COULOMB CORRECTIONS IN ELECTRON SCATTERING IN HEAVY NUCLEI

J.M.U., Universidad Complutense de Madrid (UCM)

Collaborators:

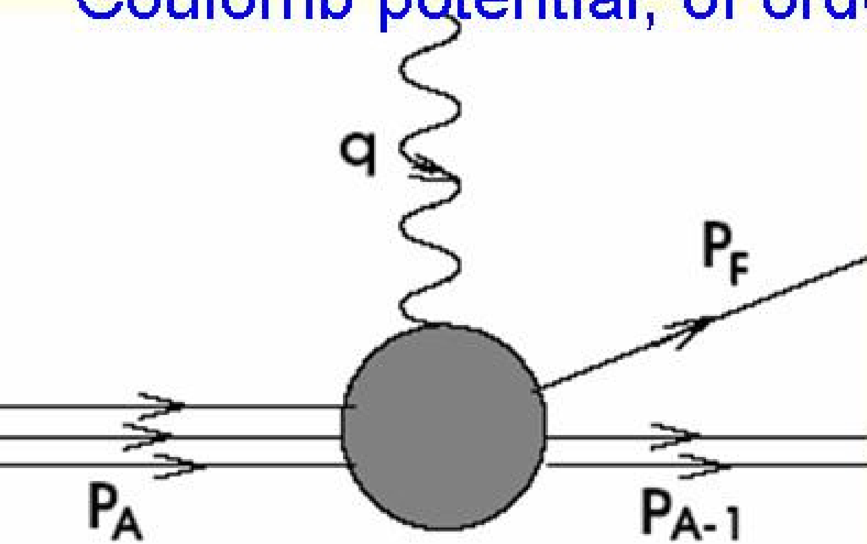
César Fernández Ramírez (UCM-CSIC)

Javier Rodríguez Vignote (UCM)

Joaquín López Herráiz (UCM)

OVERVIEW OF THE MODEL (ingredients)

- Electron-Nucleus Interactions are usually kept at the simplest possible level: one photon exchange approximation (OPE)
- Not a bad approximation as α' 1/137 is small
- However, incoming and outgoing electrons see the static Coulomb potential, of order $Z\alpha$



General expression within one photon exchange (Born Approximation)

$$\frac{d^4\sigma}{d\epsilon_f d\Omega_f dE_F d\Omega_F} = \frac{\delta(\epsilon_i + E_A - \epsilon_f - E_F - E_{A-1})}{(2\pi)^5} \times 4\alpha^2 \epsilon_f^2 E_F |\mathbf{P}_F| \overline{\sum} |W_{if}|^2 ,$$

$$W_{if} = \int d\mathbf{x} \int d\mathbf{y} \int \frac{d\mathbf{q}}{(2\pi)^2} j_\mu^e(\mathbf{x}) e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \frac{(-1)}{q_\mu^2} J_N^\mu(\mathbf{y})$$

One-photon exchange approximation yields, if Coulomb interraction is neglected (PWBA for the electrons):

$$j^{\mu}_{\mathrm{e,free}}(\mathbf{r}) = \frac{m}{\sqrt{\epsilon_i \epsilon_f}} e^{i(\mathbf{k}_i-\mathbf{k}_f)\cdot\mathbf{r}} \bar{u}(\mathbf{k}_f,\sigma_f)\gamma^{\mu} u(\mathbf{k}_i,\sigma_i)$$

$$\begin{aligned}\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F} = \frac{E_F p_F}{(2\pi)^3}\sigma_M f_{rec}\frac{1}{2}\Big\{ & v_L\left(R^L + R^L_n \widehat{S}_n\right) + v_T\left(R^T + R^T_n \widehat{S}_n\right) \\ & + v_{TL}\left[\left(R^{TL} + R^{TL}_n \widehat{S}_n\right)\cos\phi_F + \left(R^{TL}_l \widehat{S}_l + R^{TL}_s \widehat{S}_s\right)\sin\phi_F\right] \\ & + v_{TT}\left[\left(R^{TT} + R^{TT}_n \widehat{S}_n\right)\cos 2\phi_F + \left(R^{TT}_l \widehat{S}_l + R^{TT}_s \widehat{S}_s\right)\sin 2\phi_F\right] \\ & + h\Big\{v_{TL'}\left[\left(R^{TL'}_l \widehat{S}_l + R^{TL'}_s \widehat{S}_s\right)\cos\phi_F + \left(R^{TL'} + R^{TL'}_n \widehat{S}_n\right)\sin\phi_F\right] \\ & + v_{T'}\left[R^{T'}_l \widehat{S}_l + R^{T'}_s \widehat{S}_s\right]\Big\}\Big\},\end{aligned}$$

R's proportional to $W^{\mu\nu}$:

$$W^{\mu\nu} = \frac{1}{2j_b+1}\sum_{\mu_b} J^{\mu *}(\omega,\mathbf{q})J^{\nu}(\omega,\mathbf{q}).$$

If Coulomb effects of the electron wave functions are considered (DWBA), or beyond first order Born Approximation

$$j_e^{\mu}(\mathbf{r}) = \bar{\psi}_f^e(\mathbf{r})\gamma^{\mu}\psi_i^e(\mathbf{r})$$

Electron wave functions computed as solutions of Dirac eq. in the Coulomb field. *The effect of the static Coulomb potential is included nonperturbatively*

- Many numerical integrations and partial wave expansions are needed, particularly for (e,e') scattering
- But, in principle, it is a solvable problem. Just technically involved
- Several groups have developed a full DWBA formalism: OHIO (L. Wright and collaborators), GHENT (no longer in the market), MADRID

Effects of Coulomb distortion on the electron

- A shift in the momentun of the electron as seen by the nucleus
- A focussing effect, or modification of the flux near the vicinity of the nucleus

These effects are qualitatively simple to understand. The Coulomb distortion also implies

- Electron kinematics cannot be ‘separated’. Rosenbluth separation is no longer valid
- Azimuthal ϕ_F angle cannot be integrated analytically

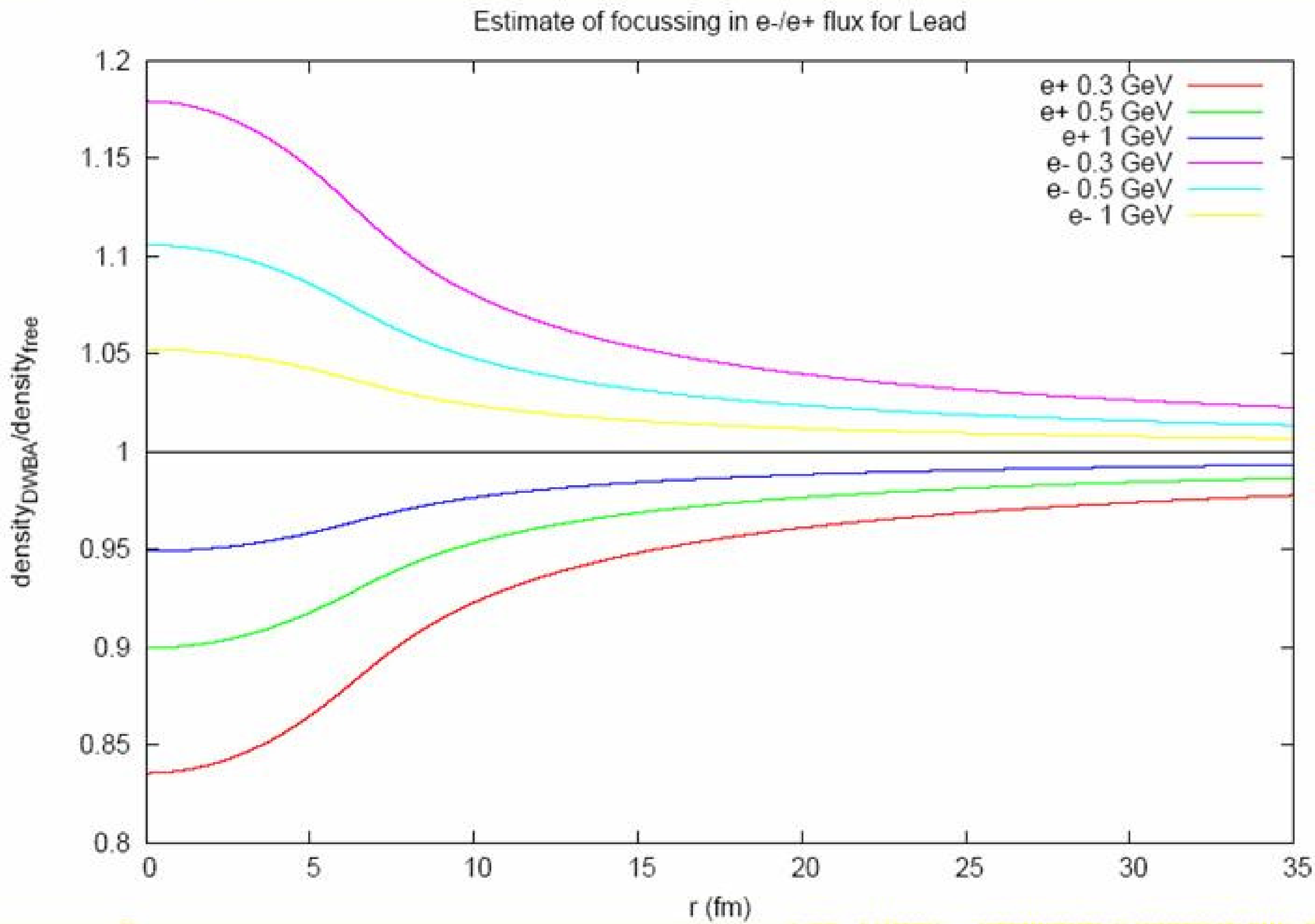
Coulomb effects (1): Focussing

- There is an easy quantity to compute:

$$\rho_e(r) = s\, d\Omega\, \Psi_e^*(\mathbf{r})\, \Psi_e(\mathbf{r})$$

- This is the r-density of the electron. It is 1/V for a free electron and will asset the value of focussing for DWBA
- There is a simple, analytical result that can be employed as a test:
- $\rho_e(r)_{DWBA} = (1+V_{Coul}(r)/|\mathbf{k}_e|)^2$, with $\mathbf{k}_e$ the electron momentum
- There will be a different focussing for the incoming and the outcoming electron / positron

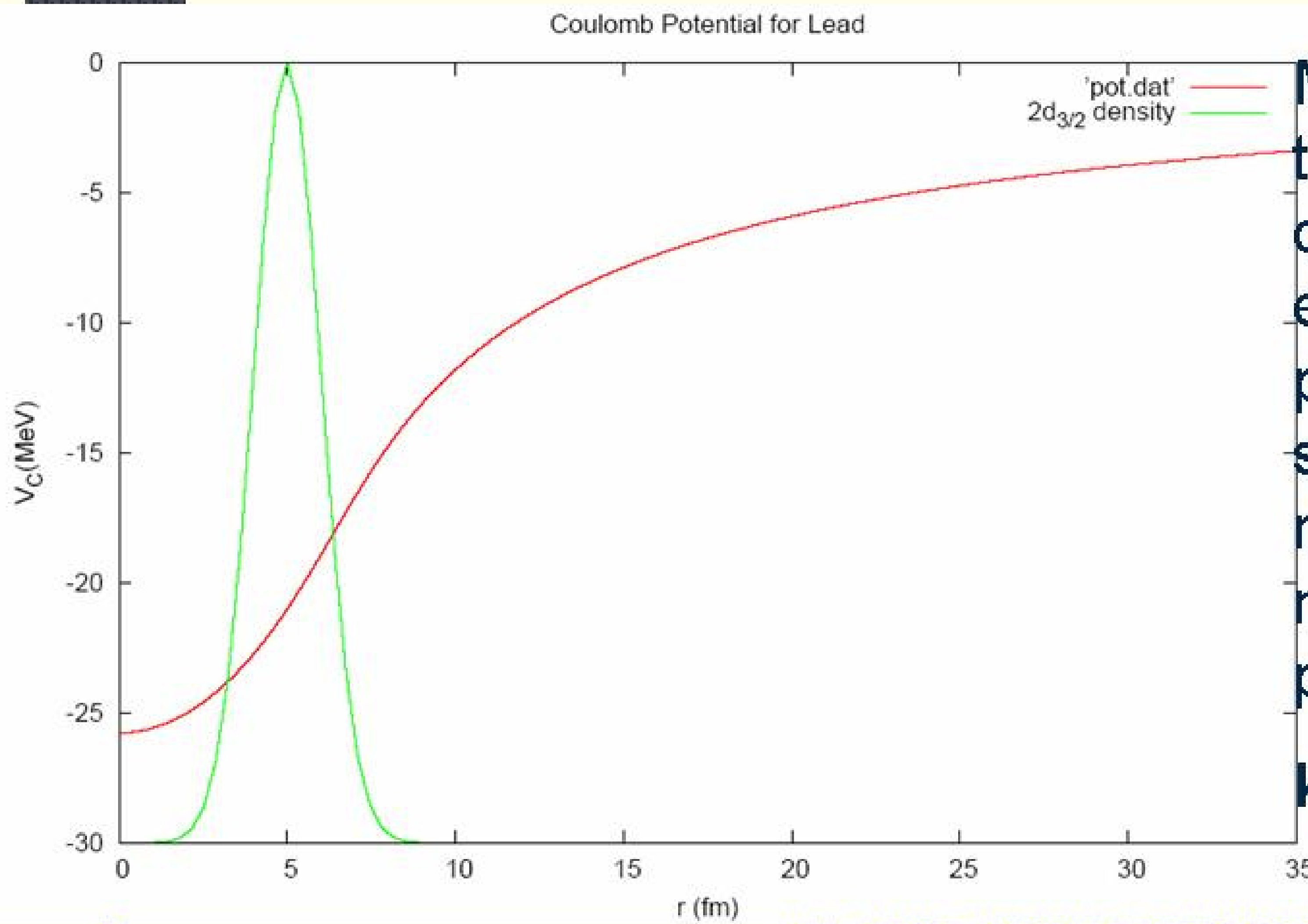
An estimate for the focussing



Focussing effect is different (symmetric, oposite effect with regards to no focussing) for e^- or e^+

It is <u>nonlinear</u> in ε

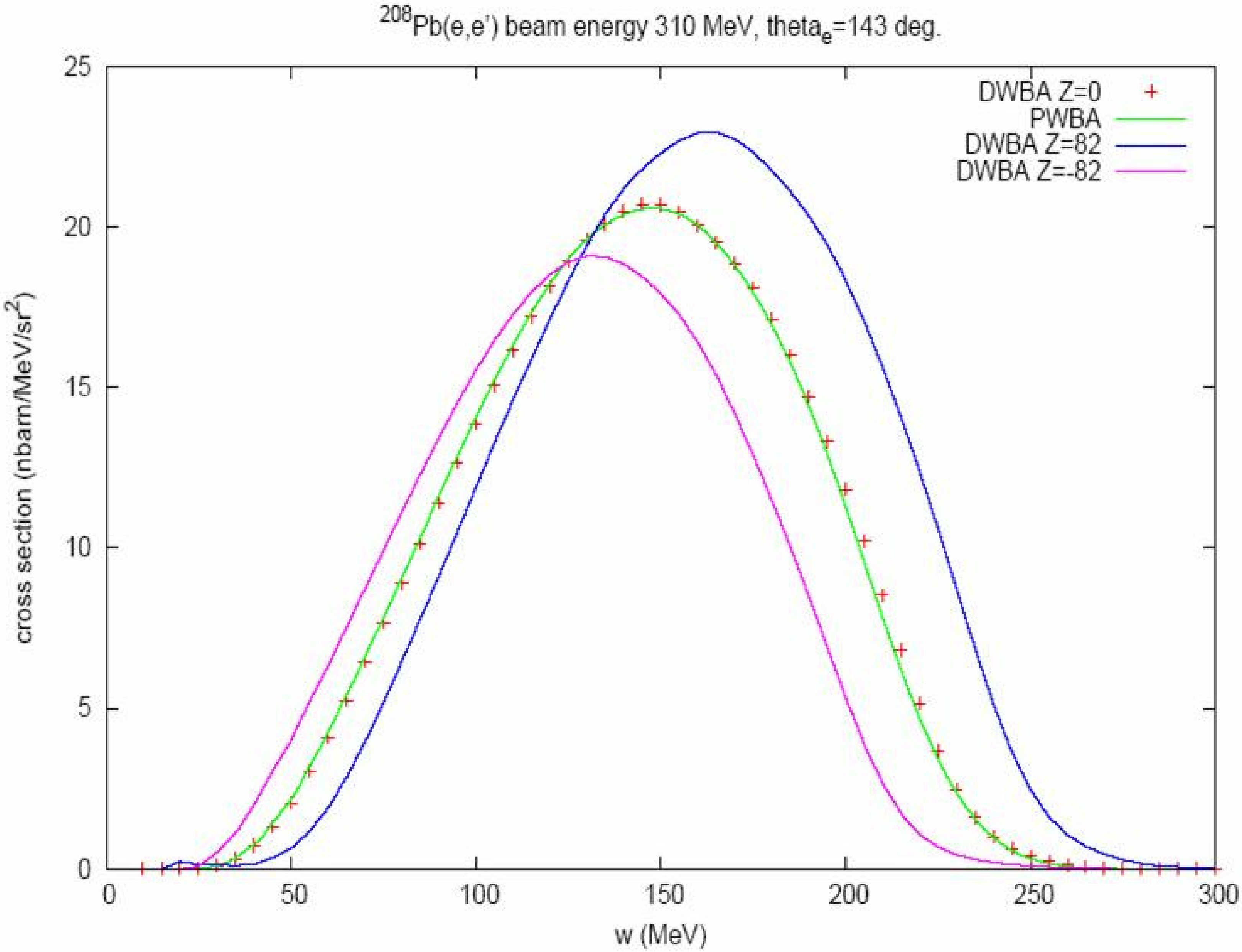
Coulomb effects (2): Shift in momenta



Momentum of the incoming / outgoing electron (or positron) as seen by the nucleus will be modified by the potential:

$\mathbf{k}_{eff}=\mathbf{k}(1+V_{coul}/|\mathbf{k}|)$

Example: backwards kinematics (for Saclay experiment P. Gueye et al., PHYSICAL REVIEW C, VOLUME 60, 044308)



- A 15% effect of increase in the cross-section for electrons (and decrease for positrons) is seen.
- A shift of the position of the QE peak is also noticed
- Consistent with estimations

Approximate treatmens

- Aim to preserve a Rosenbluth-like separation simplifying the calculation wherever possible

- Effective momentum approach (EMA). Simplest recipe, but not completely established

- Eikonal expansion of the electron wave functions (Knoll, Traini, also S. Boffi et al.). It seems not to converge. First order severely overestimate the corrections

Approximate treatmens (EMA) (many authors and flavours)

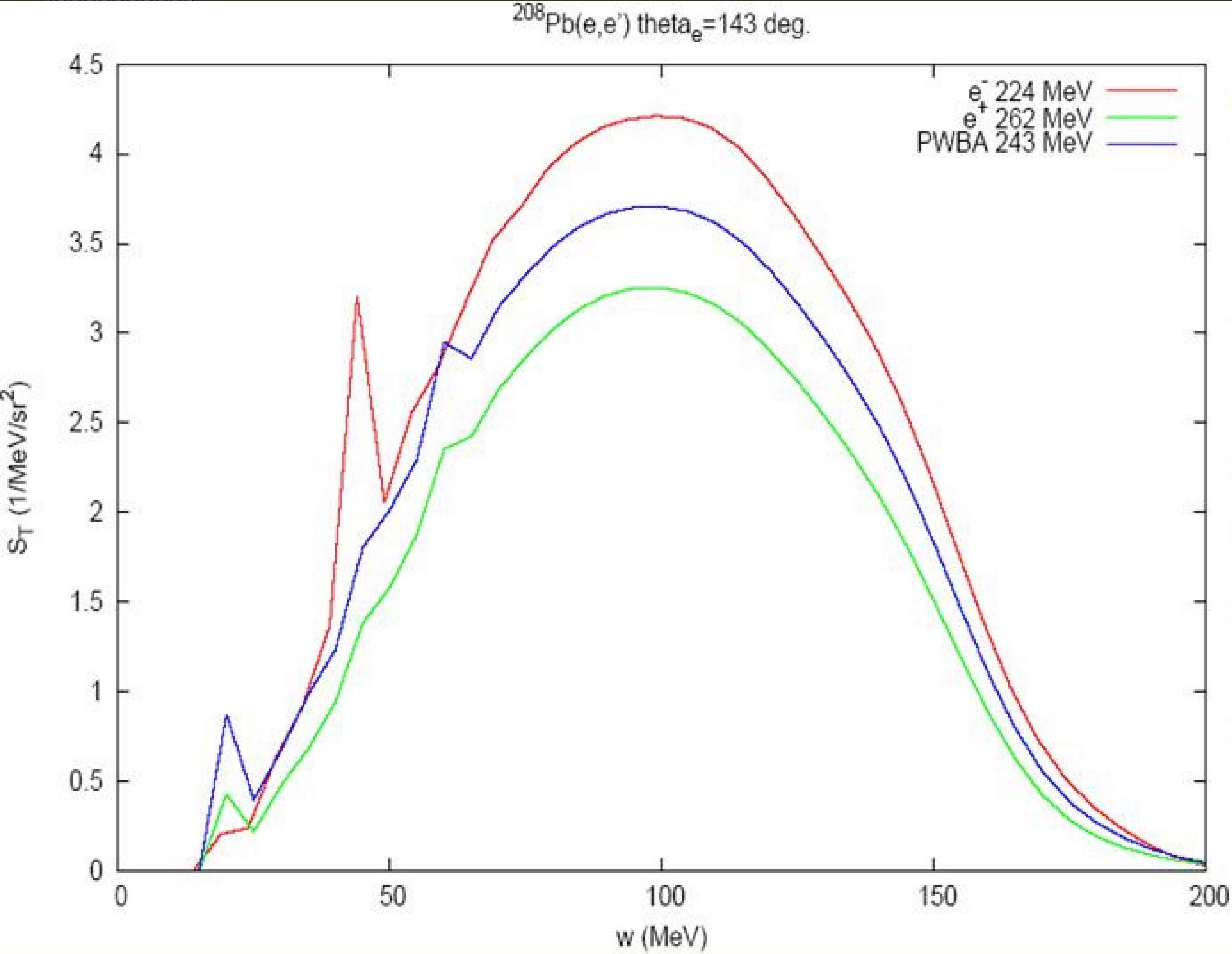
$$\left.\frac{d^3\sigma}{d\Omega_{e'}\,dE_{e'}}\right|^{\mathrm{EMA}} = \sigma_{\mathrm{Mott}}\left\{\left(\frac{Q^2_{\mathrm{eff}}}{\mathbf{q}^2_{\mathrm{eff}}}\right)^2 R_L(|\mathbf{q}_{\mathrm{eff}}|,\omega) + \left(\frac{Q^2_{\mathrm{eff}}}{2\,\mathbf{q}^2_{\mathrm{eff}}} + \tan^2\frac{\theta}{2}\right) R_T(|\mathbf{q}_{\mathrm{eff}}|,\omega)\right\} = \sigma_{\mathrm{Mott}} \times S_{\mathrm{total}}(|\mathbf{q}_{\mathrm{eff}}|,\omega,\theta),$$

$$\sigma_{\mathrm{Mott}} = 4\alpha^2\cos^2(\theta/2)E^2_{e'}/Q^4,$$

- Neglects factor $1/(\rho_e(f))^{1/2}$?

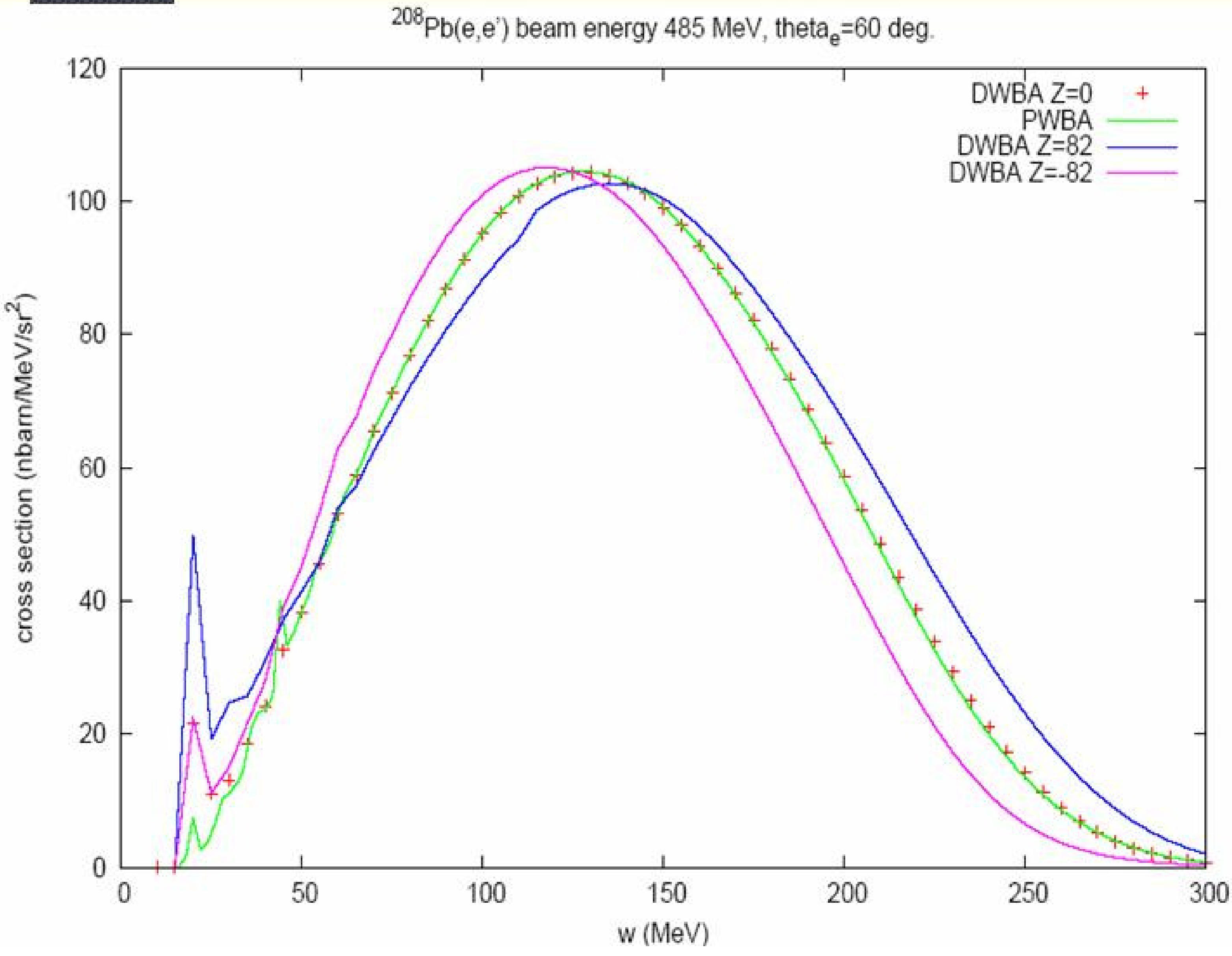
- Obtain effective variables from effective kinematics
- Include also focussing estimation via normalization of the effective espinors
- Suggest that a total DWBA ‘aware’ response S_{total} can be extracted for the effective kinematics

How good is EMA?: Compare the total S_T response at equivalent effective kinematics



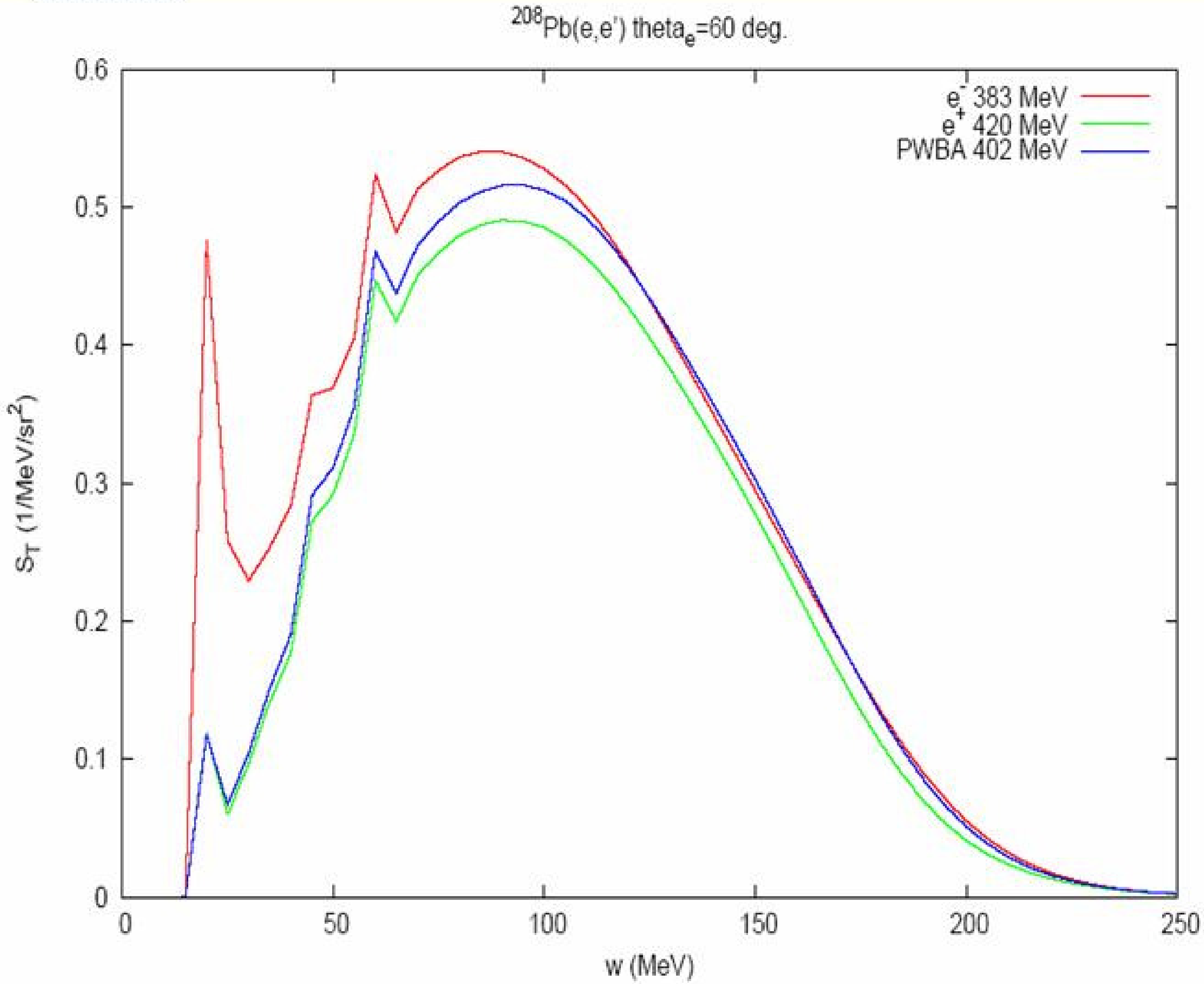
- Values of <V_C> can be obtained averaging V_C in the different shells or with the nuclear density
- Position of the peak is well taken care of
- Focussing is underestimated within EMA for this case

Forward kinematics (I)



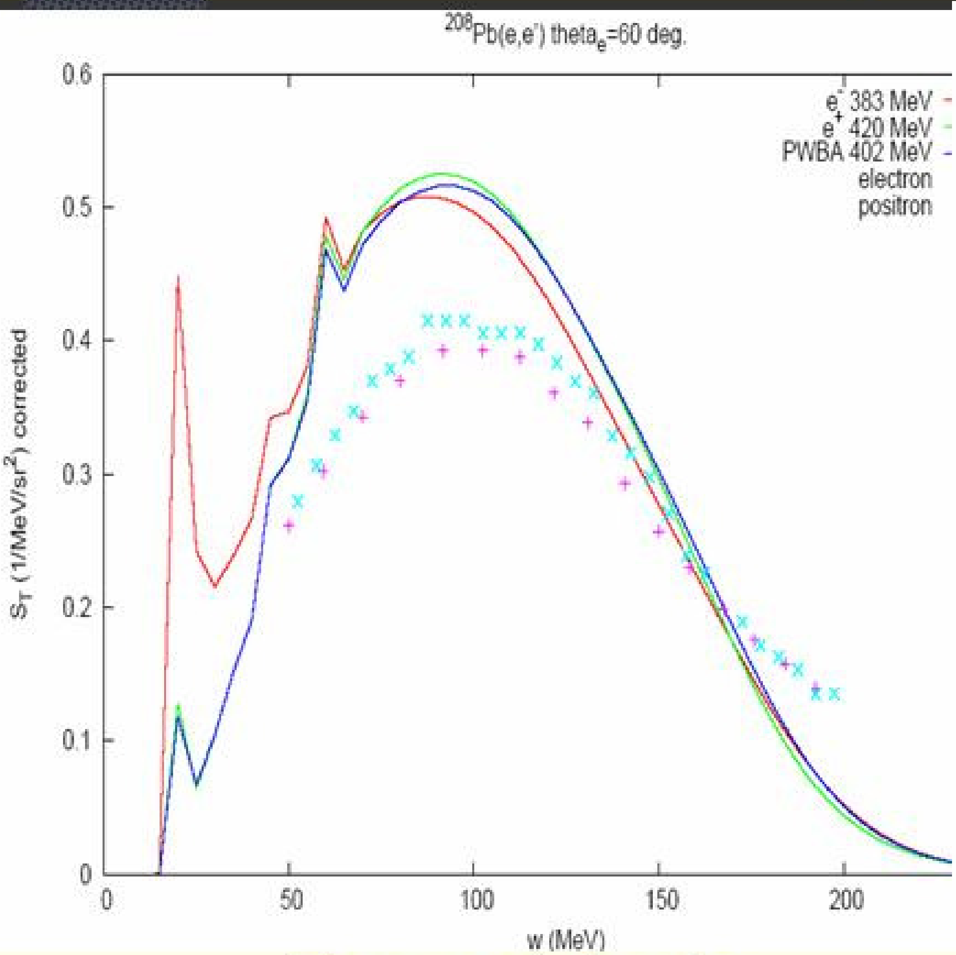
- Shift in the momenta seems to agree with the expectations
- Focusing, however, looks **oposite to the naive expectation**: complex effects revealed in the full calculation

Forward kinematics (II): EMA analysis of the total response

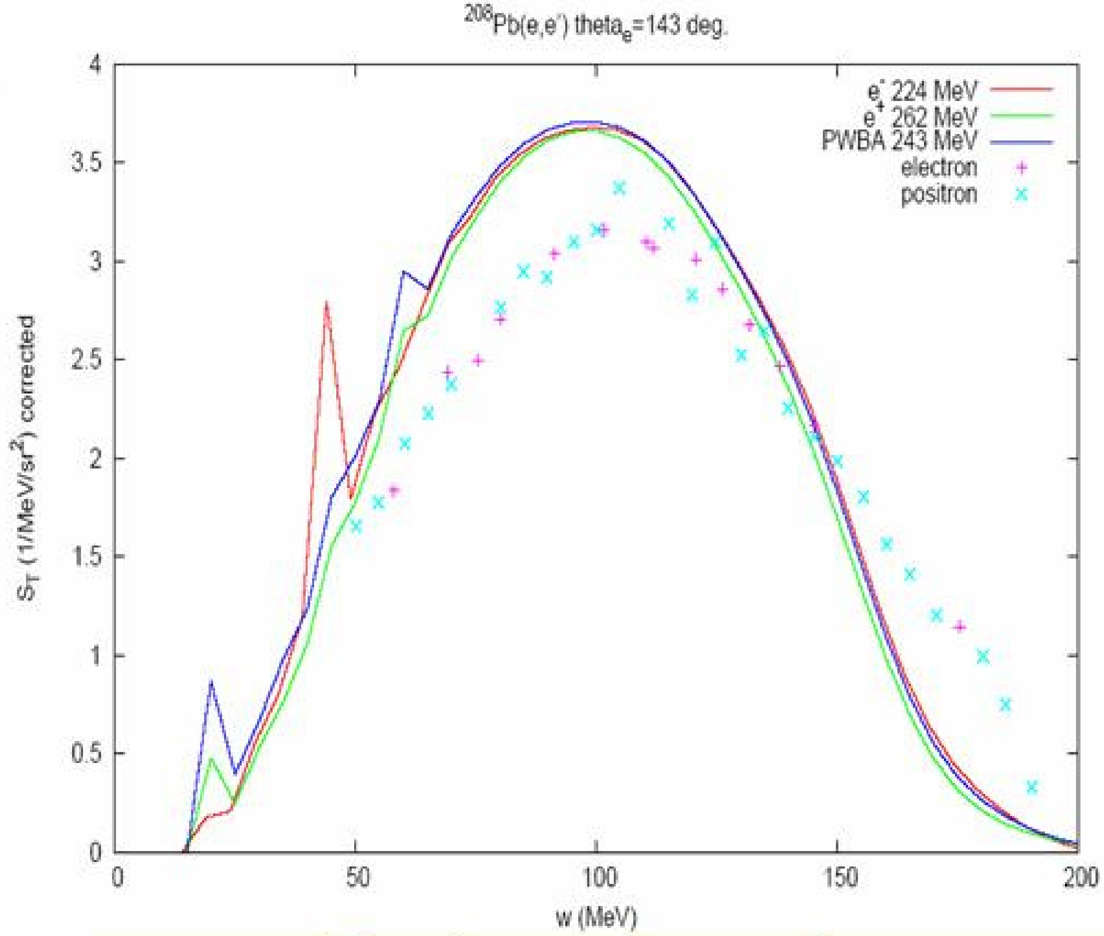


- Again, the position of the peak seems to be well taken care of by EMA
- Focussing seems to be underestimated again. In this case, as the electron energy is larger, the underestimation is smaller
- Needs factor $1/(\rho_e(f))^{1/2}$?

Including factor for the final electron (just a constant $(1.+\langle V_c\rangle/|\mathbf{k}_f|)$)

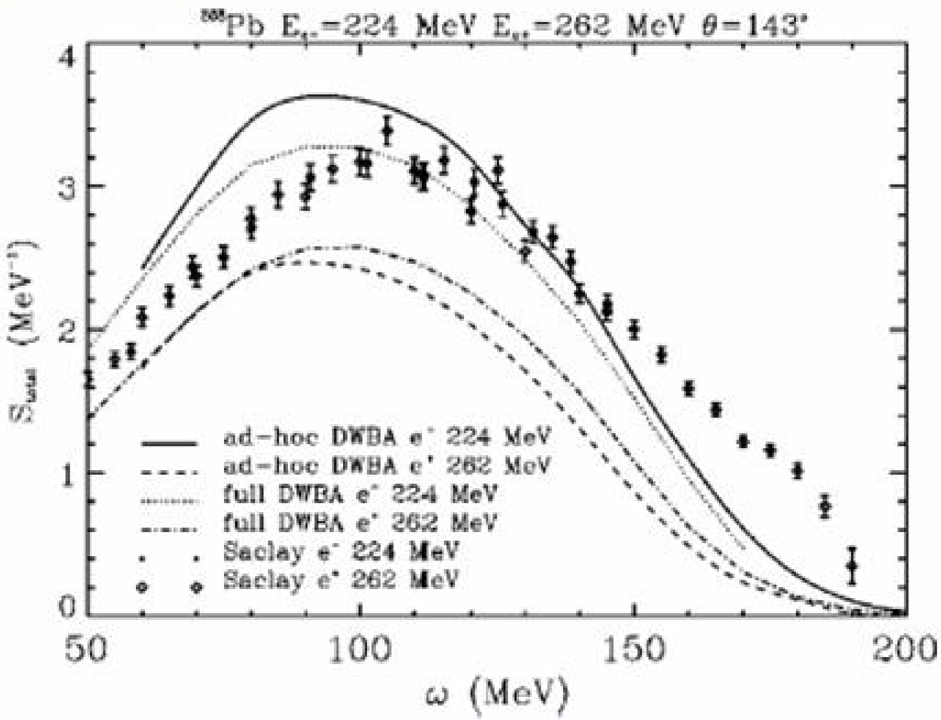


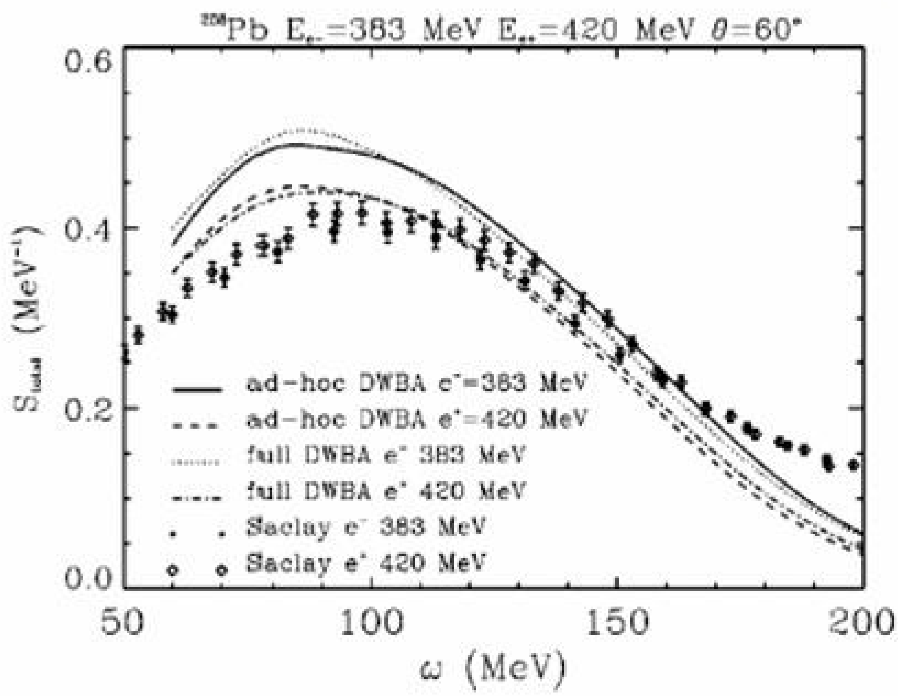
Forward Angle



Backwards angle

Comparison with other DWBA calculations (K. S. KIM, L. E. WRIGHT, AND D. A. RESLER PHYSICAL REVIEW C 64 044607)





- Good qualitative agreement between both theoretical calculations
- Quantitative comparisons difficult: different current operator and bound state wave functions in both cases, but it looks fine

Some (preliminary) conclusions

- Two independent full DWBA calculations compared (preliminary result) satisfactorily

- Comparison to combined electron / positron data from Saclay is not good. The shift in the momenta is well reproduced (is under control), but the focussing effect is overestimated with regards to experiment. EMA requires a focusing factor for final electron additional to the one employed in the experimental analysis. Further comparison with the remaining data sets is on its way

- Approximations to Coulomb effects, like EMA, must be contrasted against DWBA before being employed reliabily

- There is high hope: If the theory bears good resemblance with the experiment, focussing can be deduced from the theory and the experimental data can be corrected for it

Inclusive (e,e') reactions: RDWIA

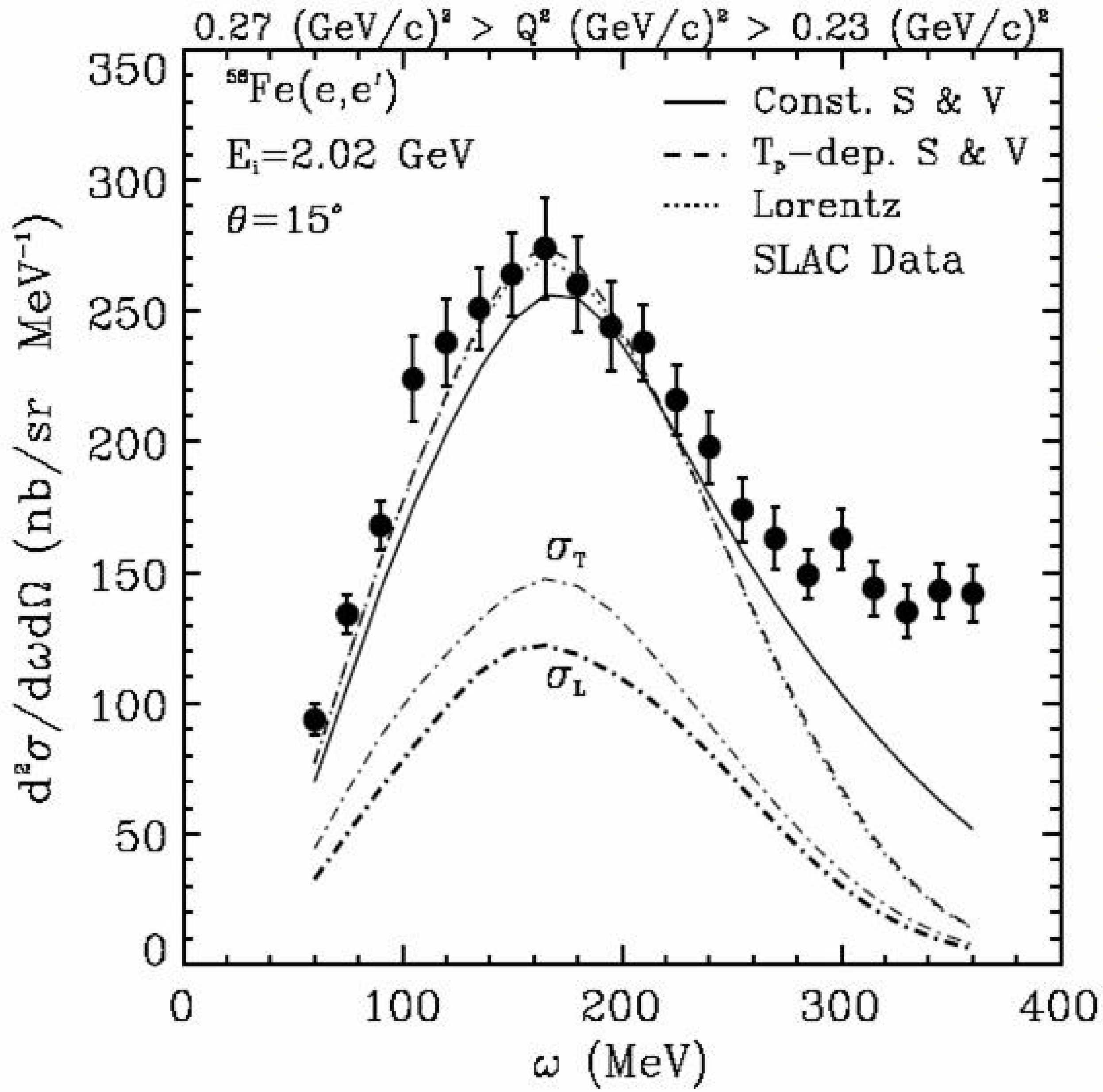
- Over the last years, the 'simple' relativistic impulse approximation has been employed with success to describe inclusive (e,e') scattering at the quasielastic peak provided Q^2 is large enough, say greater than 0.2 $(Gev/c)^2$ (K.S. Kim and L.E. Wright PRC 68 (2003) 027601, PRC 67, 054604 (2003) , Y. Jin, D.S. Onley and L.E. Wright, PRC 45 (1992) 1333, C. Maieron et al, PRC68 (2003) 048501)

- Inclusive scattering: no lost of flux into inelasticity

Inclusive (e,e') reactions: RDWIA

- If FSI <u>fitted to elastic channel only</u>, we need to recover the lost flux, setting to zero the imaginary part of the potentials, for instance. This provides an E-dependent real potential and thus violates the dispersion relationship **(Mahaux and Sartor, Adv. Nucl. Phys. 20 (1991) 1, Horikawa et al, PRC22 (1980) 1680)**
- Or we use the same (E-independent) mean field for the final as for the bound proton so no lost flux in the calculation
  - It preserves orthogonalization
  - It verifies continuity equation and Siegert's theorem
  - It fulfils the dispersion relationship

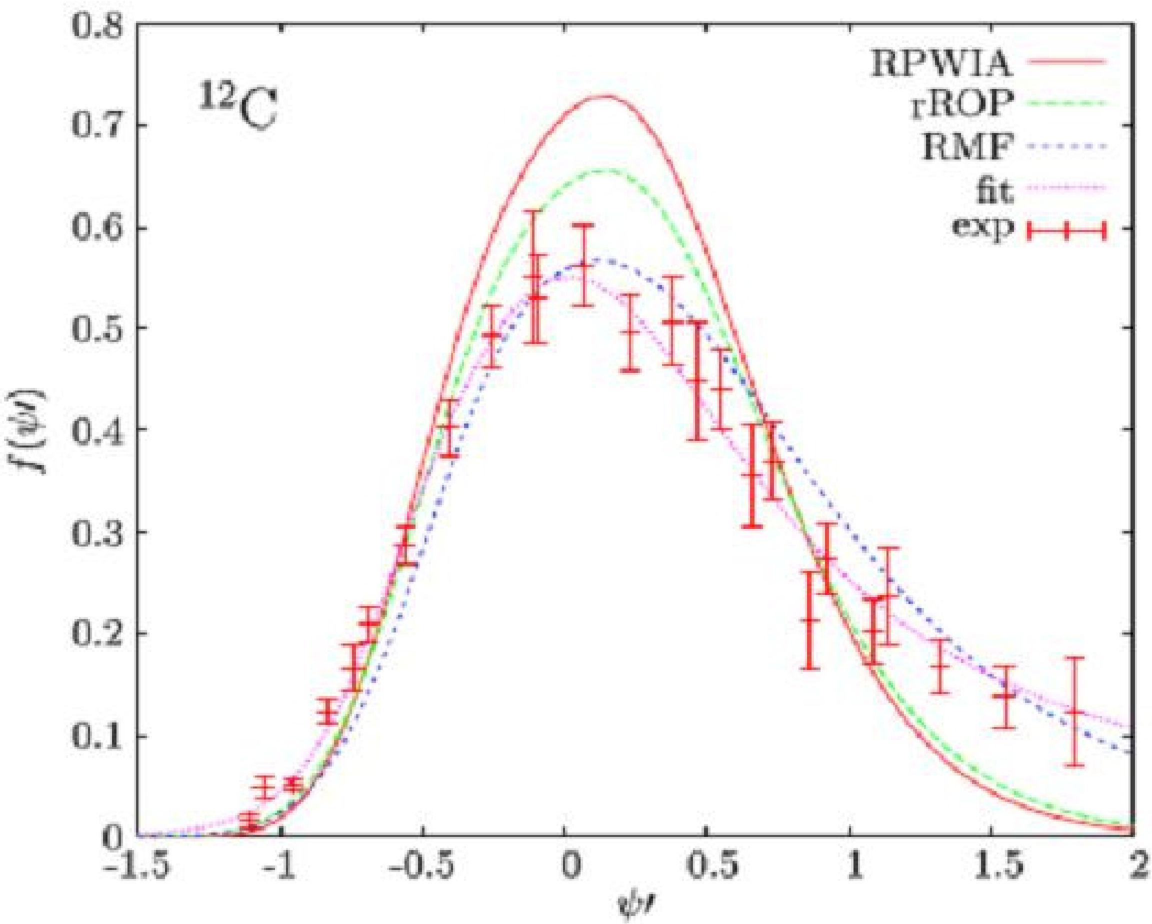
Inclusive (e,e') reactions: comparison with the data



Constant potentials (RMF) produce an asymetric cross-section with increased strength (tail) at large ω (**K.S. Kim and L.E. Wright PRC 68 (2003) 027601)**

Comparison with data is not conclusive due to the delta peak contributing into the quasielastic region

Comparison to inclusive data: Scaling analyses (J.A. Caballero et al, nucl-th/0504040)



- |q|=1 GeV/c
- We get rid of the delta by comparing to the scaling function
- Symmetrical responses are ruled out. Only RMF compares favourably with data

Conclusions

- To compare simple IA models for scattering at the quasielastic peak, it is more convenient (if possible) to use the scaling function than 'raw' data
- The RDWIA-RMF, in spite of its simplicity, compares with data favourably
- It can be used as a first estimate to compute reliablyfocusing factors at similar conditions