

II The Role of Direct ν Mass Measurements

Summary

There is strong evidence that neutrinos oscillate.

ν oscillation \Rightarrow ν mass

We would like to know the ν masses.

ν oscillation measures only ν mass splittings.

Direct mass measurements can perhaps tell us about the actual masses.

However, these masses may all be $\ll 1$ eV.

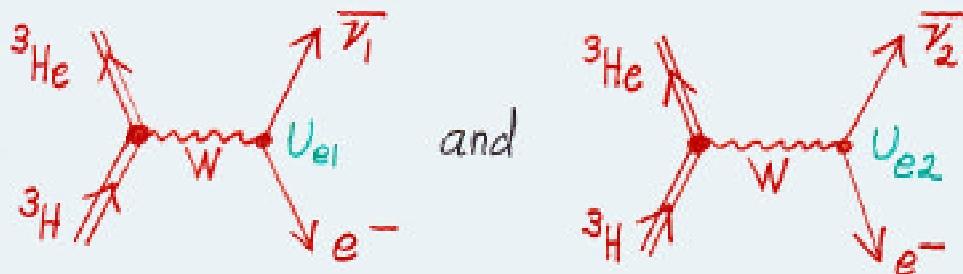
On the other hand, they may not all be $\ll 1$ eV.

§1 Neutrino Mass and Mixing

Suppose there are massive neutrinos ν_1, ν_2, \dots , with

$$\text{Mass } (\nu_m) \equiv M_m .$$

If leptons behave as quarks do,
we can have



and ...

$$\sum_m U_{em} \bar{\nu}_m \equiv \bar{\nu}_e$$

More generally,

$$\sum_m U_{\ell m} \bar{\nu}_m \equiv \bar{\nu}_{\ell}$$

and (CPT)

$$\sum_m U_{\ell m}^* \nu_m \equiv \nu_{\ell}$$

③ U is the Leptonic Mixing Matrix.

$$\text{Standard Model} \Rightarrow UU^\dagger = U^\dagger U = 1$$

Well below the E_β endpoint,

$$BR({}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e) = |U_{e\text{ml}}|^2.$$

A Cosmological Constraint on ν Mass

There are 115 neutrinos/species/cm³ in the universe.

Even with tiny masses, neutrinos can contribute significantly to

$$\frac{\text{Mass Density}}{\text{Critical Density}} \equiv \Omega .$$

Evidence from several sources implies

$$\Omega_{\text{all matter}} \lesssim 0.4 .$$

To allow the observed galactic structure to have formed,

$$\Omega_{\nu} \lesssim 0.2 .$$

(N. Bahcall, Primack, Steigman)

(c.2)

If ν mass eigenstate ν_m has mass M_{ν_m} ,

$$\Omega_\nu = (M_{\nu_1} + M_{\nu_2} + M_{\nu_3}) / 40 \text{ eV}.$$

$$\therefore M_{\nu_1} + M_{\nu_2} + M_{\nu_3} \lesssim 8 \text{ eV}.$$

4) Neutrino Oscillation

The neutrino-mass search technique sensitive to the smallest ν masses is the search for ν oscillation.

Distance ν travels →

$$\text{Amp}(\nu_l \rightarrow \nu_{l'}, j \frac{L}{E_\nu}) = \sum_m U_{lm}^* e^{-i \frac{M_m^2}{2} \frac{L}{E_\nu}} U_{l'm}$$

↑
ν energy ↑

If only 2 neutrinos need be considered,

$$U = \begin{pmatrix} \cos\theta & e^{i\alpha} \sin\theta \\ -e^{-i\alpha} \sin\theta & \cos\theta \end{pmatrix}.$$

Then

$$P(\nu_l \rightarrow \nu_{l' \neq l}) = \sin^2 2\theta \sin^2 \left[1.27 \cdot 8 M^2 (\text{eV}^2) \frac{L (\text{km})}{E_\nu (\text{GeV})} \right].$$

↑
 ν_2 $M_2^2 - M_1^2$ ↑

$\frac{\nu_2}{\nu_1}$ looks like 2 neutrinos.

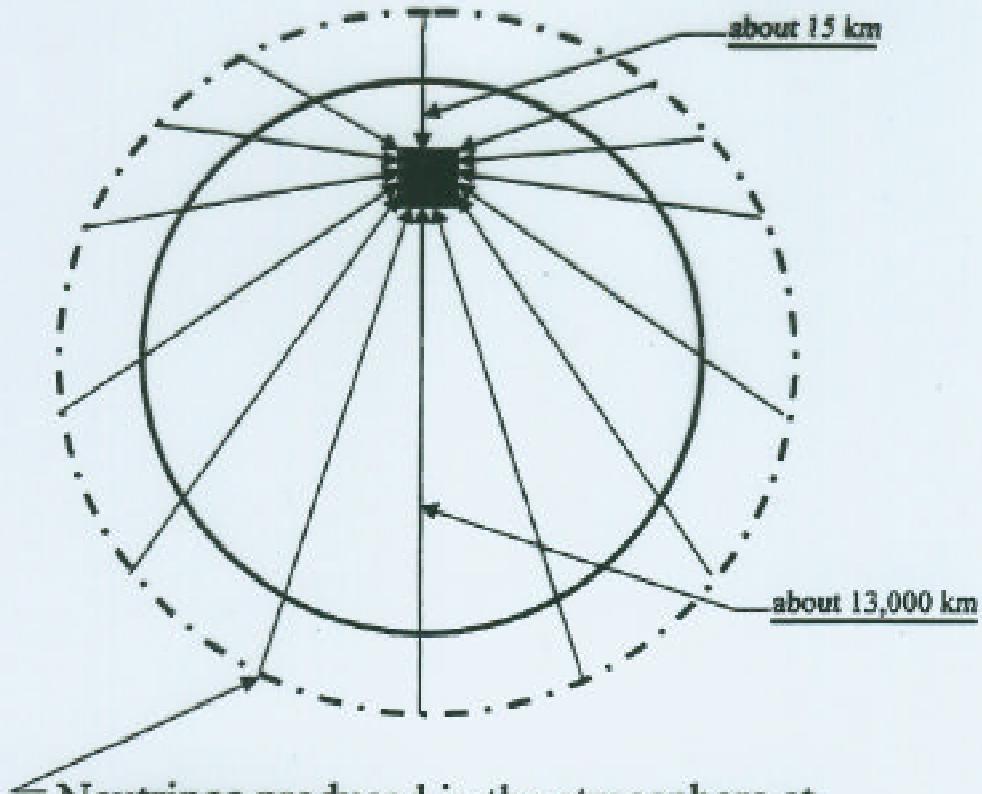
3.1] Neutrino Oscillation

There are 3 pieces of evidence that neutrinos oscillate, which implies that they have mass.

<u>Neutrinos</u>	<u>Evidence of Oscillation</u>
Atmospheric	Compelling
Solar	Strongish
LSND	Unconfirmed

Recent review of the current situation:
[hep-ph/9906244](https://arxiv.org/abs/hep-ph/9906244)
(Fisher, B.K., McFarland)

5 Atmospheric Neutrinos



— Neutrinos produced in the atmosphere at ~15 km altitude, by *cosmic rays*, travel through the earth and interact in the detector.

④ Suppose neutrinos do not change flavor or decay away.

Then, assuming only that —

- The earth is round
- The cosmic ray flux is isotropic

we must have —

$$\gamma_\mu \text{ Flux Up} = \gamma_\mu \text{ Flux Down}.$$

But SuperKamiokande finds that —

$$\gamma_\mu \text{ Flux Up} = (0.53 \pm 0.05) \gamma_\mu \text{ Flux Down}.$$

(Multi-GeV sample, for which progenitor cosmic ray flux is isotropic.)

7] The interpretation: $\nu_\mu \rightarrow \nu_e$ (or ν_s).

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2 \left[1.27 \delta M^2 (\text{eV}^2) \frac{L(\text{km})}{E_\nu(\text{GeV})} \right]$$

For this sample, $\langle E_\nu \rangle \sim \text{GeV}$.

If $\delta M^2 \sim 10^{-3} \text{ eV}^2$, then

Neutrinos	$L(\text{km})/E_\nu(\text{GeV})$	Oscillate?
Up-Going	$\sim 10^4$	Yes
Down-Going	~ 10	No

Not $\nu_\mu \rightarrow \nu_e$ because $\nu_e(\text{Up})/\nu_e(\text{Down}) = .92 \pm .11$.

⑧] The detailed SuperK ν_{Atmos} data are well described by —

$$\nu_a \rightarrow \nu_e \quad \text{with}$$

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \delta M^2 \lesssim 6 \times 10^{-3} \text{ eV}^2$$

and

$$\sin^2 2\theta \approx 1.$$

$\nu_a \rightarrow \nu_s$ is disfavored by 20 when the effect of matter on the oscillation is taken into account.

I.2] The hints of neutrino mass

Solar Neutrinos

Experimental reports:

Target	Exp.	E_ν Threshold (MeV)	Event Rate Std. Solar Model*
$H_2O(e)$	KAM	~7	.54 ± .12
»	SuperK	~6	.47 ± .07
^{37}Cl	HMSTK	0.8	.33 ± .06
^{71}Ga	SAGE	0.2	.52 ± .07
»	GALLEX	0.2	.60 ± .07

* Bahcall & Pinsonneault, 1998

It has proved very difficult to explain those departures from theory via solar and nuclear physics, without invoking neutrino mass.

(Updated analysis:
(Hata & Langacker)

$\nu_e \rightarrow \nu_{\mu, \tau, \text{or} s}$ [which implies ν mass]

explains all the ν_\odot event rates.

[MSW]

Flavor Conversion Mechanism	Requirements	
	$\delta M^2 (\text{eV}^2)$	$\sin^2 2\theta$
MSW Effect	$(0.4-1.0) \times 10^{-5}$	$10^{-3}-10^{-2}$
" "	$(2-8) \times 10^{-5}$	0.7-0.9
Vacuum Oscillation	$10^{-11}-10^{-9}$	0.7-1.0

(Hata & Langacker)
(Krauss & Petcov)

Note: If HMSTK Event Rate/Std. Solar Model is really ~ 0.5 as in all other ν_\odot experiments, vacuum oscillation will explain the rates with δM^2 as large as desired.

2.5) The Mikheyev, Smirnov, Wolfenstein (MSW) Effect

This converts a ν_e into some ν_x ($= \nu_\mu, \nu_\tau$, or ν_s) as a result of a $\nu_e - \nu_x$ level crossing:

Somewhere in the sun, we must have —

$$\sqrt{P_\nu^2 + M_{\nu_x}^2} \cong \sqrt{P_\nu^2 + M_{\nu_e}^2} + \underbrace{\sqrt{2} G_{\text{Fermi}} N_e}_{\nu_e - e \text{ interaction energy}}$$

Since $N_e(R=0) \sim 10^{26}/\text{cc}$, and $P_\nu \sim 1 \text{ MeV}$, we require

$$M_{\nu_x}^2 - M_{\nu_e}^2 \equiv \delta M_{\nu_x \nu_e}^2 \sim 10^{-5} \text{ eV}^2$$

5)

LSND

A so-far unconfirmed further hint
of ν mass.

$$\overline{\nu}_\mu \rightarrow \overline{\nu}_e$$

From $\mu^+ \rightarrow e^+ \nu_e \overline{\nu}_\mu$ ↑ ↑ Appearance

$$L/E_\nu \sim 30\text{ m}/30\text{ MeV}, \text{ so } \delta M^2 \gtrsim 1\text{ eV}^2.$$

Taking into account constraints from
other experiments (KARMEN, Bugey, NOMAD, ...):

$$0.2\text{ eV}^2 \lesssim \delta M^2 \lesssim 10\text{ eV}^2$$

$$0.002 \lesssim \sin^2 2\theta \lesssim 0.03$$

¶

Neutrino Oscillation and Neutrino Mass

Neutrino oscillation experiments can measure only ν mass splittings.

This is true even for experiments sensitive to matter (MSW) effects.

To learn the actual masses, we must appeal to direct mass measurements and double beta decay.

Where Does the ν Oscillation Evidence Point?

ν_e , ν_{Atmos} , and ν_{LSND} oscillations cannot all be explained in terms of just 3 neutrinos:

With only 3 neutrinos,

$$\sum \delta M^2 = (M_{\nu_3}^2 - M_{\nu_2}^2) + (M_{\nu_2}^2 - M_{\nu_1}^2) + (M_{\nu_1}^2 - M_{\nu_3}^2) = 0.$$

$\uparrow \quad \uparrow \quad \uparrow$ Mass eigenstates

But—

<u>Oscillating Neutrinos</u>	<u>Required $\delta M ^2$ (eV2)</u>
Solar	10^{-10} or 10^{-5}
Atmospheric	10^{-3}
LSND	1

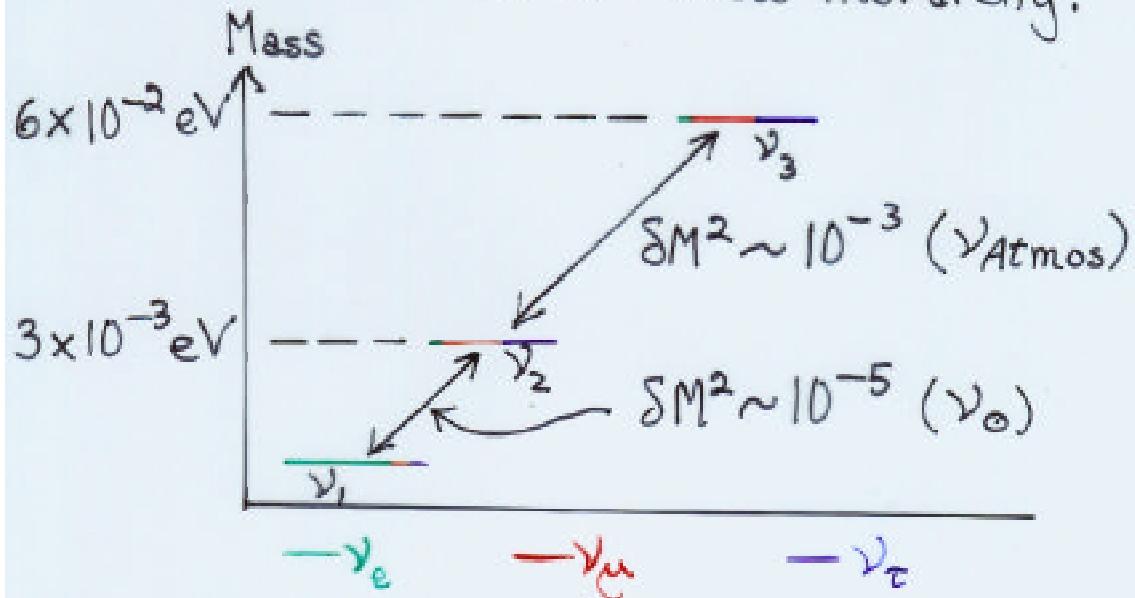
$$\sum \delta M^2 \neq 0$$

\therefore Must add a sterile neutrino ν_s .

Must not contribute to $\Gamma_{\bar{\nu}_e}$.

III To Explain Just ν_{Atmos} and ν_0

We can have a neutrino mass hierarchy.



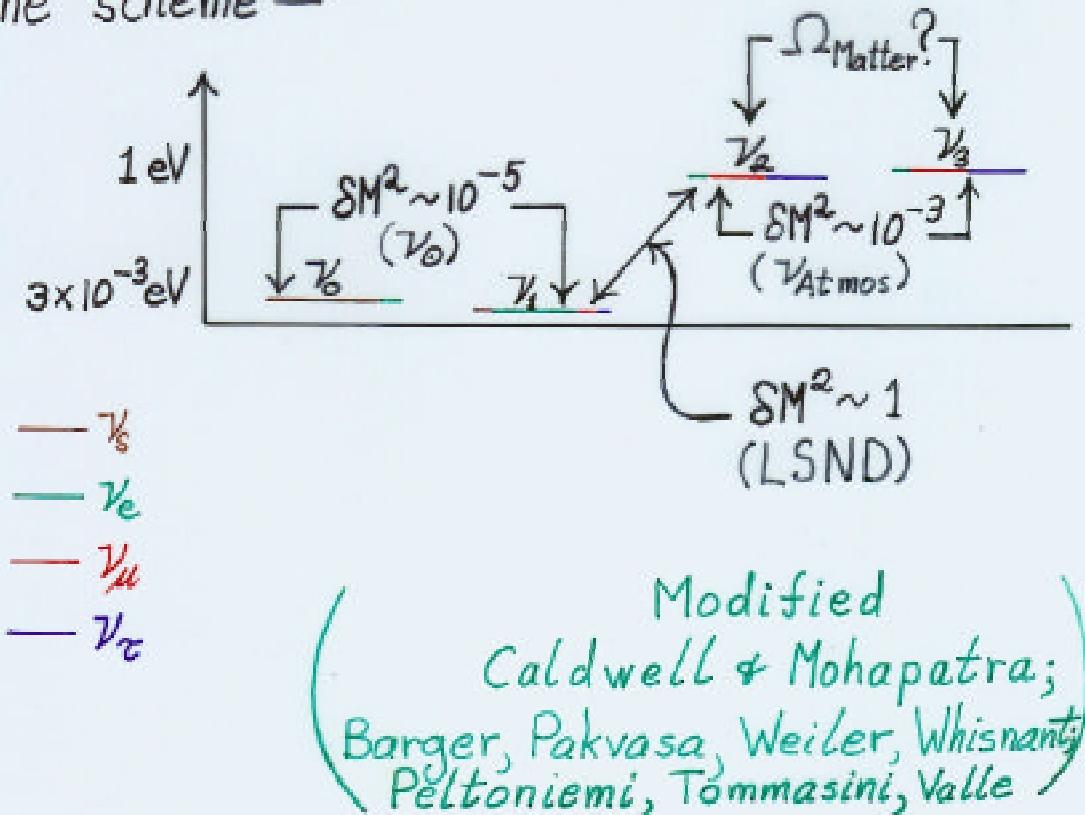
The data give us only mass splittings, not absolute masses.

The actual masses could be heavier than in this diagram.

If neutrinos contribute significantly to Ω_{Matter} , maybe $M_{\nu_1} \simeq M_{\nu_2} \simeq M_{\nu_3} \simeq (1-2) \text{ eV}$.

7) Add LSND, and include a ν_s

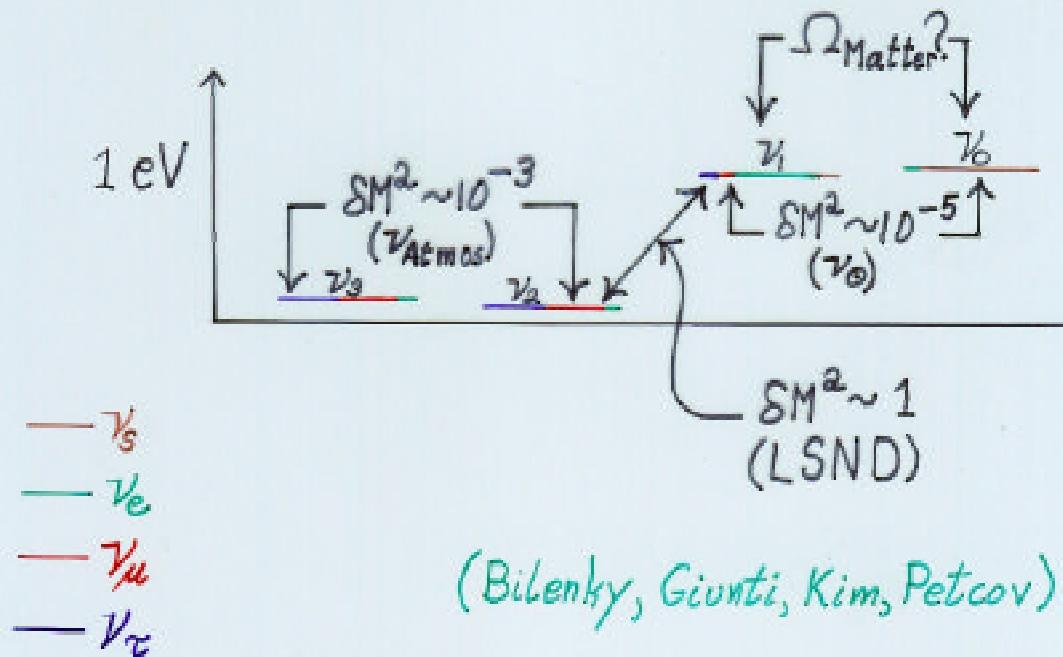
One scheme -



Discovery of a light sterile neutrino would be as groundbreaking as the discovery of neutrino mass.

④

Interchange Heavy and Light

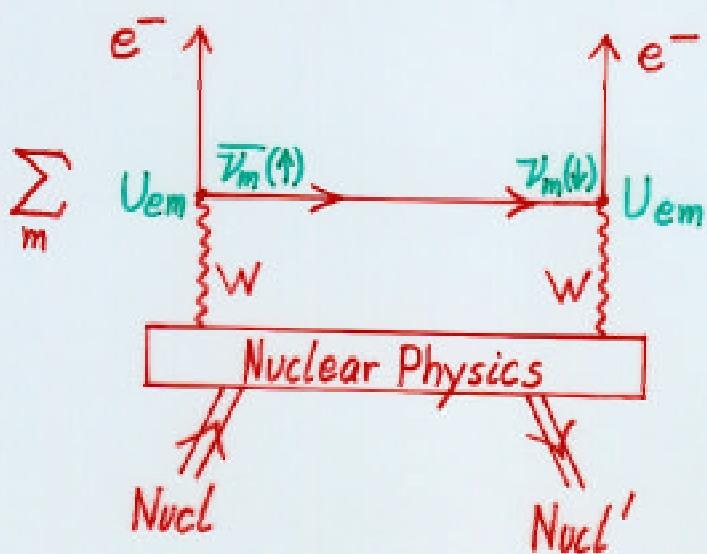


10 The Role of Direct Mass Measurements and Double Beta Decay

Some scenarios have a $\sim 1 \text{ eV}$ mass eigenstate with large ν_e content.

Some don't.

Neutrinoless Double Beta Decay ($\beta\beta_{0\nu}$)



III If the ν_m are Majorana particles,

$$|\text{Amp}| \propto M_{\beta\beta} \equiv \left| \sum_m U_{em}^2 M_m \right|$$

$$= \left| \sum_m \underbrace{\omega_m |U_{em}|^2 M_m}_{\text{Phase factor}} \right|$$

If CP is not violated,

$$\omega_m = \text{CP}(\nu_m) = \pm 1.$$

(Wolfenstein)

Otherwise, ω_m is complex.

$M_{\beta\beta}$ can be smaller than the M_m ,
but not bigger!

$$M_{\beta\beta} \leq \sum_m |U_{em}|^2 M_m \leq \left(\sum_m |U_{em}|^2 \right) M_{\text{Heaviest}}$$
$$= M_{\text{Heaviest}}$$

$\beta\beta_0\nu$ experiments presently exclude
 $M_{\beta\beta} \gtrsim 0.5 \text{ eV}$, and dream of going to
 $\sim 10^{-2} \text{ eV}$.

[2]

Scenario	$\sim 1 \text{ eV}$ Mass in ${}^3\text{H}$ Decay	$M_{\beta\beta} (\text{eV})$
3ν Light Hierarchy	No	$\lesssim 0.6 \times 10^{-2}$
3ν Quasi- Degenerate at $M^0 \sim 1 \text{ eV}$	Yes	$M_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \alpha_{\text{CP}}}$
4ν with Light ν_e	No	$\lesssim 10^{-2}$
4ν with Heavy ν_e	Yes	$\sqrt{\delta M_{\text{LSND}}^2} \sqrt{1 - \sin^2 2\theta_\odot \sin^2 \beta_{\text{CP}}}$
$(\text{Bilenky, Giunti, Grimus,})$ B.K., Petcov		

5

Conclusion

Direct ν mass measurements
(in laboratory experiments or
supernova ν detection) and
 $\beta\beta\nu\nu$ can play important roles.
