# Radiative Corrections to Parity-Violating Electron Scattering Experiments


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# Plan of talk

**Overview of radiative corrections for parity-violating experiments**

**Role of Two-Photon Exchange effects in in** $e{+}p \to e{+}p$

- Factorizable and non-factorizable corrections
- GPD-based calculation

# Atomic Parity Violation

- Review by M. Kuchiev, V. Flambaum, hep-ph/0305053
  - Atomic PNC experiments reached accuracy of ~0.3% (Boulder group)
  - Including full QED correction is crucial. Outstanding problems with self-energy correction (~0.7%) were resolved, bringing suggested 2$\sigma$-deviation into agreement with SM

# Leptonic Process: Moller Scattering

- PV in Moller scattering, SLAC E158, hep-ph/0504049
  $A_{PV}$=-131+/-14+/-10 ppb
- Radiative Corrections are be calculated within Standard Model: Czarnecki, Marciano, PRD 53, 1066 (1996); Petriello, PRD 68, 033006 (2003).
- QED correction for asymmetry is ~1%, with 2$\gamma$ and $\gamma$Z-boxes included Zykunov, Phys.Atom.Nucl. 67, 1342 (2004)

# Semi-Leptonic Processes involving nucleons

- Neutrino-nucleon scattering
  - Per cent level reached by NuTeV. Radiative corrections for DIS calculated at a partonic level (D. Bardin et al.)
- Neutron beta-decay: Important for $V_{ud}$ measurements; axial-vector coupling $g_A$

  Marciano, Sirlin, PRL 56, 22 (1986); Ando et al., Phys.Lett.B595:250-259,2004; Hardy, Towner, PRL94:092502,2005
- Parity-violating DIS: Bardin, Fedorenko, Shumeiko, Sov.J.Nucl.Phys.32:403,1980; J.Phys.G7:1331,1981, up to 10% effect from rad.corrections
- Parity-violating elastic ep (strange quark effects, weak mixing angle)

# Elastic Nucleon Form Factors

- Based on one-photon exchange approximation

$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = e^2 \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

- Two techniques to measure

$$\sigma = \sigma_0 (G_M^2 \tau + \varepsilon \cdot G_E^2) \quad : Rosenbluth \quad technique$$

$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau}\sqrt{2\varepsilon(1-\varepsilon)}}{G_M \ \tau\sqrt{1-\varepsilon^2}} \quad : Polarization \quad technique$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross

# Bethe-Heitler corrections to polarization transfer and cross sections

*AA, Akushevich, Merenkov Phys.Rev.D64:113009,2001;*
*AA, Akushevich, Ilychev, Merenkov, PL B514, 269 (2001)*

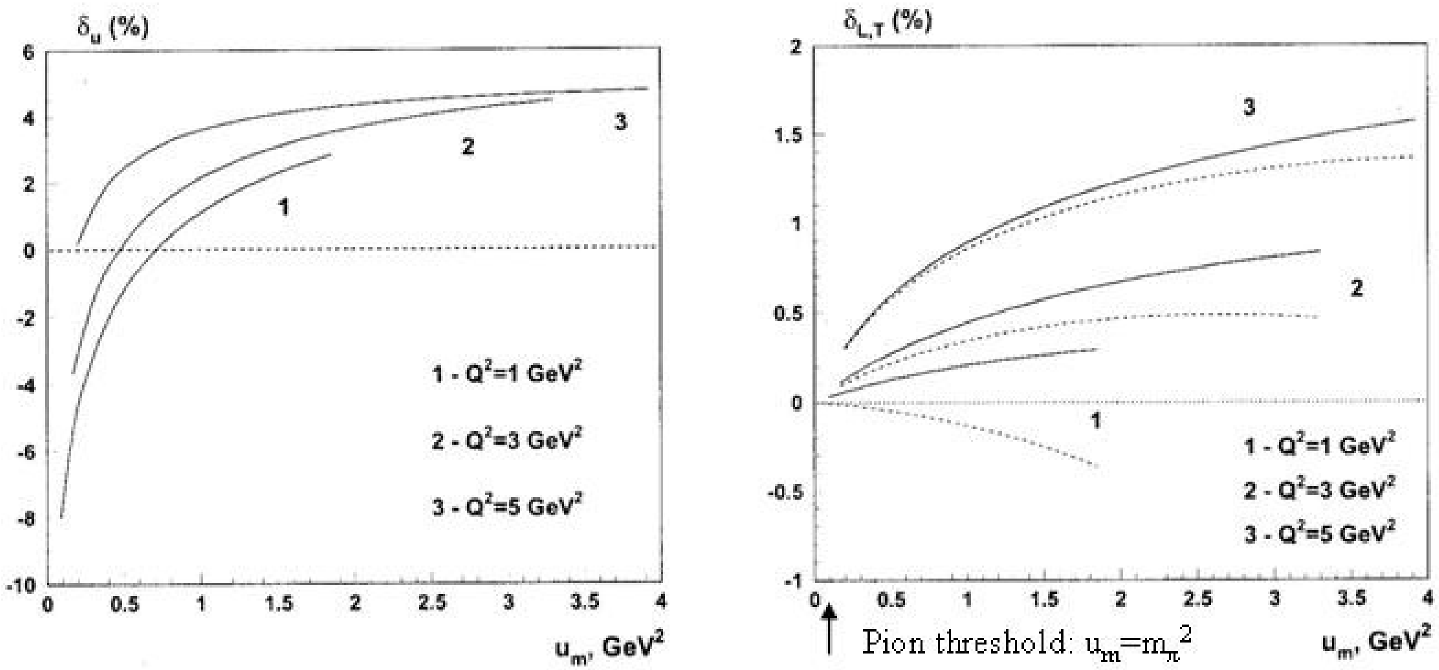



Figure 2: Radiative corrections to the unpolarized cross section (left plot) and polarization asymmetries (right plot) defined in (41). Solid and dashed lines corresponds to longitudinal and transverse cases. $S$=8 GeV$^2$.

In kinematics of elastic ep-scattering measurements, cross sections are more sensitive to RC

# Full Calculation of Bethe-Heitler Contribution

*Additional work by AA et al., using MASCARAD (**Phys.Rev.D64:113009,2001**)*

*Full calculation including soft and hard bremsstrahlung*

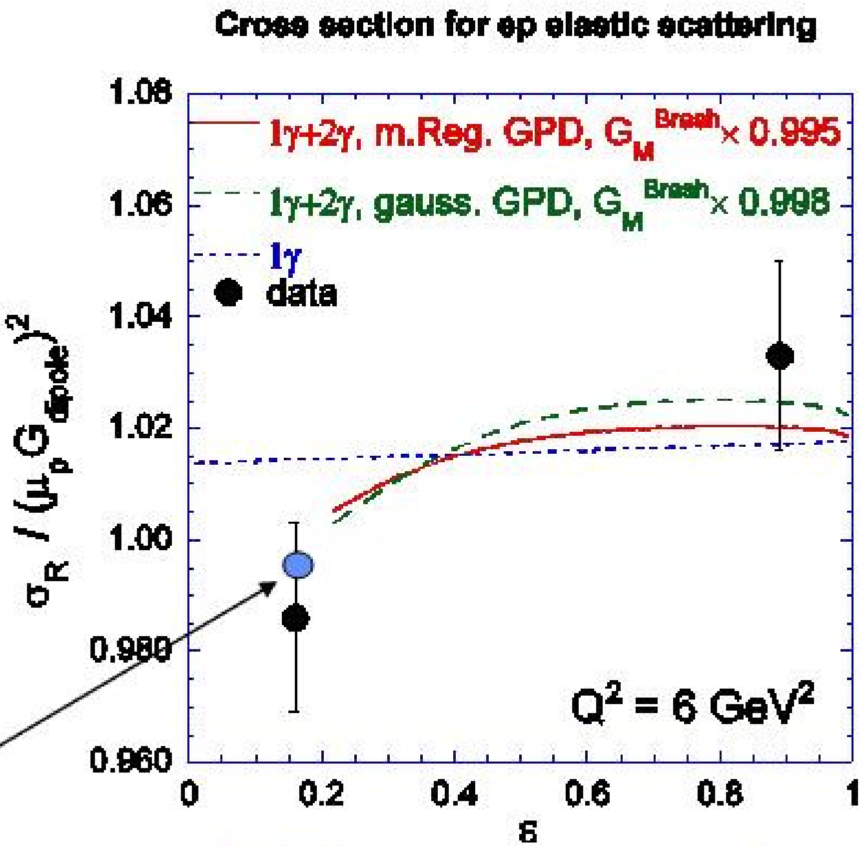
Radiative leptonic tensor in full form

AA et al, ***PLB 514, 269 (2001)***

$$L^{r}_{\mu\nu} = -\frac{1}{2} Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1+\gamma_5\hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\mu - \frac{\gamma_\mu \hat{k}\gamma_\alpha}{2k\cdot k_1} - \frac{\gamma_\alpha \hat{k}\gamma_\mu}{2k\cdot k_2}$$

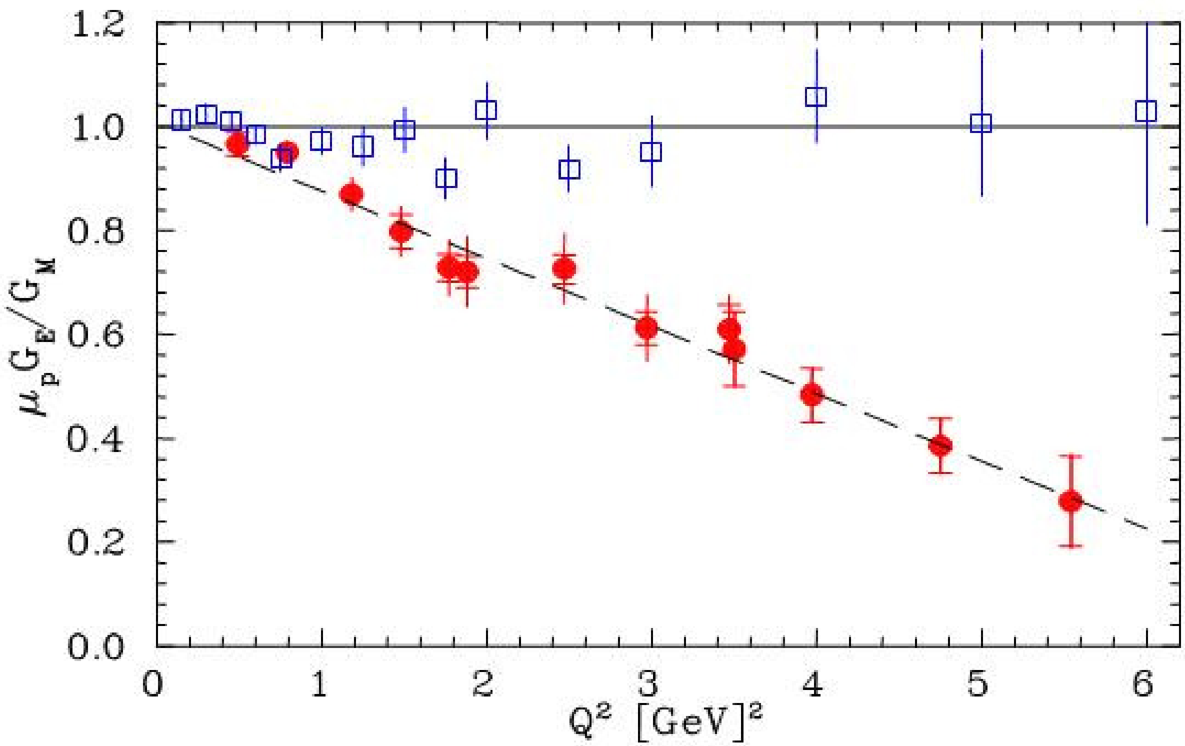
$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\nu - \frac{\gamma_\alpha \hat{k}\gamma_\nu}{2k\cdot k_1} - \frac{\gamma_\nu \hat{k}\gamma_\alpha}{2k\cdot k_2}$$




Additional effect of full soft+hard brem → +1.2% correction to ε-slope

**Resolves additional ~25% of Rosenbluth/polarization discrepancy!**

# Do the techniques agree?

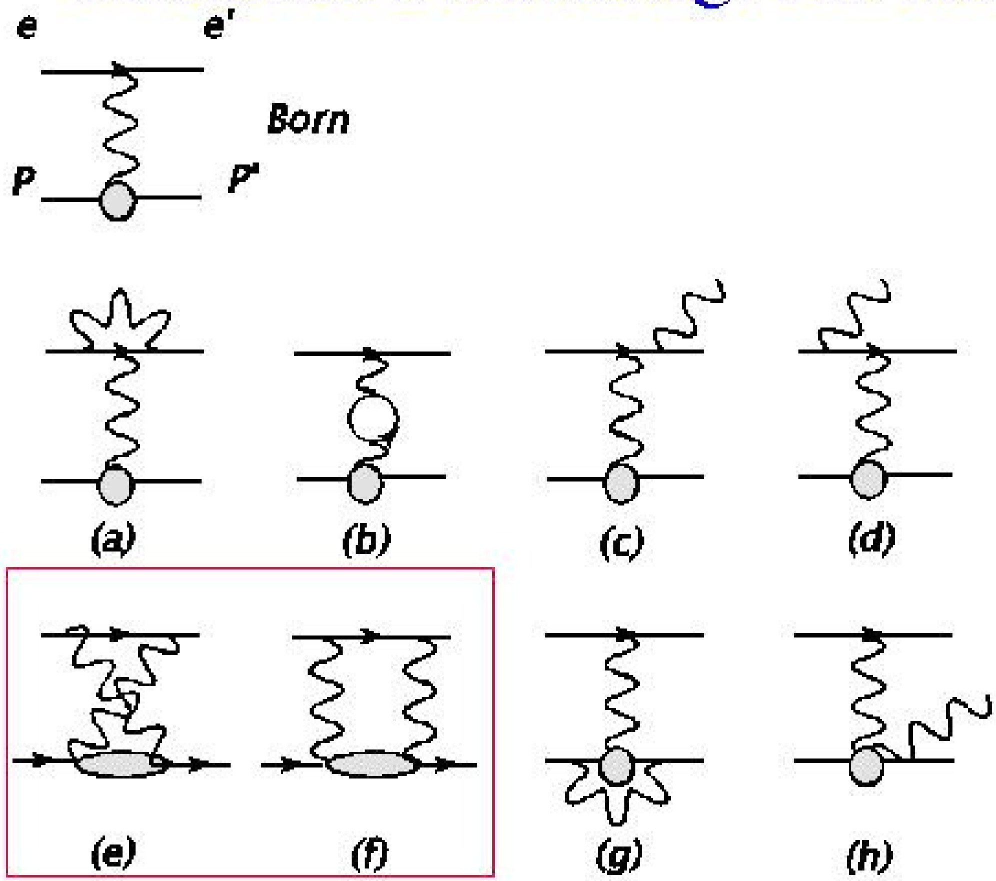





- **Both early SLAC and Recent JLab experiments on (super)Rosenbluth separations followed Ge/Gm~const**
- **JLab measurements using polarization transfer technique give different results (Jones'00, Gayou'02)**

***Radiative corrections, in particular, a short-range part of 2-photon exchange is a likely origin of the discrepancy***

# Electron Scattering: LO and NLO in $\alpha_{em}$




Radiative Corrections:
- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Guichon&Vanderhaeghen'03:
*Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3%x2 ...*

**Main issue: Corrections dependent on nucleon structure**

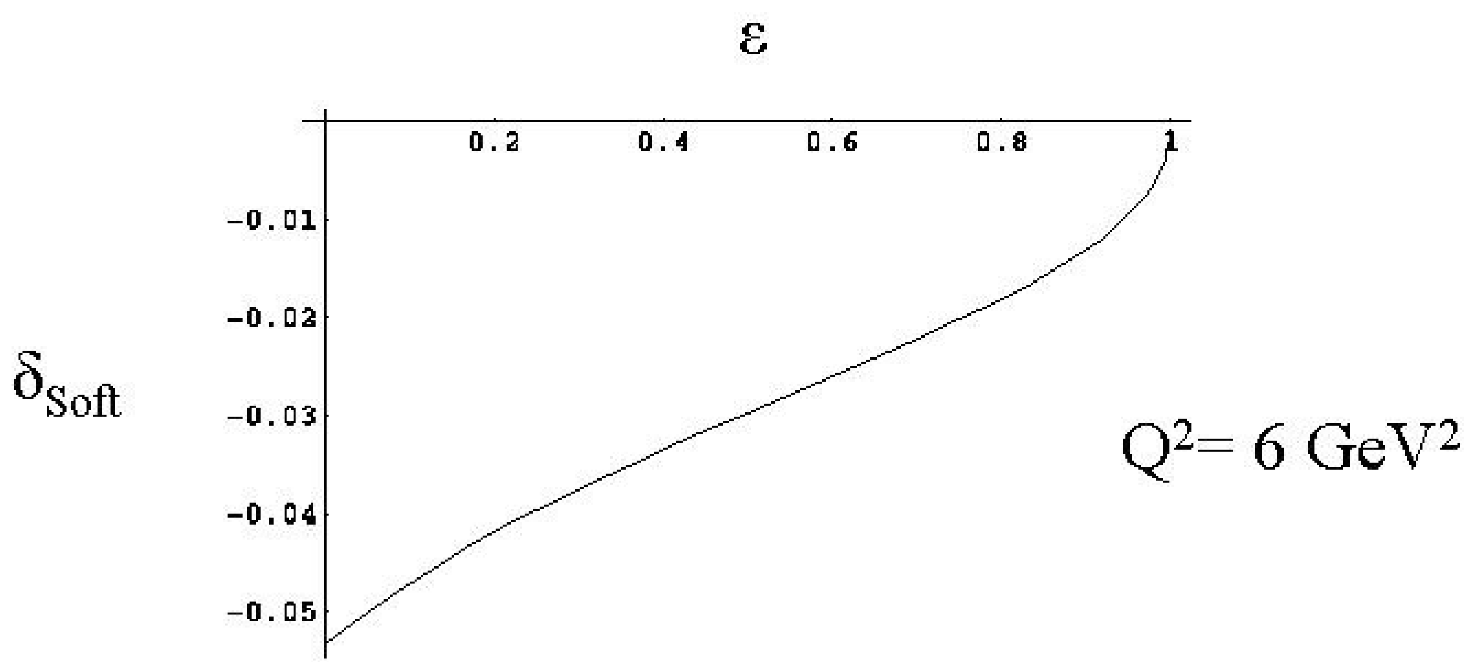
Model calculations:

•Blunden, Melnitchuk,Tjon, Phys.Rev.Lett.**91**:142304,2003

•Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

# Separating soft photon exchange

- Tsai; Maximon & Tjon
- We used Grammer & Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to the cross section
- NB: Corresponding effect to polarization transfer and/or spin asymmetry is **zero**



# Lorentz Structure of 2-γ amplitude

Generalized form factors are functions of two Mandelstam invariants;
Specific dependence is determined by nucleon structure

$$M_{fi} = M_{fi}^{1\gamma} + M_{fi}^{2\gamma}$$

$$M_{fi}^{1\gamma} = \bar{u}_e \gamma_\mu u_e \bar{u}_p (F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

$$M_{fi}^{2\gamma} = V_e \otimes V_p + A_e \otimes A_p$$

$$V_e = \bar{u}_e \gamma_\mu u_e, V_p = \bar{u}_p (F_1'(s,u)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2'(s,u)) u_p$$

$$A_e = \bar{u}_e \gamma_\mu \gamma_5 u_e, A_p = \bar{u}_p G_A(s,u) \gamma_\mu \gamma_5 u_p$$

# New Expressions for Observables

We can formally define ep-scattering observables in terms of the new form factors:

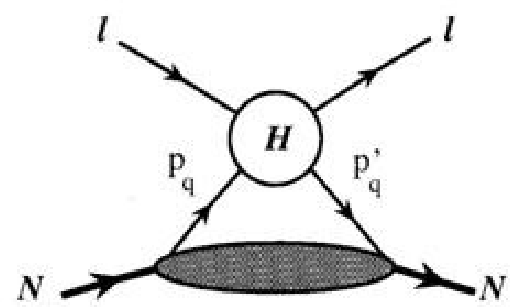
$$\sigma = \sigma_0 (|G_M|^2 \tau + \varepsilon \cdot |G_E|^2 + 2\sqrt{\tau(1+\tau)(1-\varepsilon^2)}\,\mathrm{Re}\,G_M^* G_A)$$

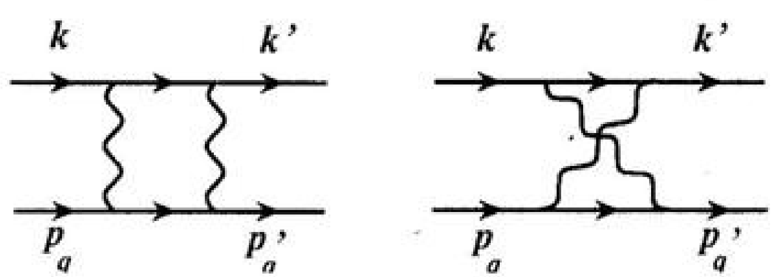
$$\frac{P_x}{P_z} = -\frac{\mathrm{Re}(G_E^* G_M)\sqrt{\tau}\sqrt{2\varepsilon(1-\varepsilon)} + \mathrm{Re}(G_E^* G_A)\sqrt{1+\tau}\sqrt{2\varepsilon(1+\varepsilon)}}{|G_M|^2 \tau\sqrt{1-\varepsilon^2} + \mathrm{Re}(G_M^* G_A) 2\sqrt{\tau(1+\tau)}}$$

$$P_y = \frac{\mathrm{Im}(G_E^* G_M)\sqrt{\tau}\sqrt{2\varepsilon(1+\varepsilon)} + \mathrm{Im}(G_E^* G_A)\sqrt{1+\tau}\sqrt{2\varepsilon(1-\varepsilon)}}{|G_M|^2 \tau + \varepsilon \cdot |G_E|^2 + 2\sqrt{\tau(1+\tau)(1-\varepsilon^2)}\,\mathrm{Re}\,G_M^* G_A}$$

For the target asymmetries: $A_x = P_x$, $A_y = P_y$, $A_z = -P_z$

# Calculations using Generalized Parton Distributions




Model schematics:
- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
  - Use Grammer-Yennie prescription

Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.93:122301,2004; Phys.Rev.D72:013008,2005 **(hep-ph/0502013)**

# Short-range effects; on-mass-shell quark (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

Two-photon probe directly interacts with a (massless) quark
Emission/reabsorption of the quark is described by GPDs

$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V^e_\mu \otimes V^q_\mu \times f_V + A^e_\mu \otimes A^q_\mu \times f_A),$$

$$V^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu u_{e,q},\ A^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$
$$+ \frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2 - s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$
$$+ \frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

Note the additional effective (axial-vector)$^2$ interaction; absence of mass terms

Cf. I.Khriplovich, Sov.J.Nucl.Phys. 17, 298 (1973)

# `Hard' contributions to generalized form factors

GPD integrals

$$A \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 \left(H^q + E^q\right),$$

$$B \equiv \int_{-1}^{1} \frac{dx}{x} \frac{\left[(\hat{s}-\hat{u})\tilde{f}_1^{hard} - \hat{s}\hat{u}\tilde{f}_3\right]}{(s-u)} \sum_q e_q^2 \left(H^q - \tau E^q\right)$$

$$C \equiv \int_{-1}^{1} \frac{dx}{x} \tilde{f}_1^{hard} \,\mathrm{sgn}(x) \sum_q e_q^2 \tilde{H}^q$$

Two-photon-exchange form factors from GPDs

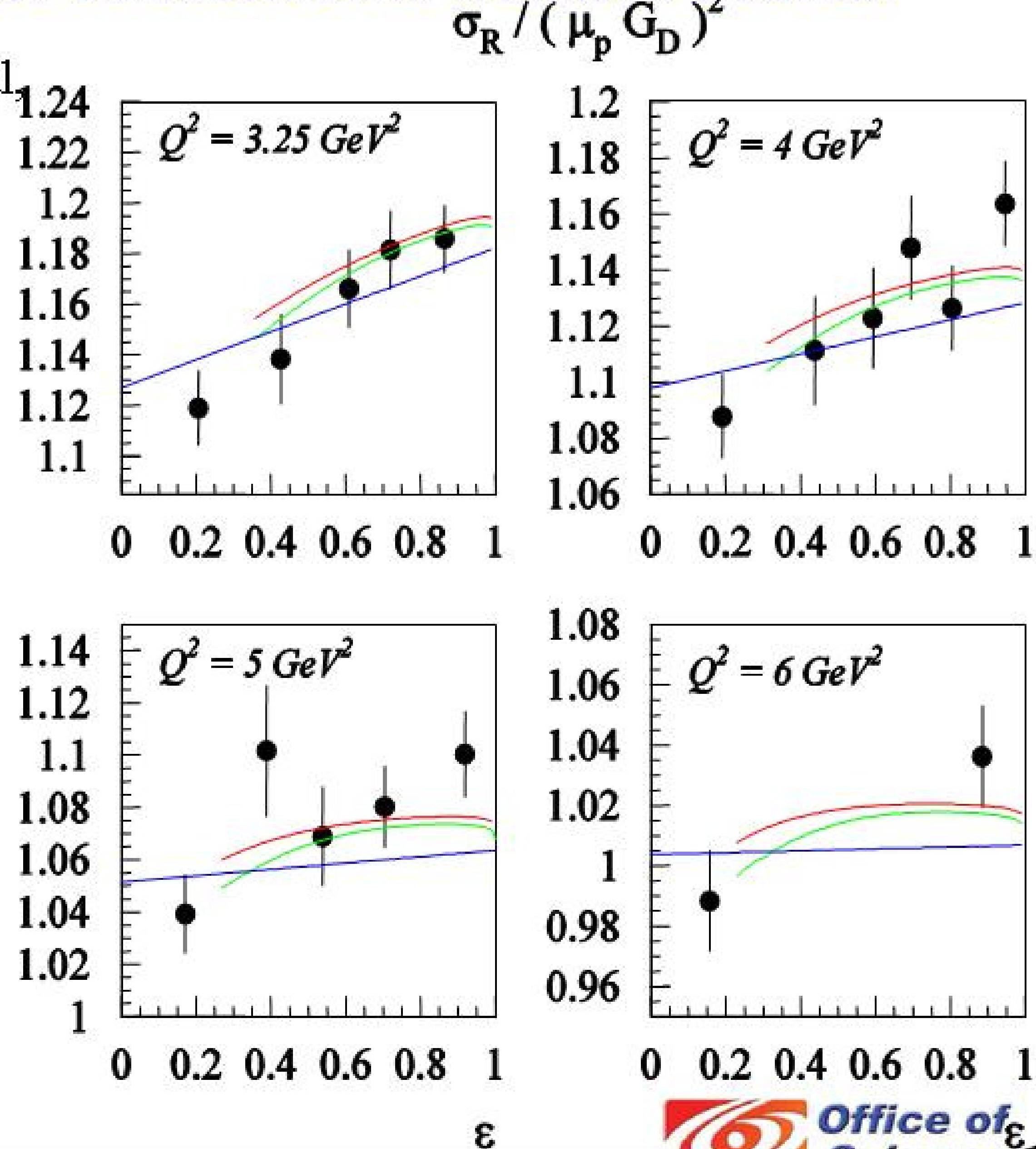
$$\delta\tilde{G}_M^{hard} = C$$

$$\delta\tilde{G}_E^{hard} = -\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C) + \sqrt{\frac{1+\varepsilon}{2\varepsilon}}\,B$$

$$\tilde{F}_3 = \frac{M^2}{\nu}\left(\frac{1+\varepsilon}{2\varepsilon}\right)(A-C)$$

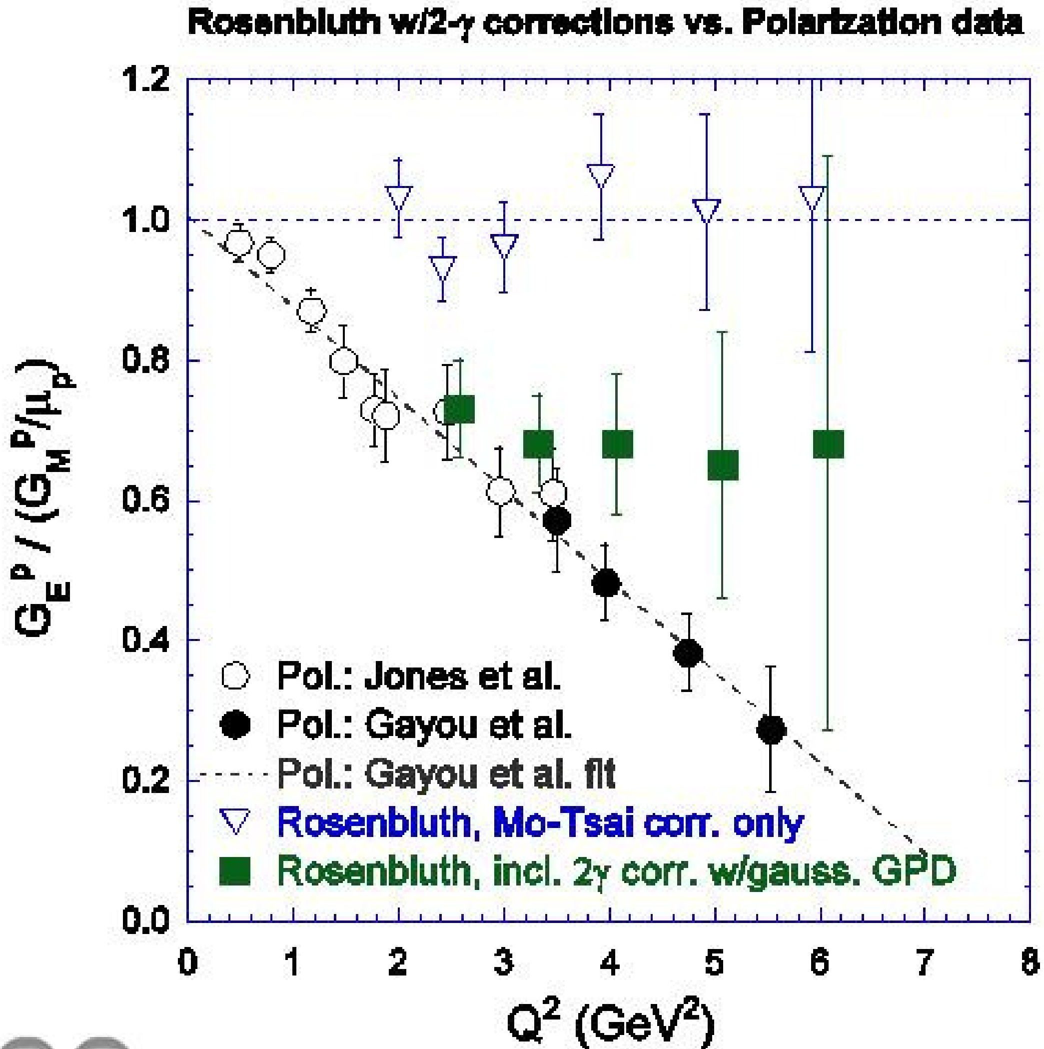
# Two-Photon Effect for Rosenbluth Cross Sections

- Data shown are from Andivahis et al, PRD 50, 5491 (1994)
- Included GPD calculation of two-photon-exchange effect
- Qualitative agreement with data:
- Discrepancy likely reconciled



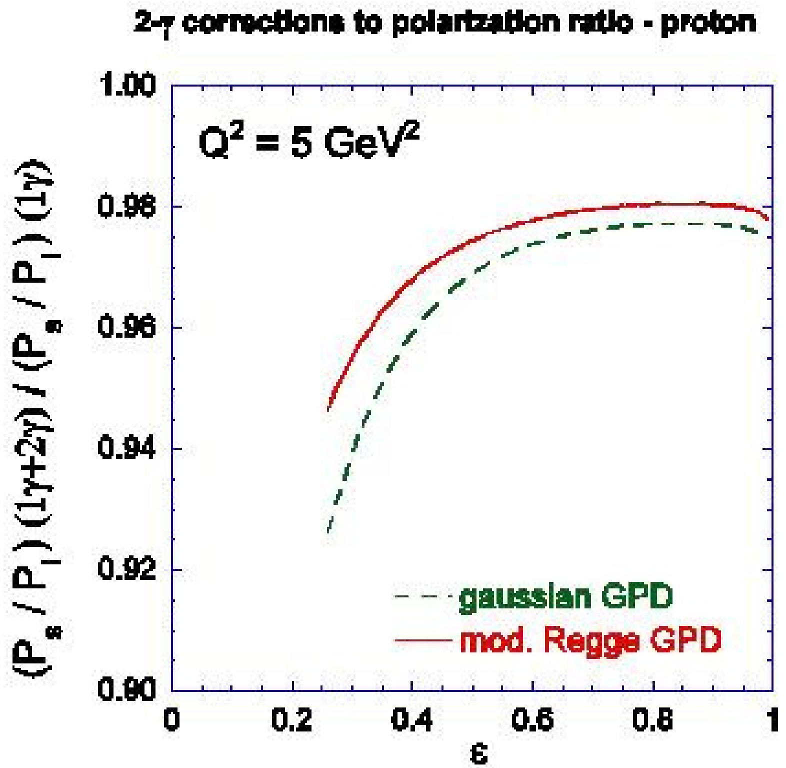
# Updated Ge/Gm plot

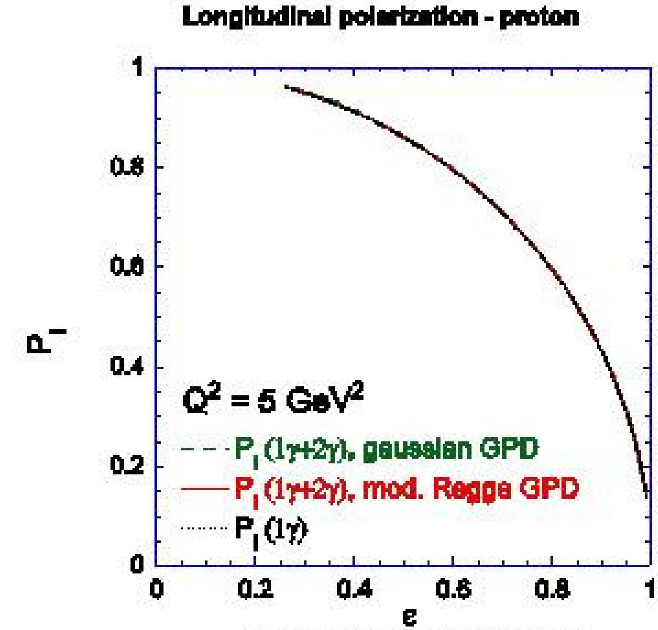
AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Phys.Rev.Lett.93:122301,2004; **hep-ph/0502013**

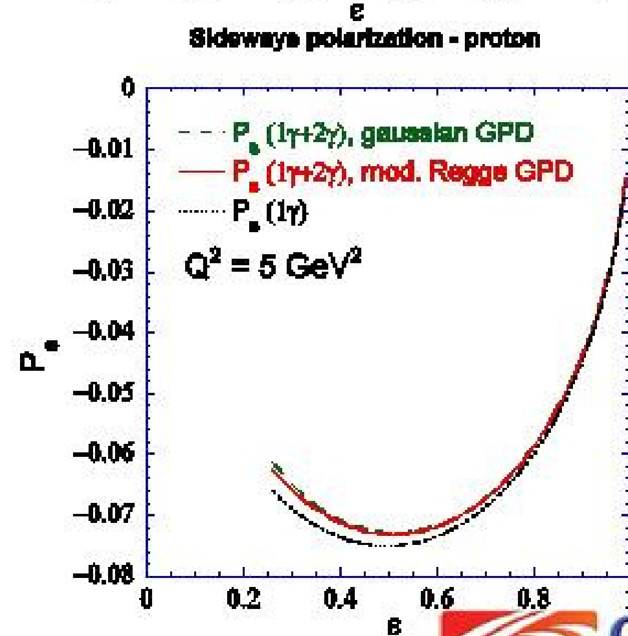


# Polarization transfer

- Also corrected by two-photon exchange, but with little impact on Gep/Gmp extracted ratio

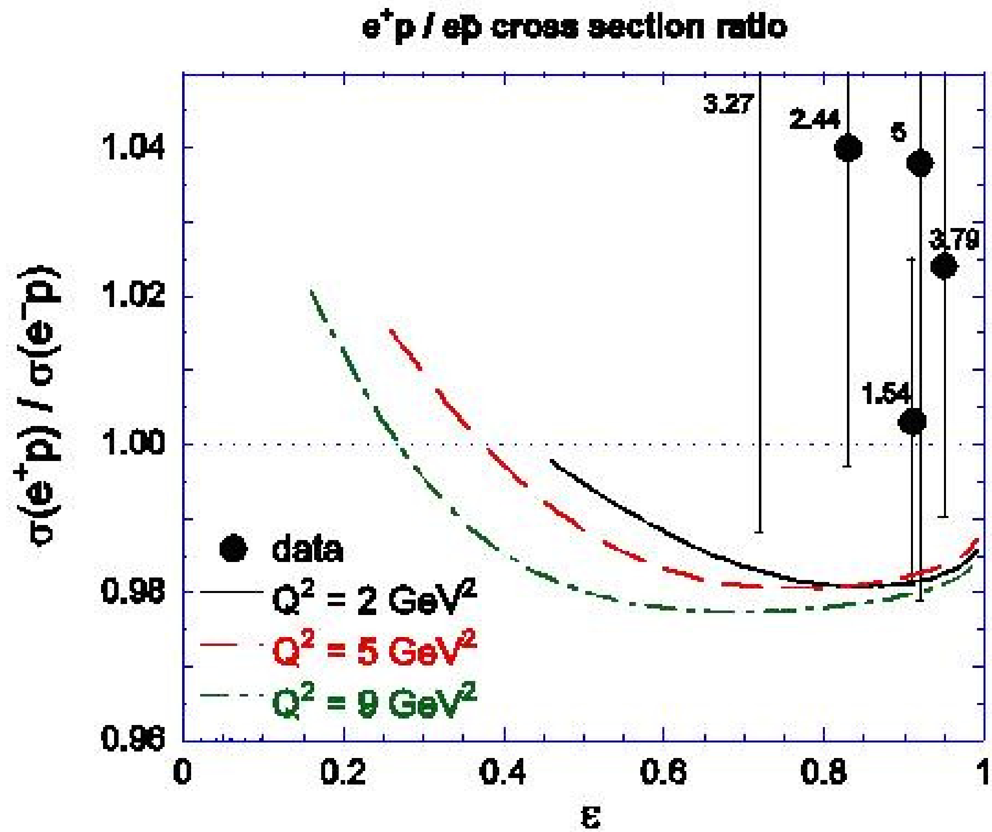


# Charge asymmetry

- Cross sections of electron-proton scattering and positron-proton scattering are equal in one-photon exchange approximation
  - Different for two- or more photon exchange

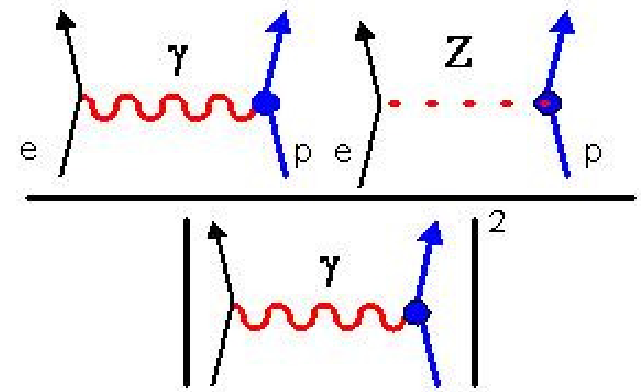



To be measured in JLab Experiment 04-116, Spokepersons W. Brooks et al.

# Parity Violating elastic e-N scattering

Longitudinally polarized electrons,
unpolarized target

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{4\pi\alpha\sqrt{2}}\right] \frac{A_E + A_M + A_A}{2\sigma_{unpol}}$$




$$A_E = \varepsilon(\theta) G_E^Z G_E^\gamma$$
$$A_M = \tau G_M^Z G_M^\gamma$$
$$A_A = -(1 - 4\sin^2\theta_W)\varepsilon' G_A^e G_M^\gamma$$

$\tau = Q^2/4M^2$
$\varepsilon = [1+2(1+\tau)\tan^2(\theta/2)]^{-1}$
$\varepsilon' = [\tau(\tau+1)(1-\varepsilon^2)]^{1/2}$

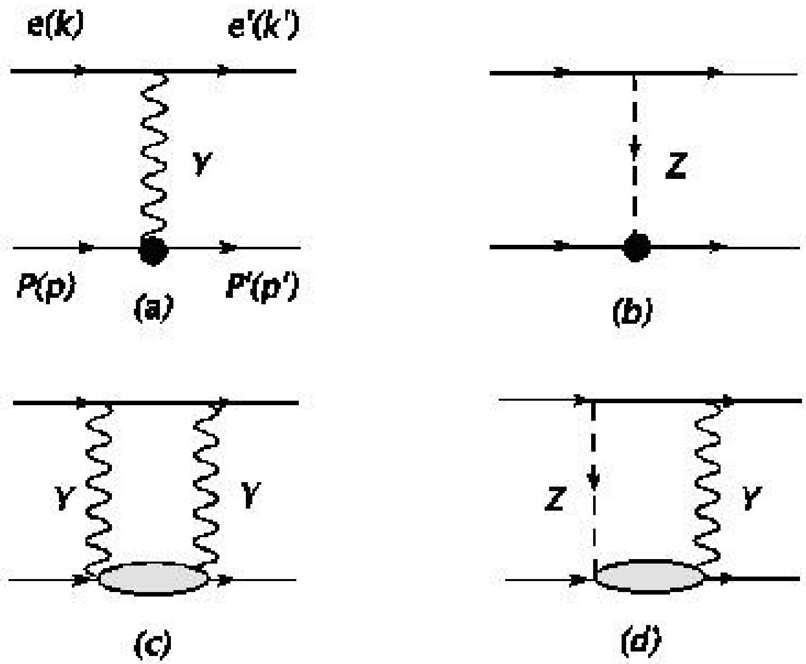
Neutral weak form factors contain explicit contributions from strange sea

$$G_{E,M}^Z(Q^2) = (1 - 4\sin^2\theta_W)(1 + R_A^p) G_{E,M}^p - (1 + R_A^n) G_{E,M}^n - G_{E,M}^s$$
$$G_A^e(Q^2) = -G_A^Z + (\eta F_A^\gamma + R^e) + \Delta s$$

$$\eta = \frac{8\pi\alpha\sqrt{2}}{1 - 4\sin^2\theta_W} = 3.45$$

$G^Z_A(0) = 1.2673 \pm 0.0035$ (from β decay)

# Born and Box diagrams for elastic ep-scattering




- (d) Computed by Marciano,Sirlin, Phys.Rev.D29:75,1984, Erratum-ibid.D31:213,1985 for atomic PV
- (c) Presumed small, e.g., M. Ramsey-Musolf, Phys.Rev. C60 (1999) 015501

# New Expressions for PV asymmetry

PV-asymmetry, Born Approximation

$$A_{PV}^{Born} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E^{Born} + A_M^{Born} + A_A^{Born}}{\tau(G_{Mp}^{\gamma})^2 + \varepsilon(G_{Ep}^{\gamma})^2}$$

$$A_E^{Born} = -2g_A^e \varepsilon G_{Ep}^Z G_{Ep}^{\gamma}, \quad A_M^{Born} = -2g_A^e \tau G_{Mp}^Z G_{Ep}^{\gamma},$$

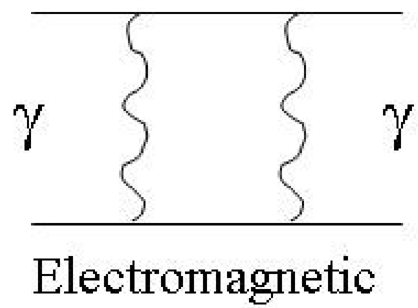
$$A_A^{Born} = 2g_V^e \varepsilon \sqrt{\tau(1+\tau)(1-\varepsilon^2)} G_A^Z G_{Mp}^{\gamma},$$

$$g_V^e = -(1-4\sin^2\theta_W)/2, \quad g_A^e = -1/2$$

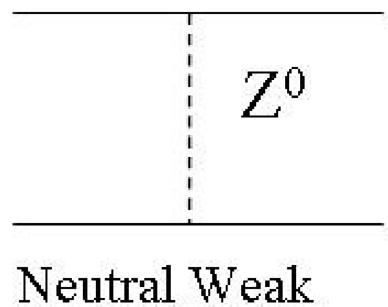
- PV asymmetry in terms of generalized form factors including multi-photon exchange

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E + A_M + A_A + A'_M + A'_A}{\tau |G'_{Mp}|^2 + \varepsilon |G_{Ep}^{\gamma}|^2 + 2\sqrt{\tau(1+\tau)} G'_{Ap}}$$

# 2γ-exchange Correction to Parity-Violating Electron Scattering




x



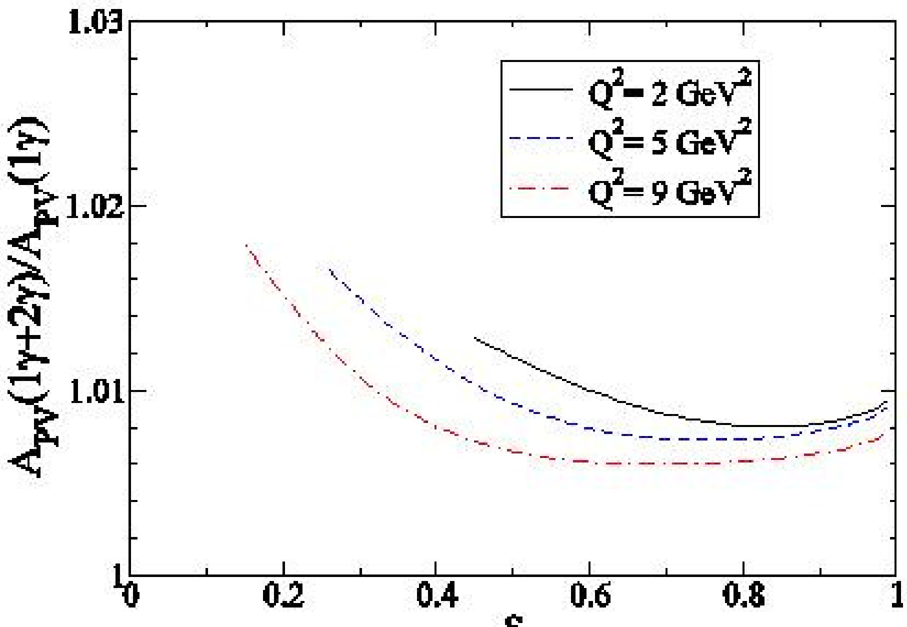

- **New** parity violating terms due to (2gamma)x($Z^0$) interference should be added:

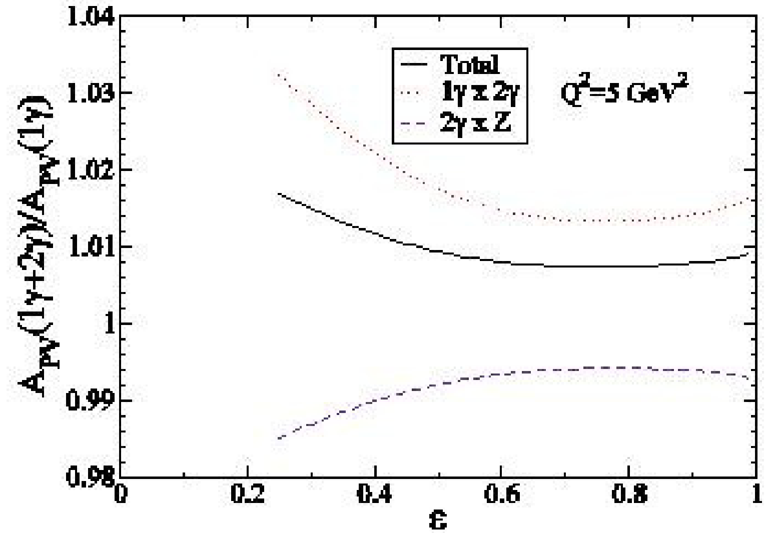
$$A'_M = -2g_A^e\sqrt{\tau(1+\tau)(1-\varepsilon^2)}G_M^Z\,\mathrm{Re}(G'_{Ap})$$

$$A'_A = 2g_V^e(1+\tau)G_A^Z\,\mathrm{Re}(G'_{Ap})$$

# GPD Calculation of 2γ×Z-interference

- Can be used at higher $Q^2$, but points at a problem of additional systematic corrections for parity-violating electron scattering. The effect evaluated in GPD formalism is the largest for backward angles:




AA & Carlson, hep-ph/0502128, ***Phys. Rev. Lett. 94, 212301 (2005):*** measurements of strange-quark content of the nucleon are affected, Δs may shift by ~10%

Important note: (nonsoft) 2γ-exchange amplitude has no $1/Q^2$ singularity; 1γ-exchange is $1/Q^2$ singular => At low Q2, 2γ-corrections is suppressed as $Q^2$

P. Blunden used this formalism and evaluated correction of 0.16% for $Q_{weak}$

# Two-photon exchange for electron-proton scattering

- Quark-level short-range contributions are substantial (3-4%)
- Structure-dependent radiative corrections calculated using GPDs bring into agreement the results of polarization transfer and Rosenbluth techniques for Gep measurements
- 2$\gamma$-correction to parity-violating asymmetry **does not cancel**. May reach a few per cent for GeV momentum transfers
- Corrections are angular-dependent, not reducible to re-definition of coupling constants
  - Revision of $\gamma$Z-box contribution and extension of model calculations to lower $Q^2$ is necessary
- Experimentally measurable directly by comparing electrons vs positrons on a spin-0 target- it is difficult => in the meantime need to rely on the studies of 2$\gamma$-effect for parity-conserving observables