Two-photon-exchange effects in electromagnetic N to $\Delta(1232)$ transition

Vladimir Pascalutsa*

In collaboration with

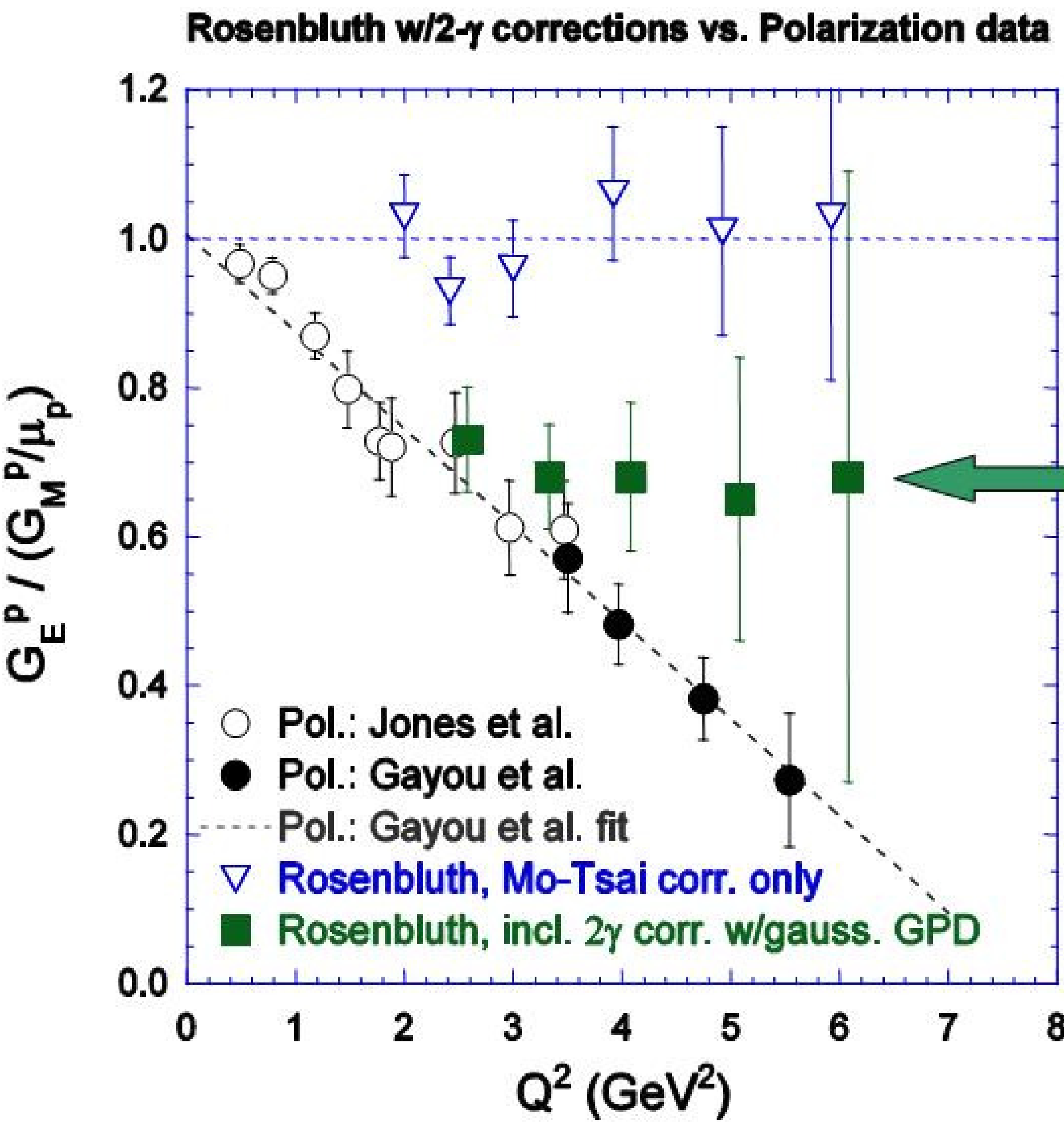
Carl Carlson & Marc Vanderhaeghen

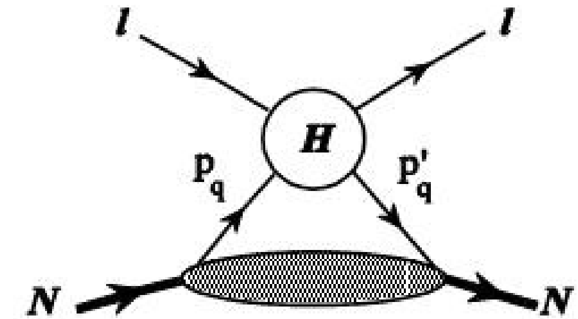
Department of Physics, The College of William & Mary
and
Theory Group, JLab

* *Supported by the U.S. Department of Energy*

JLab/INT Workshop, Williamsburg, August 15-17, 2005

Two-photon exchange effect in nucleon form factors

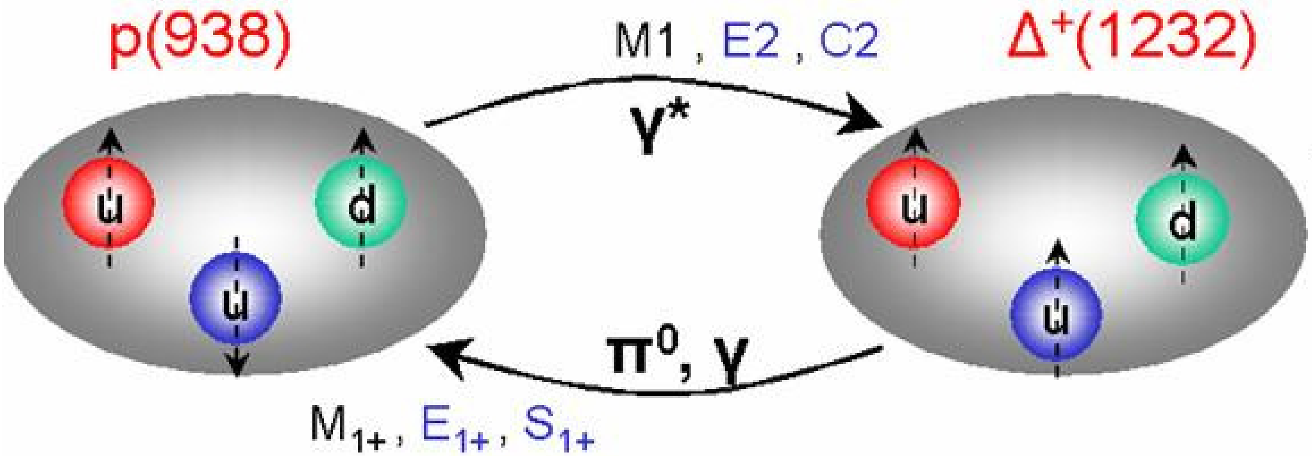




Partonic calculation

[Chen, Afanasev, Brodsky, Carlson, Vanderhaeghen, PRL (2004)]

Electromagnetic Nucleon to Delta transition

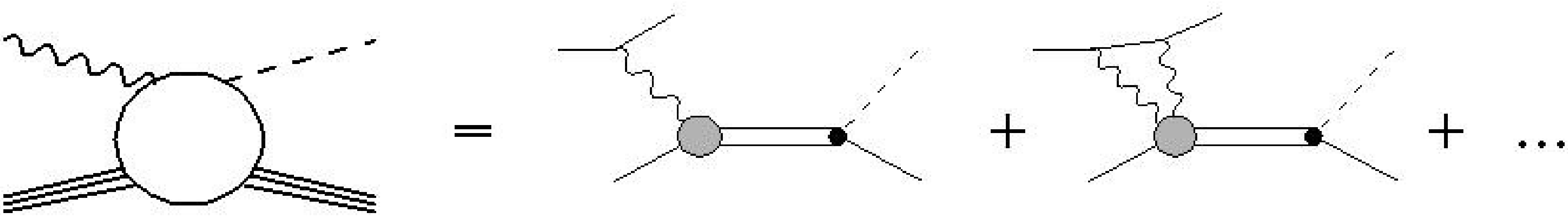


$J^P = 3/2+$, $M_\Delta = 1.232$, $\Gamma_\Delta = 0.2$ GeV

N ! Δ transition:

π N ! Δ (99%), γ N ! Δ (< 1%)

- $\gamma^*N\Delta$ is specified by three form-factors: $G_M^*(Q^2)$ [M1], $G_E^*(Q^2)$ [E2], $G_C^*(Q^2)$ [C2]
- studied in *pion electroproduction on the nucleon* at the resonance kinematics, $s = M_\Delta^2$



EM nucleon to Delta transition: general formalism

- In general (any # of exchanged photons), the e N! e Δ transition can be described by 16 form factors, or, 16 independent helicity ampl $T^{h',h}_{\lambda_\Delta,\lambda_N} \equiv \langle k',h';p_\Delta,\lambda_\Delta | T | k,h;p,\lambda_N \rangle$.
- $m_e = 0$, cuts this number down to ! $(\varepsilon_\pm = \sqrt{1 \pm \varepsilon})$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* + G_E^*) \xrightarrow{\text{PQCD}} \frac{1}{Q^6}$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N + M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^* \xrightarrow{\text{PQCD}} \frac{1}{Q^5}$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} - 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* - 3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* - 3G_E^*) \longrightarrow \frac{1}{Q^4}$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N + M_\Delta)\,Q_-}{2M_N M_\Delta}\,G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-} + 1\right)\frac{(M_N + M_\Delta)\,Q_-}{4M_N\,Q}\,(G_M^* + G_E^*)$$

$$R_{EM} = \frac{E2}{M1} = -\frac{G_E^*}{G_M^*} \xrightarrow{\text{PQCD}} 1$$

$$R_{SM} = \frac{C2}{M1} = -\frac{Q_+ Q_-}{4M_\Delta^2}\frac{G_C^*}{G_M^*} \longrightarrow \text{const}$$

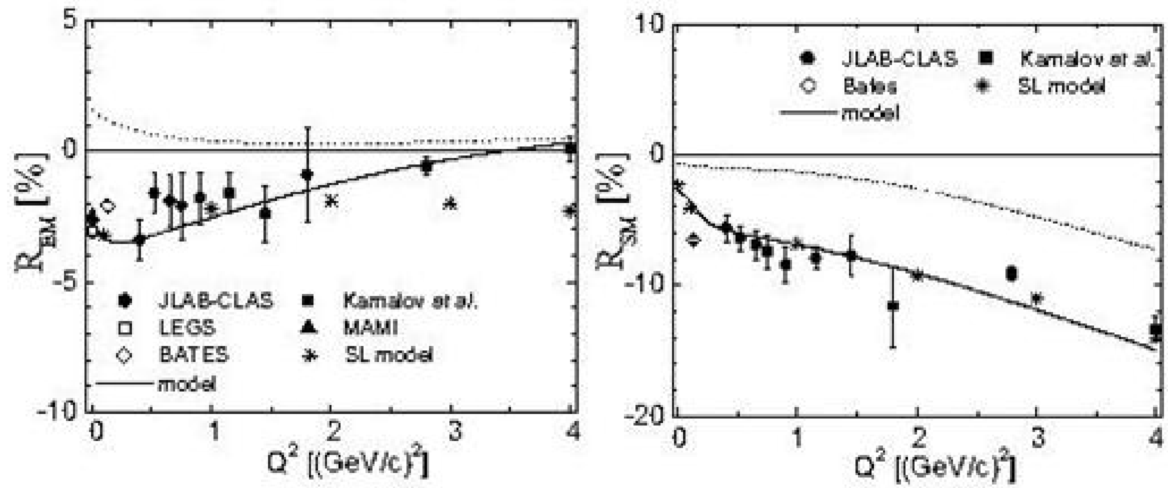
[Carlson, PRD (1986)]

one-photon exchange approximation

Experimental status of E2/M1 and C2/M1 ratios

$R_{EM}=-G_E/G_M$

$R_{SM}=-(q/2M_\Delta)\, G_C/G_M$



EM Nucleon to Delta transition: general formalism

❖ The unpolarized pion electroproduction cross-section is, *in general*, written as:

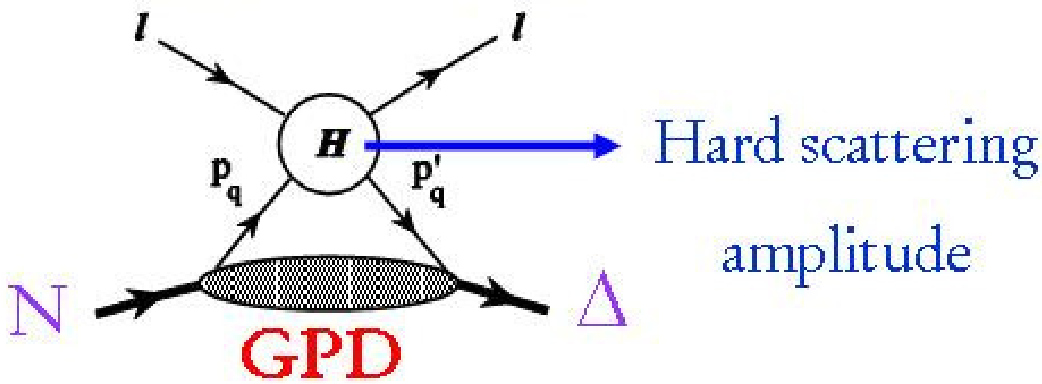
$$\frac{d\sigma}{(dE'_e\,d\Omega'_e)^{lab}\,d\Omega_\pi} \equiv \Gamma_v \frac{d\sigma}{d\Omega_\pi}, \qquad \text{Flux: } \Gamma_v = \frac{e^2}{(2\pi)^3}\left(\frac{E'_e}{E_e}\right)^{lab} \frac{(s_{\pi N} - M_N^2)/(2M_N)}{Q^2\,(1-\varepsilon)}.$$

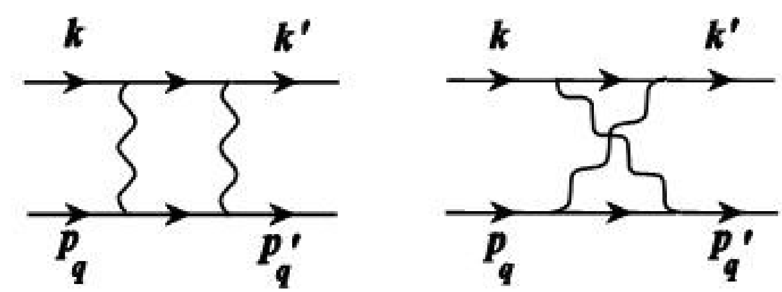
$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\cos(2\Phi)\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\Phi\frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\sin(2\Phi)\frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\varepsilon(1-\varepsilon)}\sin\Phi\frac{d\sigma_{LTi}}{d\Omega_\pi}.$$

❖ At the Δ-resonance, these cross-sections are expressed in terms of e N! e Δ helicity amplitudes

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\pi} &= \frac{1}{\pi}\frac{9\,Q^2\,(1-\varepsilon)}{16\,M_\Delta\,(M_\Delta^2 - M_N^2)\,\Gamma_\Delta} \\
&\times \left\{ \frac{1}{2}\sin^2\theta_\pi\,\left[|T_1|^2 + |T_2|^2 + |T_7|^2 + |T_8|^2\right] + \frac{1}{6}(1 + 3\cos^2\theta_\pi)\,\left[|T_3|^2 + |T_4|^2 + |T_5|^2 + |T_6|^2\right] \right. \\
&+ \cos\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Re\left[T_1T_3^* + T_2T_4^* - T_7T_5^* - T_8T_6^*\right] - \cos(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Re\left[T_1T_5^* + T_2T_6^* + T_7T_3^* + T_8T_4^*\right] \\
&\left. + \sin\Phi\,\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Im\left[T_1T_3^* + T_2T_4^* + T_7T_5^* + T_8T_6^*\right] - \sin(2\Phi)\,\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Im\left[T_1T_5^* + T_2T_6^* - T_7T_3^* - T_8T_4^*\right] \right\}.
\end{aligned}$$

Two-photon exchange : partonic calculation





$$A^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s}-\hat{u}}{Q^2} g_M^{hard} + g_A^{(2\gamma)} \right) \sqrt{\frac{2}{3}} \frac{1}{6} H_M^{(3)}$$ "magnetic" GPD

$$C^* = \int_{-1}^{1} \frac{dx}{x} \left(\frac{\hat{s}-\hat{u}}{Q^2} g_A^{(2\gamma)} + g_M^{hard} \right) \mathrm{sgm}(x) \frac{1}{6} C_1^{(3)}$$ "axial" GPD

Large N_C

$$H_M^{(3)}(x,0,Q^2) = 2 \frac{G_M^*(0)}{\kappa_V} \left[E^u(x,0,Q^2) - E^d(x,0,Q^2) \right]$$

$$C_1^{(3)}(x,0,Q^2) = \sqrt{3} \left[\tilde{H}^u(x,0,Q^2) - \tilde{H}^d(x,0,Q^2) \right]$$

Nucleon GPD's

Nucleon GPDs

Modified Regge model [Guidal, Polyakov, Radyushkin, Vanderhaeghen, PRD (2005)]:

$$H^q(x,0,q^2) = q_v(x)\, x^{\alpha_1' (1-x) Q^2}$$

$$E^q(x,0,q^2) = \frac{\kappa_q}{N_q} (1-x)^{\eta_q}\, q_v(x)\, x^{\alpha_2' (1-x) Q^2}$$

➡ $q_v(x)$ -- forward parton distributions at $\mu^2 = 1\ \mathrm{GeV}^2$

MRST2002 NNLO

$$u_v = 0.262\, x^{-0.69} (1-x)^{3.50} (1 + 3.83\, x^{0.5} + 37.65\, x)$$

$$d_v = 0.061\, x^{-0.65} (1-x)^{4.03} (1 + 49.05\, x^{0.5} + 8.65\, x)$$

Leader, Sidorov, Stamenov (2002)

$$\Delta u_v = 0.505 x^{-0.33} (1-x)^{3.428} (1 + 2.179 x^{0.5} + 14.57 x)$$

$$\Delta d_v = -0.0185 x^{-0.73} (1-x)^{3.864} (1 + 35.47 x^{0.5} + 28.97 x)$$

➡ Regge slopes : α_1' , α_2' determined from rms radii

$$r_{1,p}^2 = -6\, \alpha_1' \int_0^1 dx \left\{ e_u\, u_v(x) + e_d\, d_v(x) \right\} \ln x$$

$$r_{1,n}^2 = -6\, \alpha_1' \int_0^1 dx \left\{ e_u\, d_v(x) + e_d\, u_v(x) \right\} \ln x$$

$$\alpha_1' = 1.098\, \mathrm{GeV}^{-2}$$

$$\alpha_2' = 1.158\, \mathrm{GeV}^{-2}$$

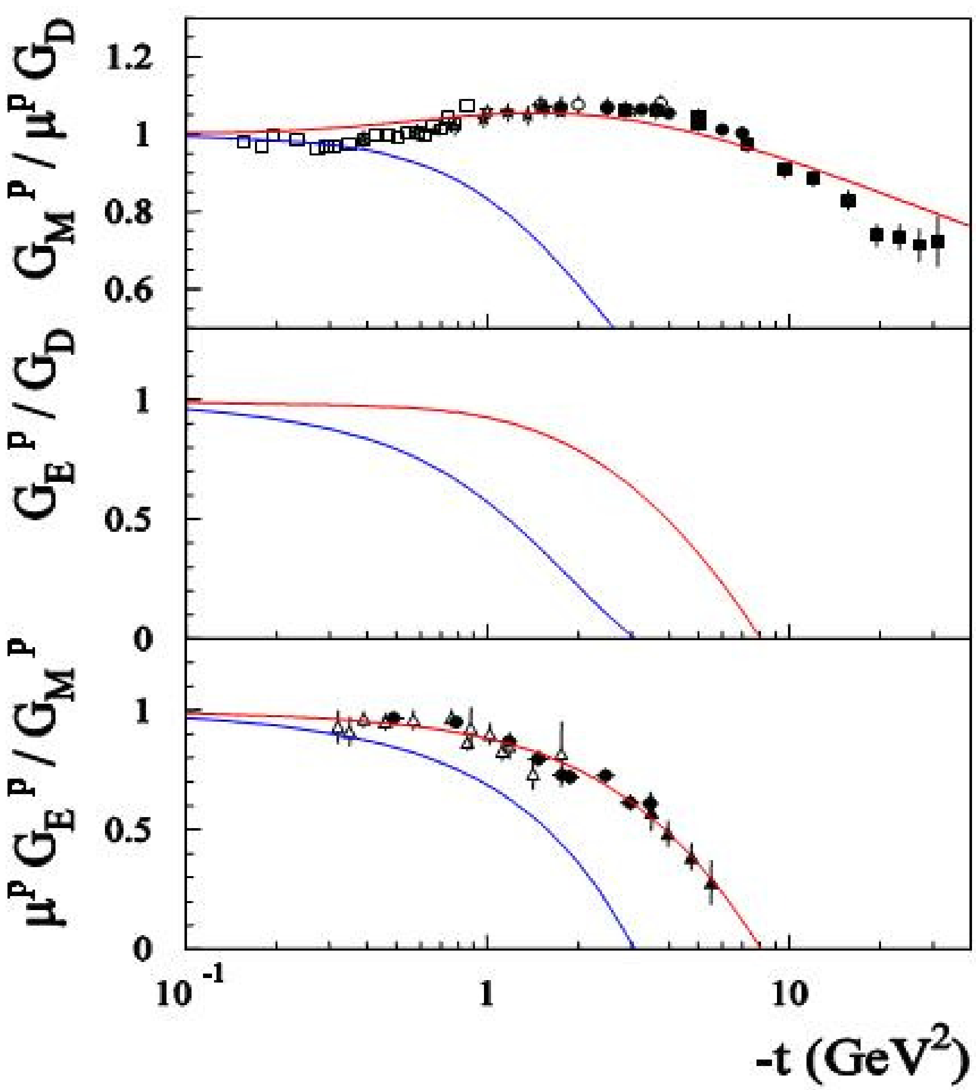
➡ η_u , η_d determined from F_2 / F_1 at large Q^2

$$\eta_u = 1.52$$

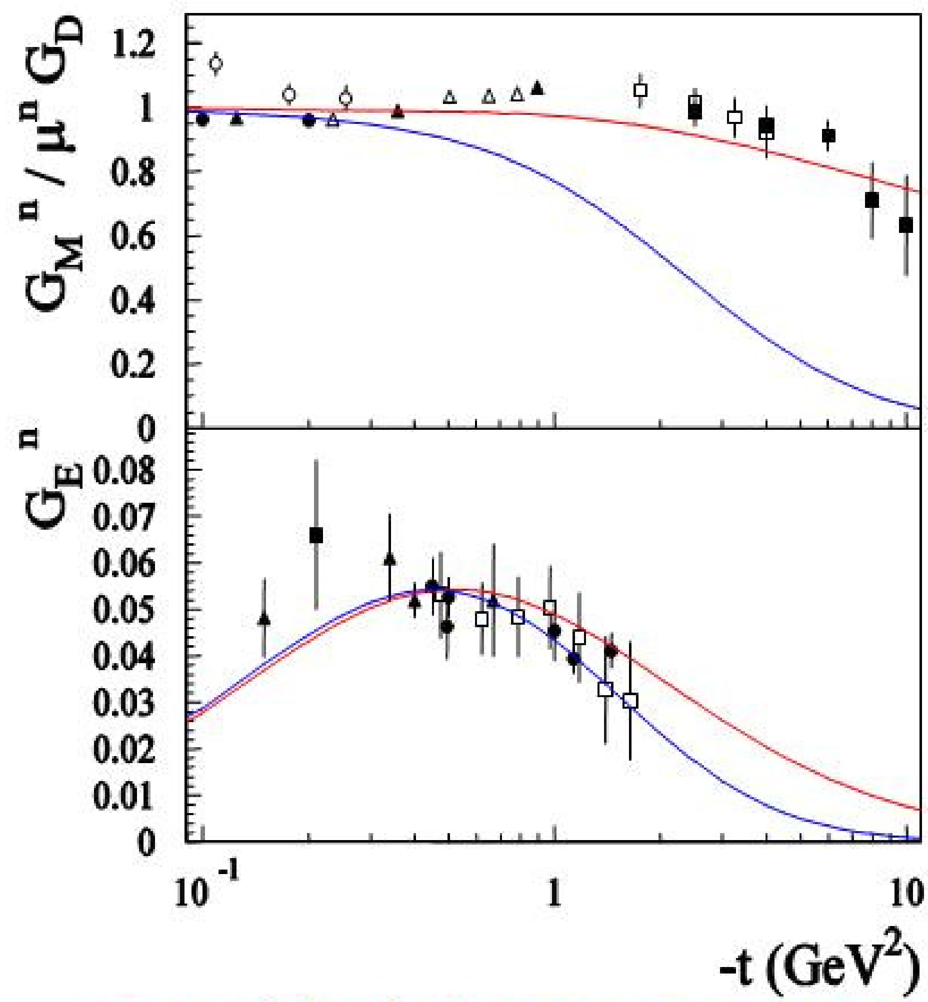
$$\eta_d = 0.31$$

Nucleon electromagnetic form factors

PROTON



NEUTRON



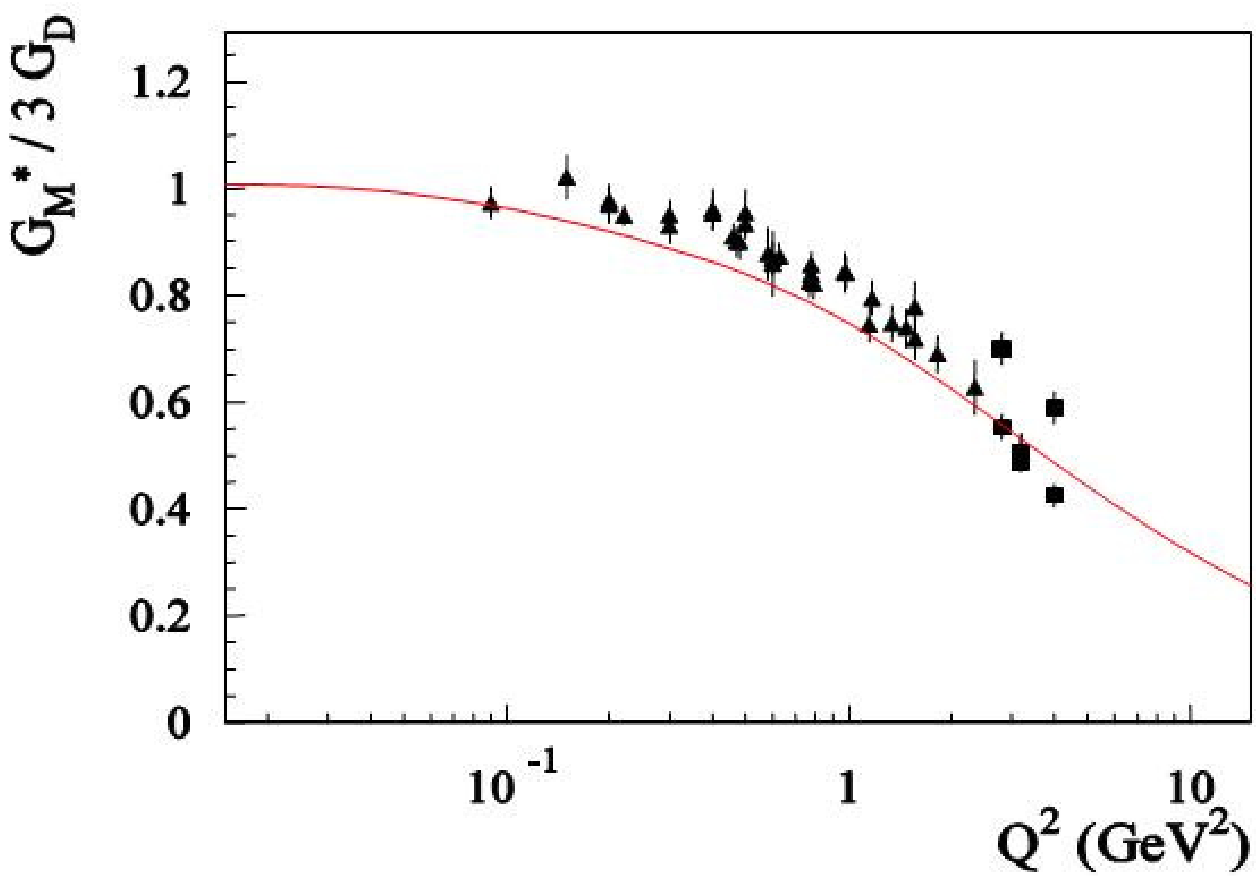
→ modified Regge parametrization

→ Regge parametrization

N -> Δ transition form factors from GPDs

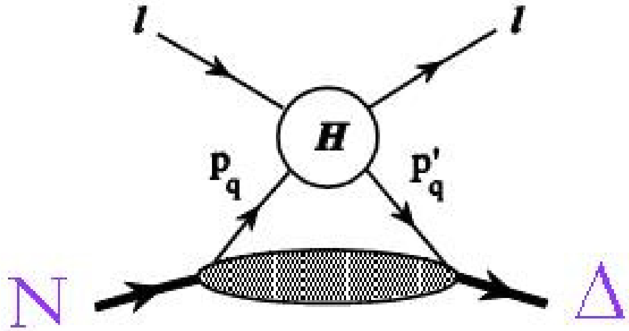
large N_c

$$G^*_M(t) = \frac{G^*_M(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x,\xi,t) - E^d(x,\xi,t) \right\} = \frac{G^*_M(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$



``modified Regge model"

Two-photon exchange contribution to helicity amplitudes



$$
\begin{aligned}
T_1^{2\gamma} &= \frac{e^2}{2}\left\{ A^* \frac{3(M_\Delta + M_N)Q}{2\sqrt{2}M_N Q_+} - C^* \frac{\sqrt{2}Q}{Q_-} \right\}, \\
T_2^{2\gamma} &= 0, \\
T_3^{2\gamma} &= -\frac{e^2}{2} C^* \sqrt{\frac{2}{3}} \frac{M_\Delta^2 - M_N^2 - Q^2}{M_\Delta Q_-} \\
T_4^{2\gamma} &= \sqrt{\frac{1}{3}} T_1^{2\gamma}, \\
T_5^{2\gamma} &= \sqrt{\frac{1}{3}} T_8^{2\gamma}, \\
T_6^{2\gamma} &= T_3^{2\gamma}, \\
T_7^{2\gamma} &= 0, \\
T_8^{2\gamma} &= \frac{e^2}{2}\left\{ A^* \frac{3(M_\Delta + M_N)Q}{2\sqrt{2}M_N Q_+} + C^* \frac{\sqrt{2}Q}{Q_-} \right\}.
\end{aligned}
$$

One + two-photon-exchange in observables

- The unpolarized pion electroproduction cross-section:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma}{d\Omega_\pi} = \frac{d\sigma_0}{d\Omega_\pi} + \varepsilon\cos(2\Phi)\frac{d\sigma_{TT}}{d\Omega_\pi} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\Phi\frac{d\sigma_{LT}}{d\Omega_\pi} + \varepsilon\sin(2\Phi)\frac{d\sigma_{TTi}}{d\Omega_\pi} + \sqrt{2\varepsilon(1-\varepsilon)}\sin\Phi\frac{d\sigma_{LTi}}{d\Omega_\pi} = \frac{1}{\pi}\frac{9Q^2(1-\varepsilon)}{16M_\Delta(M_\Delta^2-M_N^2)\Gamma_\Delta}$$

$$\times\left\{\frac{1}{2}\sin^2\theta_\pi\left[|T_1|^2+|T_2|^2+|T_7|^2+|T_8|^2\right]+\frac{1}{6}(1+3\cos^2\theta_\pi)\left[|T_3|^2+|T_4|^2+|T_5|^2+|T_6|^2\right]\right.$$

$$+\cos\Phi\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Re\left[T_1T_3^*+T_2T_4^*-T_7T_5^*-T_8T_6^*\right]-\cos(2\Phi)\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Re\left[T_1T_5^*+T_2T_6^*+T_7T_3^*+T_8T_4^*\right]$$

$$\left.+\sin\Phi\frac{1}{\sqrt{3}}\sin(2\theta_\pi)\,\Im\left[T_1T_3^*+T_2T_4^*+T_7T_5^*+T_8T_6^*\right]-\sin(2\Phi)\frac{1}{\sqrt{3}}\sin^2\theta_\pi\,\Im\left[T_1T_5^*+T_2T_6^*-T_7T_3^*-T_8T_4^*\right]\right\}.$$

$$T_1 \equiv T_{+3/2,+1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)Q_-}{4M_NQ}(G_M^*+G_E^*)$$

$$T_2 \equiv T_{+3/2,-1/2} = 0$$

$$T_3 \equiv T_{+1/2,+1/2} = -\frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)Q_-}{2M_NM_\Delta}G_C^*$$

$$T_4 \equiv T_{+1/2,-1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}-1\right)\frac{(M_N+M_\Delta)Q_-}{4M_NQ}(G_M^*-3G_E^*)$$

$$T_5 \equiv T_{-1/2,+1/2} = \left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)Q_-}{4M_NQ}(G_M^*-3G_E^*)$$

$$T_6 \equiv T_{-1/2,-1/2} = \frac{\sqrt{2\varepsilon}}{\varepsilon_-}\frac{(M_N+M_\Delta)Q_-}{2M_NM_\Delta}G_C^*$$

$$T_7 \equiv T_{-3/2,+1/2} = 0$$

$$T_8 \equiv T_{-3/2,-1/2} = \sqrt{3}\left(\frac{\varepsilon_+}{\varepsilon_-}+1\right)\frac{(M_N+M_\Delta)Q_-}{4M_NQ}(G_M^*+G_E^*)$$

$$T_1^{2\gamma} = \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_NQ_+}-C^*\frac{\sqrt{2}Q}{Q_-}\right\},$$

$$T_2^{2\gamma} = 0,$$

$$T_3^{2\gamma} = -\frac{e^2}{2}C^*\sqrt{\frac{2}{3}}\frac{M_\Delta^2-M_N^2-Q^2}{M_\Delta Q_-}$$

$$T_4^{2\gamma} = \sqrt{\frac{1}{3}}T_1^{2\gamma},$$

$$T_5^{2\gamma} = \sqrt{\frac{1}{3}}T_8^{2\gamma},$$

$$T_6^{2\gamma} = T_3^{2\gamma},$$

$$T_7^{2\gamma} = 0,$$

$$T_8^{2\gamma} = \frac{e^2}{2}\left\{A^*\frac{3(M_\Delta+M_N)Q}{2\sqrt{2}M_NQ_+}+C^*\frac{\sqrt{2}Q}{Q_-}\right\}.$$

Two-photon effects on observables

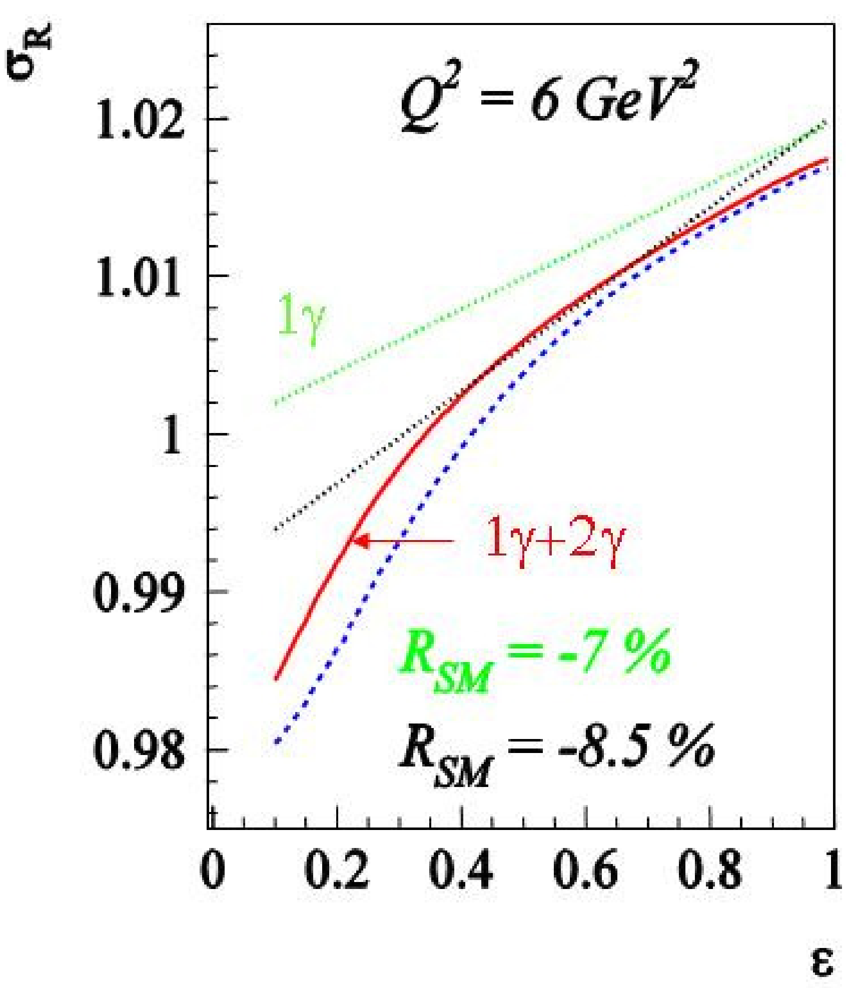
$$\sigma_R = 1 + 3\,(R_{EM}^{1\gamma})^2 + \varepsilon\,\frac{16\,M_\Delta^2\,Q^2}{Q_+^2\,Q_-^2}\,(R_{SM}^{1\gamma})^2 + \frac{1}{G_M^*}\sqrt{\frac{2}{3}}\left[\frac{1}{2}\,A^*\,\frac{Q^2}{Q_+\,Q_-}\,\varepsilon_+\,\varepsilon_- + 2\,C^*\,\frac{Q^2}{Q_-^2}\,\varepsilon_-^2\,\frac{M_N}{M_N+M_\Delta}\right].$$

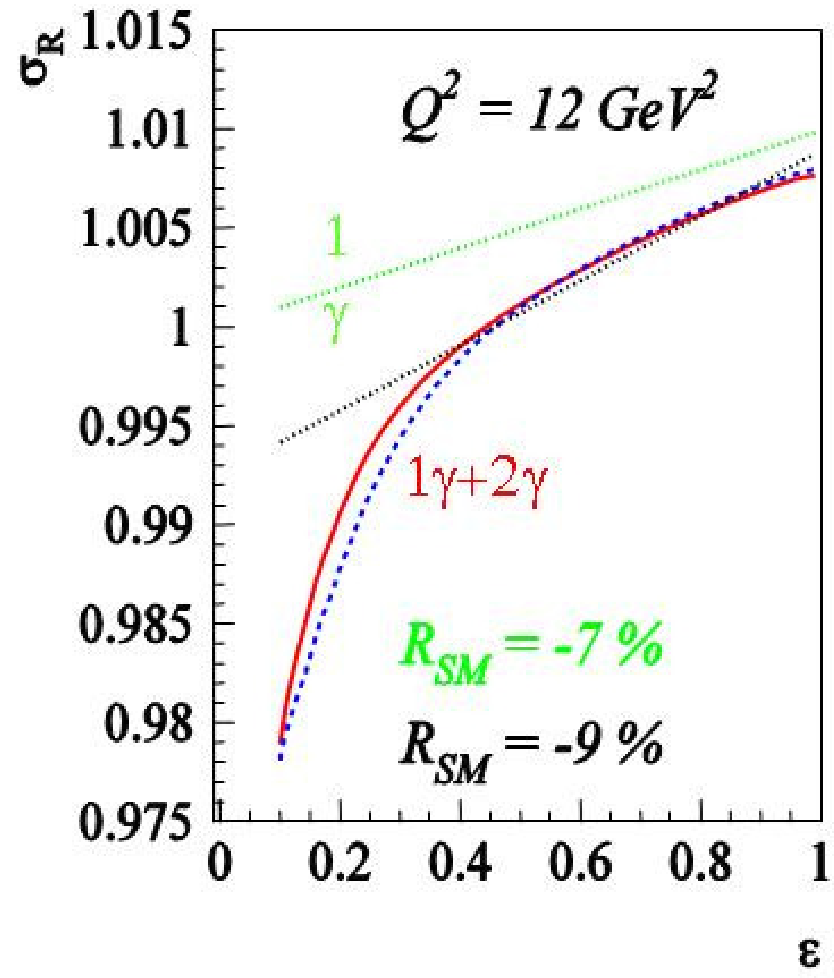
$$R_{EM} = R_{EM}^{1\gamma} + \frac{1}{8}\sqrt{\frac{3}{2}}\frac{Q^2}{Q_+\,Q_-}\,\frac{\varepsilon_-^3\,\varepsilon_+}{\varepsilon}\,\frac{1}{G_M^*}\,A^* - \frac{1}{4}\sqrt{\frac{2}{3}}\,\frac{Q^2}{Q_-^2}\,\frac{\varepsilon_-^2\,\varepsilon_+^2}{\varepsilon}\,\frac{M_N}{(M_N+M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

$$R_{SM} = R_{SM}^{1\gamma} - \sqrt{\frac{2}{3}}\,\frac{(Q^2 - M_\Delta^2 + M_N^2)}{4\,M_\Delta^2}\,\frac{Q_+}{Q_-}\,\frac{1}{\sqrt{2\,\varepsilon}}\,\frac{\varepsilon_-^2}{\varepsilon_+}\,\frac{M_N}{(M_N+M_\Delta)}\,\frac{1}{G_M^*}\,C^*$$

Two-photon effects on cross sections

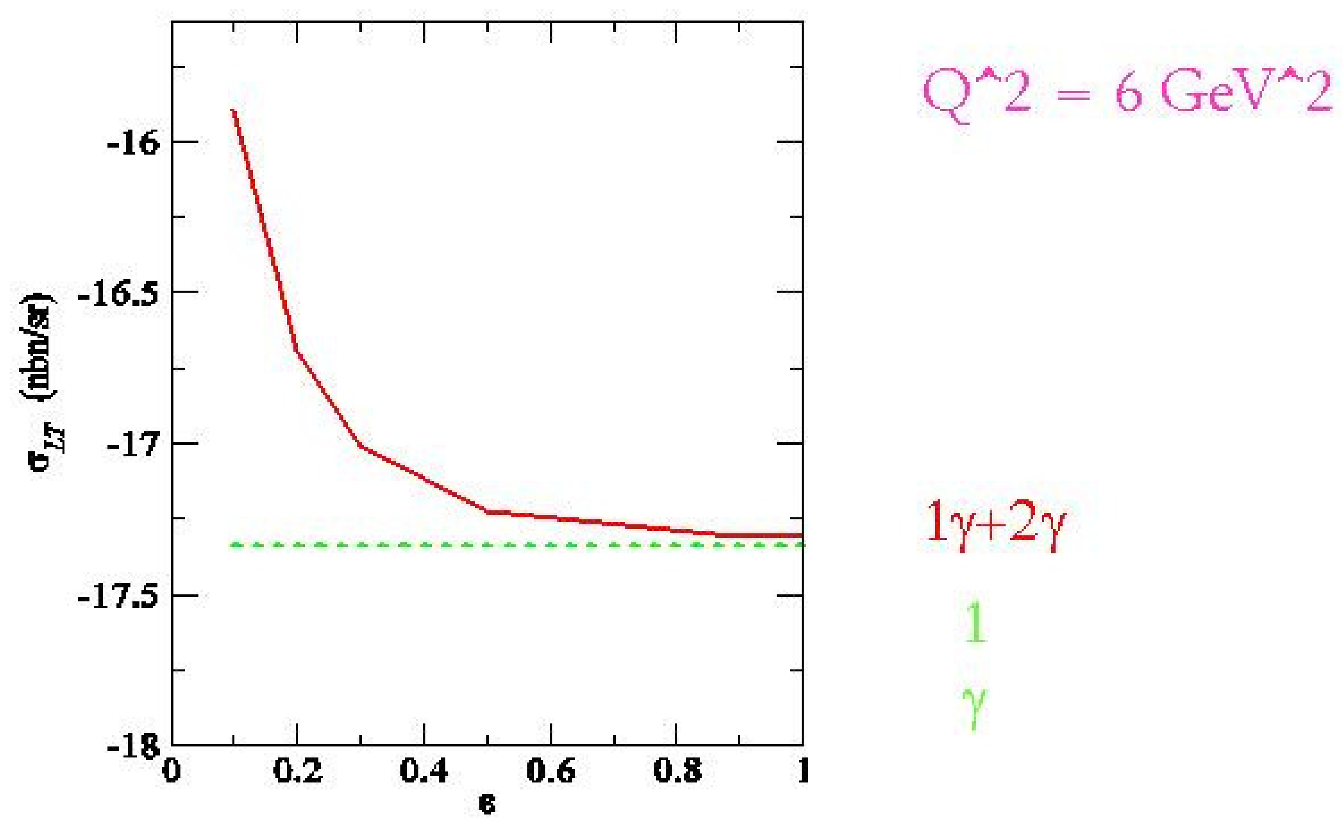
$$\sigma_R \sim \sigma_0 = \sigma_T + \varepsilon\,\sigma_L, \qquad \sigma_L \sim R_{SM}^2$$





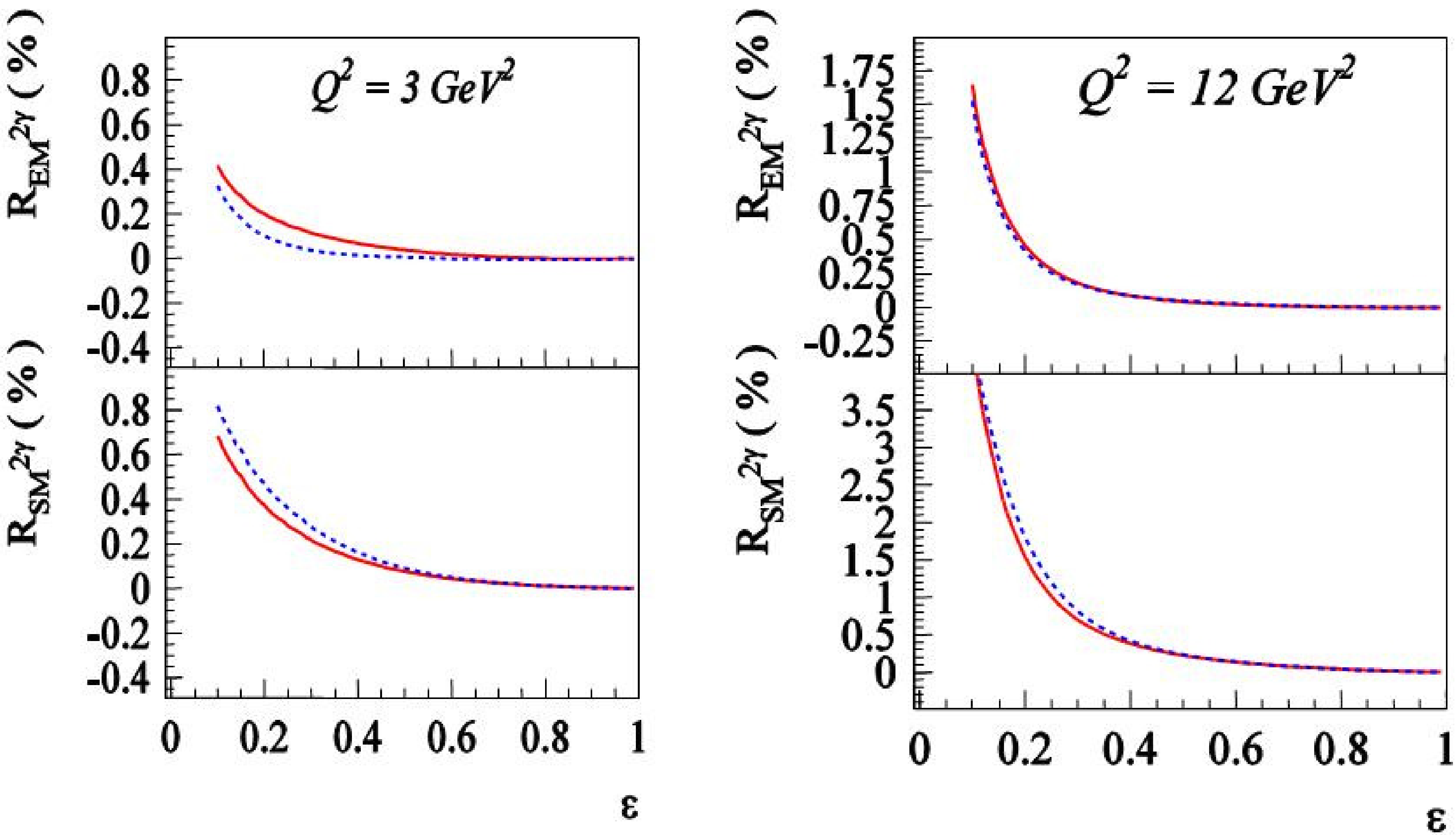
Two-photon effects on cross sections

$\sigma_{LT} \sim R_{SM}$, linear relation, in contrast to $\sigma_L \sim R_{SM}^2$.

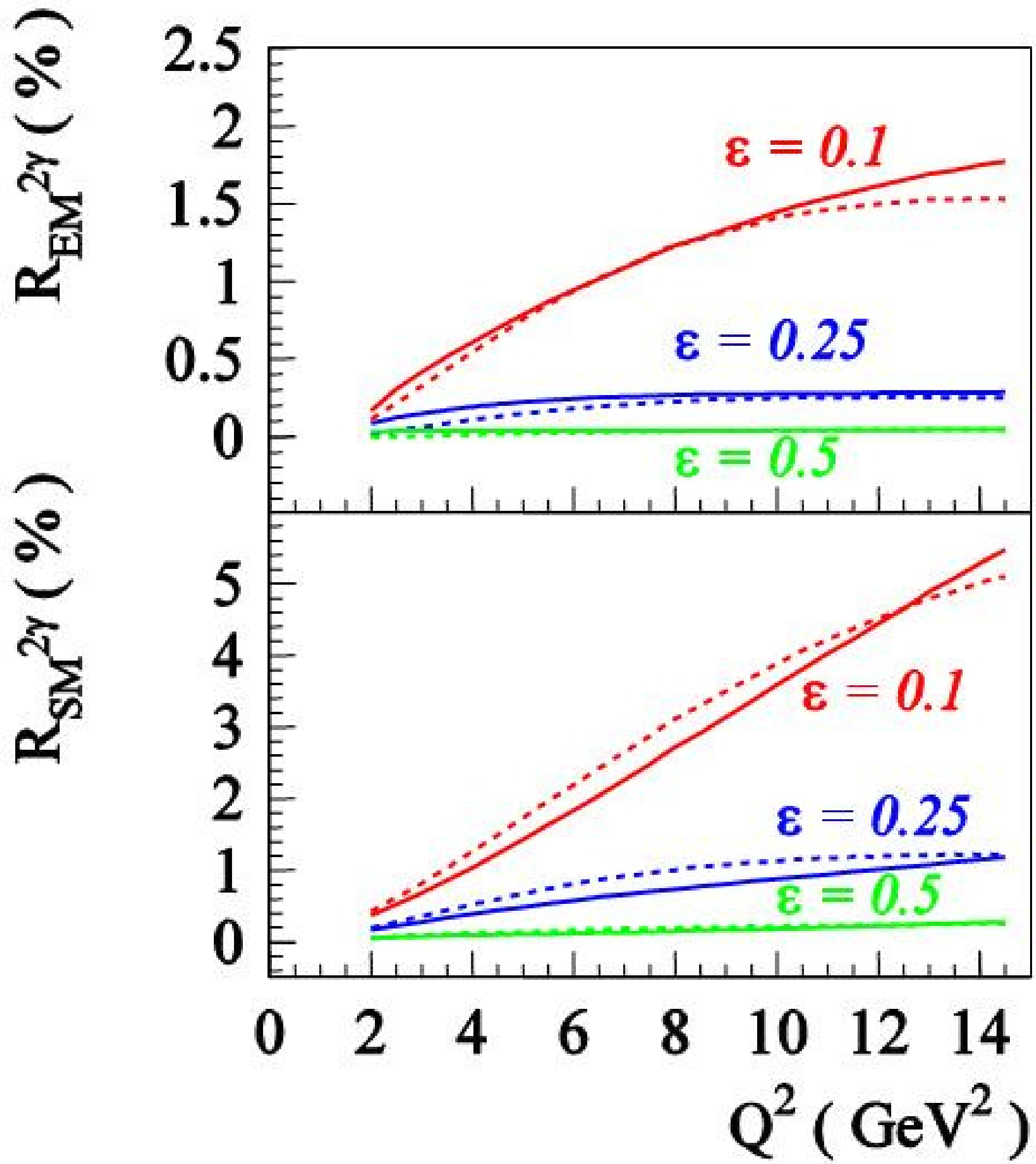


$\sigma_{TTi}, \sigma_{LTi} \sim 1$ nbn/sr.

Two-photon effects on the ratios



Two-photon effects on the ratios (Q^2 dependence)



Conclusions

✓ General formalism for the unpolarized pion electroproduction cross-sections in terms of N!Δ helicity amplitudes.

✓New responses σ_{TTi} and σ_{LTi}.

✓Parton-model calculation of two-photon exchange N!Δ helicity amplitudes.

✓A first glance at two-photon effects on inelastic observables.

❖ Corrections on R_{EM} are small, analogous to corrections in the nucleon polarization-transfer observables.

❖ Substantial (up to 10%) corrections to the L responses (σ_L and σ_{LT}). Therefore, mainly R_{SM} is affected.

❖Effects increasing with increasing $1/\varepsilon$ and Q^2