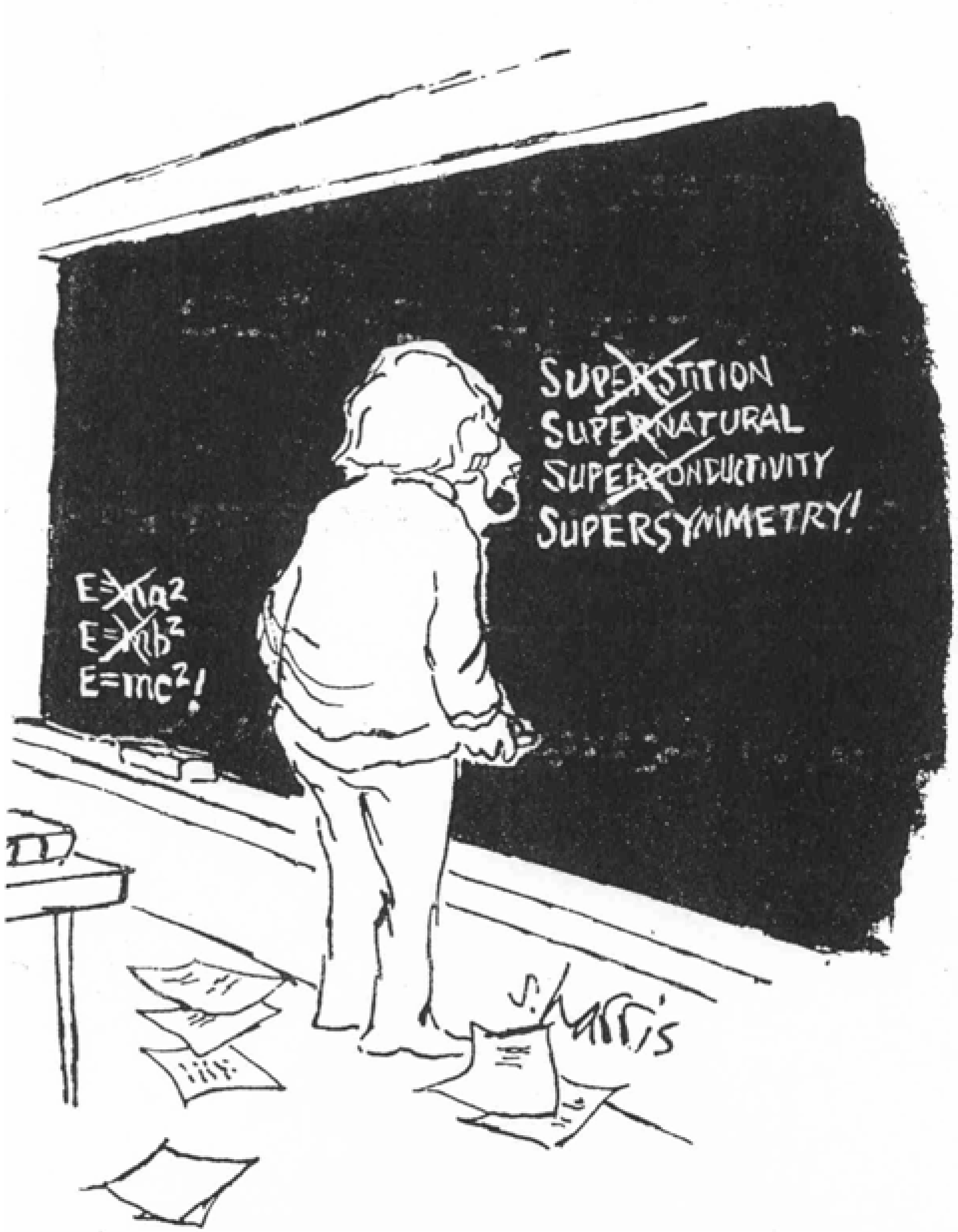


SUPERSTITION
SUPERNATURAL
SUPERCONDUCTIVITY
SUPERSYMMETRY!
E=ma²
E=mb²
E=mc²!
S. Harris

New Developments in Nuclear Supersymmetry

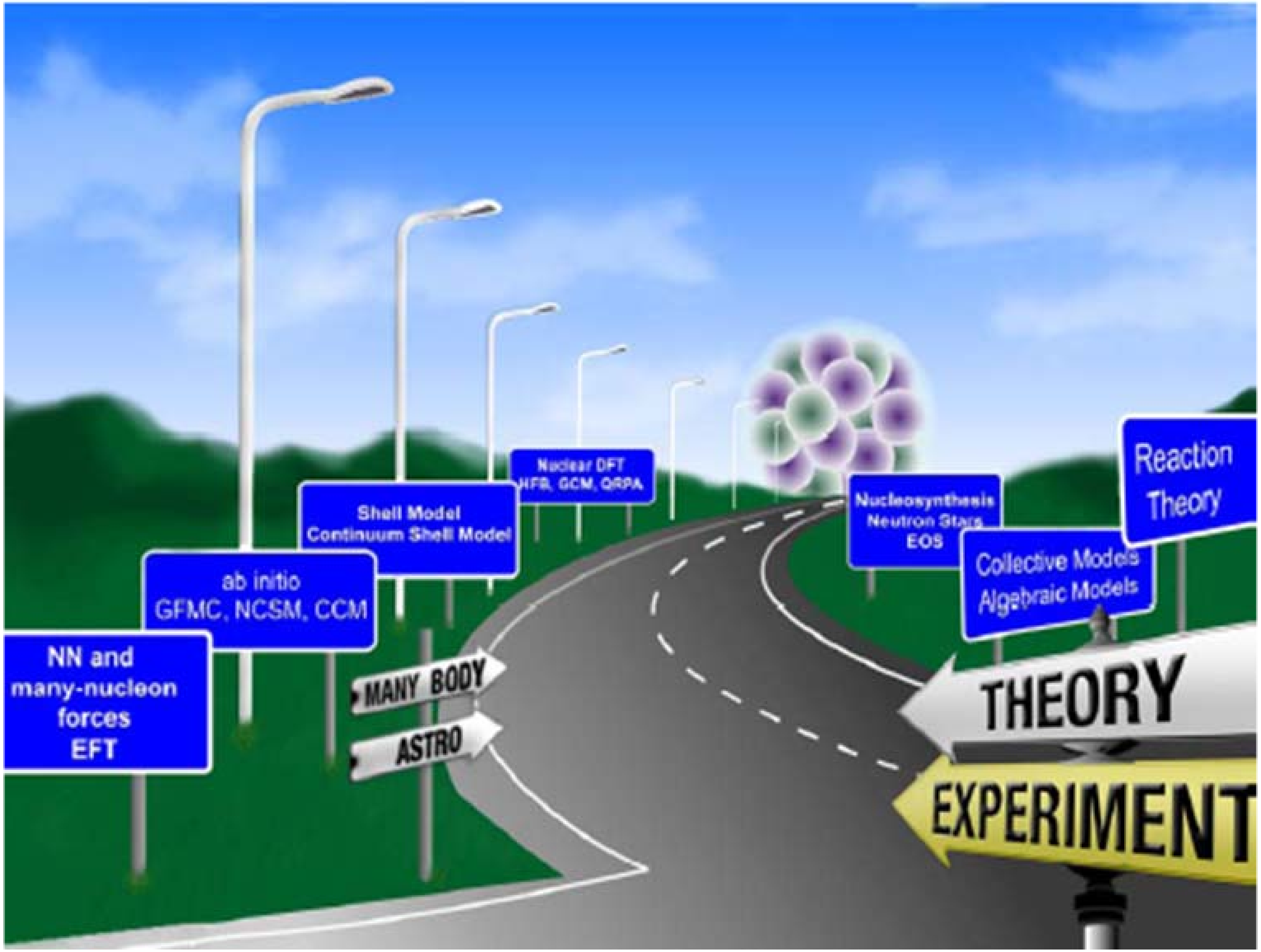
Roelof Bijker
ICN-UNAM
Mexico
bijker@nucleares.unam.mx

José Barea
Alejandro Frank

Jan Jolie (Köln)
Gerhard Graw (München)

The Nuclear Many-Body Problem

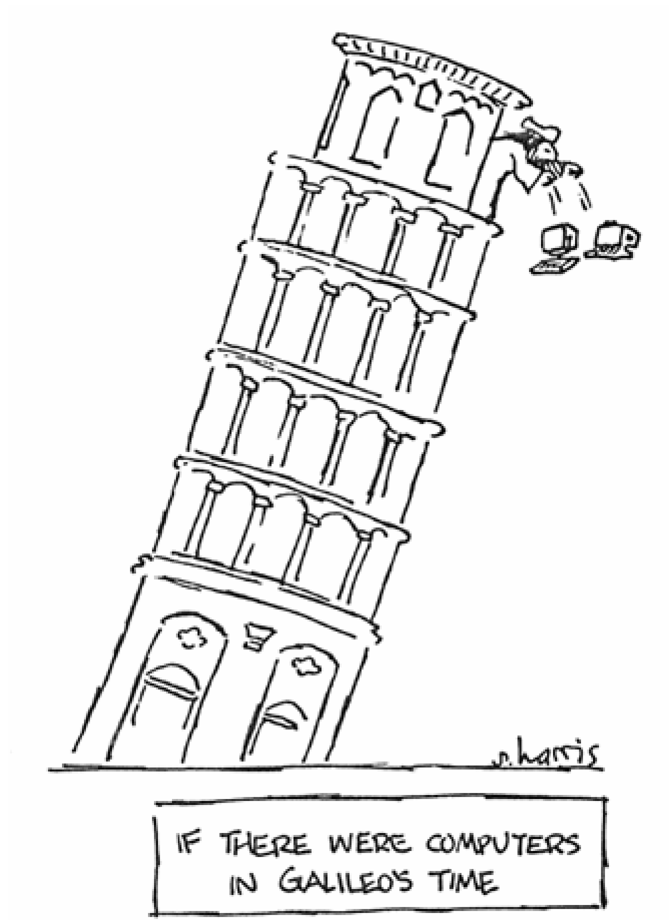
- Ab initio methods: GFMC, NCSM, CCM, …
- Effective field theory
- Shell model: Monte Carlo, continuum SM, …
- Mean-field methods: DFT, QRPA, HFB, GCM, …
- Phenomelogical models of collective motion: IBM and its extensions, …
- Dynamical (super)symmetries
- …

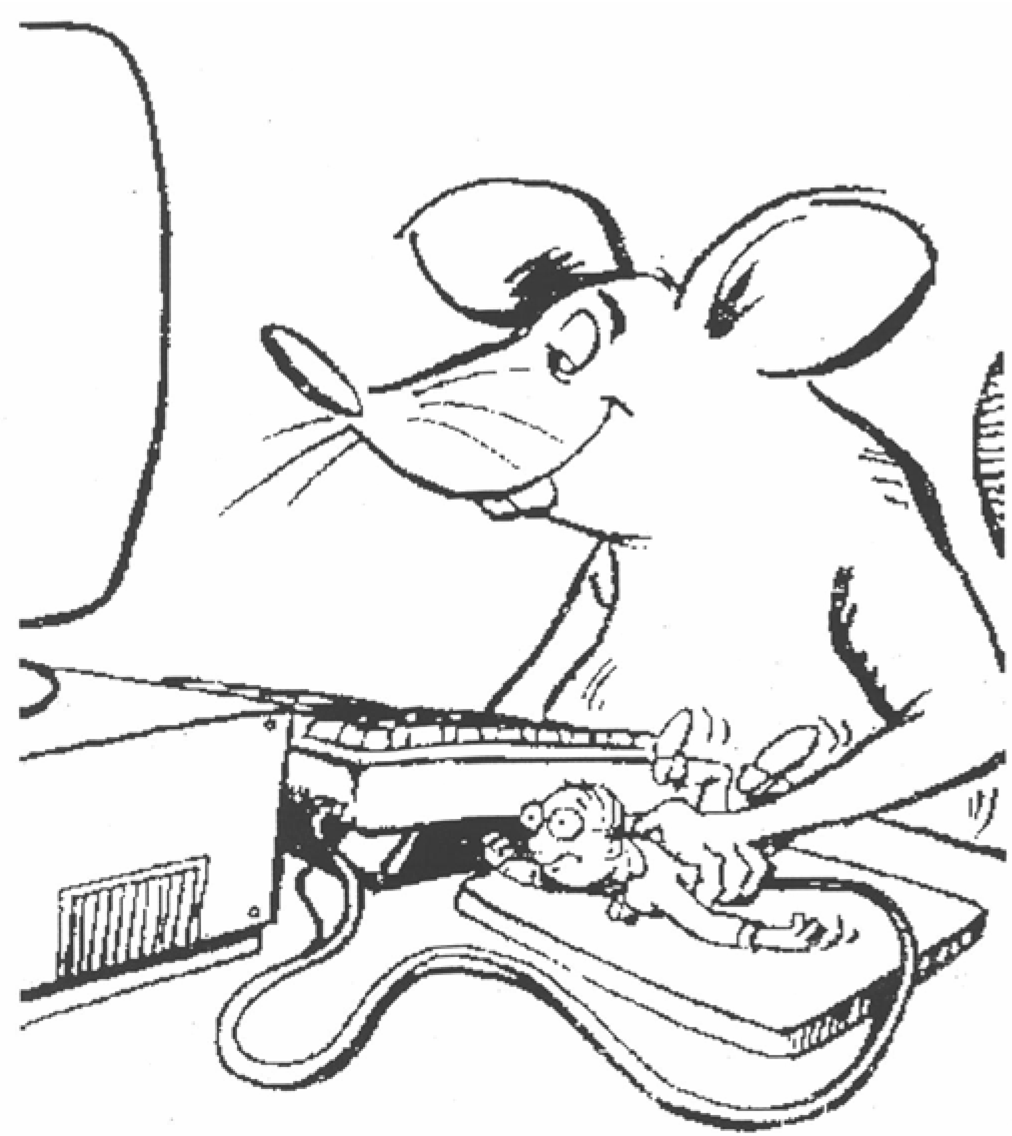


NN and
many-nucleon
forces
EFT
ab initio
GFMC, NCSM, CCM
Shell Model
Continuum Shell Model
Nuclear DFT
HFB, GCM, QRPA
MANY BODY
ASTRO
Nucleosynthesis
Neutron Stars
EOS
Collective Models
Algebraic Models
Reaction
Theory
THEORY
EXPERIMENT

Motivation

- Large scale calculations
  - Ab initio
  - Shell model
  - Mean field
- Symmetry methods
  - IBM and IBFM with isospin
  - Nuclear supersymmetry





I am very happy to learn that the computer understands the problem, but I would like to understand it too

Eugene Wigner

Motivation

- What are the “effective” degrees of freedom?
- Are there “effective” symmetries?
- Symmetries provide benchmarks
- Examples:

  special solutions to the Bohr Hamiltonian,
  dynamical symmetries of the IBM,

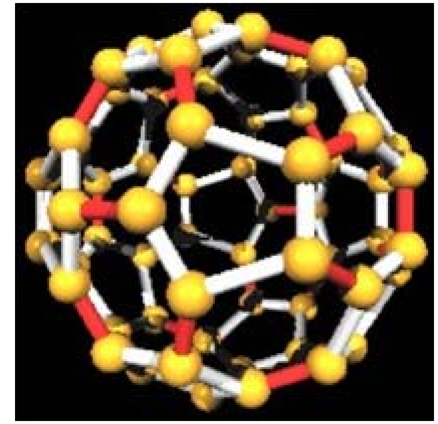
  pseudo-spin symmetries

Smaller and smaller
M.C. Escher

Outline

- Introduction
- Interacting boson models
- Dynamical supersymmetries
- Heavy nuclei: the $A \sim 190$ mass region
- Light nuclei: sd- and pf-shell
- Summary and conclusions

Symmetries



- Geometric symmetries

  Buckyball with icosahedral symmetry
- Permutation symmetries

  Fermi-Dirac and Bose-Einstein statistics
- Space-time symmetries

  Rotational invariance in nonrelativistic QM,
  Lorentz invariance in relativistic QM
- Gauge symmetries

  Dirac equation with external electromagnetic field

Dynamical Symmetries

- Hydrogen atom (Pauli, 1926)
- Isospin symmetry (Heisenberg, 1932)
- Spin-isospin symmetry (Wigner, 1937)
- Pairing, seniority (Racah, 1943)
- Elliott model (Elliott, 1958)
- Flavor symmetry (Gell-Mann, Ne'eman, 1962)
- Interacting boson model (Arima, Iachello, 1974)
- Nuclear supersymmetry (Iachello, 1980)

Interacting Boson Model

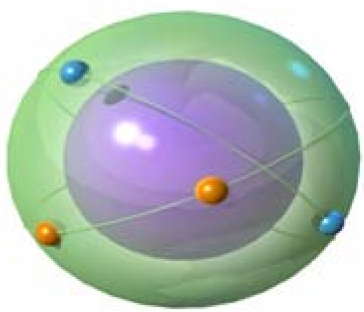
- The IBM describes collective excitations in even-even nuclei in terms of a system of correlated pairs of nucleons with angular momentum L=0 and L=2 which are treated as bosons (s and d bosons) (Arima and Iachello, 1974)
- The number of bosons N is half the number of valence nucleons
- Introduce boson creation and annihilation operators

$$b_i^\dagger, \; b_i, \quad i = l, m \quad (l = 0, 2 \quad -l \leq m \leq l)$$

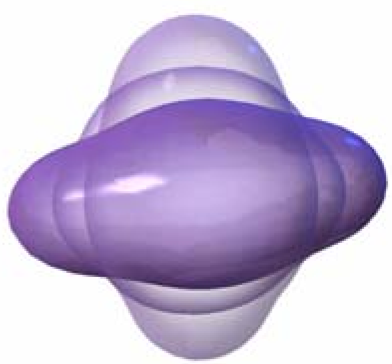
which satisfy the commutation relations

$$\left[b_i, b_j^\dagger\right] = \delta_{ij}, \qquad \left[b_i^\dagger, b_j^\dagger\right] = [b_i, b_j] = 0$$

Shell structure:
valence nucleons



Cooper pairing:
s, d boson system



Collective motion:
nuclear shapes

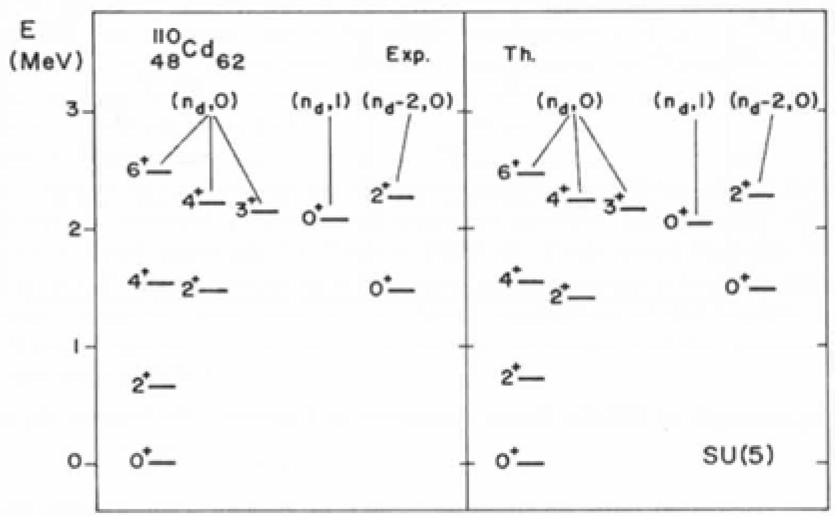
Dynamical Symmetries

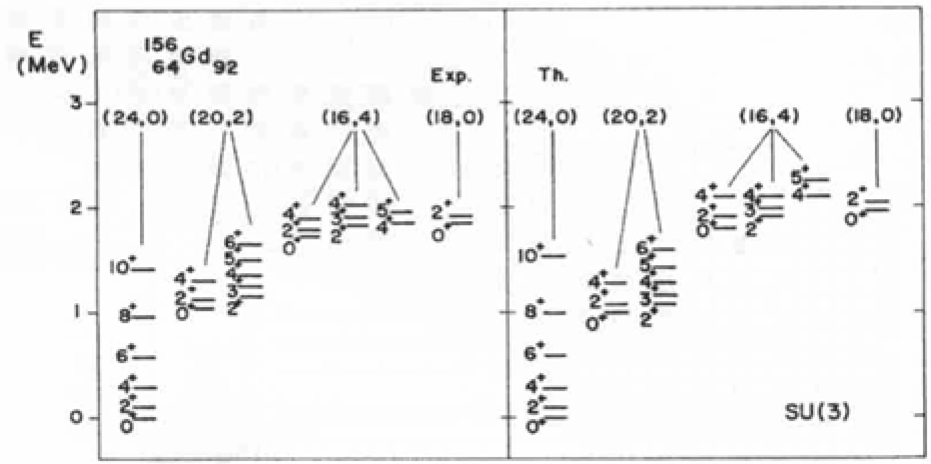
$$U(6) \supset \begin{cases} U(5) \supset SO(5) \supset SO(3) & \text{vibrational} \\ \quad n_d \qquad\quad \tau \qquad\quad L & \\ SU(3) \supset SO(3) & \text{rotational} \\ \quad (\lambda,\mu) \qquad L & \\ SO(6) \supset SO(5) \supset SO(3) & \gamma-\text{unstable} \\ \quad \sigma \qquad\quad \tau \qquad\quad L & \end{cases}$$

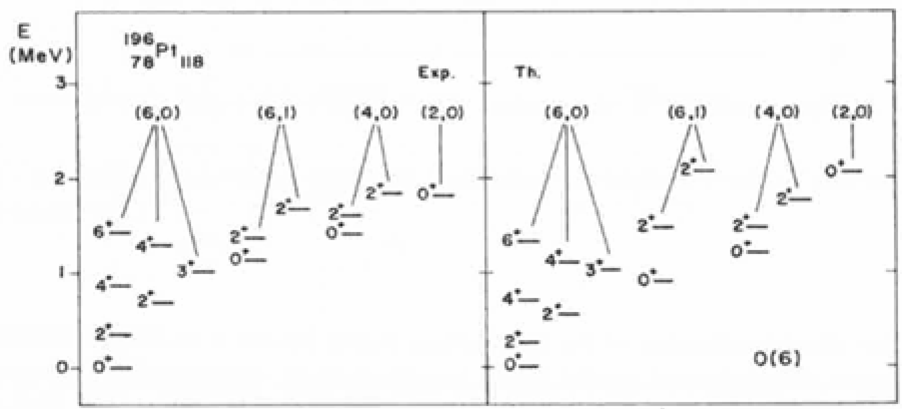
with $U(6)$ labelled by N.

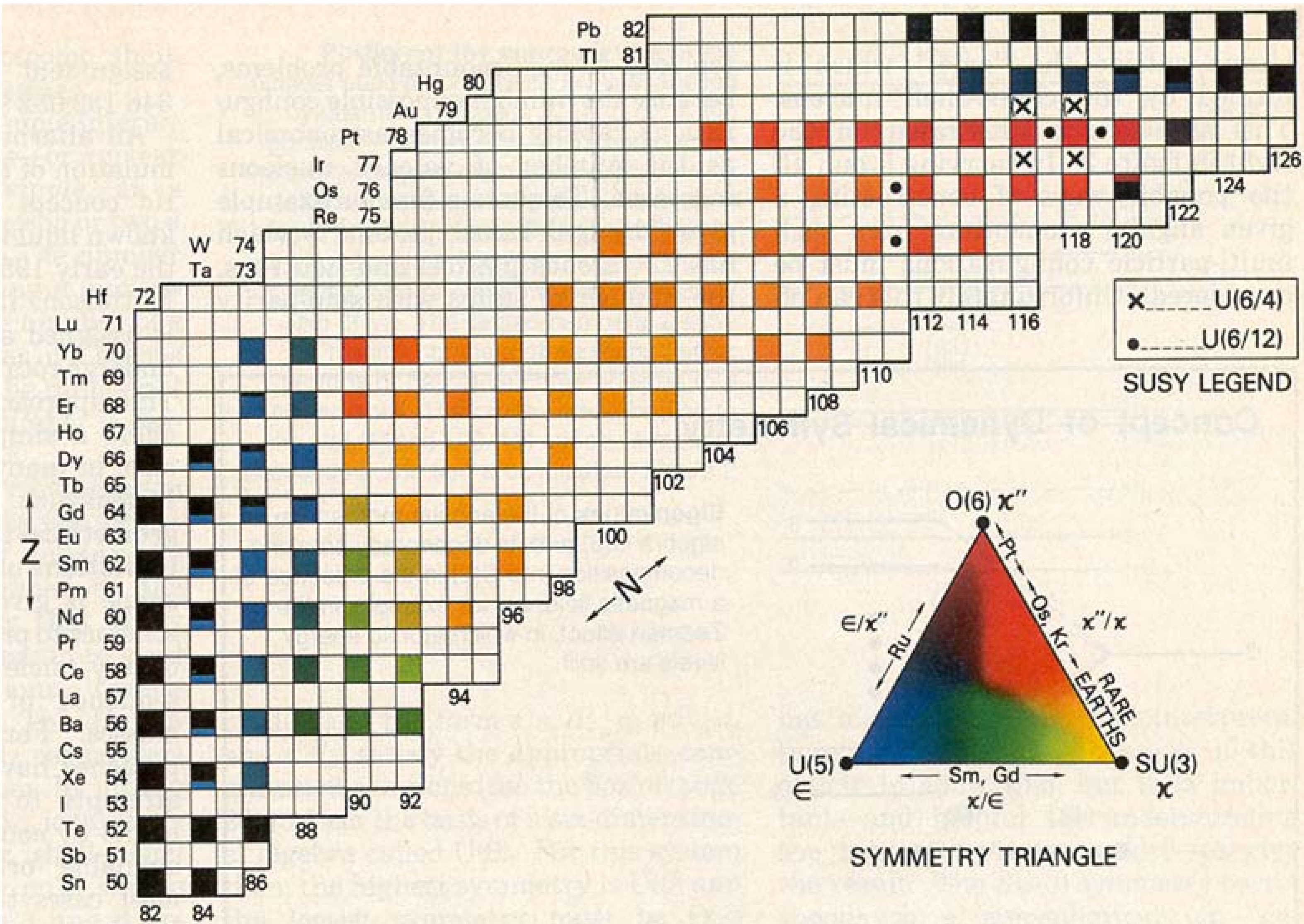
Schematic Hamiltonian :

$$\begin{aligned} H &= \epsilon\,\hat{n}_d - \kappa\,\hat{Q}(\chi)\cdot\hat{Q}(\chi) \\ \hat{n}_d &= \sum_m d_m^\dagger d_m \\ \hat{Q}(\chi) &= (s^\dagger\tilde{d} + d^\dagger\tilde{s} + \chi d^\dagger\tilde{d})^{(2)} \end{aligned}$$


E
(MeV)
$^{110}_{48}Cd_{62}$
Exp.
Th.
$(n_d,0)$
$(n_d,1)$
$(n_d-2,0)$
SU(5)


E
(MeV)
$^{156}_{64}Gd_{92}$
Exp.
Th.
(24,0)
(20,2)
(16,4)
(18,0)
SU(3)


E
(MeV)
$^{196}_{78}Pt_{118}$
Exp.
Th.
(6,0)
(6,1)
(4,0)
(2,0)
O(6)



Pb 82
Tl 81
Hg 80
Au 79
Pt 78
Ir 77
Os 76
Re 75
W 74
Ta 73
Hf 72
Lu 71
Yb 70
Tm 69
Er 68
Ho 67
Dy 66
Tb 65
Gd 64
Eu 63
Sm 62
Pm 61
Nd 60
Pr 59
Ce 58
La 57
Ba 56
Cs 55
Xe 54
I 53
Te 52
Sb 51
Sn 50
Z
N
82
84
86
88
90
92
94
96
98
100
102
104
106
108
110
112
114
116
118
120
122
124
126
× U(6/4)
• U(6/12)
SUSY LEGEND
O(6) ϰ″
Pt
Os, Kr
RARE EARTHS
ϰ″/ϰ
∈/ϰ″
Ru
U(5)
∈
SU(3)
ϰ
Sm, Gd
ϰ/∈
SYMMETRY TRIANGLE

Nuclear Supersymmetry

- Consider an extension of the IBM which includes, in addition to the collective degrees of freedom (bosons), single-particle degrees of freedom of an extra unpaired proton or neutron (fermion with angular momentum $j=j_1, j_2, \ldots$)
- For the extra nucleon, introduce fermion creation and annihilation operators satisfy anticommutation relations

$$\{a_\mu, a_\nu^\dagger\} = \delta_{\mu\nu}, \qquad \{a_\mu^\dagger, a_\nu^\dagger\} = \{a_\mu, a_\nu\} = 0$$

Building Blocks

bosons	$l = 0, 2$	$\sum_l (2l+1) = 6$
fermions	$j = j_1, j_2, \ldots$	$\sum_j (2j+1) = \Omega$

Model	Generators	Invariant	Algebra
IBM	$b_i^\dagger b_j$	N	$U(6)$
IBFM	$b_i^\dagger b_j,\ a_\mu^\dagger a_\nu$	$N,\ M$	$U(6) \otimes U(\Omega)$
SUSY	$b_i^\dagger b_j,\ a_\mu^\dagger a_\nu,\ b_i^\dagger a_\mu,\ a_\mu^\dagger b_i$	$\mathcal{N}$	$U(6/\Omega)$

$N = \sum_i b_i^\dagger b_i$	total number of bosons
$M = \sum_\mu a_\mu^\dagger a_\mu$	total number of fermions
$\mathcal{N} = N + M$	total number of bosons plus fermions

Algebraic Structure

$$
\begin{array}{lllcl}
B_{ij} = b_i^\dagger b_j & \text{boson} & \rightarrow & \text{boson} \\
A_{\mu\nu} = a_\mu^\dagger a_\nu & \text{fermion} & \rightarrow & \text{fermion} \\
F_{i\mu} = b_i^\dagger a_\mu & \text{fermion} & \rightarrow & \text{boson} \\
G_{\mu i} = a_\mu^\dagger b_i & \text{boson} & \rightarrow & \text{fermion}
\end{array}
$$

Supersymmetry: the total number of bosons AND fermions is conserved

$$\mathcal{N} = N + M$$

Hamiltonian

$$H = H_B + H_F + V_{BF}$$

$$H_B = \sum_{ij} \epsilon_{ij} B_{ij} + \sum_{ijkl} u_{ijkl} B_{ij} B_{kl}$$

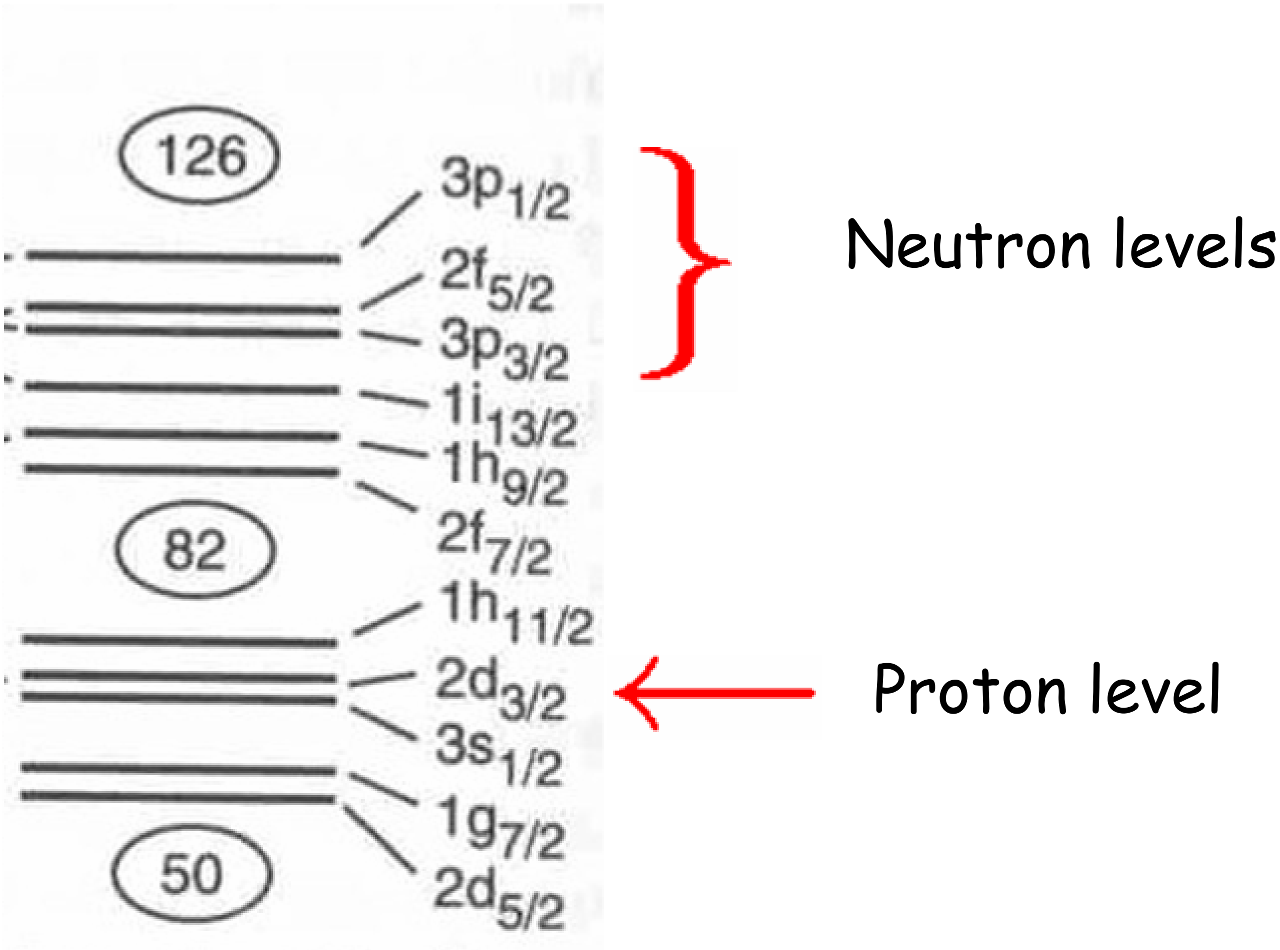
$$H_F = \sum_{\mu\nu} \epsilon_{\mu\nu} A_{\mu\nu} + \sum_{\mu\nu\rho\sigma} v_{\mu\nu\rho\sigma} A_{\mu\nu} A_{\rho\sigma}$$

$$V_{BF} = \sum_{ij\mu\nu} w_{ij\mu\nu} B_{ij} A_{\mu\nu}$$

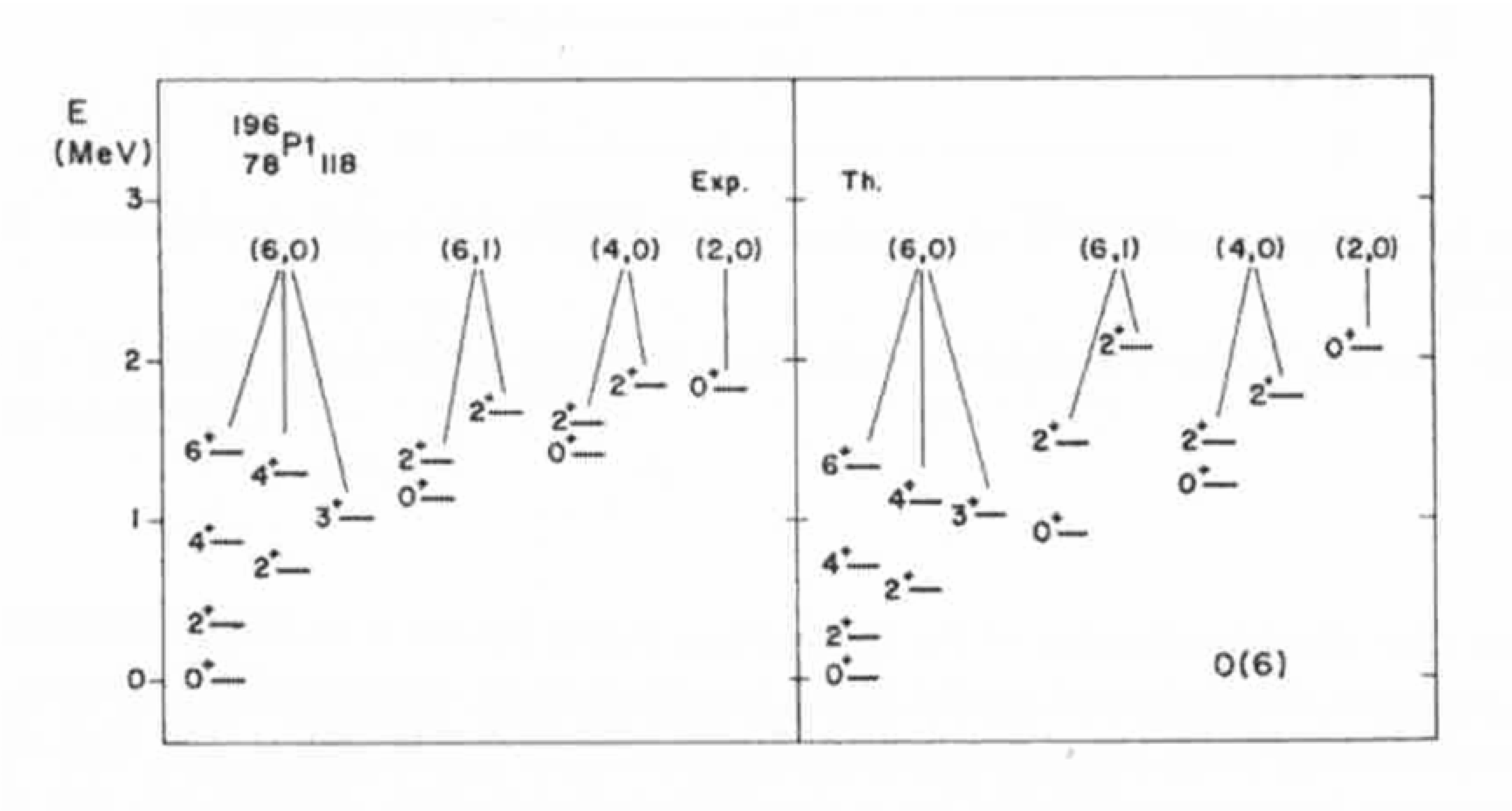
Examples

even-even	s, d	$U_B(6) \supset SO_B(6)$
odd-proton	$j = 2d_{3/2}$	$SU_F(4) \sim SO_B(6)$ spinor
odd-neutron	$j = 3p_{1/2}, 3p_{3/2}, 2f_{5/2}$ $= (\tilde{l} = 0, 2) \otimes (\tilde{s} = \frac{1}{2})$	$U_F(12) \supset U_F(6) \otimes U_F(2)$ $\supset SO_F(6) \otimes U_F(2)$ pseudo-spin

Supersymmetry in Heavy Nuclei

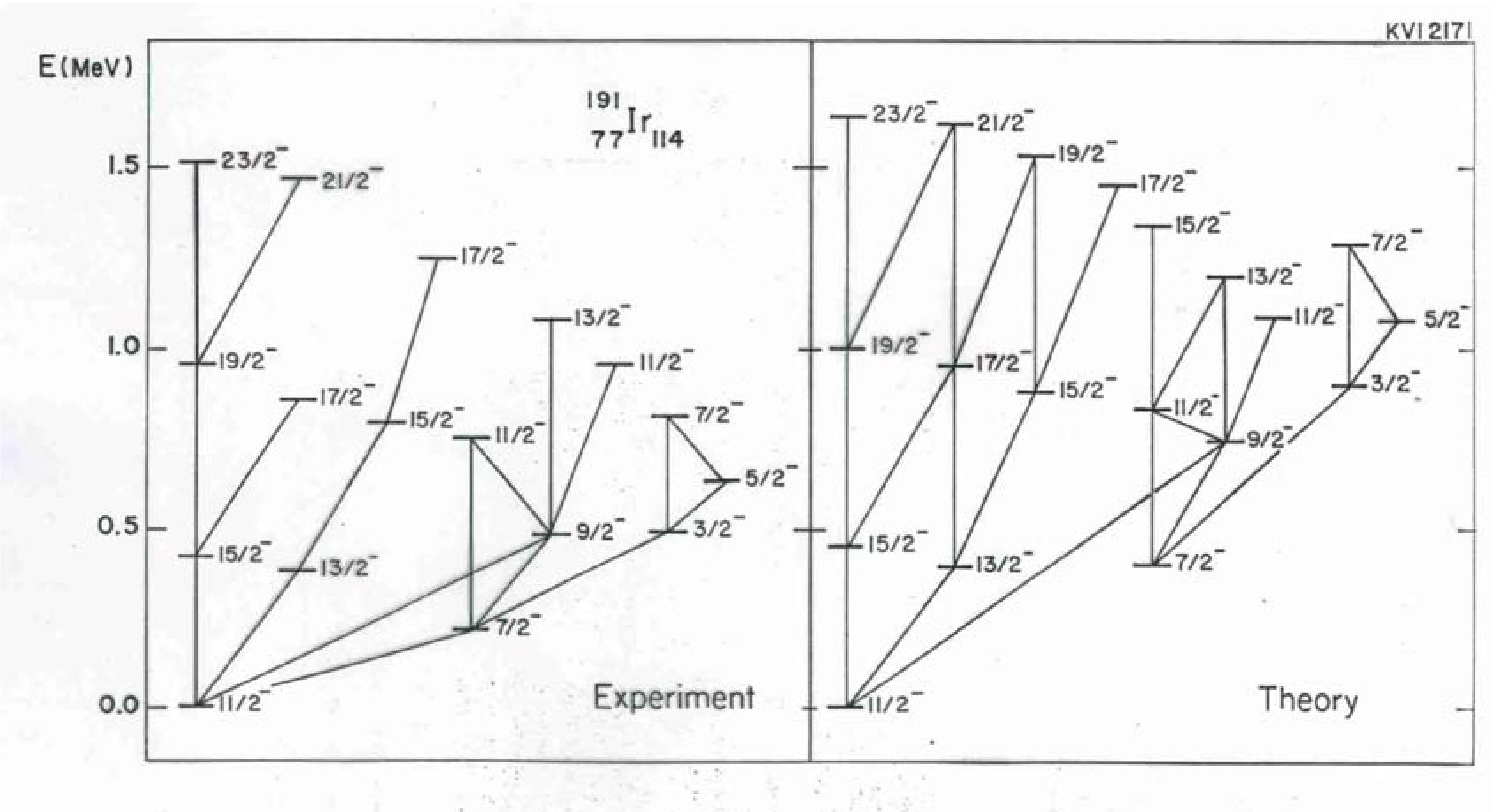


Even-even nucleus



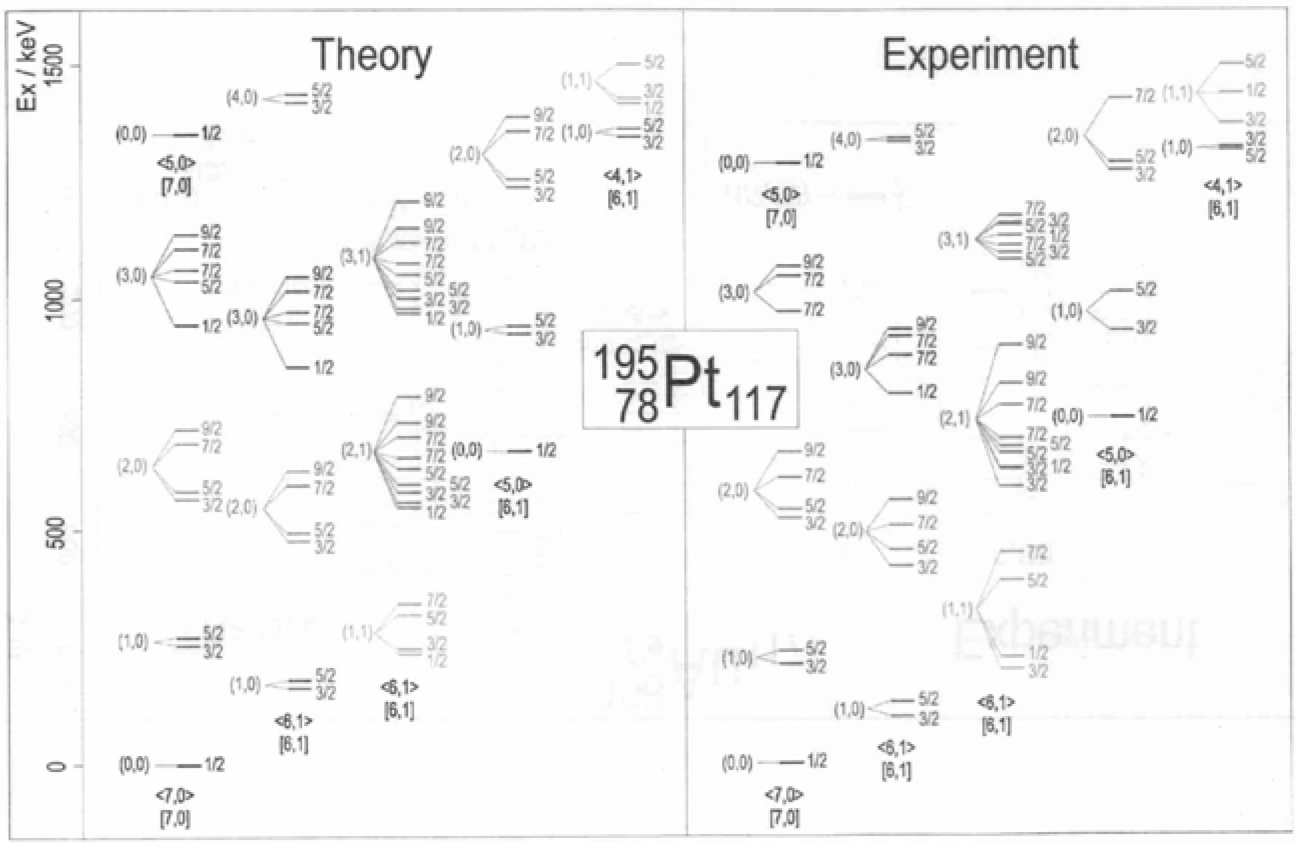
Cizewski et al, PRL 40, 167 (1978)
Arima, Iachello, PRL 40, 385 (1978)

Odd-proton nucleus



Iachello, PRL 44, 772 (1980)

Odd-neutron nucleus



Balantekin, Bars, Bijker, Iachello, PRC 27, 1761 (1983)

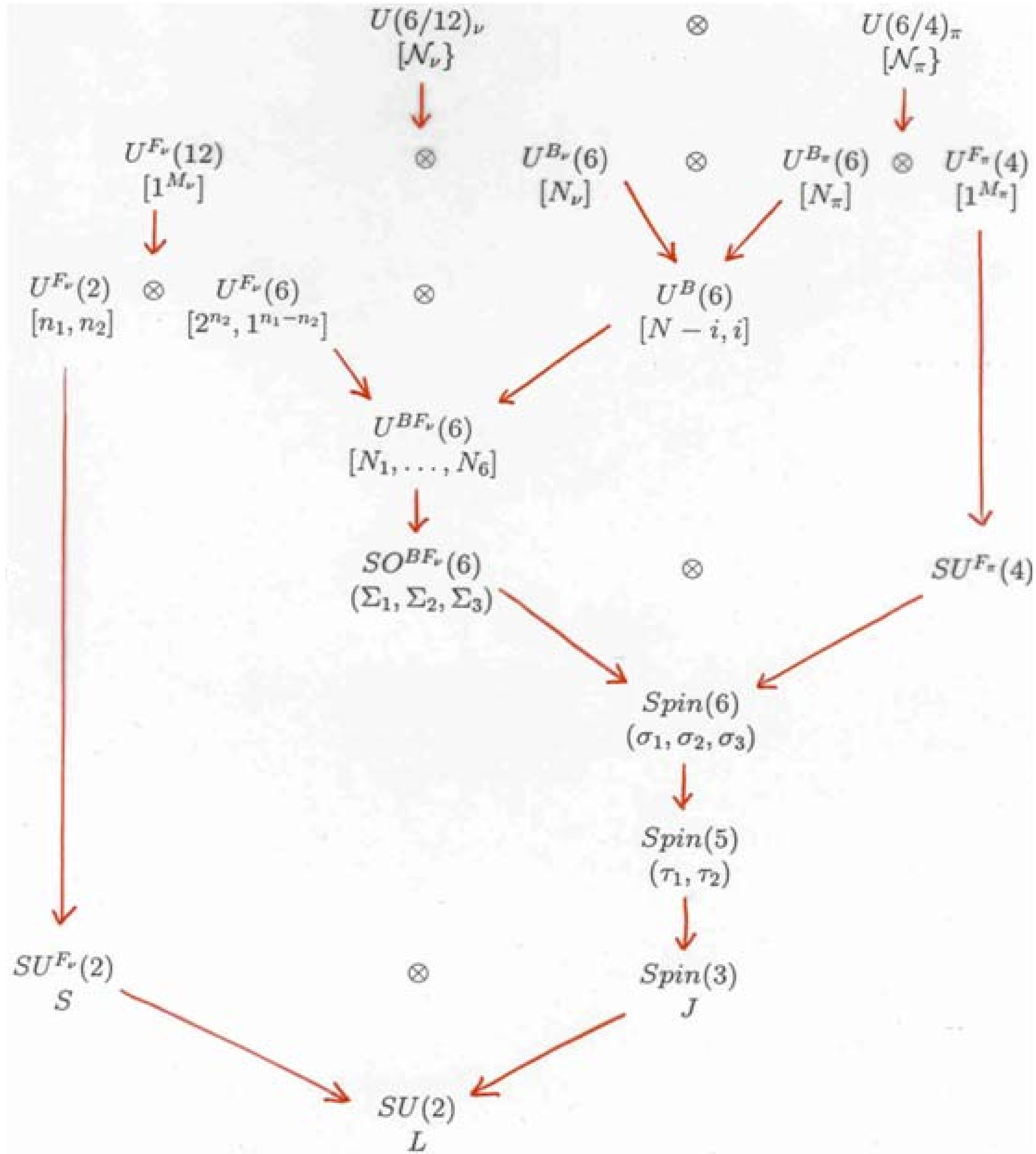
Supersymmetric Quartet of Nuclei

Neutron-proton SUSY : $U(6/12)_\nu \otimes U(6/4)_\pi$

odd-proton		odd-odd
${}^{195}_{79}\mathrm{Au}_{116}$	$\leftrightarrow$	${}^{196}_{79}\mathrm{Au}_{117}$
$\updownarrow$		$\updownarrow$
${}^{194}_{78}\mathrm{Pt}_{116}$	$\leftrightarrow$	${}^{195}_{78}\mathrm{Pt}_{117}$
even-even		odd-neutron

Van Isacker, Jolie, Heyde, Frank, PRL 54, 653 (1985)

Group chain



$U(6/12)_\nu$
$\{\mathcal{N}_\nu\}$
$\otimes$
$U(6/4)_\pi$
$\{\mathcal{N}_\pi\}$
$U^{F_\nu}(12)$
$[1^{M_\nu}]$
$U^{B_\nu}(6)$
$[N_\nu]$
$U^{B_\pi}(6)$
$[N_\pi]$
$U^{F_\pi}(4)$
$[1^{M_\pi}]$
$U^{F_\nu}(2)$
$[n_1, n_2]$
$U^{F_\nu}(6)$
$[2^{n_2}, 1^{n_1-n_2}]$
$U^B(6)$
$[N-i, i]$
$U^{BF_\nu}(6)$
$[N_1, \ldots, N_6]$
$SO^{BF_\nu}(6)$
$(\Sigma_1, \Sigma_2, \Sigma_3)$
$SU^{F_\pi}(4)$
$Spin(6)$
$(\sigma_1, \sigma_2, \sigma_3)$
$Spin(5)$
(τ_1, τ_2)
$SU^{F_\nu}(2)$
S
$Spin(3)$
J
$SU(2)$
L

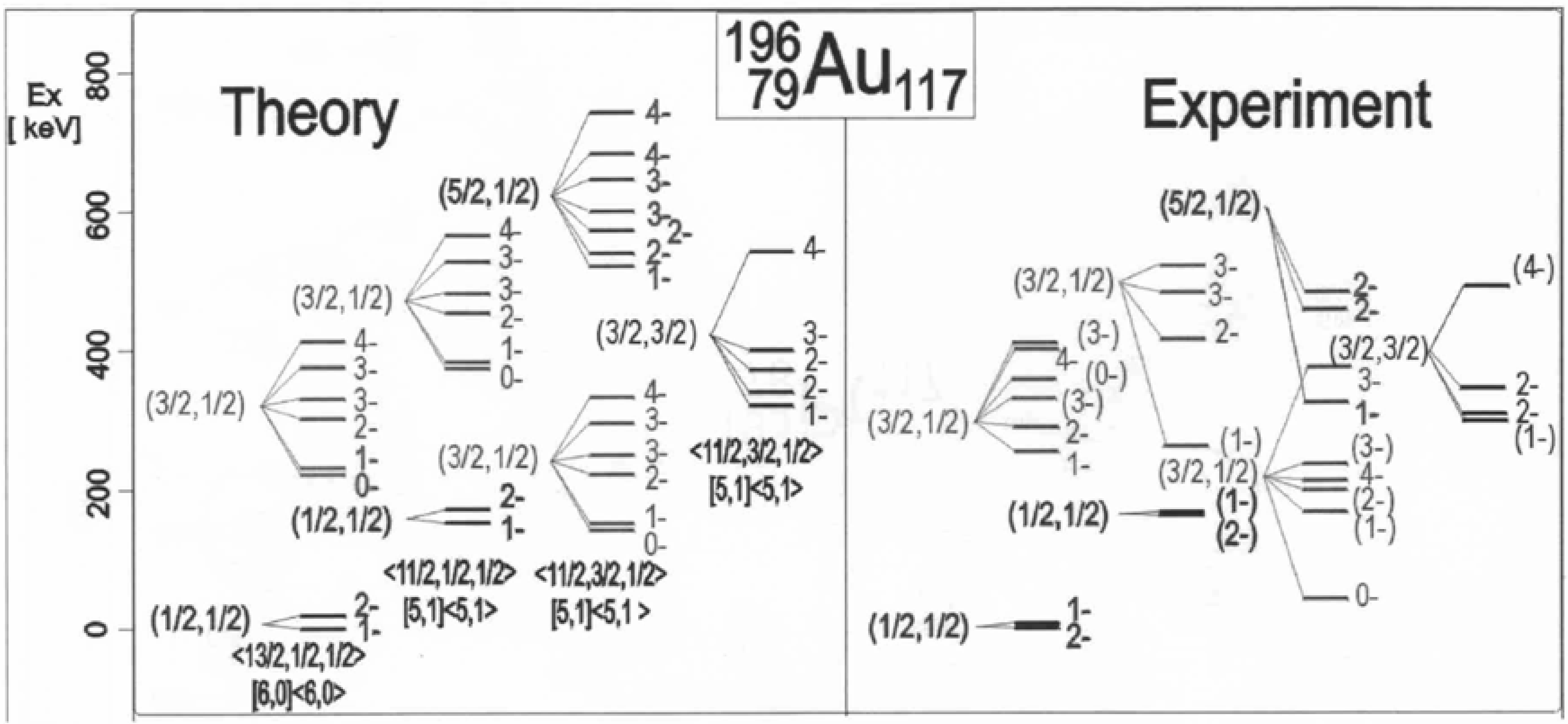
Hamiltonian

$$H = a\,C_{2U^{BF\nu}(6)} + b\,C_{2SO^{BF\nu}(6)} + c\,C_{2Spin(6)} + d\,C_{2Spin(5)} + e\,C_{2Spin(3)} + f\,C_{2SU(2)}$$

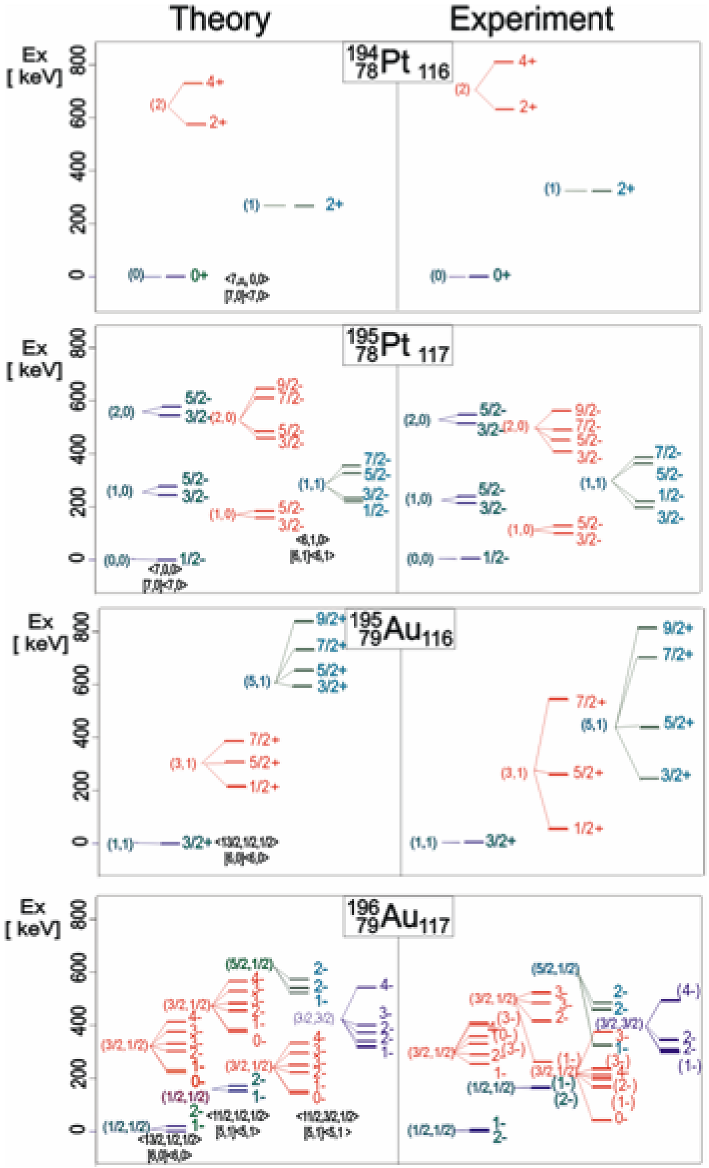
Energies

$$\begin{aligned} E = {} & a\,[N_1(N_1+5) + N_2(N_2+3) + N_3(N_3+1)] \\ & + b\left[\Sigma_1(\Sigma_1+4) + \Sigma_2(\Sigma_2+2) + \Sigma_3^2\right] \\ & + c\left[\sigma_1(\sigma_1+4) + \sigma_2(\sigma_2+2) + \sigma_3^2\right] \\ & + d\,[\tau_1(\tau_1+3) + \tau_2(\tau_2+1)] \\ & + e\,J(J+1) + f\,L(L+1) \end{aligned}$$

Odd-odd nucleus



Metz et al, PRL 83, 1542 (1999)


Theory
Experiment
Ex [keV]
$^{194}_{78}Pt_{116}$
$^{195}_{78}Pt_{117}$
$^{195}_{79}Au_{116}$
$^{196}_{79}Au_{117}$

One-proton transfer

Test of the fermionic generators of the superalgebra !

$$P^{(\frac{3}{2})\dagger}_{\pi,1} = -\sqrt{\frac{1}{6}}\left(\tilde{s}_\pi \times a^\dagger_{\pi,\frac{3}{2}}\right)^{(\frac{3}{2})} + \sqrt{\frac{5}{6}}\left(\tilde{d}_\pi \times a^\dagger_{\pi,\frac{3}{2}}\right)^{(\frac{3}{2})}$$

$$P^{(\frac{3}{2})\dagger}_{\pi,2} = +\sqrt{\frac{5}{6}}\left(\tilde{s}_\pi \times a^\dagger_{\pi,\frac{3}{2}}\right)^{(\frac{3}{2})} + \sqrt{\frac{1}{6}}\left(\tilde{d}_\pi \times a^\dagger_{\pi,\frac{3}{2}}\right)^{(\frac{3}{2})}$$

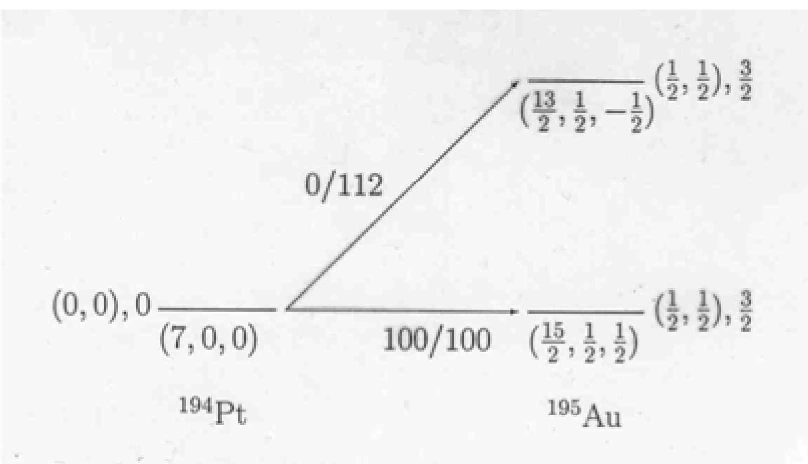
Barea, Bijker, Frank, JPA 37, 10251 (2004)

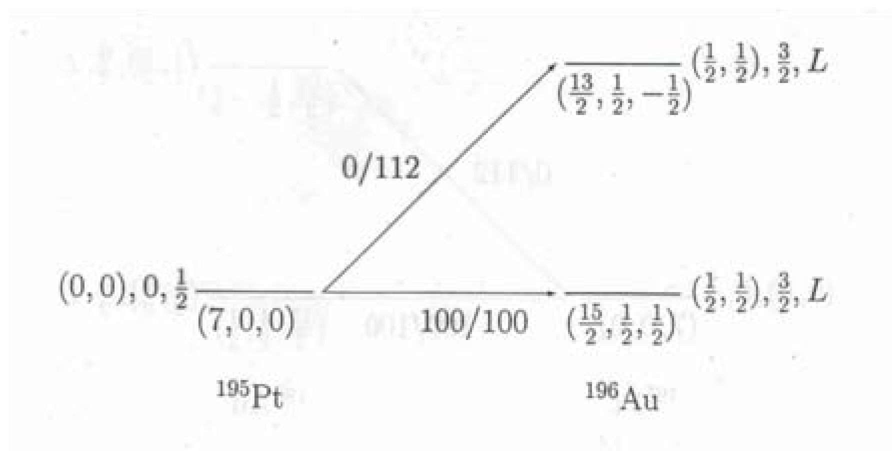
^{194}Pt $\rightarrow$ ^{195}Au	$\mid \langle f \| P_{\pi,1}^{(\frac{3}{2})\dagger} \| i \rangle \mid^2$	$\mid \langle f \| P_{\pi,2}^{(\frac{3}{2})\dagger} \| i \rangle \mid^2$
$\langle ((N+\frac{3}{2},\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2} \mid$	$\frac{2(N_\pi+1)}{3}$	$\frac{8(N+6)^2(N_\pi+1)}{15(N+3)^2}$
$\langle ((N+\frac{1}{2},\frac{1}{2},-\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2} \mid$	0	$\frac{6(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}$
^{195}Pt $\rightarrow$ ^{196}Au	$\mid \langle f \| P_{\pi,1}^{(\frac{3}{2})\dagger} \| i \rangle \mid^2$	$\mid \langle f \| P_{\pi,2}^{(\frac{3}{2})\dagger} \| i \rangle \mid^2$
$\langle ((N+\frac{3}{2},\frac{1}{2},\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2},L \mid$	$\frac{2(N_\pi+1)}{3}\frac{2L+1}{4}$	$\frac{8(N+6)^2(N_\pi+1)}{15(N+3)^2}\frac{2L+1}{4}$
$\langle ((N+\frac{1}{2},\frac{1}{2},-\frac{1}{2}),(\frac{1}{2},\frac{1}{2}),\frac{3}{2},L \mid$	0	$\frac{6(N+1)(N+5)(N_\pi+1)}{5(N+3)^2}\frac{2L+1}{4}$

Correlations

$$R_1 = \frac{I_{gs \to exc}}{I_{gs \to gs}} = 0$$

$$R_2 = \frac{I_{gs \to exc}}{I_{gs \to gs}} = \frac{9(N+1)(N+5)}{4(N+6)^2}$$





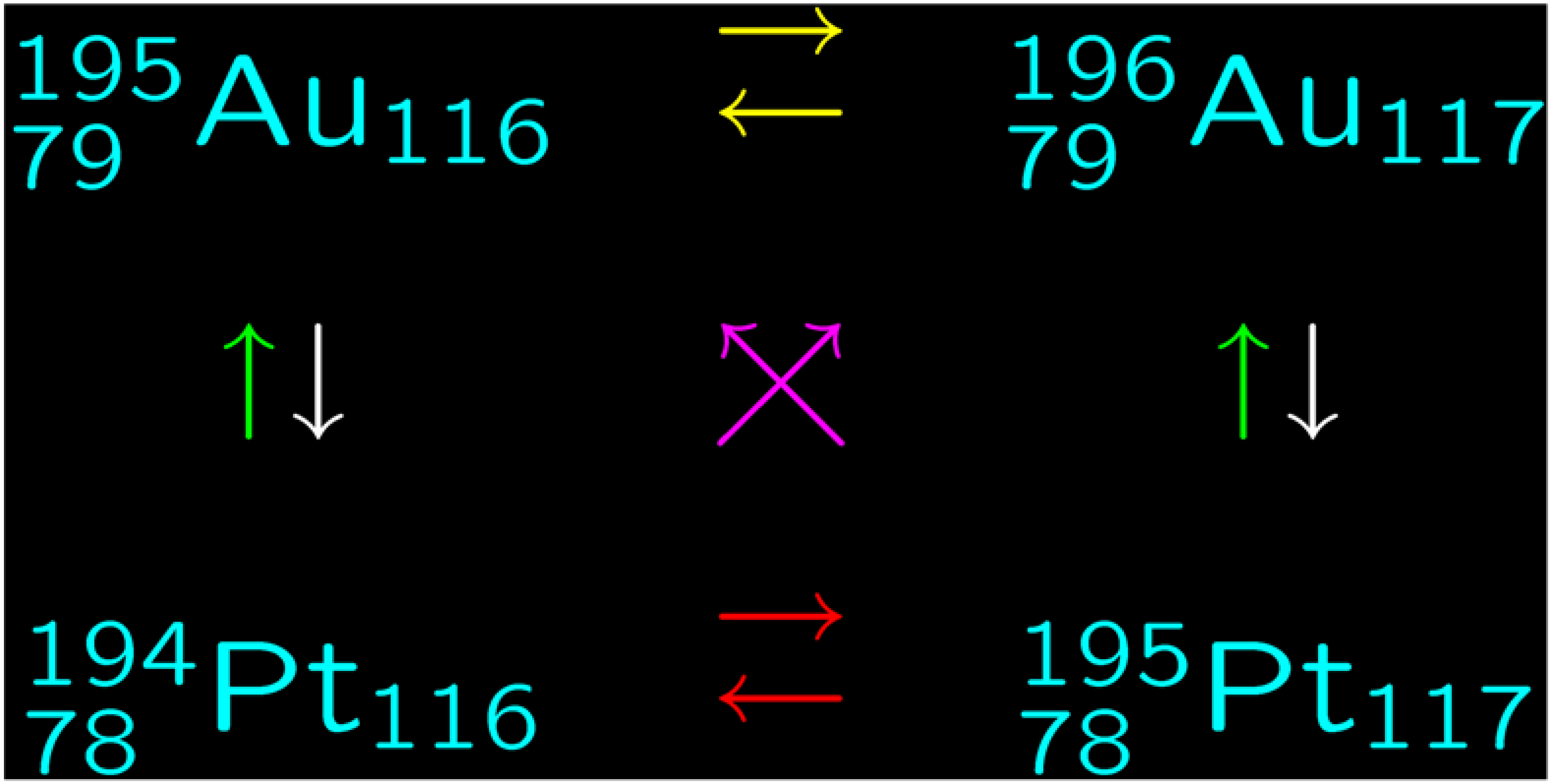
Correlations

- One-proton transfer reactions

$$\frac{S_i(^{195}\mathrm{Pt} \rightarrow ^{196}\mathrm{Au})}{S_i(^{194}\mathrm{Pt} \rightarrow ^{195}\mathrm{Au})} = \frac{2L+1}{8}$$

- One-neutron transfer reactions

$$\frac{S_i(^{195}\mathrm{Pt} \rightarrow ^{194}\mathrm{Pt})}{S_i(^{194}\mathrm{Pt} \rightarrow ^{195}\mathrm{Pt})} = \begin{cases} \frac{1}{2} \\ \\ \frac{N_\pi+1}{2(N+1)(N_\nu+1)} \end{cases}$$



$^{195}_{79}Au_{116}$
$^{196}_{79}Au_{117}$
$^{194}_{78}Pt_{116}$
$^{195}_{78}Pt_{117}$

Two-nucleon transfer

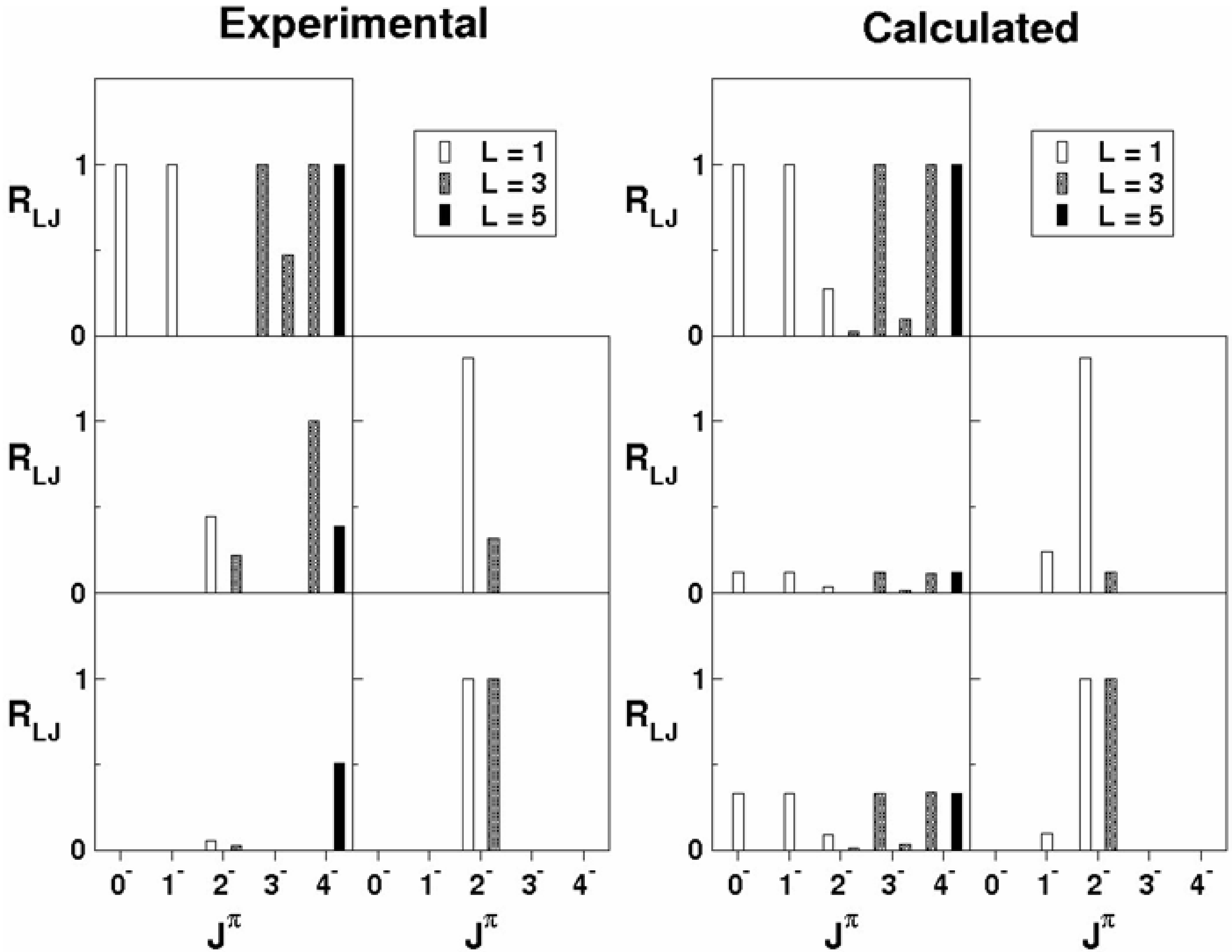
Reaction $^{198}\mathrm{Hg}\,(\vec{d},\alpha)\,^{196}\mathrm{Au}$

Spectroscopic factors

$$G_{LJ} = \left| \sum_{j_\nu j_\pi} g^{LJ}_{j_\nu j_\pi} \left\langle {}^{196}\mathrm{Au} \right\| (a^\dagger_{j_\nu} a^\dagger_{j_\pi})^{(\lambda)} \left\| {}^{198}\mathrm{Hg} \right\rangle \right|^2$$

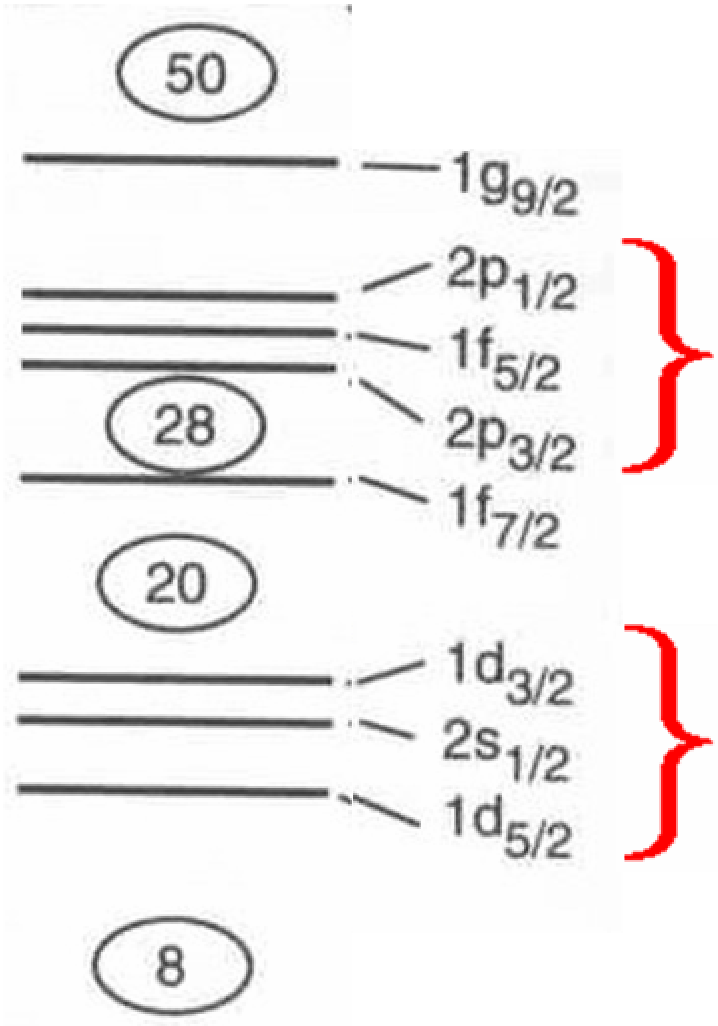
Relative strength

$$R_{LJ} = \frac{G_{LJ}}{G_{LJ}(\mathrm{ref})} = \begin{cases} \dfrac{N+4}{15N} & = \; 0.12 \\[2ex] \dfrac{2(N+4)(N+6)}{15N(N+3)} & = \; 0.33 \end{cases}$$



Barea, Bijker, Frank, PRL 94, 152501 (2005)

Supersymmetry in Light Nuclei



Pseudo sd-shell

Pseudo-SU(4) symmetry

Van Isacker et al,

PRL 82, 2060 (1999)

sd-shell

Wigner SU(4) symmetry

Interacting Boson Models

- Heavy nuclei: protons and neutrons in different major shells

IBM-1	$s^\dagger, d^\dagger$	$U(6)$
IBM-2	$(s_\nu^\dagger, d_\nu^\dagger) \otimes (s_\pi^\dagger, d_\pi^\dagger)$ $T = 1, M_T = \pm 1$	$U_\nu(6) \otimes U_\pi(6)$

- Light nuclei: protons and neutrons occupy same major shells $\Rightarrow$ isospin invariant IBM

IBM-3	$s^\dagger, d^\dagger$ $(S, T) = (0, 1)$	$U(18)$
IBM-4	$s^\dagger, d^\dagger$ $(S, T) = (0, 1), (1, 0)$	$U(36)$

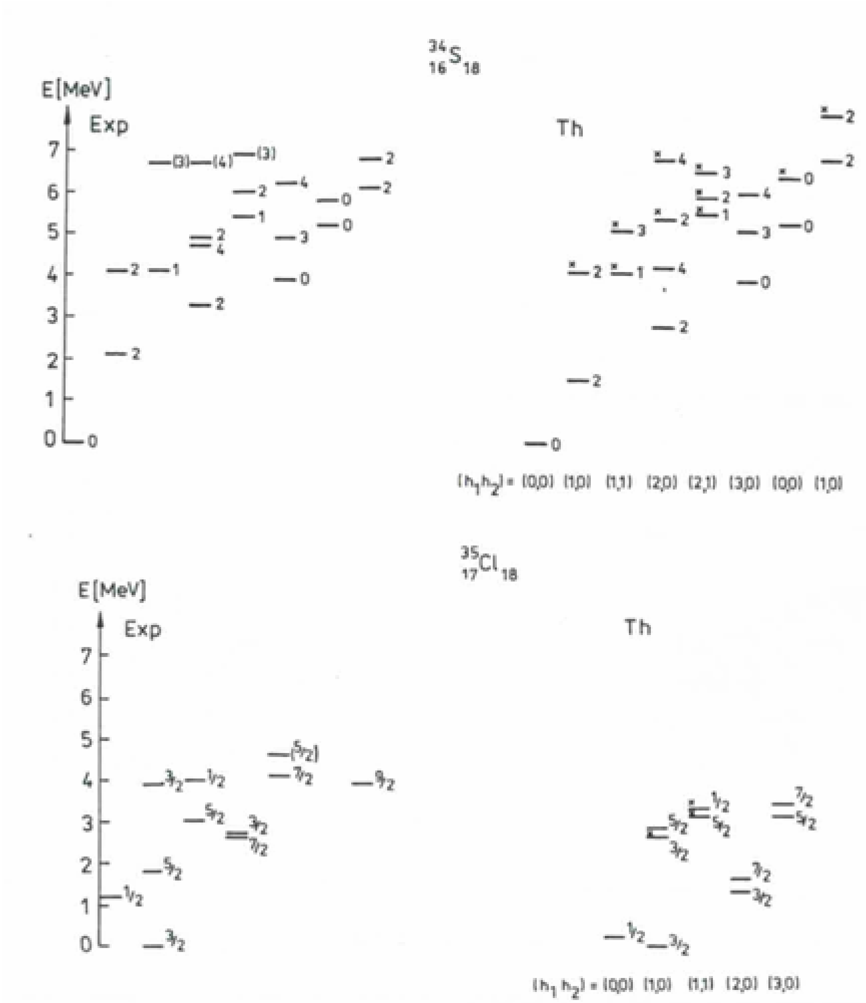
Elliott, White, PLB 97, 169 (1980)
Elliott, Evans, PLB 101, 216 (1981)

Dynamical Supersymmetry

$$\begin{array}{llll}
\text{bosons}: & U^B(36) & \supset & U_L^B(6) \otimes SU_{ST}^B(6) \\
 & & \supset & U_L^B(6) \otimes SU_{ST}^B(4) \\
 & & & \quad\wr \qquad\qquad\quad \wr \\
\text{fermions}: & U^F(24) & \supset & U_L^F(6) \otimes SU_{ST}^F(4)
\end{array}$$

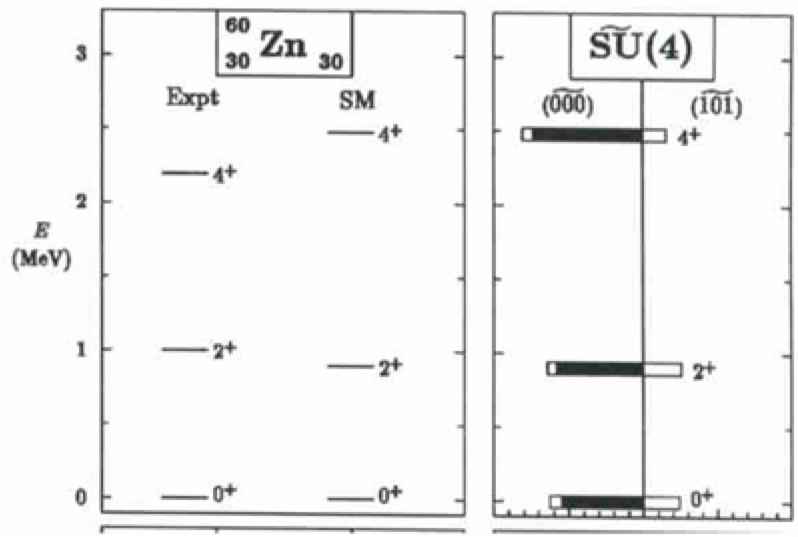
$$\begin{array}{lll}
U(36/24) & \supset & U^B(36) \otimes U^F(24) \\
 & \supset & U_L^B(6) \otimes SU_{ST}^B(4) \otimes U_L^F(6) \otimes SU_{ST}^F(4) \\
 & \supset & U_L^{BF}(6) \otimes SU_{ST}^{BF}(4) \\
 & \supset & \cdots
\end{array}$$

Example in the sd-shell

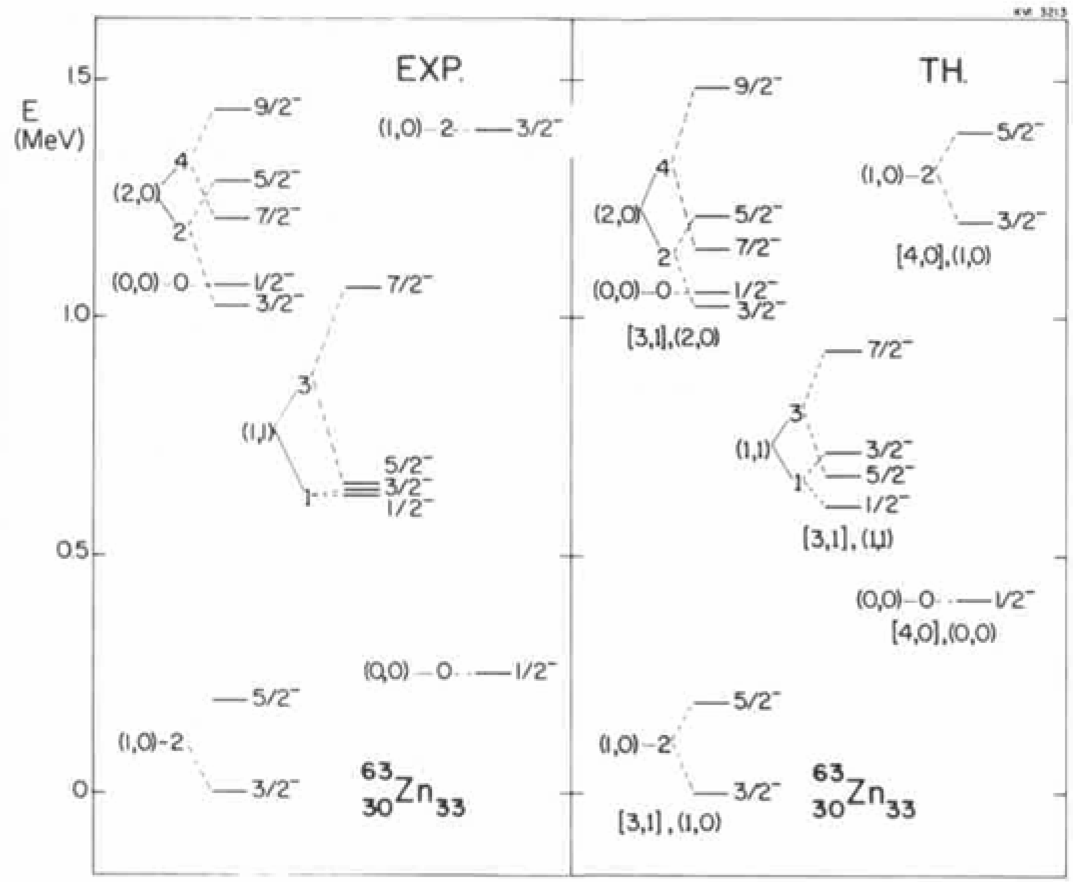


Szpikowski et al,
NPA 487, 301 (1988)

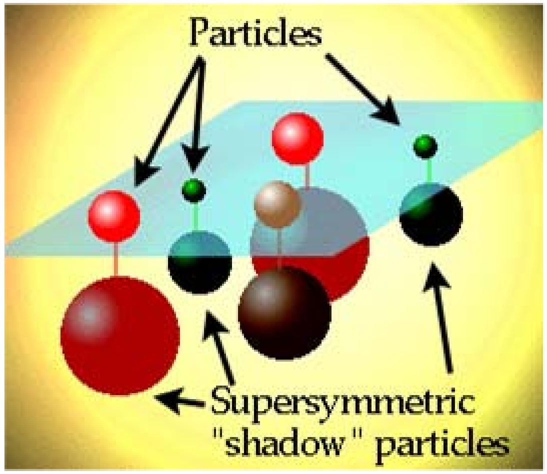
Example in the pf-shell

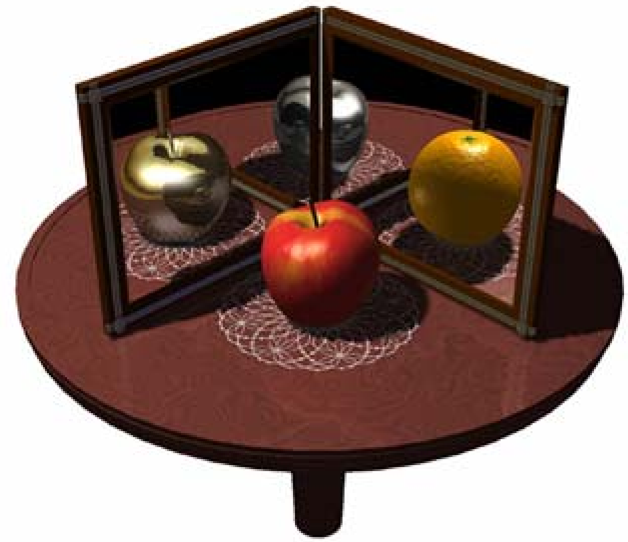


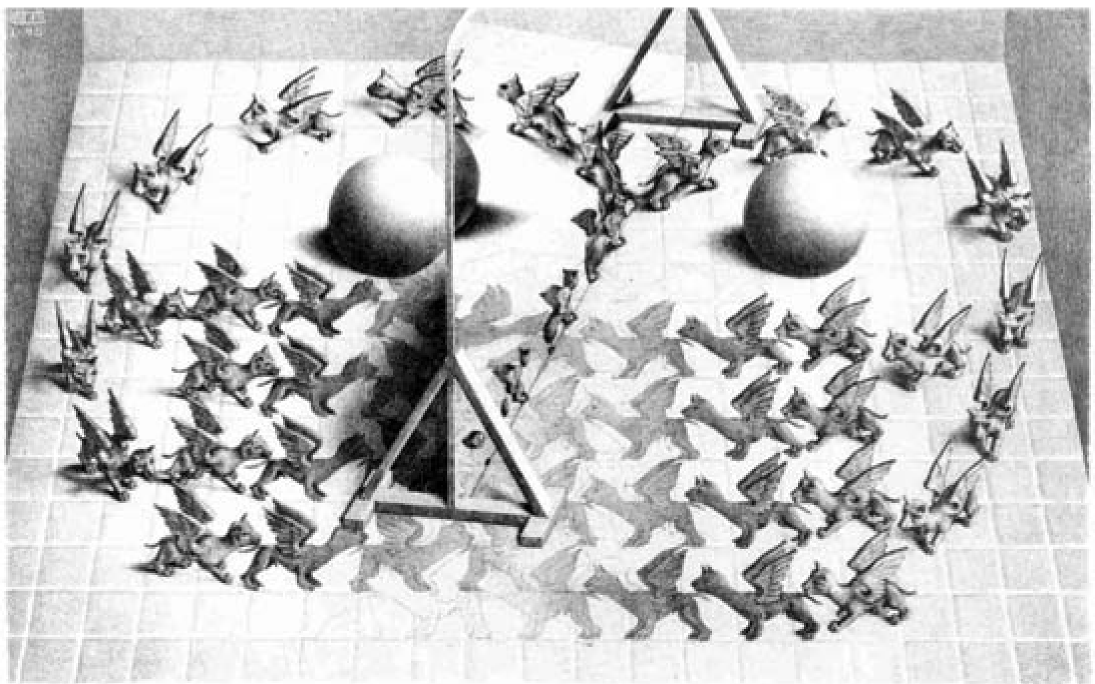
Van Isacker et al,
PRL 82, 2060 (1999)



R. Bijker,
Ph.D. Thesis, 1984







Magic mirror
M.C. Escher

Summary and Conclusions

- Nuclear supersymmetry: energy formulas, selection rules, transition rates, etc.
- Supersymmetry leads to correlations between different transfer reactions
- Applications in both heavy and light nuclei
- Proton-rich nuclei: dynamical (super)symmetries of isospin invariant IBM and IBFM?
- Neutron-rich nuclei: are there additional degrees of freedom (valence protons, valence neutrons, skins), what are the corresponding symmetries?
- SUSY without dynamical symmetry
- Predictability