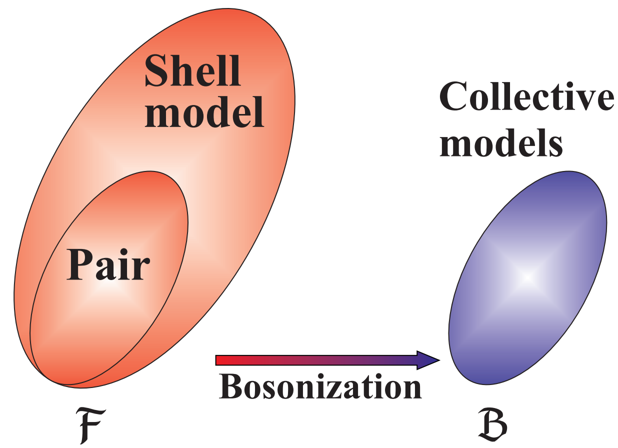
Where might asymmetric structure be expected?

Collective structure depends upon underlying single-particle structure

- Energy spacing (subshell gaps?)
- Ordering of orbitals (low j? high j?)
- Radial wave functions (compact? diffuse?)

Manifested in effective interactions

- Pairing interaction (s-wave, d-wave, ...)
- Multipole interaction (quadrupole, ...)
- Symmetry energy (Majorana)

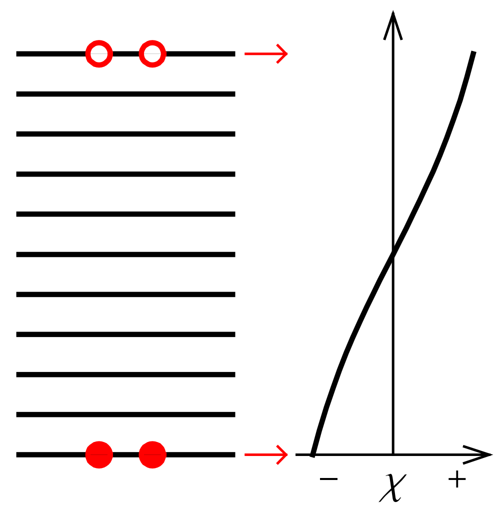


Qualitative estimate

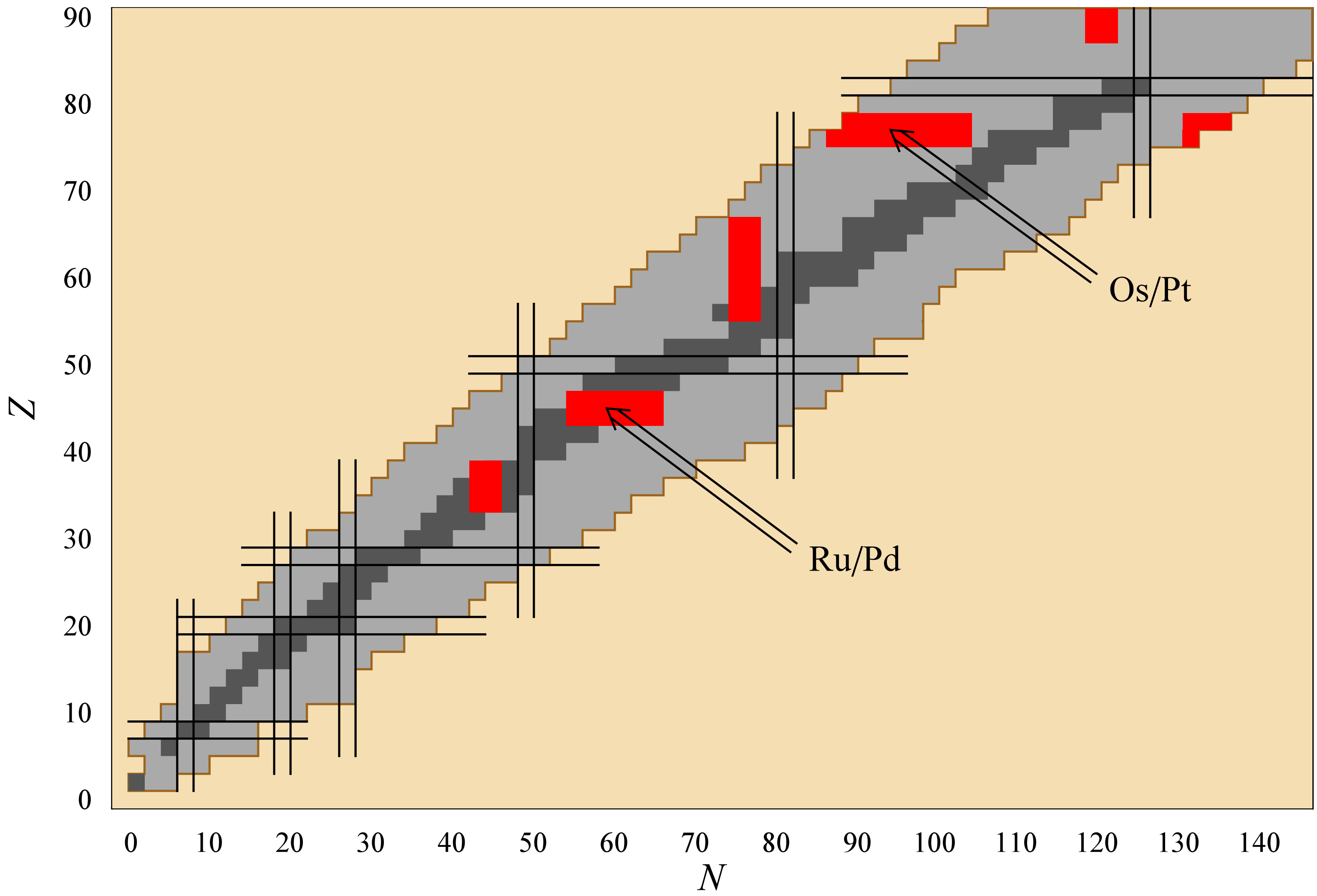
Particle-like bosons $\Rightarrow$ Prolate tendency
Hole-like bosons $\Rightarrow$ Oblate tendency

But very sensitive to underlying shell structure

A. van Egmond and K. Allaart, Nucl. Phys. A **425**, 275 (1984).
T. Otsuka, Nucl. Phys. A **557**, 531c (1993).



Prospective regions for $\mathrm{SU}^{*}_{\pi\nu}(3)$ triaxial structure



Conclusions

In preparation for exotic beam facility...

Have investigated proton-neutron asymmetric collective structure, within framework of IBM-2

Proton-neutron asymmetry

- Suppressed by Majorana interaction
- But could play role for nuclei far from stability

$SU^{*}_{\pi\nu}(3)$ *dynamical symmetry*

- Ideal limit, not likely to be reached
- Illustrates basic characteristics of proton-neutron triaxiality

Full collective analysis of two-fluid system

- Phase diagram
- Nature of phase transitions
- Signatures of asymmetric structure

Bose-Fermi system

- Odd mass or odd-odd nuclei

  bosonic core + unpaired nucleons
- Odd nuclei will play major role in shell structure studies
- Coupling to unpaired nucleon significantly influences collective structure of even-even core (core polarization)
- Interacting boson fermion model (IBFM)