Massive neutrinos can couple to EM fields through dipole moments

$$\bar{\nu}_i^{(R)}(\mu_{ij} + i d_{ij}\gamma_5)\nu_j^{(L)}\sigma_{\alpha\beta}F^{\alpha\beta}$$

neutrino mass eigenstates

we will assume $d_{ij} = 0$
and measure μ_{ij} in units of $\mu_B = \frac{e\hbar}{2m_e c}$

How to find out whether $\mu_{ij} \neq 0$?

$\Rightarrow$ measure $\nu + e \to \nu + e$

Review of Particle Physics: (above the dotted line)

ν_e : $\mu_1 < 1.8 \times 10^{-10}$ ... reactor $\bar{\nu}_e e$, Derbin 94

ν_μ : $\mu_2 < 7.4 \times 10^{-10}$ -- LAMPF $\nu_\mu, \bar{\nu}_\mu + e$, Krakauer 90

ν_τ : $\mu_3 < 5.4 \times 10^{-7}$ ... BEBC $\nu_\tau e$, Cooper 92

Plan:

1) Introduction – μ_ν effects in $e\nu$ scattering

2) What about neutrino mixing?

   a) in vacuum

   b) in matter

3) How to deduce a limit on μ_ν from the observation of solar neutrinos?

All is based on a work in progress with John Beacom

(thanks to Boris Kayser for consultation in the early stages)

Why $\mu_\nu \sim 10^{-10}$ is the experimental limit?

$\nu + e \to \nu + e$

$$\sigma_{weak} \sim \frac{2 G_F^2 m_e E_\nu}{2\pi}$$

$$\left(G_F = \frac{\pi \alpha}{\sqrt{2}\, M_W^2 \cdot \sin^2\theta_W} \right)$$

$$\sigma_{Mag} \sim \mu_\nu^2 \frac{\pi \alpha^2}{m_e^2} \qquad (\mu_\nu \text{ in } \mu_B)$$

When is $\sigma_{Mag} \sim \sigma_{weak}$?

$$\frac{2\pi \alpha^2 m_e E_\nu}{4 \sin^4\theta_W M_W^4} = \mu_\nu^2 \frac{\pi \alpha^2}{m_e^2}$$

$$\Rightarrow \mu_\nu^2 = \frac{m_e^3 E_\nu}{2 \sin^4\theta_W \cdot M_W^4}$$

$$\mu_\nu \sim \left(\frac{E_\nu}{2 m_e}\right)^{1/2} \cdot \left(\frac{2 m_e}{M_W}\right)^2 \sim \sqrt{\frac{E_\nu}{\text{MeV}}}\; 10^{-10}$$

Weak-magnetic interference

Grimus + Stockinger, PRD 57, 1762 (1998)

$$\sigma_{w-m} \sim \frac{\alpha G_F}{\sqrt{2}\, E_\nu} m_\nu \mu_\nu$$

$$\sim \frac{\pi \alpha^2}{2 M_W^2 \sin^2\theta_W E_\nu} m_\nu \mu_\nu$$

now substitute $\mu_\nu \sim O\left[\left(\frac{m_e}{M_W}\right)^2\right]$ ... present limit

$$\sigma_{w-m} \sim \frac{m_\nu}{E_\nu} \frac{\pi \alpha^2 m_e^2}{2 M_W^4 \sin^2\theta_W} \sim \frac{m_\nu}{E_\nu} \sigma_{weak}$$

totally negligible

$$\nu + e \to \nu + e$$

Electron recoil spectrum

$$\frac{d\sigma(E_\nu)}{dT} = \frac{G_F^2 m_e}{2\pi}\left[(g_V+g_A)^2 + (g_V-g_A)^2\left(1-\frac{T}{E_\nu}\right)^2 + (g_A^2-g_V^2)\frac{m_e T}{E_\nu}\right]$$

$$+ \frac{\pi\alpha^2\mu_\nu^2}{m_e^2}\,\frac{1-T/E_\nu}{T}$$

$$\cos\chi = \frac{E_\nu+m_e}{E_\nu}\left[\frac{T}{T+2m_e}\right]^{1/2} \sim 1 - m_e\left(\frac{1}{T}-\frac{1}{E_\nu}\right)$$

this is incoherent (explain)

$g_V, g_A \sim 1, \frac{1}{2}$ for ν_e $\quad \sim 0, -1/2$ for ν_μ, ν_τ

$1, -1/2$ for $\bar{\nu}_e$ $\quad 0, 1/2$ for $\bar{\nu}_\mu, \bar{\nu}_\tau$

The magnetic scattering is recognized by the shape of the recoil spectrum, which is universal. It is characterized by the magnitude of the parameter μ_ν^2.

What about oscillations in vacuum

$$|\nu_e(L)\rangle = \sum_k U_{ek}\, e^{-iE_k L}\, |\nu_k\rangle$$

in magnetic scattering $|\nu_k^{(L)}\rangle \to \sum_j \mu_{kj} |\nu_j^{(R)}\rangle$

and the resulting $\underline{\underline{j}}$ states are <u>incoherent</u>

$$\mu_e^2 = \sum_j \left| \sum_k U_{ek}\mu_{kj}\, e^{-iE_k L} \right|^2$$

$$= \sum_j \left[\sum_k (U_{ek}\mu_{kj})^2 + 2\sum_{k<k'} U_{ek}\mu_{kj} U_{ek'}\mu_{k'j} \cos \frac{2\pi L}{L_{kk'}} \right]$$

$$L_{kk'} = \frac{4\pi E_\nu}{\Delta m^2_{k,k'}}$$ ... oscillation length

(for simplicity I assumed U_{ek}, μ_{kj} real)

Simplifications

a) Dirac ν, only diagonal $\mu_{kj} = \delta_{kj}\mu_j$

$$\mu_e^2 = \sum_j (U_{ej}\mu_j)^2 \quad \text{does not oscillate}$$

$$\mu_e^2 = \cos^2\Theta\,\mu_1^2 + \sin^2\Theta\,\mu_2^2 \quad \text{for two flavors}$$

b) Majorana (or Dirac) with only one $\mu_{12} \neq 0$

$$\mu_e^2 = \mu_{12}^2\,(U_{e1}^2 + U_{e2}^2) = \mu_{12}^2 \quad \text{(for two flavors)}$$

So, when does $\overline{\mu}$ oscillate?

only when $\mu_{jk}\mu_{jk'} \neq 0$ for $k \neq k'$

and moreover, for noticeable oscillations

$|\mu_{jk}| \sim |\mu_{jk'}|$ for $k \neq k'$

Oscillations in matter

it is no longer true that $|\nu_1(L)\rangle = e^{-iE_1 L}|\nu_1\rangle$ always

Instead when going through the resonance

$$|\nu_1(L)\rangle \sim c_1|\nu_1\rangle + c_2\, e^{\frac{-i\Delta m^2}{2E_\nu}(L-L_{res})}|\nu_2\rangle$$

$$|\nu_2(L)\rangle \sim -c_2^*|\nu_1\rangle + c_1^*\, e^{\frac{i\Delta m^2}{2E_\nu}(L-L_{res})}|\nu_2\rangle$$

where $|c_1|^2 + |c_2|^2 = 1$

$$|c_2|^2 = P_{hop} \simeq \exp\left(-\frac{\pi\Delta m^2}{2E_\nu}\cdot r_s\cdot(1-\cos 2\Theta_{vac})\right)$$

scale height

The initial state is

$$|\nu_e(0)\rangle = \cos\Theta_i|\nu_1\rangle + \sin\Theta_i|\nu_2\rangle$$

effective mixing angle at the initial point

In vacuum (or low density)

$c_2 = 0, \quad c_1 = 1, \quad \Theta_i = \Theta_{vac}$

$$|\nu_e(L)\rangle = \cos\Theta_{vac}\,|\nu_1\rangle + e^{-\frac{i\Delta m^2 L}{2E_\nu}} \sin\Theta_{vac}\,|\nu_2\rangle$$

$$\mu_e^2 = \cos^2\Theta_{vac}\,\mu_1^2 + \sin^2\Theta_{vac}\,\mu_2^2 \quad \ldots \text{Dirac}$$

$$= \mu_{12}^2 \quad \ldots \text{Majorana}$$

In the Sun (high density limit)

$\Theta_i = \frac{\pi}{2}, \quad c_2 \neq 0$ is allowed

$$|\nu_e(L)\rangle = -c_2^*\,|\nu_1\rangle + c_1^*\,e^{\frac{i\Delta m^2 L}{2E_\nu}}\,|\nu_2\rangle$$

$$\mu_e^2 = |c_2|^2\mu_1^2 + |c_1|^2\mu_2^2 \quad \ldots \text{Dirac}$$

$$= \mu_{12}^2 \quad \ldots \text{Majorana}$$

Derbin et al., JETP Lett. 57, 769 (1993)

75 kg silicon detector
15 m from the reactor core

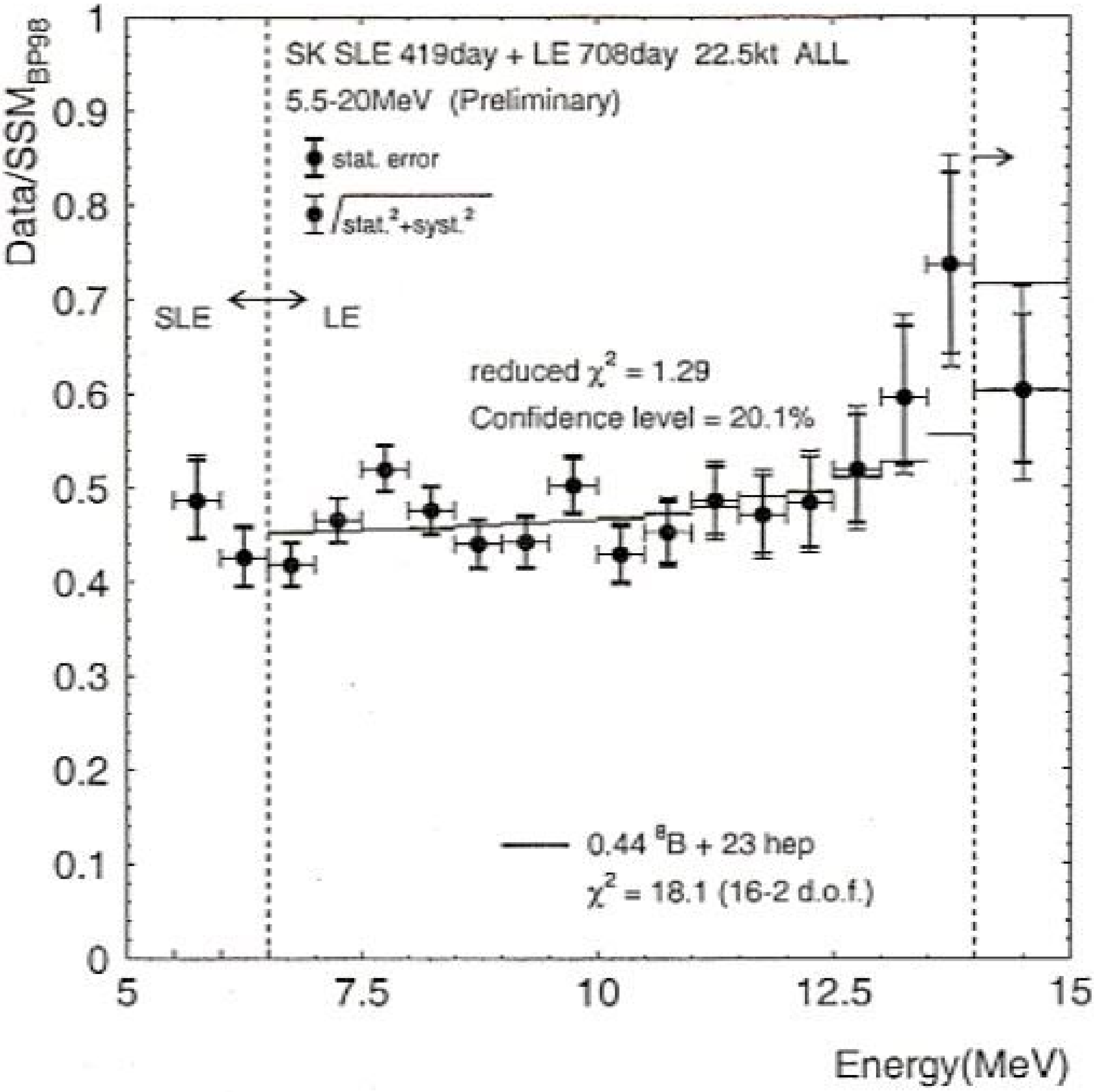
29.6 days reactor on
16.7 days reactor off

TABLE I. Count rates of the 37.5-kg detector with the reactor in operation and shut down.

Interval MeV	Operating	Shut down	Open.-Down	Weak scattering	Magnetic scattering
0.2–2.0	15 327±92	14 878±90	449±130	62	178
0.3–2.0	11 193±70	10 908±70	285±98	53	124
0.6–2.0	4962±12	4921± 16	41±20	32	54
1.3–2.0	508.5±4.0	503.3 ±5.6	5.2±6.8	8.9	10

The last two columns give the expected difference in counts according to the standard theory ($\sin^2\theta_W=0.22$) and for the case in which the scattering is due to a magnetic moment of $2\times10^{-10}\mu_B$.

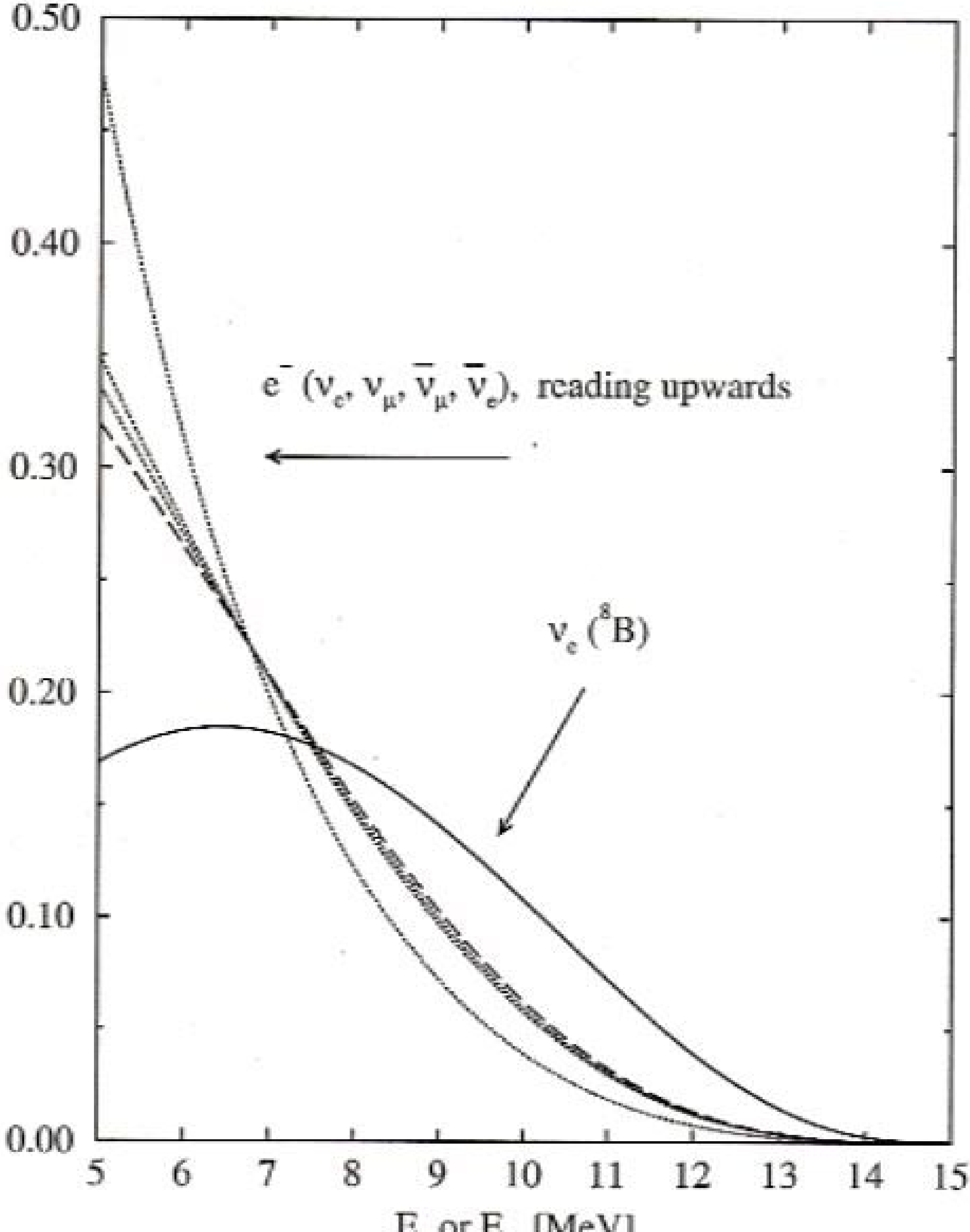
Smy, hep-ex/9903034



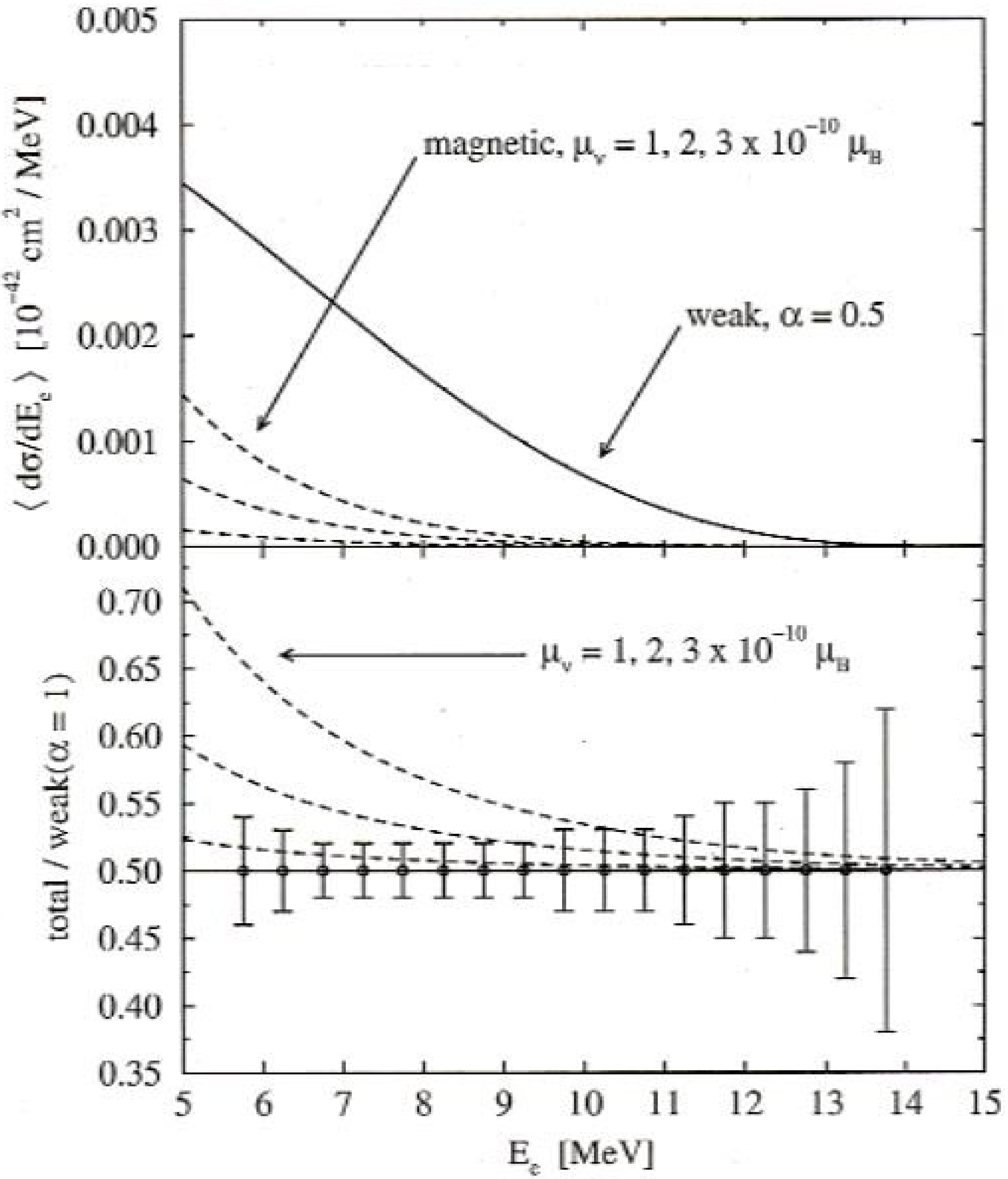
Fit to a constant: my estimated $\chi^2/dof \simeq 25/17$, for $E_e < 13$ MeV $\sim 16/14$

published 504 days data: $\chi^2/dof = 25.3/15$, for $E_e < 13$ MeV $\sim 14/12$

Neutrino spectrum of ^8B decay (full line)
Folded and normalized electron spectra
for the indicated flavors



(13)


magnetic, μ_ν = 1, 2, 3 x 10^{-10} μ_B
weak, α = 0.5
$\langle d\sigma/dE_e \rangle$ [10^{-42} cm^2 / MeV]
μ_ν = 1, 2, 3 x 10^{-10} μ_B
total / weak(α = 1)
E_e [MeV]
0.000
0.001
0.002
0.003
0.004
0.005
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
5
6
7
8
9
10
11
12
13
14
15

Approach :

1) Calculate the expected electron spectrum due to the weak scattering $n_W(i)$

2) Calculate the expected electron spectrum due to the magnetic scattering $n_M(i)$

3) Choose some reference values for the overall reduction α_{ref} and magnetic moment μ_{ref}

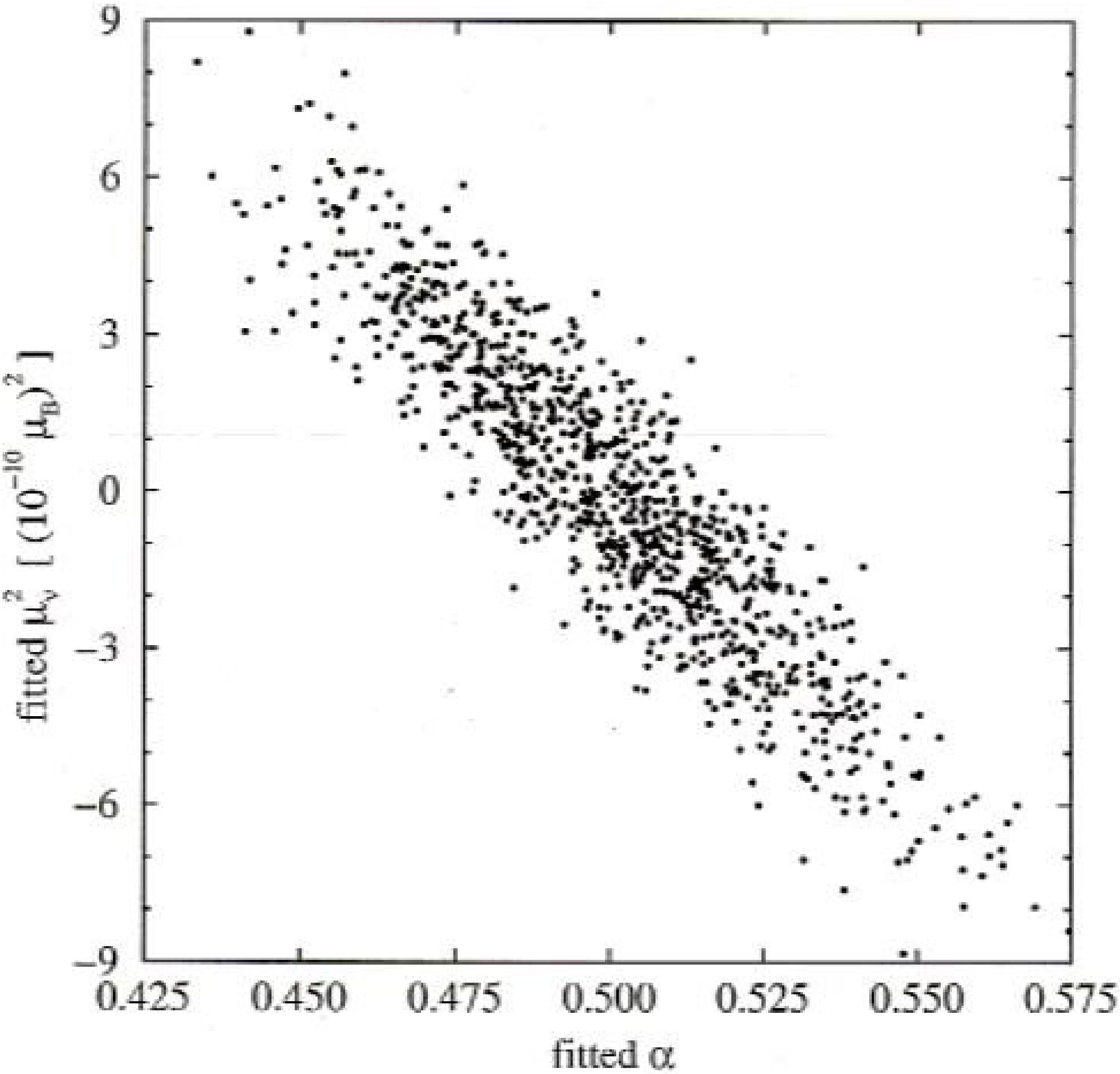
4) Construct a simulated spectrum $n_S(i)$ which is Gaussian distributed around $\alpha_{ref}\, n_W(i) + \mu_{ref}^2\, n_M(i)$ with the relative error bars $\sigma(i)$ given by SK

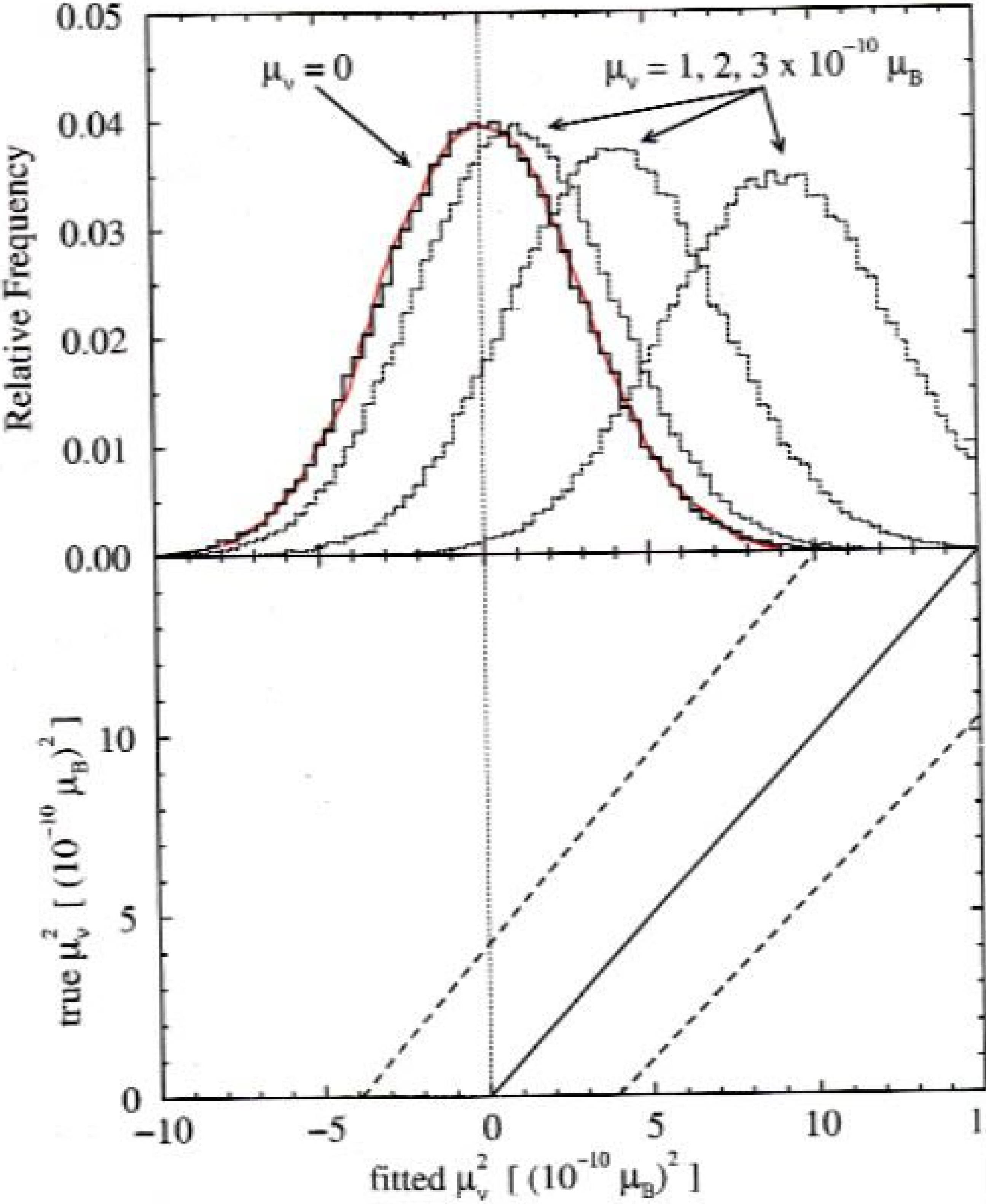
5) minimize

$$\chi^2 = \sum_i \left[\frac{\alpha\, n_W(i) + \mu_\nu^2\, n_M(i) - n_S(i)}{(\sigma(i)\, n_S(i))^2} \right]^2$$

with respect to α and μ_ν^2

scatter plot for $\alpha_{ref} = 0.5$, $\mu_{ref} = 0.0$
using the relative error bars of SK





$\mu_\nu = 0$
$\mu_\nu = 1, 2, 3 \times 10^{-10}\ \mu_B$
Relative Frequency
0.05
0.04
0.03
0.02
0.01
0.00
true μ_ν^2 [$(10^{-10}\ \mu_B)^2$]
10
5
0
−10
−5
0
5
10
fitted μ_ν^2 [$(10^{-10}\ \mu_B)^2$]

TABLE I. Limits on the magnetic moment (at 90% CL, in units of $10^{-10}\mu_B$) and statistical errors (1σ) on α for various assumed data sets: 504 days [5] (I), the same just up to 13.0 MeV (II), 708 days [6], projected 1000 days. In all cases we ignore the bin from 14.0 – 20.0 MeV.

case	$\lvert\mu_e^{sol}\rvert^2$	$\lvert\mu_e^{sol}\rvert$	$\delta\alpha$
504 days, I	≤ 3.9	≤ 2.0	0.025
504 days, II	≤ 4.2	≤ 2.0	0.027
708 days	≤ 2.5	≤ 1.6	0.018
1000 days	≤ 1.7	≤ 1.3	0.014

Assume that the reduction factor α is independently and accurately known. Then the upper limit on $\lvert\mu_e^{sol}\rvert^2$ would be improved by $\sqrt{1-r^2} \sim 0.4$

Conclusions

There are three new results in
Beacom & Vogel hep-ph/9907383

1, In several cases of practical interest μ_ν do not oscillate, i.e., you get the same result close and far

2) Matter oscillations, however, can affect μ_ν. Hence, the μ_ν measured with reactor $\bar{\nu}_e$ and with solar ν_e could be different.

3, The <u>shape</u> of the electron spectrum currently measured by SK can be used to place a <u>robust</u> <u>new</u> limit on μ_ν. From the 708 days data set we get $\mu_\nu \leq 1.6 \times 10^{-10} \mu_B$ (90% CL), the best limit of any direct measurement